(По учебнику Виноградовой “Математический анализ в задачах и упражнениях” (примерно с 246 стр) Номера: 6.3, 6.7, 6.9, 6.12, 6.17, 6.24, 6.25, 6.28, 6.30, 6.38, 6.40, 6.42, 6.43, 6.44, 6.47, 6.61, 6.66, 6.75, 6.81, 6.82, 6.83, 6.84, 6.89, 6.98, 6.99, 6.271, 6.269, 6.272, 6.288

6.3 $$\int \frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt[4]{x}}\,dx = \int \frac{x^{\frac12}+x^{\frac13}}{x^{\frac14}}\,dx \quad \Big| \; t^{12}=x \; \Big| \quad = \int \frac{t^6+t^4}{t^3}\cdot 12t^{11}dt = \int (12t^{14}+12t^{12})\,dt = 12\cdot\frac{t^{15}}{15}+12\cdot\frac{t^{13}}{13}+C = \frac45\cdot x\cdot\sqrt[4]{x}+\frac{12}{13}\cdot x^{\frac{13}{12}}+C$$

6.7 $$\int \frac{4-x^2}{3+x^2}\,dx = -\int\frac{x^2-4}{x^2+3}\,dx = -\int dx+\int\frac{7}{x^2+3}\,dx = \frac{7}{\sqrt3}\,\text{arctg}\frac{x}{\sqrt3}-x+C$$

6.9 $$\int\frac{dx}{x^4-1} = \int\frac{dx}{(x^2-1)(x^2+1)} = \int\left(\frac{Ax+B}{x^2-1}+\frac{Cx+D}{x^2+1}\right)dx = \frac12\int\left(\frac{1}{x^2-1}-\frac{1}{x^2+1}\right)dx = \frac12\int\frac{dx}{x^2-1}-\frac12\int\frac{dx}{x^2+1} = \frac12\left[\frac12\ln\left|\frac{x-1}{x+1}\right|-\text{arctg}\,x\right]+C$$

6.12 $$\int\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{(1+x^2)(1-x^2)}}\,dx = \int\frac{dx}{\sqrt{1-x^2}}+\int\frac{dx}{\sqrt{1+x^2}} = \arcsin x+\ln\left|x+\sqrt{1+x^2}\right|+C$$

6.17 $$\int\frac{dx}{\sqrt{2x^2+5}} = \int\frac{dx}{\sqrt2\cdot\sqrt{x^2+\frac52}} = \frac{\sqrt2}{2}\cdot\ln\left|x+\sqrt{x^2+\frac52}\right|+C$$

6.24 $$\int\frac{2^{2x}-1}{\sqrt{2^x}}\,dx \ominus$$

$t^2=2^x \qquad x=\log_2 t^2 \qquad dx=\frac{\ln t^2}{\ln 2}dt=\frac{2t}{t^2\ln2}dt=\frac{2}{t\ln2}dt$

$\sqrt{2^x}=t \quad 2^{2x}=t^4$

$$\ominus\int\frac{t^4-1}{t}\cdot\frac{2}{t\ln2}\,dt = \frac{2}{\ln2}\left[\int t^2dt-\int\frac{1}{t^2}dt\right] = \frac{2}{\ln2}\cdot\left[\frac{t^3}{3}+\frac1t\right]+C = \frac{2}{\ln2}\left[\frac{2^{\frac{3x}{2}}}{3}+\frac{1}{\sqrt{2^x}}\right]+C$$

6.25 $$\int\sin^2\frac x2\,dx = \frac12\int 1-\cos x\,dx = \frac x2-\sin x+C$$

6.28 $$\int\frac{\cos2x\,dx}{\sin^2 2x} = \frac12\int\frac{\cos2x\,d2x}{\sin^2 2x} = \frac12\int\frac{d\sin2x}{\sin^2 2x} = -\frac{1}{2\sin2x}+C$$

6.30 $$\int\text{ctg}^2x\,dx = \int\frac{dx}{\sin^2x}-\int dx = -\text{ctg}\,x-x+C$$

6.38 $$\int\frac{x+3}{(x+2)(x-1)}\,dx = \int\frac{A}{x+2}+\frac{B}{x-1}\,dx = \frac13\int\frac{dx}{x+2}+\frac13\int\frac{3}{x-1}\,dx = \frac{-\ln|x+2|}{3}+\frac{4\ln|x-1|}{3}+C$$

6.40 $$\int(2x+5)^{17}dx = \frac{(2x+5)^{18}}{36}+C$$

6.43 $$\int(1-2x)^{-\frac12}dx = -\sqrt{1-2x}+C$$

6.47 $$\int\frac{dx}{\sin^2 7x} = -\frac{\text{ctg}\,7x}{7}$$

6.42 $$\int(4x+1)^{\frac12}dx = \frac{(4x+1)^{\frac32}}{6}+C$$

6.44 $$\int(3x+1)^{-\frac45}dx = \frac53\cdot\sqrt[5]{3x+1}+C$$

6.61 $$\int x\sqrt{1-2x}\,dx = \frac12\int(t^2-1)\,t^2\,dt = \frac{t^5}{10}-\frac{t^3}{6}+C = \frac{(\sqrt{1-2x})^5}{10}-\frac{(\sqrt{1-2x})^3}{6}+C$$

$t^2=1-2x$

$2x=1-t^2$

$x=\frac{1-t^2}{2}$

$dx=-\frac{2t\,dt}{2}=-t\,dt$

6.66 $$\int\frac{x^2dx}{x^6-5} = \frac13\int\frac{dx^3}{x^6-5} = \frac13\int\frac{dt}{t^2-5} = -\frac13\int\frac{dt}{5-t^2} = -\frac{1}{6\sqrt5}\ln\left|\frac{\sqrt5+t}{\sqrt5-t}\right|+C = \frac{1}{6\sqrt5}\cdot\ln\left|\frac{\sqrt5-x^3}{\sqrt5+x^3}\right|+C$$

6.75 $$\int\frac{dx}{x\ln^5x} = -\frac{1}{4\ln^4x}+C$$

6.81 $$\int\frac{e^x+e^{2x}}{1-e^x}\,dx = \int\frac{t+t^2}{1-t}\cdot\frac1t\cdot dt \ominus$$

$t=e^x$

$x=\ln t \qquad dx=\frac1t\,dt$

$$\ominus -\int\frac{t^2+t}{t^2-t}\,dt = -\int\frac{t^2-t+2t}{t^2-t}\,dt = -t-\int\frac{2}{t-1}\,dt = -t-2\ln|t-1|+C = -e^x-2\ln|e^x-1|+C$$

6.82 $$\int\sqrt{e^{3x}+e^{2x}}\,dx = \int e^x\sqrt{e^x+1}\,dx = \int t\sqrt{t+1}\cdot\frac1t\,dt = \frac23(t+1)^{\frac32}+C = \frac23(e^x+1)^{\frac32}+C$$

$t=e^x \qquad dx=\frac1t\,dt$

6.83 $$\int x\sin x^2\,dx = \frac12\int\sin x^2\,dx^2 = -\frac{\cos x^2}{2}+C$$

6.84 $$\int\frac{1}{\sqrt x}\cos\sqrt x\,dx = 2\int\cos\sqrt x\,d\sqrt x = 2\sin\sqrt x+C$$

6.89 $$\int\text{ctg}\,x\,dx = \int\frac{\cos x}{\sin x}\,dx = \int\frac{1}{\sin x}\,d\sin x = \ln|\sin x|+C$$

6.98 $$\int\frac{x+\sqrt{\text{arctg}\,2x}}{4x^2+1}\,dx = \frac12\int\frac{dx^2}{4x^2+1}+\int\frac{\sqrt{\text{arctg}\,2x}}{4x^2+1}\,dx = \frac18\ln|4x^2+1|+\frac13(\text{arctg}\,2x)^{\frac32}+C$$

6.99 $$\int \frac{\arcsin x - \arccos x}{\sqrt{1-x^2}}\,dx = \frac{\arcsin^2 x}{2} + \frac{\arccos^2 x}{2} + C$$

$E = \frac{1}{8}$ $C = \frac{1}{4}$ $A = D = \frac{3}{16}$ $B = \frac{1}{4}$

6.271 $$\int \frac{dx}{(1+x)^3(1-x)^2} = \int \overset{(1+x)^2(1-x)^2}{\frac{A}{1+x}} + \overset{(1+x)(1-x)^2}{\frac{B}{(1+x)^2}} + \overset{(1-x)^2}{\frac{C}{(1+x)^3}} + \overset{(1+x)^3(1-x)}{\frac{D}{(1-x)}} + \overset{(1+x)^3}{\frac{E}{(1-x)^2}} = \frac{3}{16}\ln|x+1| - \frac{1}{4(1+x)} - \frac{1}{8(1-x)^2} - \frac{3}{16}\ln|1-x| + \frac{1}{8(1-x)} + C$$

$E \quad (1+x)^3 = 1 + 3x + 3x^2 + x^3$

$A \quad ((1+x)(1-x))^2 = x^4 + 1 - 2x^2$

$B \quad (1+x)(1-x)^2 = (1+x)(1+x^2-2x) = 1 + x^2 - 2x + x + x^3 - 2x^2 = x^3 - x^2 - x + 1$

$C \quad (1-x)^2 = 1 + x^2 - 2x$

$D \quad (1+x)^3(1-x) = -x^4 - 2x^3 + 2x + 1$

4: $A - D$

3: $E + B - 2D$

2: $3E - 2A - B + C$

1: $3E - B - 2C + 2D$

0: $E + A + B + C + D = 1$

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ -2 & -1 & 1 & 0 & 3 & 0 \\ 0 & -1 & -2 & 2 & 3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & 1 & -2 & 3 & 0 \\ 0 & -1 & -2 & 2 & 3 & 0 \\ 0 & 1 & 1 & 2 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & -4 & 4 & 0 \\ 0 & 0 & -2 & 0 & 4 & 0 \\ 0 & 0 & 1 & 4 & 0 & 1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & -4 & 4 & 0 \\ 0 & 0 & 0 & -8 & 12 & 0 \\ 0 & 0 & 0 & 0 & 8 & 1 \end{pmatrix}$$

$$\begin{cases} \frac{1}{8} + B - 2D = 0 \\ \frac{3}{8} - 2D - B + \frac{2}{8} = 0 \end{cases}$$

$-\frac{2}{8} + 2B - \frac{2}{8} = 0$

$B = \frac{1}{4}$

$\frac{4}{8} - 4D + \frac{2}{8} = 0$

$\frac{6}{8} = 4D$

$D = \frac{3}{16}$

6.269 $$\int \frac{x^3+1}{x^2(1-x)}\,dx = -\int \frac{x^3+1}{x^3-x^2}\,dx = -\int \frac{x^3-x^2+x^2+1}{x^3-x^2}\,dx = -\int dx - \int \frac{1}{x-1}\,dx + \int \frac{1}{x^2(1-x)}\,dx =$$

$$\int \frac{dx}{x^2(1-x)} = \int \left(\overset{x(1-x)}{\frac{A}{x}} + \overset{(1-x)}{\frac{B}{x^2}} + \overset{x^2}{\frac{C}{1-x}} \right) dx = \int \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{1-x} \right) dx$$

$B = 1$

$C = 1$

$A = 1$

$$= -x - \ln|x-1| + \ln|x| - \frac{1}{x} - \ln|x-1| + C = -x - \frac{1}{x} - 2\ln|x-1| + \ln|x|$$

6.272 $$\int \frac{dx}{(1+x)(1+x^2)^2} = \int \overset{(1+x^2)^2}{\frac{A}{1+x}} + \overset{(1+x)(1+x^2)}{\frac{Bx+C}{1+x^2}} + \overset{(1+x)}{\frac{Dx+E}{(1+x^2)^2}} =$$

$A = \frac{1}{4}$

$B = -\frac{1}{4}$

$C = \frac{1}{4}$

$D = -\frac{1}{2}$

$E = \frac{1}{2}$

$$= \int \left(\frac{1}{4(1+x)} + \frac{1-x}{4(1+x^2)} + \frac{1-x}{2(1+x^2)^2} \right) dx =$$

$$= \frac{1}{4}\ln|1+x| + \frac{1}{4}\operatorname{arctg} x - \frac{\ln|x^2+1|}{8} + \frac{1}{4}\operatorname{arctg} x + \frac{x}{4(1+x^2)} \oplus$$

$$\oplus \frac{1}{4(1+x^2)} + C = \boxed{\frac{1}{4}\left(2\operatorname{arctg} x + \ln\frac{|1+x|}{\sqrt{x^2+1}} + \frac{x+1}{1+x^2} \right) + C}$$

Ответ!!!

A: $1 + x^2 = x^4 + 2x^2 + 1$

B: $(1+x)(1+x^2) = x^4 + x^3 + x^2 + x$

C: $x^3 + x^2 + x + 1$

D: $x^2 + x$

E: $x + 1$

$u\,dv = uv - \int v\,du$

$x = u$ $\quad dv = \frac{x\,dx}{(1+x^2)^2}$

$v = -\frac{1}{2(1+x^2)}$

$$\int \frac{1-x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx - \frac{1}{2}\int \frac{dx^2}{x^2+1} = \operatorname{arctg} x - \frac{\ln|x^2+1|}{2}$$

$$\int \frac{1-x}{(1+x^2)^2} = \int \frac{dx}{(1+x^2)^2} - \frac{1}{2}\int \frac{dx^2}{(1+x^2)^2} = \frac{\operatorname{arctg} x}{2} + \frac{x}{2(1+x^2)} + \frac{1}{2(1+x^2)} + C$$

$$\int \frac{dx}{(1+x^2)^2} = \int \frac{(1+x^2) - x^2}{(1+x^2)^2}\,dx = \int \frac{dx}{1+x^2} - \int \frac{x^2\,dx}{(1+x^2)^2} = \operatorname{arctg} x + \frac{x}{2(1+x^2)} - \frac{1}{2}\int \frac{dx}{1+x^2} = \operatorname{arctg} x - \frac{\operatorname{arctg} x}{2} + \frac{x}{2(1+x^2)} + C$$

6.288

$$\int \frac{3x^2+x-2}{(x-1)^3(x^2+1)}\,dx = \int\left(\frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{Dx+E}{x^2+1}\right)dx = -\frac{3}{2}\int\frac{dx}{x-1} + \frac{5}{2}\int\frac{dx}{(x-1)^2} + \int\frac{dx}{(x-1)^3} + \frac{3}{2}\int\frac{x\,dx}{x^2+1} - \int\frac{dx}{x^2+1} \ominus$$

$A: (x-1)^2(x^2+1) = (x^2-2x+1)(x^2+1) = x^4+x^2-2x^3-2x+x^2+1 = x^4-2x^3+2x^2-2x+1$

$B: (x-1)(x^2+1) = x^3-x^2+x-1$

$$\ominus -\frac{3}{2}\ln|x-1| - \frac{5}{2(x-1)} - \frac{1}{2(x-1)^2} + \frac{3}{4}\ln|x^2+1| - \operatorname{arctg}x + C$$

