# Symmetric Key Crypto

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Some slides are written by Mark Stamp.

## (Modern) Symmetric Key Crypto

#### Stream cipher

- generalize one-time pad
- Except that key is relatively short → works on one bit/byte at a time
- Key is stretched into a long keystream
- Keystream is used just like a one-time pad

#### Block cipher

- More popular than stream cipher
- Works on larger chunks of data (blocks) at a time
- Can even combine blocks for additional security

### Stream Ciphers

Generates keystream of any length from random seed

- Keystream is pseudorandom
- Key is truly uniformly random
- Key is only used once, ever

$$Enc_{seed}(M) = K_{seed} \oplus M$$

### Stream Ciphers

- Stream ciphers were the king of crypto
- Today, not as popular as block ciphers
- We'll discuss two stream ciphers:
- A5/1
  - Based on shift registers → hardware
  - Used in GSM mobile phone system
- RC4 → Rivest Cipher 4
  - Based on a changing lookup table
  - Used many places
  - Note: RC5 is a block cipher
  - RSA
    - Shamir



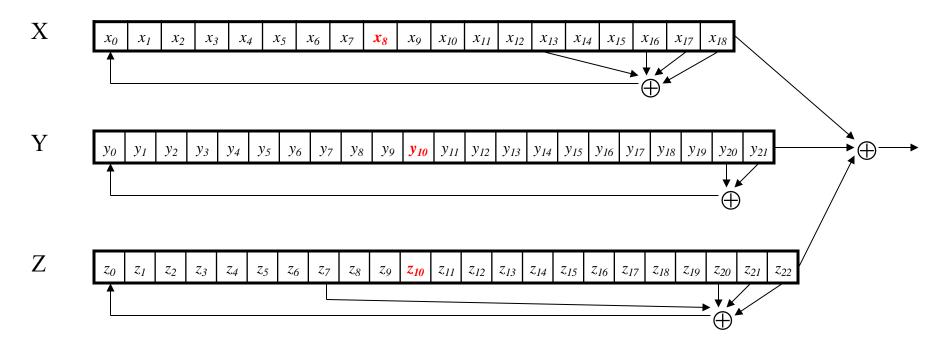
## A5/1: Shift Registers

- A5/1 uses 3 shift registers
  - X: 19 bits  $(x_0, x_1, x_2, ..., x_{18})$
  - Y: 22 bits  $(y_0, y_1, y_2, ..., y_{21})$
  - Z: 23 bits  $(z_0,z_1,z_2,...,z_{22})$
- Use these bits to generate as many bits as we want to serve as One Time Pad.

## A5/1: Keystream

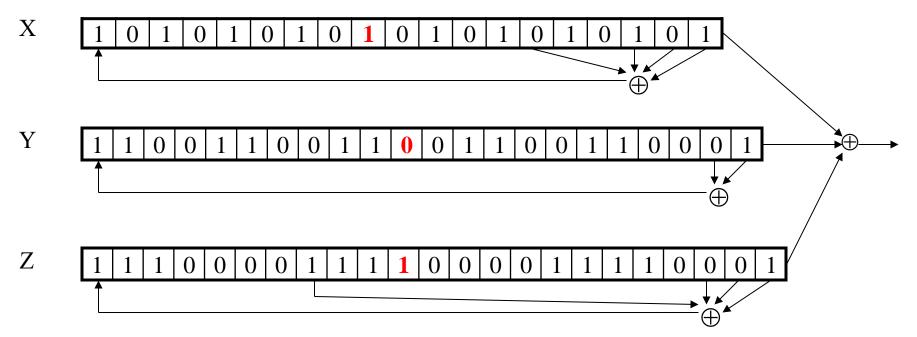
- At each iteration:  $m = \text{maj}(x_8, y_{10}, z_{10})$ 
  - Examples: maj(0,1,0) = 0 and maj(1,1,0) = 1
- If  $x_8 = m$  then X steps
  - $t = x_{13} \oplus x_{16} \oplus x_{17} \oplus x_{18}$
  - $x_i = x_{i-1}$  for i = 18, 17, ..., 1 and  $x_0 = t$
- If  $y_{10} = m$  then Y steps
  - $t = y_{20} \oplus y_{21}$
  - $y_i = y_{i-1}$  for i = 21,20,...,1 and  $y_0 = t$
- If  $z_{10} = m$  then Z steps
  - $t = \mathbf{z}_7 \oplus \mathbf{z}_{20} \oplus \mathbf{z}_{21} \oplus \mathbf{z}_{22}$
  - $z_i = z_{i-1}$  for i = 22,21,...,1 and  $z_0 = t$
- Keystream bit is  $x_{18} \oplus y_{21} \oplus z_{22}$

