

# Computational core (constraints & goals — the what/why)

## 1. Stay near-critical ( $\Psi$ -band) while performing

$$\max_{\pi} \mathbb{E} \left[ \sum_t \gamma^t r_t \right] \text{ s.t. } \Omega \Psi(\pi)_{\text{time in } \Psi\text{-band}} \geq \theta. \quad \max_{\pi} \mathbb{E} \left[ \sum_t \gamma^t r_t \right] \quad \text{underbrace} \{ \Omega \Psi(\pi)_{\text{time in } \Psi\text{-band}} \} \geq \theta.$$

## 2. Non-zero free-energy principle (don't collapse uncertainty)

Maintain **target, non-zero** predictive free energy FF (or entropy/uncertainty proxy) to preserve exploration capacity:

$$F_{\min} \leq \mathbb{E}[F_t] \leq F_{\max} \quad \text{with } F_{\min} > 0. \quad F_{\min} \leq \mathbb{E}[F_t] \leq F_{\max} \quad \text{with } F_{\min} > 0.$$

## 3. Risk discipline under heavy tails

Bound downside when shocks are fat-tailed:

$$\mathbb{E} \left[ \Delta F \right] \leq c, \quad \mathbb{E} \left[ \Delta F \right] \leq c,$$

where  $\mathbb{E} \left[ \Delta F \right]$  is Expected Shortfall (CVaR) of performance residuals  $\Delta F_t$ .

## 4. Competence growth without destabilisation

$$\mathbb{E}[\Delta \eta_t] \geq \alpha > 0 \text{ and } \Pr(\Psi\text{-exit}) \leq \varepsilon. \quad \mathbb{E}[\Delta \eta_t] \geq \alpha > 0 \text{ and } \Pr(\Psi\text{-exit}) \leq \varepsilon.$$

## 5. Precision/effort budget

Keep control effort bounded (a soft “conservation” of precision/energy):

$$\sum_{k \in \{b, T, \lambda\}} \mathbb{E}[\Delta k^2] \leq B. \quad \sum_{k \in \{b, T, \lambda\}} \mathbb{E}[\Delta k^2] \leq B.$$

A compact Lagrangian you can optimise online:

$$L(\pi) = E[\sum_t \gamma^t r_t] - \lambda \Psi[\theta - \Omega \Psi] + -\lambda E[Sq(\Delta F) - c] + -\lambda F \Phi(F_{\min}, F_{\max}) + \lambda \eta (\alpha - E[\Delta \eta t]) + , \mathcal{L}(\pi) = \mathbb{E}[\sum_t \gamma^t r_t] - \lambda \Psi[\theta - \Omega \Psi] - \lambda \mathbb{E}[Sq(\Delta F) - c] + -\lambda F \Phi(F_{\min}, F_{\max}) + \lambda \eta (\alpha - E[\Delta \eta t]) + ,$$

with  $\Phi$  any barrier that penalises  $F$  outside  $[F_{\min}, F_{\max}]$ .

## Algorithmic/representational core (state spaces & update rules — the how in principle)

Minimal state:  $s_t = (\phi_t, F_t^*, b_t, T_t, \lambda_t, \eta_t, \chi_t)$

Order parameter(s) for  $\Psi$ : e.g., synchrony/metastability  $mtm_t$  and band indicator  $1\Psi_t$ .

### 1. Policy with $\Psi$ -sieve + tail penalty

$$\pi_t(a|s_t) \propto \exp\left(\frac{Q_t(a) - \lambda_r \text{CVaR}_q[L_t(a)]}{T_t}\right) \mathbb{1}_{a \in A\Psi(s_t)}, \pi_t(a|s_t) \propto \exp\left(\frac{Q_t(a) - \lambda_r \text{CVaR}_q[L_t(a)]}{T_t}\right) \mathbb{1}_{a \in \mathcal{A}_\Psi(s_t)},$$

where  $A\Psi = \{a: \Pr(\Psi\text{-exit}|s_t, a) \leq \varepsilon\}$

(Use QQ or -GEFE-G<sub>r</sub> EFE;  $\lambda_r$  rises only on tail alarms.)

### 2. Difficulty set-point servo (keep at the branch)

$$F_{t+1}^* = F_t^* + \kappa_F (E_t - F_t^*) - \rho_F \partial \Phi / \partial F^* (\text{project to keep } F \in [F_{\min}, F_{\max}]), F_{t+1}^* = F_t^* + \kappa_F (E_t - F_t^*) - \rho_F \partial \Phi / \partial F^* (\text{project to keep } F \in [F_{\min}, F_{\max}]).$$

### 3. Temperature / uncertainty controller

$$T_{t+1} = T_t + \kappa_T (\chi_t - \chi^*) - \rho_T T_t, T_{t+1} = T_t + \kappa_T (\chi_t - \chi^*) - \rho_T T_t,$$

with  $T_{\min} \leq T_t \leq T_{\max}$ ;  $\chi^*$  is your target meta-uncertainty.

#### 4. Stability–flexibility bias

$$b_{t+1} = b_t - \kappa_b (m_t - m^*) - \rho_b b_t, \quad b_{t+1} = b_t - \kappa_b (m_t - m^*) - \rho_b b_t,$$

pulling toward a metastability set-point  $m^*$  (avoid lock-in or fragmentation).

#### 5. Representation/map update with bounded step (non-zero F)

$$\phi_{t+1} = \arg\min_{\phi} \{ F(\phi; s_t)_{\text{variational free energy}} + \beta \text{DKL}(p_{\phi} \parallel p_{\phi}^{\text{trust region}}), \phi_{t+1} \} \\ = \arg\min_{\phi} \{ F(\phi; s_t)_{\text{variational free energy}} + \beta \text{DKL}(p_{\phi} \parallel p_{\phi}^{\text{trust region}}) \};$$

so each learning step reduces FF **a bit** but never to zero (trust-region keeps you in-band).

#### 6. Mode arbitration (creative vs control)

$$m_t = \begin{cases} \text{Creative} & \text{if } \chi_t > \chi_{hi} \text{ and } |\Delta F^t| < \delta \\ \text{Control} & \text{otherwise} \end{cases} \quad \Delta F^t = \kappa (E_t - F^*_t). \quad m_t = \begin{cases} \text{Creative} & \text{if } \chi_t > \chi_{hi} \\ \text{Control} & \text{otherwise} \end{cases} \quad \widehat{\Delta F}_t = \kappa (E_t - F^*_t).$$

#### 7. Tail-alarm reflex (k-step schedule)

If  $\zeta < \zeta^*$  or  $J > J^*$  or  $ES_q > c$ : for next  $k$  steps,

$$T \uparrow, \lambda \downarrow, b \rightarrow b_{mid}, \quad T \uparrow, \lambda \downarrow, b \rightarrow b_{mid}, \quad T \uparrow, \lambda \downarrow, b \rightarrow b_{mid},$$

then decay back via #3–#4 when alarms clear.

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five computational constraints (what/why) + seven lean update rules (how-in-principle). They're modular: you can run 1–3 + 6–7 for a tiny agent, or plug all of them into a single online Lagrangian optimiser.