$C: x^2+1$

$D: x(x-1)^3 = x(x^3-3x^2+3x-1) = x^4-3x^3+3x^2-x$

$E: x^3-3x^2+3x-1$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 0 & 0 \\ -2 & 1 & 0 & -3 & 1 & 0 \\ 2 & -1 & 1 & 3 & -3 & 3 \\ -2 & 1 & 0 & -1 & 3 & 1 \\ 1 & -1 & 1 & 0 & -1 & -2 \end{array}\right) \sim \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 1 & -3 & 3 \\ 0 & 1 & 0 & 1 & 3 & 1 \\ 0 & -1 & 1 & -1 & -1 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 3 \\ 0 & 0 & 0 & 2 & 2 & 1 \\ 0 & 0 & 1 & -2 & 0 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 3 \\ 0 & 0 & 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 4 & -4 \end{pmatrix}$$

$E = -1 \quad C = 1 \quad A = -\frac{3}{2}$

$D = \frac{3}{2} \quad B = \frac{5}{2}$

2. Интегрирование тригонометрических функций (по сборнику Демидовича):
1992, 1994, 1998, 2001, 2003, 2004, 2006, 2011, 2014, 2022, 2026, 2027, 2029, 2032, 2035, 2039, 2040, 2041, 2043.1

1992 $\int \sin^6 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^3 dx = \frac{1}{8}\int\left(\underset{\text{отв}}{1} - \underset{\text{отв}}{3\cos 2x} + \underset{\text{отв}}{3\cos^2 2x} - \cos^3 2x\right)dx = \frac{x}{8} - \frac{3}{16}\sin 2x + \frac{3}{16}x + \frac{3}{64}\sin 4x - \frac{\sin 2x}{16} + \frac{\sin^3 2x}{48}$

$\int \cos 2x\,dx = \frac{\sin 2x}{2} + C$

$\int \cos^2 2x\,dx = \frac{1}{2}\int(1+\cos 4x)dx = \frac{x}{2} + \frac{\sin 4x}{8} + C$

$\int \cos^3 2x\,dx = \frac{1}{2}\int \cos^2 2x \cdot \cos 2x\,d2x = \frac{1}{2}\int \cos^2 2x\,d\sin 2x = \frac{1}{2}\int(1-\sin^2 2x)\,d\sin 2x = \frac{\sin 2x}{2} - \frac{\sin^3 2x}{6} + C$

1994 $\int \sin^2 x \cos^4 x\,dx = \int \sin^2 x\,(1-\sin^2 x)^2\,dx = \int \sin^2 x\,dx - 2\int \sin^4 x\,dx + \int \sin^6 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} - x + \sin 2x - \frac{x}{2} - \frac{\sin 4x}{8} + (1992)$

$\int \sin^2 x\,dx = \frac{1}{2}\int(1-\cos 2x)\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$

$\int \sin^4 x\,dx = \frac{1}{2}\int(1-\cos 2x)^2\,dx = \frac{1}{2}\left[\int dx - 2\int \cos 2x\,dx + \int \cos^2 2x\,dx\right] = \frac{x}{2} - \frac{\sin 2x}{2} + \frac{x}{4} + \frac{\sin 4x}{16} + C$

$\int \cos^2 2x\,dx = \frac{x}{2} + \frac{\sin 4x}{8} + C$

$\int \sin^6 x\,dx = (1992)$

1998 $\int \frac{\cos^4 x}{\sin^3 x}dx = \int \frac{\sin x \cdot \cos^4 x}{\sin^4 x}dx = -\int \frac{\cos^4 x}{(1-\cos^2 x)^2}d\cos x = \int \frac{t^4}{(1+t)^2(1-t)^2}dt = \int\left[\frac{A}{1+t} + \frac{B}{(1+t)^2} + \frac{C}{(1-t)} + \frac{D}{(1-t)^2}\right]dt =$

$= \frac{\ln|1+t|}{2} + \frac{1}{4(1+t)} + \frac{\ln|1-t|}{2} + \frac{1}{4(1-t)} + C$

$A: (1+t)(1-t)^2 = (1-t^2)(1-t) = 1 - t - t^2 + t^3 = t^3 - t^2 - t + 1$

$B: (1-t)^2 = t^2 - 2t + 1$

$C: (1+t)^2(1-t) = (1+t)(1-t^2) = 1 + t - t^2 - t^3 = -t^3 - t^2 + t + 1$

$D: (1+t)^2 = t^2 + 2t + 1$

$$\begin{pmatrix} 1 & 0 & -1 & 0 & 1 \\ -1 & 1 & -1 & 1 & 0 \\ -1 & -2 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$A = \frac{1}{2}$, $B = -\frac{1}{4}$, $C = -\frac{1}{2}$, $D = \frac{1}{4}$

2001 $\int \frac{dx}{\sin^4 x \cos^4 x} = \frac{1}{16}\int \frac{dx}{\sin^4 2x} = \frac{1}{16}\int \frac{4\,dx}{(1-\cos 4x)^2} = \frac{1}{4}\left[\int dx - \frac{1}{2}\int \frac{dx}{\cos 4x} + \int \frac{dx}{\cos^2 4x}\right] =$

$= \frac{x}{4} - \frac{\ln|\mathrm{tg}(2x+\frac{\pi}{4})|}{32} + \frac{1}{4}\int \frac{2\,dx}{1+\cos 8x} = \frac{x}{4} - \frac{\ln|\mathrm{tg}(2x+\frac{\pi}{4})|}{32} + \frac{x}{2} + \frac{\ln|\mathrm{tg}(4x+\frac{\pi}{4})|}{16}$

2003 $\int \frac{dx}{\sin x \cos^4 x} = \int \frac{\sin x\,dx}{\sin^2 x \cos^4 x} = -\int \frac{d\cos x}{(1-\cos^2 x)\cos^4 x} = \int \frac{dt}{(t-1)(t+1)\,t^4}$

$= \int\left[\frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{t} + \frac{D}{t^2} + \frac{E}{t^3} + \frac{F}{t^4}\right]dt = \frac{1}{2}\ln\left|\frac{\cos x - 1}{\cos x + 1}\right| + \frac{1}{\cos x} + \frac{1}{3\cos^3 x} + C$

$A: t^4(t+1) = t^5 + t^4$
$B: t^4(t-1) = t^5 - t^4$
$C: t^3(t^2-1) = t^5 - t^3$
$D: t^2(t^2-1) = t^4 - t^2$
$E: t(t^2-1) = t^3 - t$
$F: t^2 - 1$

$$\left(\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{array}\right)$$

$A = \frac{1}{2}$, $B = -\frac{1}{2}$, $C = 0$, $D = -1$, $E = 0$, $F = -1$

2004 $\mathrm{tg}\,x = t \quad dt = \sec^2 x\,dx$

$\int \mathrm{tg}^k x\,dx = \int \mathrm{tg}^{k-2} x \cdot (\sec^2 x - 1)\,dx = \int \mathrm{tg}^{k-2} x \sec^2 x\,dx - \int \mathrm{tg}^{k-2} x\,dx =$

$= \int t^{k-2}dt - \int \mathrm{tg}^{k-2} x\,dx = \frac{1}{k-1}\mathrm{tg}^{k-1} x - \int \mathrm{tg}^{k-2} x\,dx$

$\int \mathrm{tg}\,x\,dx = \int \frac{\sin x}{\cos x}dx = \ln|\sec x|$

$\int \mathrm{tg}^5 x\,dx = \frac{1}{4}\mathrm{tg}^4 x - \frac{1}{2}\mathrm{tg}^2 x + \ln|\sec x| + C$

2006 $\int \frac{\sin^4 x}{\cos^6 x}dx = \int \mathrm{tg}^4 x \cdot \sec^2 x\,dx = \frac{1}{5}\mathrm{tg}^5 x + C$

2011 $I_n = \int \sin^n x\,dx = \int \underbrace{\sin^{n-1} x}_{u} \cdot \underbrace{\sin x\,dx}_{dv} = \underbrace{-\cos x \sin^{n-1} x}_{= Z_n} + (n-1)\int \cos^2 x \cdot \sin^{n-2} x\,dx =$

$v = -\cos x$

$= Z + (n-1)\int(1-\sin^2 x)\sin^{n-2} dx = Z_n + (n-1)I_{n-2} - (n-1)I_n$

$I_{n-2} \quad - \quad I_n$

$I_n(1 + n - 1) = Z_n + (n-1)I_{n-2}$

$I_n = \frac{Z_n}{n} + \frac{n-1}{n}I_{n-2}$

$\int \sin^4 x\,dx = \underbrace{\frac{-\cos x \sin^3 x}{4}}_{=\frac{Z_4}{4}} + \underbrace{\frac{3}{4}}_{\frac{n-1}{n}}\cdot\underbrace{\left[\underbrace{\frac{-\cos x \sin x}{2}}_{=\frac{Z_2}{2}} + \frac{1}{2}\underbrace{I_0}_{=x}\right]}_{I_2} = -\frac{\cos x \sin^3 x}{4} - \frac{3\cos x \sin x}{8} + \frac{3}{8}x + C$

$\int \sin^6 dx = -\frac{\cos x \sin^5 x}{6} - \frac{5}{6\cdot 4}\cos x \sin^3 x - \frac{5}{6}\cdot\frac{3}{4\cdot 2}\cdot\cos x \sin x + \frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot x + C$

$$K_n = \int \cos^n x\,dx = \int \underbrace{\cos^{n-1}x}_{u}\,\underbrace{\cos x\,dx}_{dv} = \sin x\cos^{n-1}x + \int (n-1)\sin x\cos^{n-2}x\sin x\,dx = \sin x\cos^{n-1}x + (n-1)K_{n-2} - (n-1)K_n$$

$v = \sin x$

$$K_n(1+n-1) = \sin x\cos^{n-1}x + (n-1)K_{n-2}$$

$$K_n = \frac{\sin x\cos^{n-1}x}{n} + \frac{n-1}{n}K_{n-2}$$

$$\int \cos^8 x = \frac{\sin x\cos^7 x}{8} + \frac{7}{8\cdot 6}\sin x\cos^5 x + \frac{7\cdot 5}{8\cdot 6\cdot 4}\sin x\cos^3 x + \frac{7\cdot 5\cdot 3}{8\cdot 6\cdot 4\cdot 2}\sin x\cos x + \frac{7\cdot 5\cdot 3}{8\cdot 6\cdot 4\cdot 2}x$$

(2014) $$\int \cos x\cos 2x\cos 3x\,dx = \int \frac{1}{2}(\cos 3x + \cos x)\cos 3x\,dx = \frac{1}{6}\int \cos^2 3x\,d3x + \frac{1}{4}\int \cos 4x + \cos 2x\,dx = \frac{1}{12}\sin 3x\cos 3x + \frac{1}{4}x + \frac{\sin 4x}{16} + \frac{\sin 2x}{8} + C$$

(2022) $$\int \frac{dx}{\sin x - \sin\alpha} =$$

$\sin(\alpha-\beta) = \sin[(x+\alpha)-(x+\beta)] = \sin(x+\alpha)\cos(x+\beta) - \cos(x+\alpha)\sin(x+\beta)$

$\sin(x+a)\sin(x+b) = \cos(a-b) - \cos(x+a)\cos(x+b)$

(2026) $$\int \frac{dx}{(2+\cos x)\sin x} = \left| \begin{matrix} \cos x = \sqrt{1-t^2} \\ dx = d\arcsin x = \frac{dt}{\sqrt{1-t^2}} \\ \sin x = t \end{matrix} \right| = \int \frac{dt}{(2+\sqrt{1-t^2})\sqrt{1-t^2}\cdot t} = \frac{1}{2}\int \frac{dt^2}{(2+\sqrt{1-t^2})\sqrt{1-t^2}\,t^2} = \frac{1}{2}\int \frac{dt^2}{t^2(2\underbrace{\sqrt{1-t^2}}_{y} + \underbrace{1-t^2}_{y^2})} = -\frac{1}{2}\int \frac{dy^2}{(-y^2+1)(y^2+2y)} = \int \frac{dy}{(y-1)(y+1)(y+2)} =$$

$\underbrace{t^2}_{-y^2+1}$

$$= \int \left[\frac{1}{6(y-1)} - \frac{1}{2(y+1)} + \frac{1}{3(y+2)}\right]dy = \frac{1}{6}\ln\frac{|y-1|(y+2)^2}{|y+1|^3}$$

(2027) x3 $$\int \frac{\sin^2 x}{\sin x + 2\cos x}\,dx = \int \frac{4t^2}{(1+t^2)^2}\cdot\frac{\cancel{1+t^2}}{2t+\cancel{2}-\cancel{2}t^2}\cdot\frac{2\,dt}{\cancel{1+t^2}} = \int \frac{4t^2\,dt}{(1+t^2)^2(-t^2+t+1)}$$

(2029) $$\int \frac{\sin^2 x}{1+\sin^2 x}\,dx = \int \frac{\sin^2 x + 1 - 1}{1+\sin^2 x}\,dx = \int dx - \int \frac{dx}{1+\sin^2 x} = \int dx - \int \frac{dx}{\cos^2 x}\cdot\frac{1}{(tg^2 x + 1 + tg^2 x)} = \int dx - \int \frac{d\,tg\,x}{(\sqrt{2}\,tg\,x)^2+1} = x - arctg(\sqrt{2}\,tg\,x)\cdot\frac{1}{\sqrt{2}} + C$$

(2032) $$\int \frac{\sin x\cos x}{\sin x + \cos x}\,dx = \frac{1}{2}\int \frac{(\sin x + \cos x)^2}{\sin x + \cos x}\,dx - \frac{1}{2}\int \frac{dx}{\sin x + \cos x} = \frac{1}{2}\int(\sin x + \cos x)\,dx - \int \frac{dt}{2t+1-t^2} = \frac{1}{2}\int(\sin x + \cos x)\,dx + \int \frac{dt}{(t-1)^2-(\sqrt{2})^2} = \frac{\sin x}{2} - \frac{\cos x}{2} + \frac{1}{2\sqrt{2}}\ln\left|\frac{t-1+\sqrt{2}}{t-1-\sqrt{2}}\right| + C$$

тогд

(2035) $$\int \frac{dx}{\sin^4 x + \cos^4 x} = 4\int \frac{dx}{\cos^2 2x + \cancel{2\cos 2x} + 1 + 1 - \cancel{2\cos 2x} + \cos^2 2x} = 2\int \frac{dx}{\cos^2 2x + 1} = 2\int \frac{dx}{\cos^2 2x}\cdot\frac{1}{1+\frac{1}{\cos^2 2x}} = \frac{\sqrt{2}}{2}\int \frac{d\,\frac{tg\,2x}{\sqrt{2}}}{1+\frac{tg^2 2x}{\sqrt{2}^2}} = \sqrt{2}\,arctg\left(\frac{tg\,2x}{\sqrt{2}}\right) + C$$

$\cos 2x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

$\cos^2 x = \dfrac{\cos 2x + 1}{2}$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

(2039) $$\int \frac{dx}{\sin^6 x + \cos^6 x} = \frac{2}{2}\int \frac{d\,\frac{tg\,2x}{2}}{1+\frac{tg^2 x}{2^2}} = arctg\left(\frac{tg\,2x}{2}\right) + C$$

$$\sin^6 x + \cos^6 x = \underbrace{(\sin^2 x + \cos^2 x)}_{1}(\sin^4 x - \sin^2 x\cos^2 x + \cos^4 x) = (\sin^2 x - \cos^2 x)^2 + \sin^2 x\cos^2 x = \frac{1}{4}(4\cos^2 2x + \sin^2 2x) = \frac{\cos^2 2x}{4}(4 + tg^2 2x)$$

(2040) $$\int \frac{dx}{(\sin^2 x + 2\cos^2 x)^2} = \int \frac{dx}{\cos^2 x}\cdot\frac{1}{\cos^2 x}\cdot\frac{1}{(tg^2 x + 2)^2} = \int \frac{d\,tg\,x}{(tg^2 x+2)^2}(1+tg^2 x) = \int \frac{(1+t^2)\,dt}{(2+t^2)^2} = \int \frac{2+t^2-1}{(2+t^2)^2}\,dt = \int \frac{dt}{2+t^2} - \int \frac{dt}{(2+t^2)^2} = \frac{1}{\sqrt{2}}\,arctg\,t - ⊛$$

$$\int \frac{dt}{(2+t^2)^2} = \frac{1}{4}\int \frac{dt}{(1+(\frac{t}{\sqrt{2}})^2)^2} = \frac{1}{2\sqrt{2}}\int \frac{du}{(1+u^2)^2} = \frac{1}{2\sqrt{2}}\left(arctg\frac{t}{\sqrt{2}} + \frac{\frac{t}{\sqrt{2}}}{2(2+\frac{t^2}{2})} + C\right) ⊛$$

(2041) $$\int \frac{dx}{a\sin x + b\cos x} = \frac{1}{\sqrt{a^2+b^2}}\int \frac{d(x+\varphi)}{\sin(x+\varphi)} = \frac{1}{\sqrt{a^2+b^2}}\cdot\ln\left|tg\left(\frac{x+\varphi}{2}\right)\right| + C$$

$$\frac{dx}{\cos x} = \ln\left|tg\left(\frac{x}{2}-\frac{\pi}{4}\right)\right|$$

(2043) $$\int \frac{\sin x\,dx}{\sin x - 3\cos x} = \int \frac{A(\sin x - 3\cos x) + B(\cos x + 3\sin x)}{\sin x - 3\cos x}\,dx = \frac{x}{10} + \frac{3}{10}\ln|\sin x - 3\cos x| + C$$

$1 = A + 3B$

$0 = -3A + B$

$$\left(\begin{array}{cc|c} 1 & 3 & 1 \\ -3 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 10 & 3 \end{array}\right)$$

$B = \frac{3}{10}$

$A = \frac{1}{10}$

Исследовать на сходимость

1) $\int_1^{+\infty} \frac{x}{x^3+1}dx$ ограничивается сверху $\frac{1}{x^2}$ сходится

2) $\int_1^{+\infty} \frac{\ln(x^2+1)}{x}dx$ ограничивается снизу $\frac{1}{x}$ расх-ся

3) $\int_0^{+\infty} \frac{x\,\mathrm{arctg}\,x}{\sqrt[3]{1+x^4}}dx$ аигу $\frac{1}{x}$ рас-ся

4) $\int_0^1 \frac{1}{\sqrt{1-x^2}}dx = \arcsin x\Big|_0^1 = \frac{\pi}{2}$ сх-ся

5) $\int_1^2 \frac{dx}{x\ln x} = \int_1^2 \frac{d\ln x}{\ln x} = \ln(\ln x)\Big|_1^2 = \mathrm{const} - \infty$ расх-ся

6) $\int_1^2 \frac{1}{x\sqrt{\ln x}}dx = 2\sqrt{\ln x}\Big|_1^2 = 2\sqrt{\ln 2}$ сх-ся

7) $\int_1^2 \frac{x}{\sqrt{x-1}}dx = \left| \begin{matrix} t=\sqrt{x-1}, & x=t^2+1, & dx=2t\,dt \\ a=0 & b=1 & \end{matrix} \right| = \int_0^1 \frac{t^2+1}{\cancel{t}}\cdot 2\cancel{t}\,dt = \frac{2}{3}t^3+t\Big|_0^1 = \frac{2}{3}t\,(t^2+1)\Big|_0^1 = \frac{2}{3}\sqrt{x-1}\,(x+2)\Big|_1^2 = \frac{8}{3}$ сх-ся

8)

7.54, 7.55, 7.56, 7.62, 7.63, 7.70, 7.76
7.85, 7.86, 7.93, 7.95, 7.96

7.54 $y=x^2e^{-x}$, $y=0$, $x=2$

$$\int_2^0 \underbrace{x^2}_{4} \underbrace{e^{-x}d(-x)} = e^{-x}\cdot x^2 + 2\int_2^0 x e^{-x}d(-x) = e^{-x}\cdot x^2\Big|_2^0 + 2\cdot e^{-x}\cdot x\Big|_2^0 + 2e^{-x}\Big|_2^0 =$$

$$= -e^{-x}(\underset{4+4+2}{x^2+2x+2})\Big|_0^2 = 2-\frac{10}{e^2} \approx 0{,}6467$$

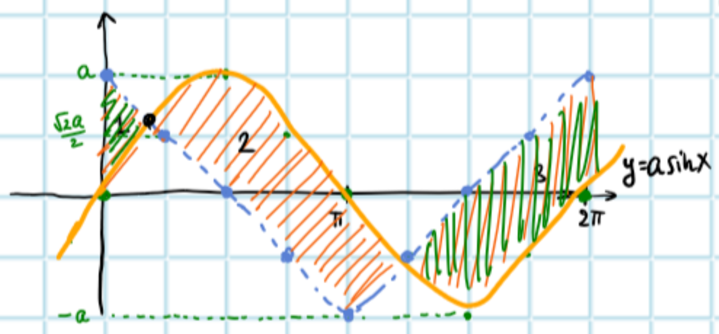
$u\,dv = uv - v\,du$

7,55 $y=a\sin x$, $y=a\cos x$ $[0;2\pi]$

1: $[0;\frac{\pi}{4})$ $a\cos x - a\sin x$

2: $[\frac{\pi}{4};\frac{5\pi}{4}]$ $a\sin x - a\cos x$

3: $[\frac{5\pi}{4}; 2\pi)$ $a\cos x - a\sin x$



$$S = a\left[\int_0^{\frac{\pi}{4}} \cos x - \sin x\,dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin x - \cos x\,dx + \int_{\frac{5\pi}{4}}^{2\pi} \cos x - \sin x\,dx\right] = a4\sqrt{2}$$

$\sin + \cos$ $\quad -\cos - \sin$ $\quad \sin + \cos$

$\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}-\cancel{1}$ $\quad \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}$ $\quad \cancel{1}+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}$

7.56

$$S = \int_0^1 x\ln x(\ln x-1)dx + \int_1^e x\ln x(1-\ln x)dx =$$

$$= \int_0^1 (x\ln x - x)\,d(x\ln x - x) - \int_1^e (x\ln x - x)\,d(x\ln x - \cancel{x}) =$$

$$= \frac{(x\ln x - x)^2}{2}\Big|_0^1 - \frac{(x\ln x - x)^2}{2}\Big|_1^e = \frac{1}{2}+\frac{1}{2} = 1$$

7.62

$$S = \frac{4}{\sqrt{a}}\int_0^a x(a^2-x^2)^{\frac{1}{4}}dx = -\frac{2}{\sqrt{a}}\int_0^a (a^2-x^2)^{\frac{1}{4}}d(a^2-x^2) = -\frac{2}{\sqrt{a}}\cdot\frac{4}{5}\cdot(a^2-x^2)^{\frac{5}{4}}\Big|_0^a$$

$$= \frac{8}{5\cancel{\sqrt{a}}}\cdot a^2\cancel{\sqrt{a}} = \frac{8}{5}a^2$$

7.63 $S =$

$$\int_0^1 \left(\sqrt{\frac{1+y^2}{2}} - y\right)dy$$

7.70. Найти площадь каждой из частей, на которые парабола $y^2=a(a-x)$ разбивает круг $x^2+y^2\leqslant a^2$. (*) $\quad x=\sqrt{a-y^2}$ $\quad y=\sqrt{a-x^2}$ $\quad y=\sqrt{a(a-x)}$ (**)

Выражение (*) описывает круг с центром в $(0,0)$ и $r=a$

Выражение (**) описывает параболу (y) $x=-\frac{y^2}{a}+a$, вершина в т. $(a,0)$

$$S_{\text{большая}} = 2\left[\int_{-a}^0 \sqrt{a-x^2}\,dx + \int_0^a \sqrt{a(a-x)}\,dx\right] \quad t=\arcsin\frac{x}{a}$$

$x=a\sin t$

$$\int_{-\frac{\pi}{2}}^0 a\cos t\cdot a\cos t\,dt = a^2\int_{-\frac{\pi}{2}}^0 \frac{\cos 2t+1}{2}dt = \frac{a^2}{4}\sin 2t\Big|_{-\frac{\pi}{2}}^0 + \frac{a^2}{2}t\Big|_{-\frac{\pi}{2}}^0 = \frac{\pi}{4}a^2$$

$$-\sqrt{a}\int_0^a \sqrt{a-x}\,d(a-x) = -\sqrt{a}\,\frac{2}{3}\sqrt{a-x}\,(a-x)\Big|_0^a = \sqrt{a}\cdot\frac{2}{3}\sqrt{a}\cdot a = \frac{2}{3}a^2$$

$$S_{\text{мал}} = \frac{\pi a^2}{2} - \left(\frac{\pi}{4}a^2+\frac{2}{3}a^2\right) = \left(\frac{\pi}{4}-\frac{2}{3}\right)a^2$$

7.76

$$S = 4\int_0^{\frac{\pi}{2}} a\sin 2t\,(a\sin t)'\,dt = 4a^2\int_0^{\frac{\pi}{2}} 2\sin t\cos t\cos t\,dt =$$

$$= -8a^2\int_0^{\frac{\pi}{2}} \cos^2 t\,d\cos t = -8a^2\frac{\cos^3 t}{3}\Big|_0^{\frac{\pi}{2}} = -8a^2\left(\quad -\frac{1}{3}\right) = \frac{8}{3}a^2$$

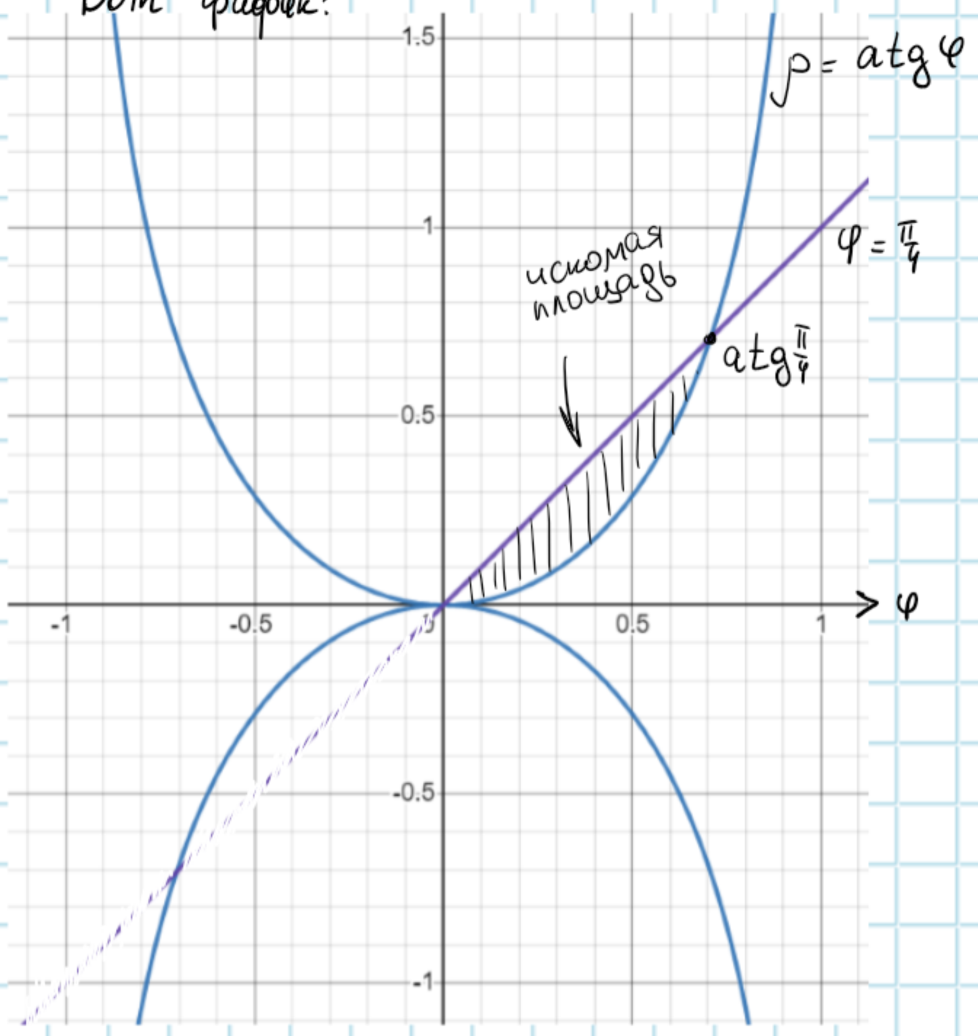
7.85

$$S = 4a^2\int_0^{\frac{\pi}{4}} \sin^2 4t\,dt = 4a^2\int_0^{\frac{\pi}{4}} \frac{1-\cos 8t}{2}dt = 2a^2t\Big|_0^{\frac{\pi}{4}} - \frac{a^2\sin 8t}{4}\Big|_0^{\frac{\pi}{4}} = \frac{a^2\pi}{2}$$

$$\int_0^{\frac{\pi}{4}} \sin^2$$

③ $r = a\,tg\varphi$, $\varphi = \frac{\pi}{4}$

Вот графек:



$$S = \frac{1}{2}\int_0^{\frac{\pi}{4}} \rho^2 d\varphi = \frac{a^2}{2}\int_0^{\pi/4} tg^2\varphi\, d\varphi = \frac{a^2}{2}\int_0^{\pi/4}\left(\frac{1}{\cos^2\varphi} - 1\right)d\varphi = \frac{a^2}{2}\cdot\left(tg\varphi - \varphi\right)\Big|_0^{\frac{\pi}{4}} = \frac{a^2}{2}\left(1 - \frac{\pi}{4}\right) = \boxed{a^2\left(\frac{1}{2} - \frac{\pi}{8}\right)}$$

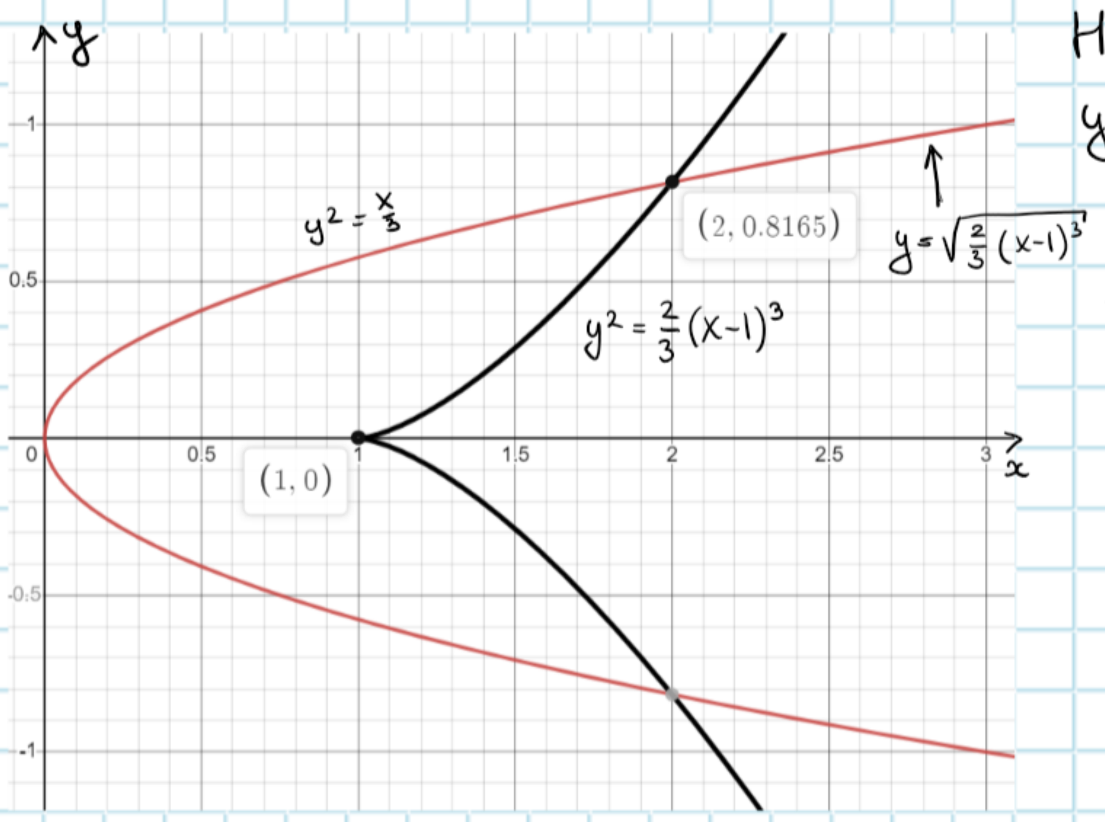
Ответ:

$\sin^2\varphi + \cos^2\varphi = 1 \quad | : \cos^2\varphi$

$tg^2\varphi + 1 = \frac{1}{\cos^2\varphi}$

$tg^2\varphi = \frac{1}{\cos^2\varphi} - 1$

④ Найти длину дуги полукубической параболы $y^2 = \frac{2}{3}(x-1)^3$, заключенной внутри параболы $y^2 = \frac{x}{3}$



Найдем длину дуги в $y > 0$ и умножим на 2

$$\frac{L}{2} = \int_1^2 \sqrt{1 + (y')^2}\,dx = \int_1^2 \sqrt{1 + \frac{2}{3}\cdot\frac{3\cdot 3}{2\cdot 2}\cdot(x-1)}\,dx = \frac{2}{3}\int_1^2\sqrt{\frac{3}{2}x - \frac{3}{2} + 1}\,d\left(\frac{3}{2}x\right) =$$

$$= \frac{2}{3}\int_1^2\sqrt{\frac{3}{2}x - \frac{1}{2}}\,d\left(\frac{3}{2}x - \frac{1}{2}\right) =$$

$$= \frac{2}{3}\cdot\frac{2}{3}\cdot\left(\frac{3}{2}x - \frac{1}{2}\right)^{\frac{3}{2}}\Big|_1^2 =$$

$$= \frac{4}{9}\cdot\left(\left(\frac{6}{2} - \frac{1}{2}\right)^{\frac{3}{2}} - \left(\frac{3}{2} - \frac{1}{2}\right)^{\frac{3}{2}}\right)$$

$$= \frac{4}{9}\cdot\left(\left(\frac{5}{2}\right)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right) =$$

$$= \frac{4}{9}\left(\frac{5\sqrt{5}}{2\sqrt{2}} - 1\right) =$$

$$= \frac{4}{9}\left(\frac{5\sqrt{10}}{4} - 1\right) =$$

$$= \frac{20\sqrt{10} - 16}{9\cdot 4} = \frac{5\sqrt{10} - 4}{9}$$

$y = \sqrt{\frac{2}{3}}\cdot(x-1)^{\frac{3}{2}}$

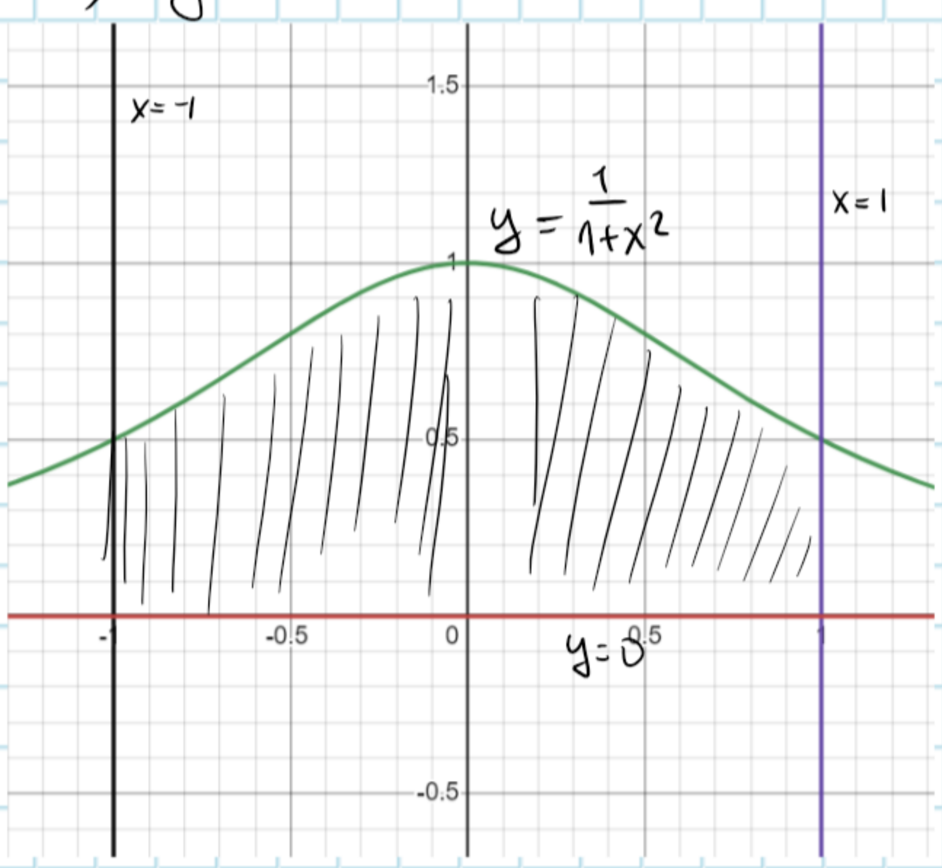
$y' = \sqrt{\frac{2}{3}}\cdot\frac{3}{2}(x-1)^{\frac{1}{2}}\cdot 1$

$\left(\frac{3x}{2} - \frac{1}{2}\right)^{\frac{1}{2}}$

$\boxed{\text{Ответ: } 2\cdot\frac{5\sqrt{10} - 4}{9} = \frac{10\sqrt{10} - 8}{9}}$

⑤ $y = \frac{1}{1+x^2}$, $x = \pm 1$, $y = 0$

а) Ox
б) Ось симметрии (O_y)
в) $y = 1$



а) $$V_x = \pi\int_a^b f^2(x)\,dx = \pi\int_{-1}^{1}\frac{1}{(1+x^2)^2}dx = \pi\int_{-1}^{1}\frac{(x^2+1) - x^2}{(x^2+1)^2}dx \ominus$$

$$\int\frac{x^2\,dx}{(x^2+1)^2} = \frac{1}{2}\int \underset{u}{x}\cdot\frac{d(x^2+1)}{(x^2+1)^2} = \frac{1}{2}\left[-\frac{x}{(x^2+1)} + \int\frac{dx}{x^2+1}\right]$$

$dv \Rightarrow v = -\frac{1}{(x^2+1)}$

$$\ominus\ \pi\int_{-1}^{1}\frac{dx}{x^2+1} + \frac{\pi}{2}\left(\frac{x}{x^2+1}\right)\Big|_{-1}^{1} - \frac{\pi}{2}\int_{-1}^{1}\frac{dx}{x^2+1} \ominus \frac{\pi}{2}\,arctg\,x\Big|_{-1}^{1} + \frac{\pi}{2}\left(\frac{x}{x^2+1}\right)\Big|_{-1}^{1} =$$

$$= \frac{\pi}{2}\left(\frac{\pi}{4} + \frac{\pi}{4}\right) + \frac{\pi}{2}\left(\frac{1}{2} + \frac{1}{2}\right) =$$

Ответ:

$$= \boxed{\frac{\pi^2}{4} + \frac{\pi}{2}}$$ Ответ.

б) $$V_y = 2\pi\int_0^1 x\cdot\frac{1}{1+x^2}dx = \pi\int_0^1\frac{dx^2+1}{x^2+1} = \pi\cdot\ln|x^2+1|\Big|_0^1 = \boxed{\pi\ln 2}$$

в) сделаем сдвиг СК на 1 вверх

$$y' = y - 1 \Rightarrow y'_{кривой} = \frac{1}{1+x^2} - 1$$

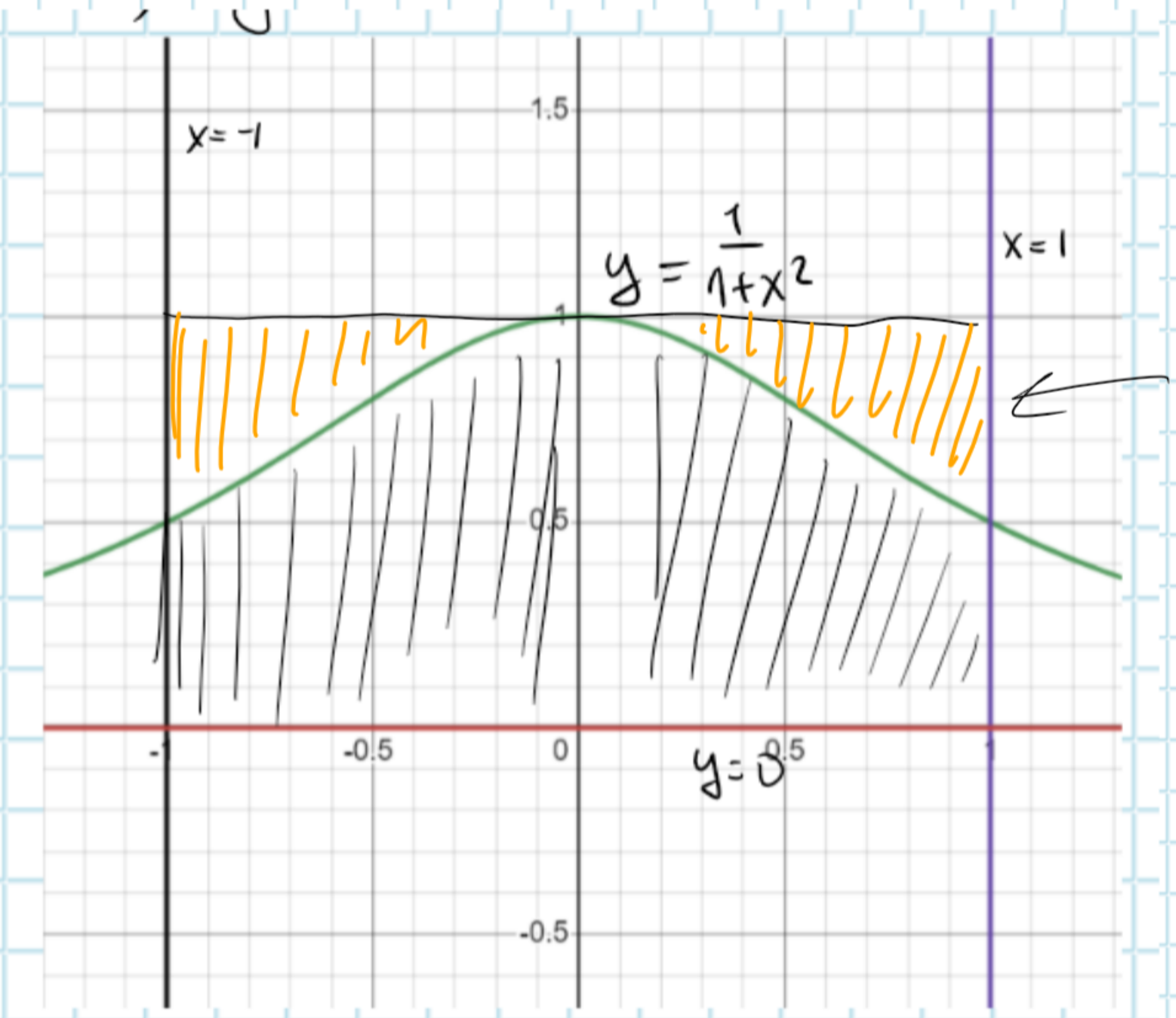
Вращаем вокруг Ox

$$V_x = \pi\int_{-1}^{1}\left(\frac{1-1-x^2}{1+x^2}\right)^2 dx = \pi\int_{-1}^{1}\left(\frac{x^2}{1+x^2}\right)^2 dx = \pi\int_{-1}^{1}\frac{x^4 \overset{+2x^2+1}{}}{x^4+2x^2+1}dx = \pi\int_{-1}^{1}\left[1-\frac{2x^2+1}{(1+x^2)^2}\right]dx = \pi x\Big|_{-1}^{1} - \pi\int_{-1}^{1}\frac{2x^2+1}{(1+x^2)^2}dx \ominus$$

решал в пункте а

$$\pi\int_{-1}^{1}\frac{2x^2+1^{+1-1}}{(x^2+1)^2}dx = \pi\int_{-1}^{1}\left[\frac{2(x^2+1)}{(x^2+1)^2} - \frac{1}{(1+x^2)^2}\right]dx = \pi 2\,\mathrm{arctg}\,x\Big|_{-1}^{1} - \frac{\pi^2}{4} - \frac{\pi}{2} = 2\pi\left(\frac{\pi}{4}+\frac{\pi}{4}\right) - \frac{\pi^2}{4} - \frac{\pi}{2} = \frac{3\pi^2}{4} - \frac{\pi}{2}$$

$$\ominus\ 2\pi - \frac{3\pi^2}{4} + \frac{\pi}{2} = \frac{5\pi}{2} - \frac{3\pi^2}{4}$$



Я нашел $V_{y=1}$ вращения такого тела

Ответ будет равен объему цилиндра с $r=1$ и $h=(1-(-1))$ минус найденный мной объем

Ответ: $\pi\cdot1\cdot2 - 2{,}5\pi + \frac{3\pi^2}{4} = \frac{3\pi^2}{4} - \frac{\pi}{2}$

(1) $\int_1^{e^3}\frac{dx}{x\sqrt{1+\ln x}}$; $\int_0^{\pi/2}\cos^3 x\sin 2x\,dx$;

$$\int_1^{e^3}\frac{dx}{x\sqrt{1+\ln x}} = \int_1^{e^3}\frac{d\ln x}{(1+\ln x)^{\frac12}} = 2\sqrt{1+\ln x}\Big|_1^{e^3} = 2(2-1) = \boxed{2}$$

Ответ:

$$\int_0^{\frac{\pi}{2}}\cos^3x\sin2x\,dx = 2\int_0^{\frac{\pi}{2}}\cos^4 x\sin x\,dx = -2\int_0^{\frac{\pi}{2}}\cos^4 x\, d\cos x = -2\,\frac{\cos^5 x}{5}\Big|_0^{\frac{\pi}{2}} = \boxed{\frac25}$$

(2)

$$\lim_{n\to\infty}\frac{\sqrt[n]{n!}}{n} = \lim_{n\to\infty}\sqrt[n]{\frac{n\cdot(n-1)\cdot\ldots\cdot2\cdot1}{\underbrace{n\cdot n\cdot\ldots\cdot n\cdot n}_{n}}} = \lim_{n\to\infty} e^{\ln\left(\sqrt[n]{\ldots}\right)} = e^{\lim\limits_{n\to\infty}\frac1n\sum_{i=1}^{n}\ln\left(\frac kn\right)}$$

$$\xi_k = a + \frac kn(b-a),\ \Delta x_k = \frac{b-a}{n} \Rightarrow b-a=1$$

Пусть $a=0$, $b=1$

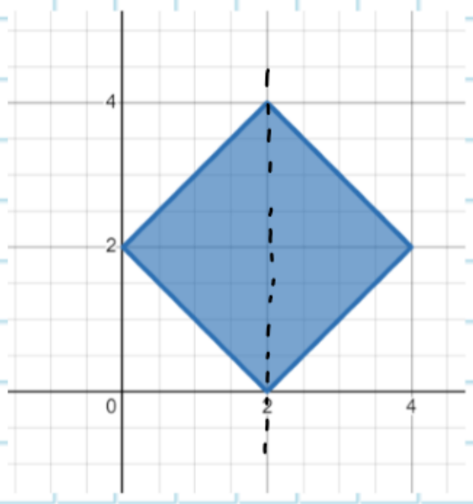
$$\lim_{n\to\infty}\sum_{k}^{n}\ln\left(\frac kn\right)\cdot\underbrace{\frac1n}_{\Delta x_k} = \lim_{n\to\infty}\sum_{k=1}^{n} f(\xi_k)\cdot\Delta x_k = \int_a^b f(\xi_k)\,dx = \int_0^1 \ln x\,dx = x\ln x\Big|_0^1 - x\Big|_0^1 =$$

$$= \left(1\cdot\ln1 - \lim_{a\to0^+} x\ln x\right) - (1-0) = -1$$

0 (по т. о произв. б.м.ф на огр.)

$$\boxed{e^{\int_0^1\ln(x)dx} = \frac1e}$$ Ответ

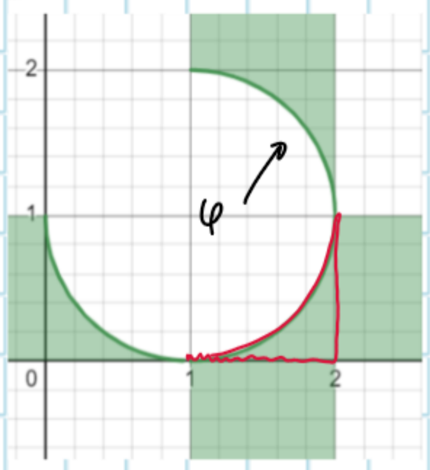
15.8 $\mathcal{D}:=\{(x,y): |x-2|+|y-2|\le 2\}$



$\int\limits_0^2 dx \int\limits_{2-x}^{2+x} dy + \int\limits_2^4 dx \int\limits_{x-2}^{-x+6} dy$

в другом порядке тоже понятно

15.11 $\mathcal{D}:=\{(x,y): x^2+y^2\ge 2x+2y-1,\ 1\le x\le 2,\ 0\le y\le 1\}$



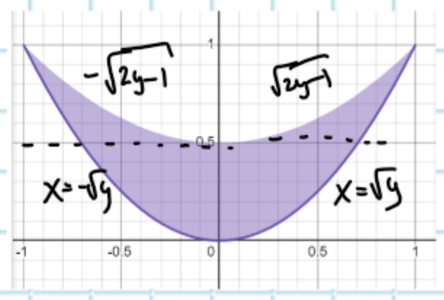
$\int\limits_1^2 dx \int\limits_0^{-\sqrt{1-(x-1)^2}+1} dy$, обратно $\int\limits_1^2 dy \int\limits_{+\sqrt{\ }+const}^2 dx$

$\varphi: (x-1)^2+(y-1)^2=1$

$y-1=\pm\sqrt{1-(x-1)^2}$

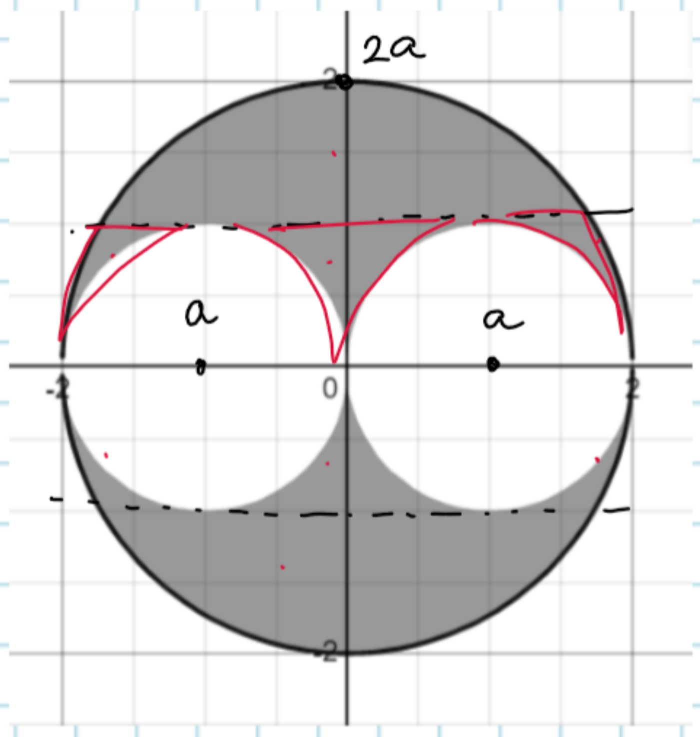
$y=\pm\sqrt{\dots}+1$

15.13 $\mathcal{D}:=\{(x,y): y\ge x^2,\ y\le \frac{x^2+1}{2}\}$



$\int\limits_{-1}^1 dx \int\limits_{x^2}^{\frac{x^2+1}{2}} dy$ или $\int\limits_0^{0.5} dy \int\limits_{-\sqrt{y}}^{\sqrt{y}} dx + \int\limits_{0.5}^1 dy \int\limits_{\sqrt{2y-1}}^{\sqrt{y}} dx + \int\limits_{0.5}^1 dy \int\limits_{-\sqrt{y}}^{-\sqrt{2y-1}} dx$

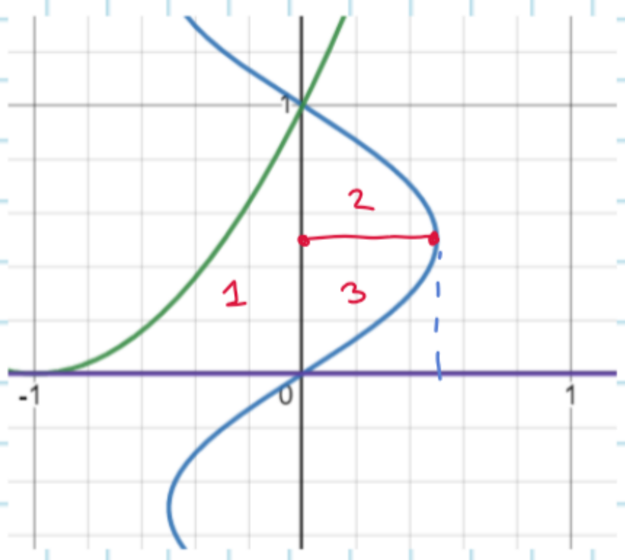
15.16 $\mathcal{D}:=\{(x,y): x^2+y^2\le 4a^2,\ (x-a)^2+y^2\ge a^2,\ (x+a)^2+y^2\ge a^2\}$



для dxdy разбить на 4 сектора и расписать

для dydx – на 8

15.18

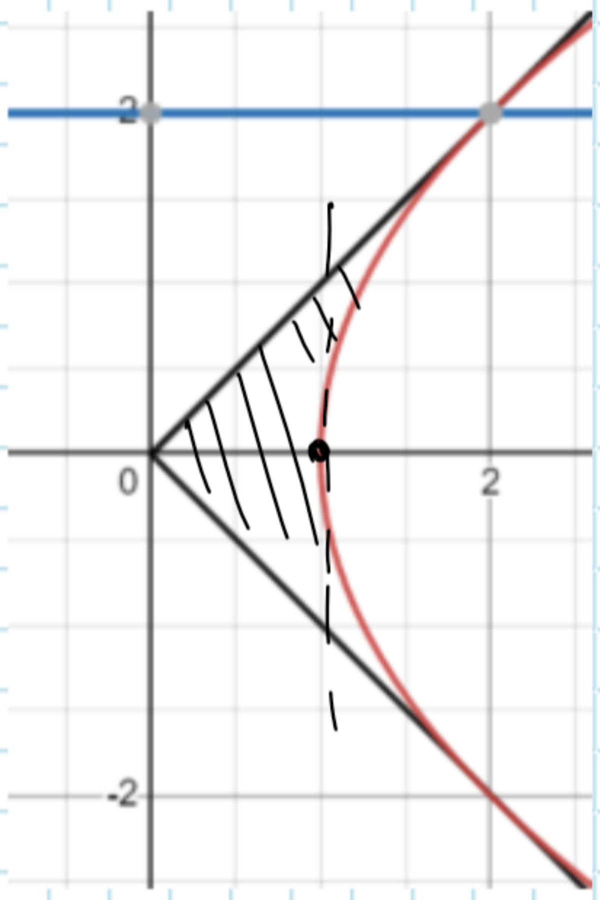


dxdy зоны 1, 2, 3

dydx хватит 1 интеграла

$\mathcal{D}:=\{(x,y): y\le(1+x)^2,\ y\ge 0,\ 2x\le \sin\pi y\}$

15.21 $\mathcal{D}:=\{(x,y): x\ge|y|,\ y^2\ge 4(x-1)\}$

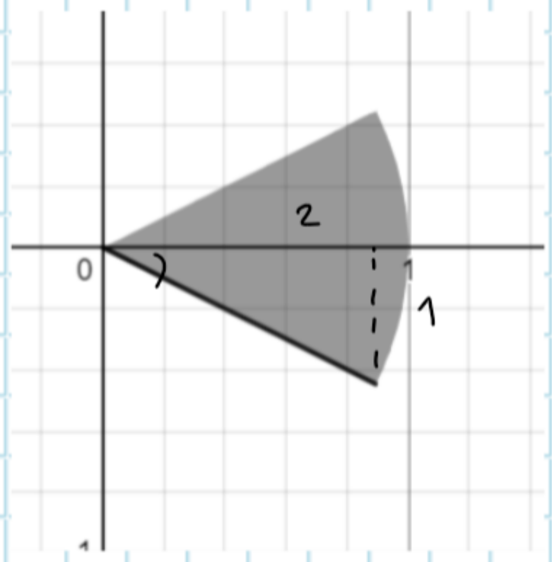


для dxdy: 3 зоны

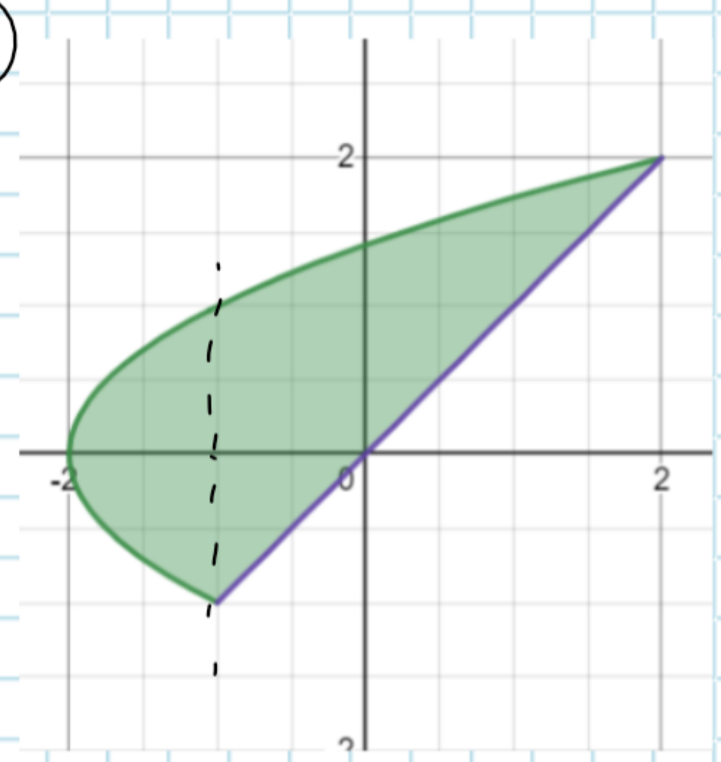
для dydx 2 зоны от $\pm y$ до $\varphi(y)$

15.32 $\mathcal{D}:=\{(x,y): x^2+y^2\le 1,\ -x\le 2y\le x\}$

$\int\limits_{-\operatorname{arctg}\frac12}^{\operatorname{arctg}\frac12} d\varphi \int\limits_0^1 \rho\, d\rho$



15.28



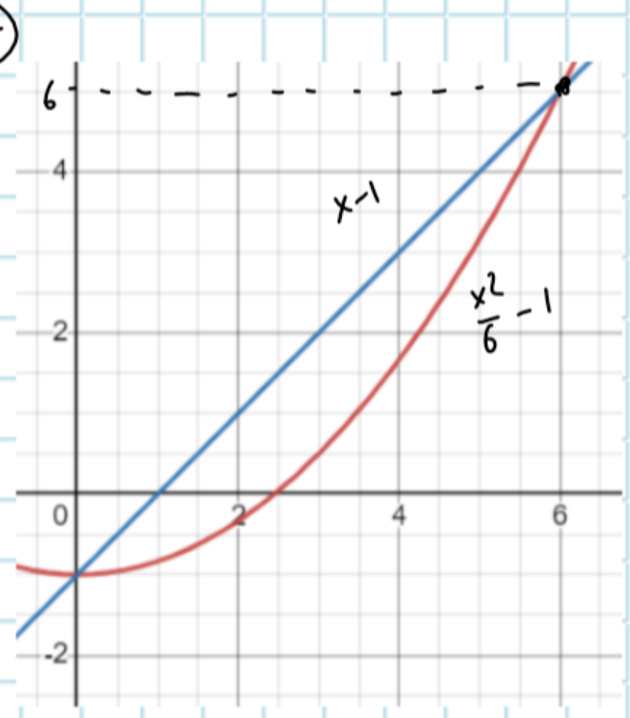
dxdy 2 шт

dydx 1 шт

$\mathcal{D}:=\{(x,y): x\ge y^2,\ y\ge x\}$

№ 2

15.37

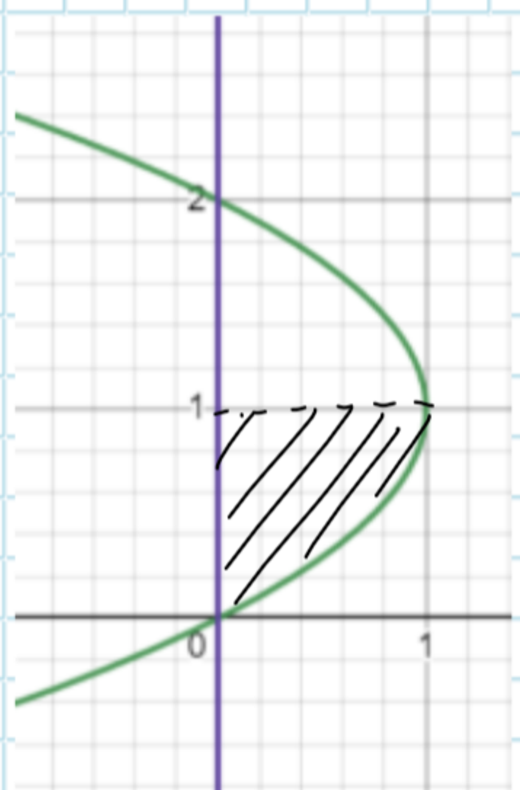


$y=\frac{x^2}{6}-1 \Rightarrow \sqrt{6y+6}=\pm x$

$y=x-1 \Rightarrow x=y+1$

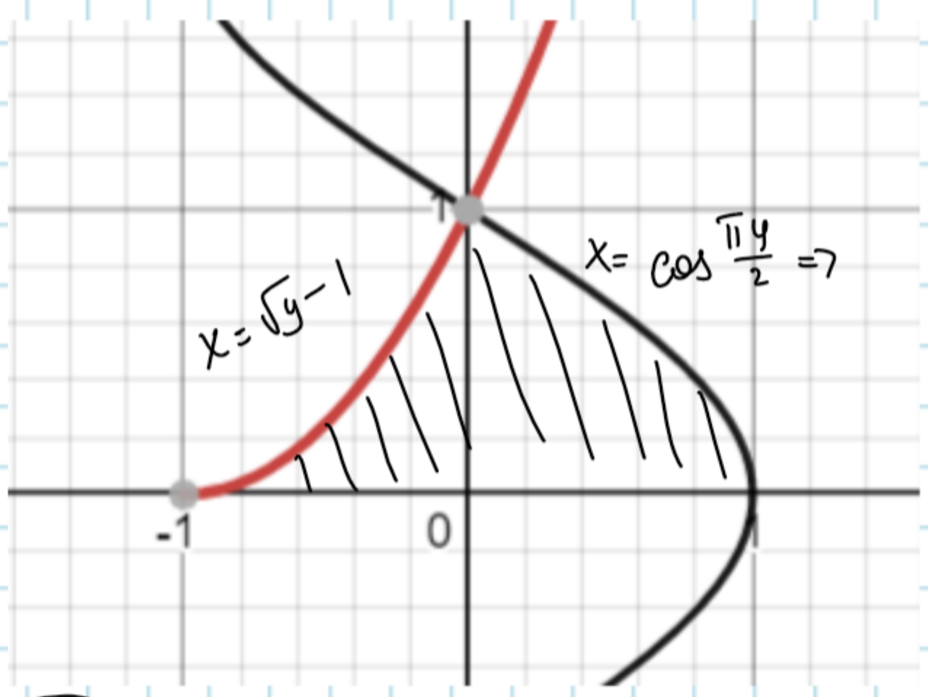
$\int\limits_0^6 dx \int\limits_{\frac{x^2}{6}-1}^{x-1} dy = \int\limits_{-1}^{6} dy \int\limits_{y+1}^{\sqrt{6y+6}} dx$

15.40



$\int\limits_0^2 dy \int\limits_0^{2y-y^2} dx = \int\limits_0^1 dx \int\limits_{-\sqrt{\dots}+1}^{1} dy +$

15.41

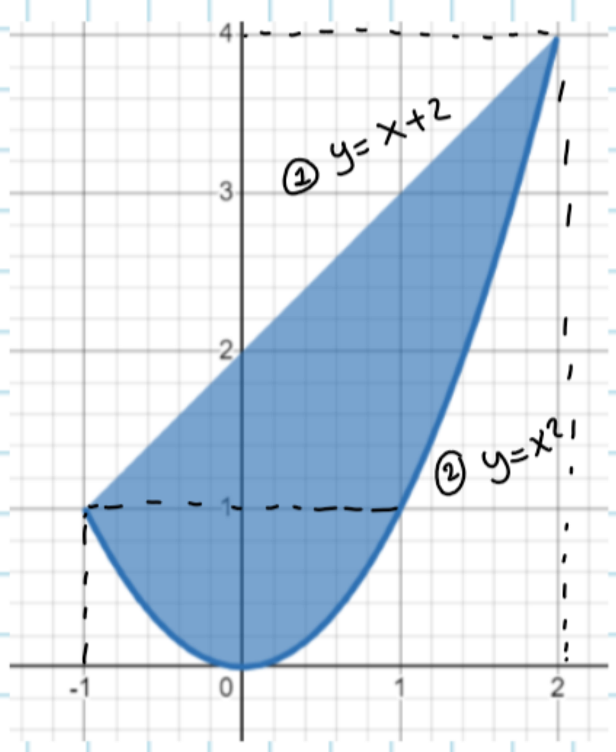


$\int\limits_0^1 dy \int\limits_{\sqrt{y-1}}^{\cos\frac{\pi y}{2}} dx = \int\limits_{-1}^0 dx \int\limits_0^{(x+1)^2} dy + \int\limits_0^1 dx \int\limits_0^{\frac{2}{\pi}\arccos x} dy$

$y=\frac{2}{\pi}\arccos x$

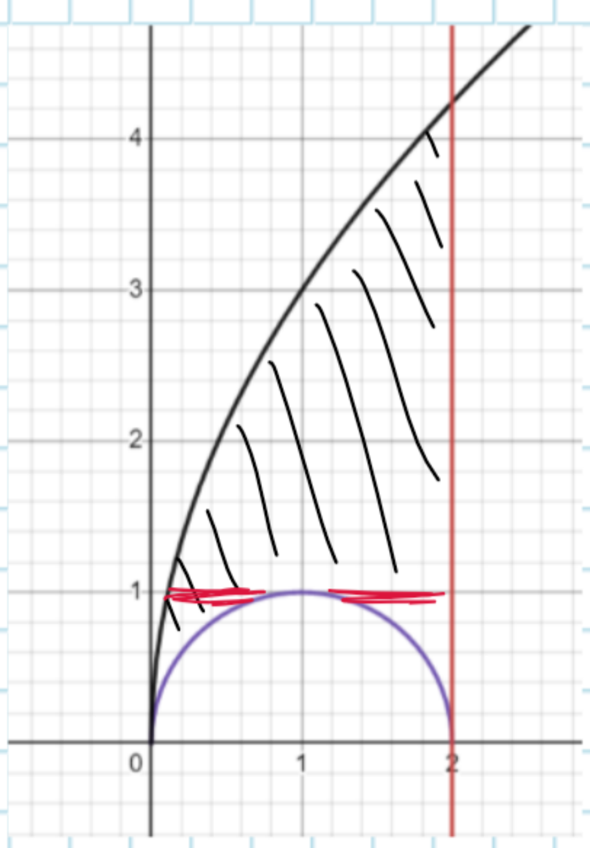
$y=(x+1)^2$

15.43



$\int\limits_1^4 dy \int\limits_{y-2}^{\sqrt{y}} dx + \int\limits_0^1 dy \int\limits_{-\sqrt{y}}^{\sqrt{y}} dx$

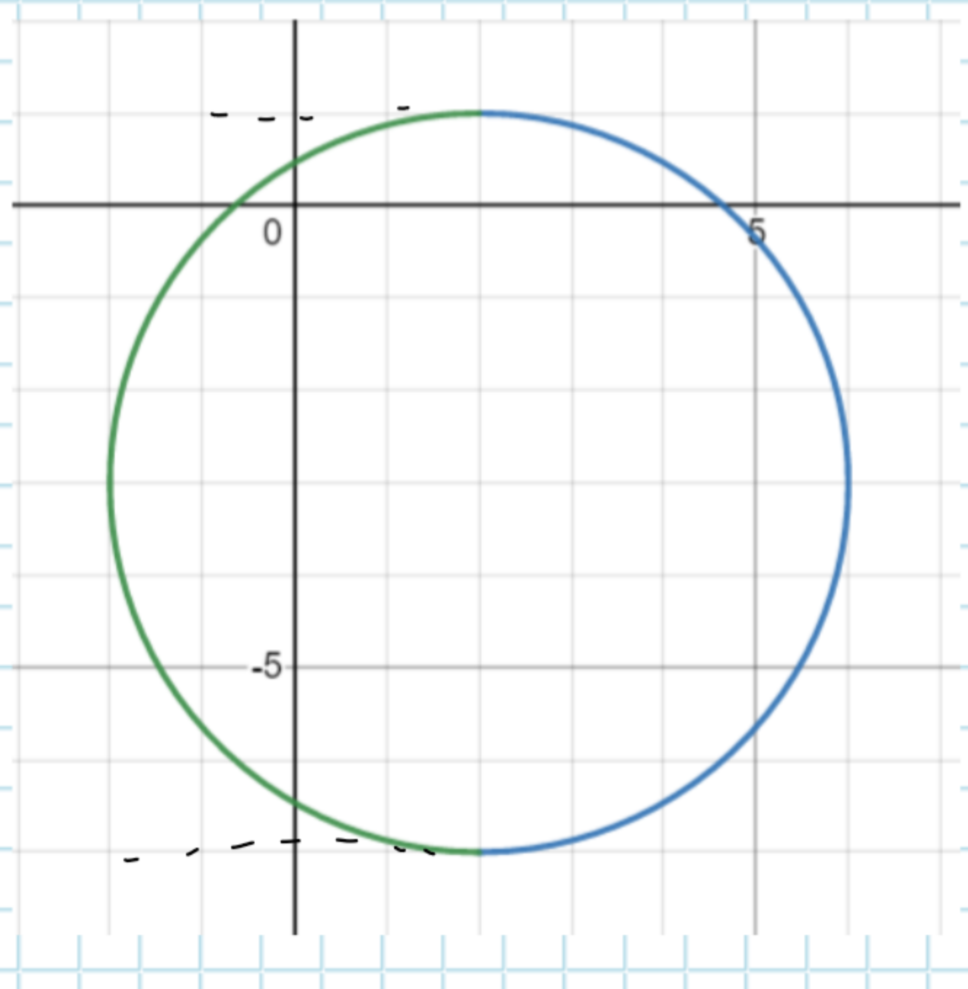
15.44



3шт интегралов для dydx

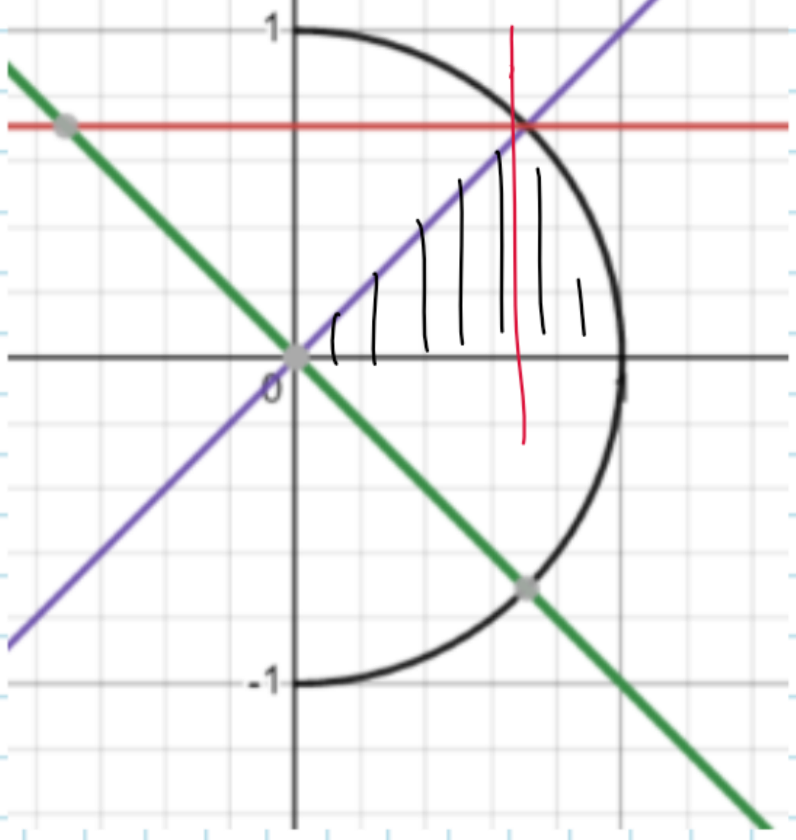
$\int\limits_0^2 dx \int\limits_{\sqrt{2x-x^2}}^{3\sqrt{x}} dy$

15.45



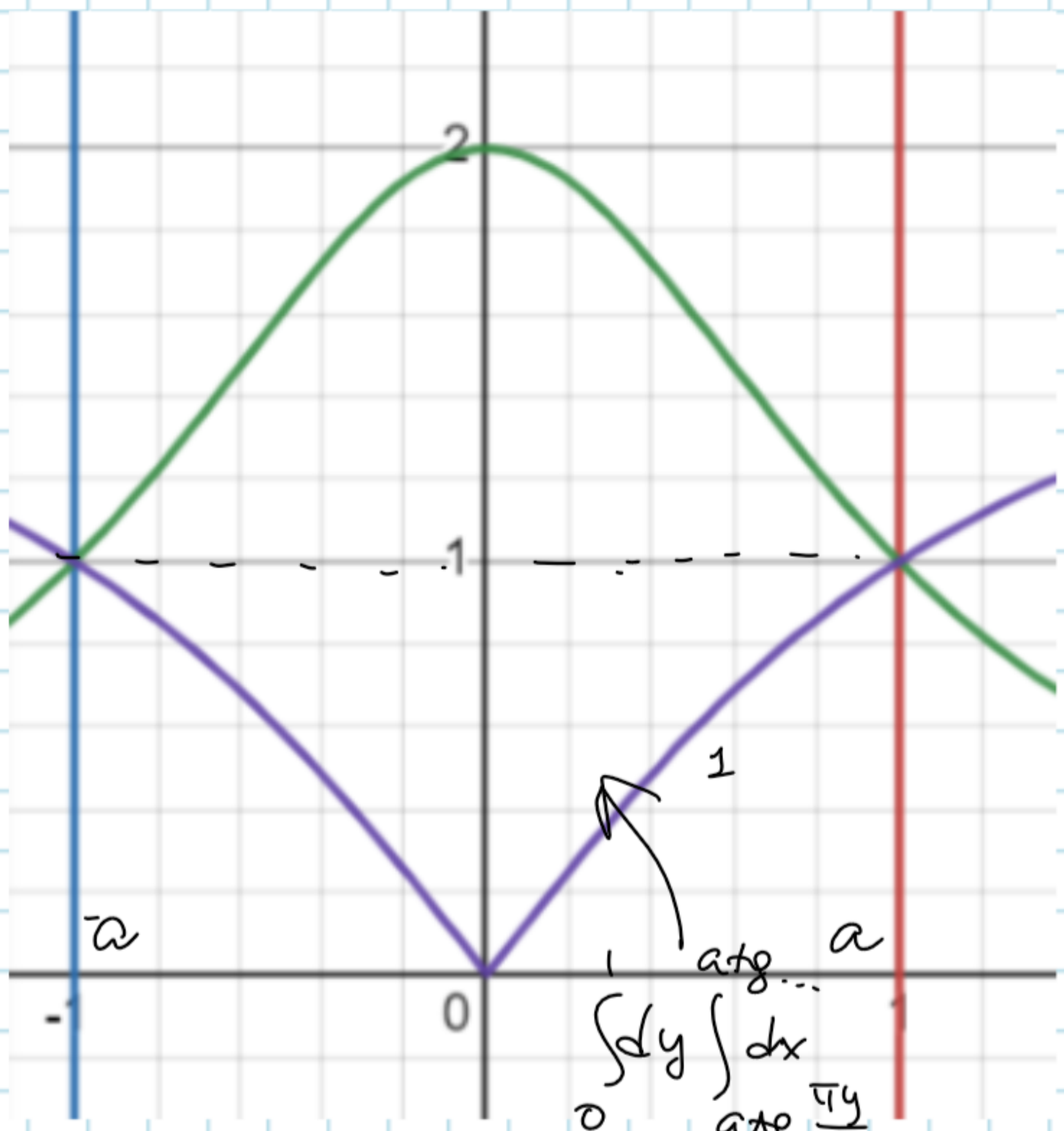
1 интеграл по окружности, можно перейти в полярную СК со смещением

15.49



разбить на 2, ~~один~~ будет [до прямой / от прямой] другой до ок-ти / от ок-ти

15.51



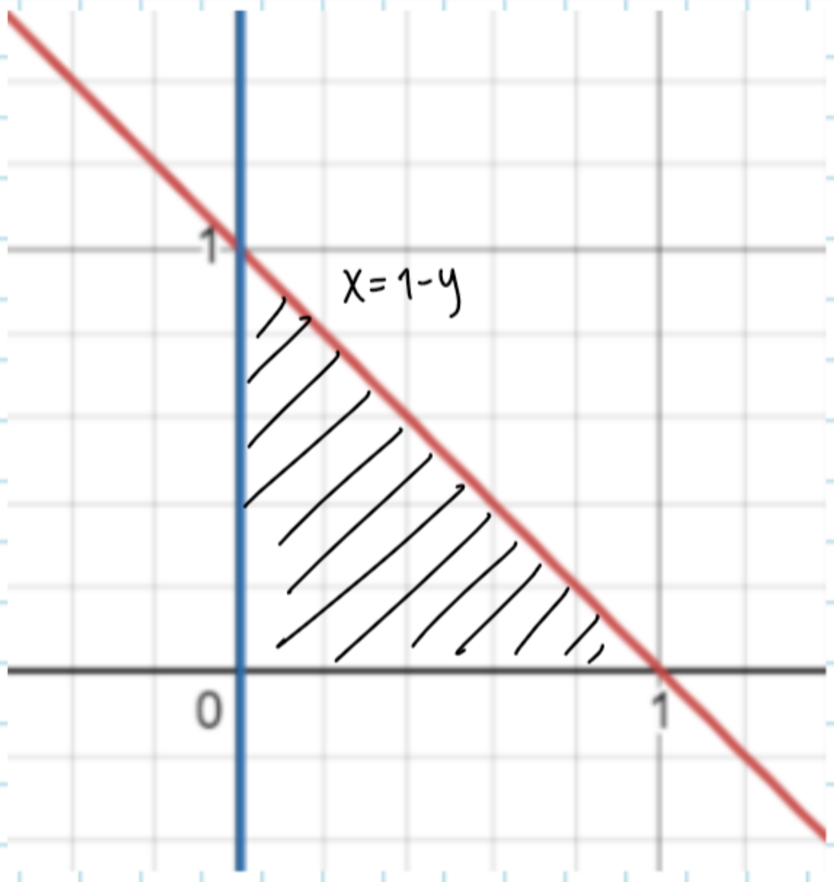
$1:\ y = \frac{4a}{\pi}\operatorname{arctg}\left|\frac{x}{a}\right|$

$\operatorname{tg}\frac{\pi}{4a}y = \left|\frac{x}{a}\right|$

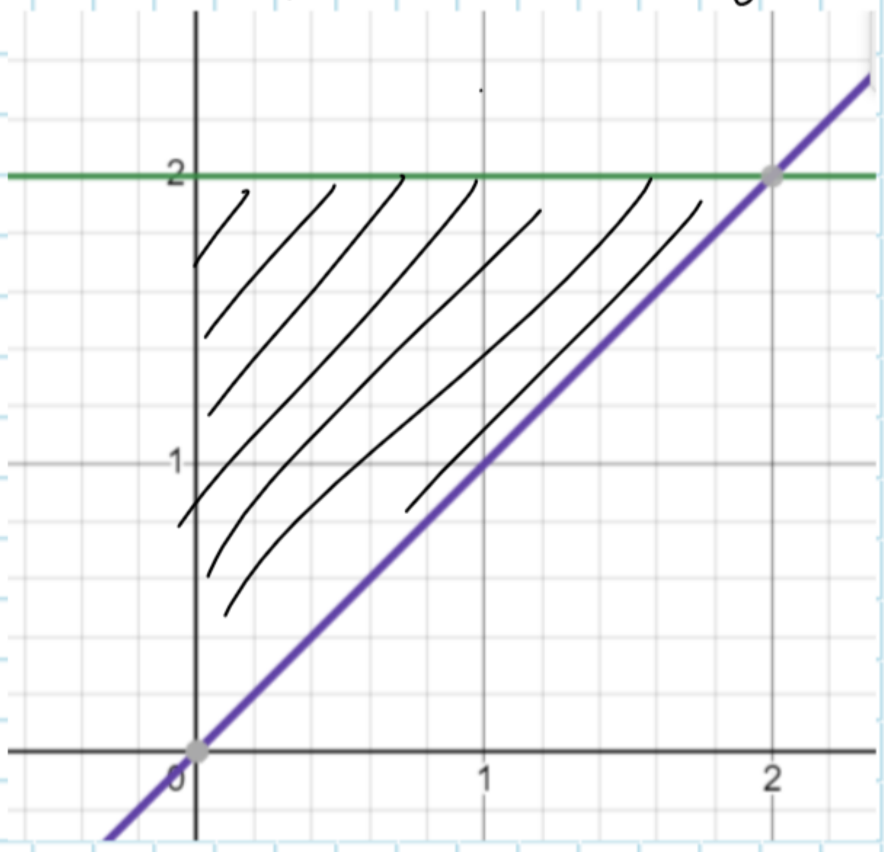
2 интеграла

$\int_0^1 dy \int_{-\operatorname{atg}\frac{\pi y}{4a}}^{\operatorname{atg}\dots} dx$

№ 3 15.60 $\int_0^1 dy \int_0^{1-y} e^{-x^2+2x+1}dx = \int_0^1 dx \int_0^{1-x} \exp(\varphi(x))\, dy = \frac{1}{2}\int_0^1 e^{-(x-1)^2+2} d\left(-(x-1)^2+2\right) = \frac{1}{2}\left[e^2 - e\right]$



15.61 $\int_0^2 x^2dx \int_x^2 \ln(1+y^2)dy = \int_0^2 \ln(1+y^2)dy \int_0^y x^2\, dx = \frac{1}{3}\int_0^2 \ln(1+y^2)\, y^3 dy$ = дальше там считается крч.



15.62 $\int_0^{\pi} x\, dx \int_x^{\pi} \frac{\sin y}{y} dy = \int_0^{\pi} \frac{\sin y}{y} dy \int_0^y x\, dx =$

$= -\frac{1}{2}\int_0^{\pi} y\, d\cos y = -\frac{1}{2} y\cos y \Big|_0^{\pi} + \int_0^{\pi} \cos y\, d\dots$

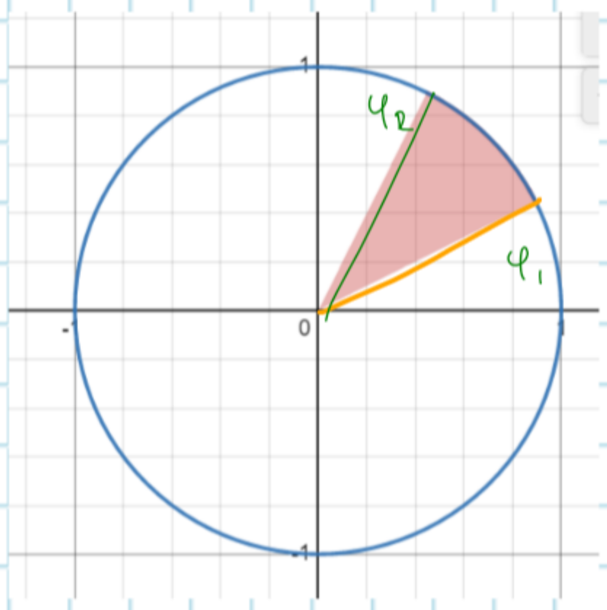
$= \frac{\pi}{2}$

№ 4 15.75 там ок-ть

$\int_0^{2\pi} d\varphi \int_0^{a} f \cdot \rho\, d\rho$

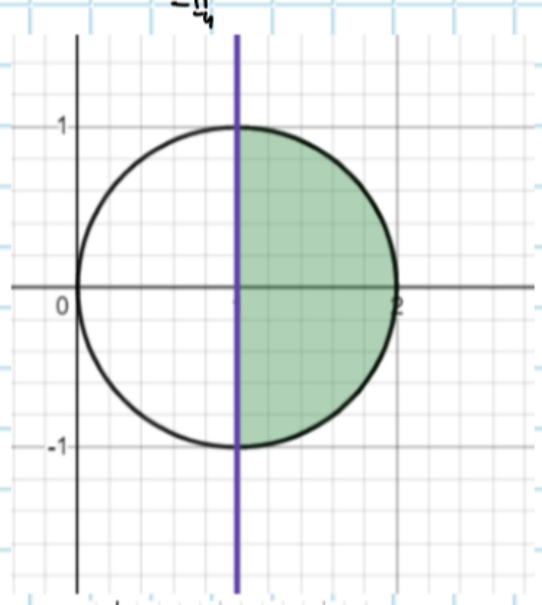
R

15.83

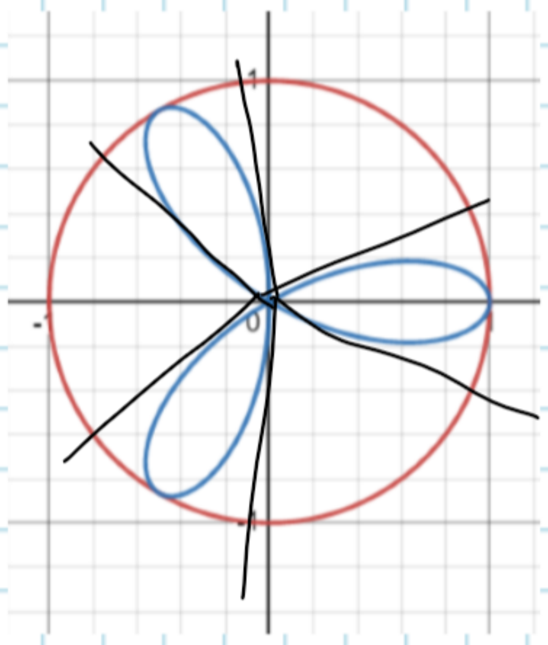


$\int_{\varphi_1}^{\varphi_2} d\varphi \int_0^{R} \rho f d\rho$

15.86 $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ очев. крч.



15.88 разбить на кучу всего

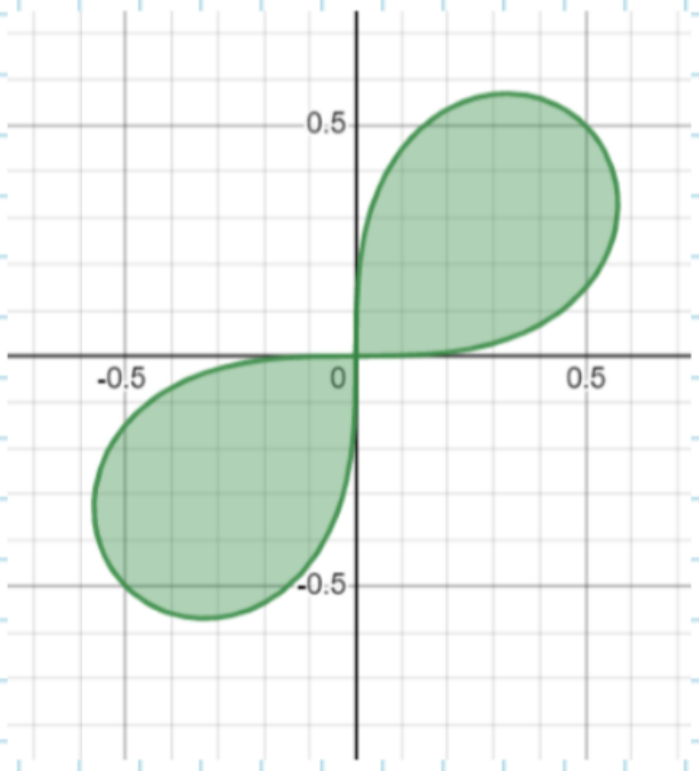


15.95 $(x^2+y^2)^2 \le a^2xy$

$\rho^4 \le a^2\rho^2$

$\rho^2 \le \frac{a^2}{2}\sin 2\varphi$

подставить, дальше очев



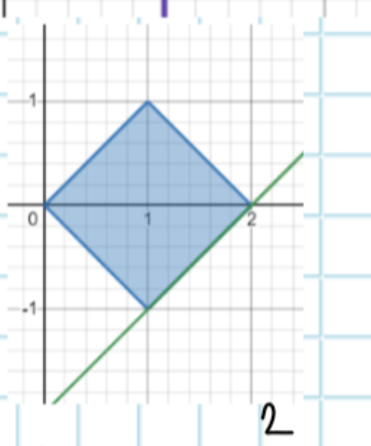
15.96 $|x-1| + |y| \le 1$

$x \geq 1,\ y \geq 0$

$x + y = 2$

$\sin\varphi + \cos\varphi = \frac{2}{\rho} \Leftrightarrow \rho = \frac{2}{\sin\varphi + \cos\varphi}$

дальше очев



$\int_0^1 dx \int_0^1 (x-y)\, dy = \int_0^1 dx \int$

№ 4 15.107

$\iint_D (x^2y^2+y^2)\,dxdy = \iint_D \frac{(u^2+v^2)}{v}\,dudv = \int_1^2 du \int_{\sqrt{u}}^{\sqrt{3u}} \left(\frac{u^2}{v}+v\right)dv = \int_1^2 du \left[\ln v \cdot u^2 + \frac{v^2}{2}\right]\Big|_{\sqrt{u}}^{\sqrt{3u}} =$

$D := \{(x,y): \frac{1}{x} \le y \le \frac{2}{x},\ x \le y \le 3x\}$

$= \int_1^2 u^2 \cdot \frac{1}{2}\ln 3\, du + \int_1^2 u^2 du =$

$= \left[\frac{8}{6} - \frac{1}{6}\right]\ln 3 + \frac{8}{3} - \frac{1}{3}$

$\begin{cases} u = yx \\ v = y \end{cases}$ $J\left(\frac{u,v}{x,y}\right) = \begin{vmatrix} \frac{\partial u}{\partial x} & \dots \\ \dots & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} y & x \\ 0 & 1 \end{vmatrix} = y$

$J\left(\frac{x,y}{u,v}\right) = \frac{1}{v}$

$\frac{u}{v} \le v \le 3\frac{u}{v}$

$\frac{v}{u} \le v \le 2\frac{v}{u}$ $\quad u \le v^2 \le 3u \quad v^2 = 3u$

$\frac{1}{u} \le 1 \le \frac{2}{u}$

15.109 $\iint_D (x^3+y^3)\,dx\,dy = \int_1^3 dv \int_{\frac{1}{2}u^{-3}}^{\frac{3}{2}u^{-3}} u^3 + v^3u^6\, dv$

$D := \{(x,y): x^2 \le y \le 3x^2,\ \frac{1}{x} \le 2y \le \frac{3}{x}\}$

$\begin{cases} u = x \\ v = \frac{y}{x^2} \Rightarrow y = vu^2 \end{cases}$ $J \begin{vmatrix} 2x & 0 \\ y & x \end{vmatrix} = 2x^2$ $\quad J\left(\frac{x,y}{u,v}\right) = \begin{vmatrix} 1 & 0 \\ x & u^2 \end{vmatrix}$

$u^2 \le vu^2 \le 3u^2$

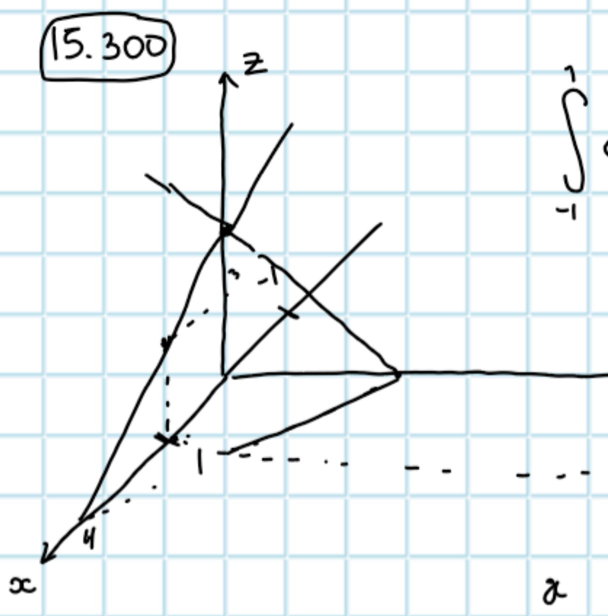
$\frac{1}{u} \le 2vu^2 \le \frac{3}{u}$

$\frac{1}{2u^3} \le v \le \frac{3}{2}\cdot\frac{1}{u^3}$

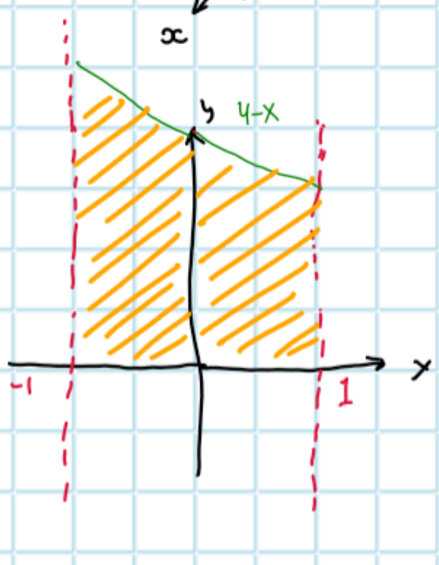
$\frac{2}{3}v = \frac{1}{u^3}$

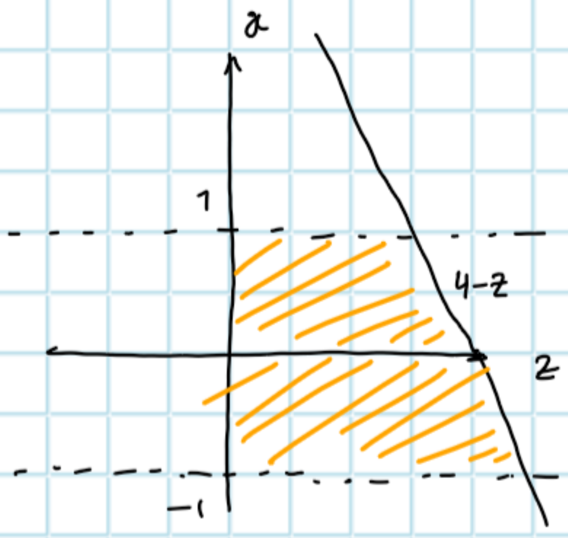
Тройной интеграл

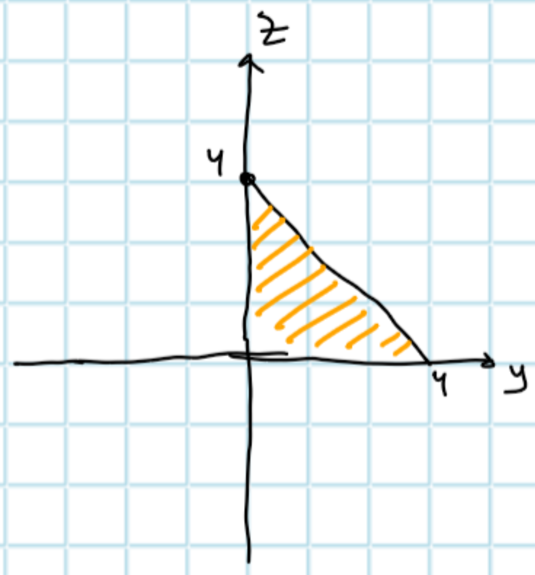
№ 1 $\boxed{15.300}$



$$\int_{-1}^{1} dx \int_{0}^{4-x} dz \int_{0}^{4-x-z} dy = \int_{0}^{3} dy \int_{-1}^{1} dx \int_{0}^{4-x-y} dz + \int_{3}^{5} dy \int_{-1}^{4-y} dx \int_{0}^{4-x-y} dz = \ldots$$

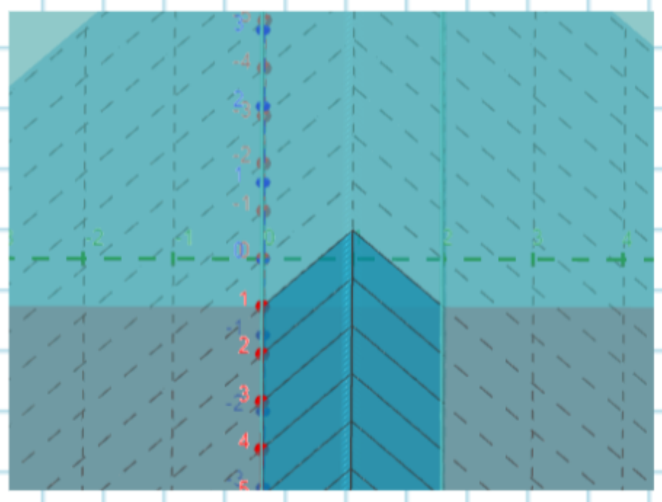






$\boxed{15.301}$ $\int_{0}^{1} dx \int_{0}^{2} dy \int_{0}^{1-|y-1|} dz =$

можно поменять местами



$\underbrace{1 \ldots 1}_{9}\ 2$ $\quad \Sigma\ 11 \quad$ Average: 1

$+ z$

$\frac{10+z}{11}$, $z=11$ Ave → 1

$\frac{10+1+z}{11}$, $z=11$ ave → 2

$m \quad \Sigma y+\gamma$

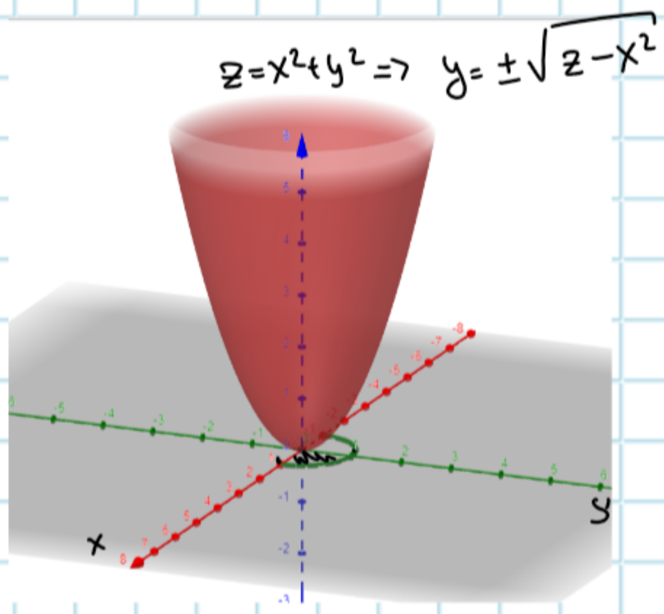
$m+1 \quad y+z+\gamma \quad \frac{x+\varepsilon}{n} \quad \frac{y+\gamma}{m}$

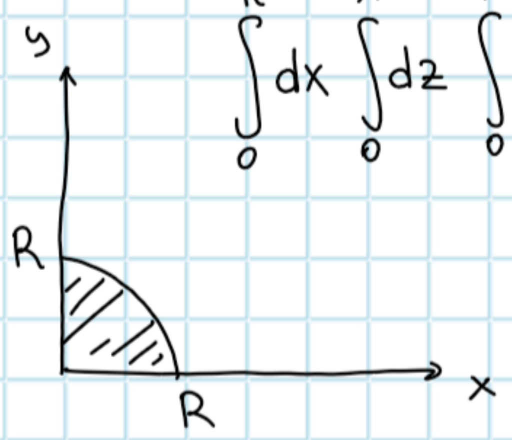
$\frac{y+z+\gamma}{m+1} = \frac{y+z}{m+1} + \frac{\gamma}{m+1}$ $\quad \frac{x+\varepsilon+\gamma+y}{n+m} = \frac{x+y}{n+m} + \frac{\varepsilon+\gamma}{n+m} \le m$

$\frac{y+k}{m+1} + \frac{\varepsilon+\gamma}{m+1} \to 1$ $\quad \frac{\varepsilon+\gamma}{n+m} \to 0$

$\frac{y+k+\varepsilon}{m+1}$ $\quad x \cdot n + y \cdot m$

$\boxed{15.303}$





$$\int_{0}^{R} dx \int_{0}^{x^2} dz \int_{0}^{\sqrt{R^2-x^2}} dy + \int_{0}^{R} dx \int_{x^2}^{R^2} dz \int_{\sqrt{z-x^2}}^{\sqrt{R^2-x^2}} dy$$

$\boxed{15.307}$ Цилиндрическая СК

коническая п-ть

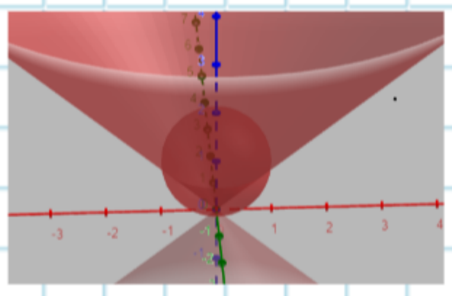
$\begin{cases} x = \rho\cos\varphi \\ y = \rho\sin\varphi \\ z = z \end{cases}$ $\quad \mathcal{D} = \rho \quad x^2+y^2 \le k^2 z^2$, $z \in [0, H]$

$\rho^2 \le k^2 \cdot z^2$

$\int_{0}^{2\pi} d\varphi \int_{0}^{kH} \rho\, d\rho \int_{\frac{\rho}{k}}^{H} dz$ дальше очев.

$\boxed{15.309}$ $x^2+y^2+z^2 \le 2az \Rightarrow \rho^2+z^2 \le 2az \Rightarrow \rho^2 \le a^2-(z-a)^2$

$x^2+y^2 \le z^2 \Rightarrow \rho^2 \le z^2$



Пересечение конуса и шара с центром в $(0,0,a)$ и $R=a$

Пределы по φ всегда $[0, 2\pi]$

дальше - дело техники

$\boxed{15.316}$ $V := \{(x,y,z) : x^2+y^2 \le z,\ 0 \le z \le H\}$

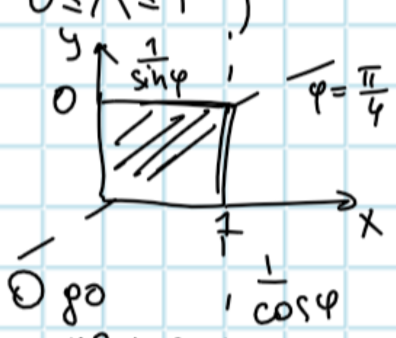
$\rho^2 \le z$

$$\int_{0}^{2\pi} d\varphi \int_{0}^{H} dz \int_{0}^{\sqrt{z}} \rho\, d\rho$$

$\boxed{15.317}$ $V := \{(x,y,z) : 0 \le x \le 1\}$

$0 \le \rho\cos\varphi \le 1$

$0 \le \rho \le \frac{1}{\cos\varphi}$



Пределы по z: от 0 до

$\boxed{15.323}$ $\int_{0}^{2\pi} d\varphi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin\theta\, d\theta \int_{R\sin\theta}^{4R\sin\theta} \rho^2\, d\rho$

$\boxed{15.348}$ $\int_{0}^{1} x\, dx \int_{0}^{2} y\, dy \int_{0}^{3} z\, dz = \frac{1}{2} \cdot \frac{4}{2} \cdot \frac{9}{2} = \frac{9}{2}$