### A5/1



- Each variable here is a single bit
- Key is used as initial fill of registers
- Each register steps (or not) based on  $\mathrm{maj}(x_8,\,y_{10},\,z_{10})$
- Keystream bit is XOR of rightmost bits of registers

### A5/1



- In this example,  $m = \text{maj}(x_8, y_{10}, z_{10}) = \text{maj}(\mathbf{1}, \mathbf{0}, \mathbf{1}) = \mathbf{1}$
- Register X steps, Y does not step, and Z steps
- Keystream bit is XOR of right bits of registers
- Here, keystream bit will be  $0 \oplus 1 \oplus 0 = 1$

### Shift Register Crypto

- Shift register crypto efficient in hardware
- Often, slow if implemented in software
- In the past, very, very popular
- Today, more is done in software due to fast processors
- Shift register crypto still used some
  - E.g. in resource-constrained devices; embedded devices.

#### RC4

- A self-modifying lookup table
- Table always contains a permutation of the byte values 0,1,...,255
- Initialize the permutation using key
- At each step, RC4 does the following
  - Swaps elements in current lookup table
  - Selects a keystream byte from table
- Each step of RC4 produces a byte
- Each step of A5/1 produces only a bit
  - Efficient in hardware

### RC4 Initialization

```
• S[] is permutation of 0,1,...,255
• key[] contains N bytes of key
       for i = 0 to 255
              S[i] = i
              K[i] = \text{key}[i \pmod{N}] \leftarrow \text{any key length!}
       next i
       \dot{J} = 0
       for i = 0 to 255
              j = (j + S[i] + K[i]) \mod 256
              swap(S[i], S[j])
       next i
       i = j = 0
```

### RC4 Keystream

 At each step, swap elements in table and select keystream byte

```
i = (i + 1) mod 256
j = (j + S[i]) mod 256
swap(S[i], S[j])
t = (S[i] + S[j]) mod 256
keystreamByte = S[t]
```

- Use keystream bytes like a one-time pad
- Note: first 256 bytes should be discarded
  - Otherwise, attack exists

### Stream Ciphers

- Stream ciphers were popular in the past
  - Efficient in hardware → speed
  - Speed was needed to keep up with voice, etc.
    - Less memory consumption as well.
  - Today, processors are fast, so software-based crypto is usually more than fast enough
- Future of stream ciphers?
  - "the death of stream ciphers"?

## Block Ciphers



### Block Cipher

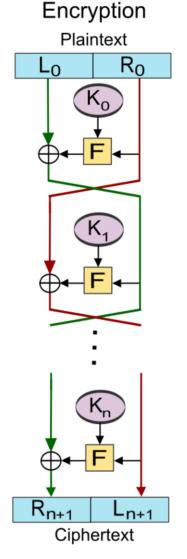
- Plaintext and ciphertext consist of fixed-sized blocks 
   bits
- Ciphertext obtained from plaintext by iterating a round function
- Input to round function consists of key and output of previous round
- Usually implemented in software
  - Used to be slow, but it's fine for modern CPUs
  - Intel has specific hardware instructions to speedup AES
    - AES-NI
    - AESENC AESDEC

### Feistel Cipher: Encryption

- Feistel cipher is a "type" of block cipher
  - Not a specific block cipher but a framework
  - Instances: DES; blowfish; RC5; TEA; Twofish...
- Split plaintext block into left and right components:  $P = (L_0, R_0)$
- For each round i = 0, 1, ..., n, compute

$$\begin{split} L_{i+1} &= R_i \\ R_{i+1} &= L_i \oplus F(R_i, K_i) \\ \text{where } F \text{ is round function and } K_i \text{ is subkey} \end{split}$$

• Ciphertext:  $C = (R_{n+1}, L_{n+1})$ 

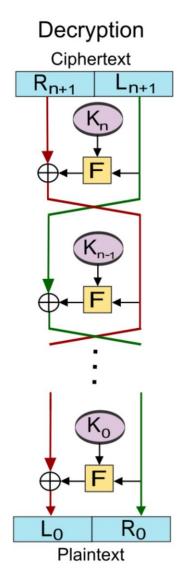


## Feistel Cipher: Decryption

- Start with ciphertext  $C = (R_{n+1}, L_{n+1})$
- For each round i = n+1, n, ..., 1, compute

$$\begin{split} R_{i-1} &= L_i \\ L_{i-1} &= R_i \oplus F(L_i, K_{i-1}) \\ \text{where } F \text{ is round function and } K_i \text{ is subkey} \end{split}$$

- Plaintext:  $P = (L_0, R_0)$
- Decryption works for any function F
  - But only secure for certain functions F
  - What about F = 0?

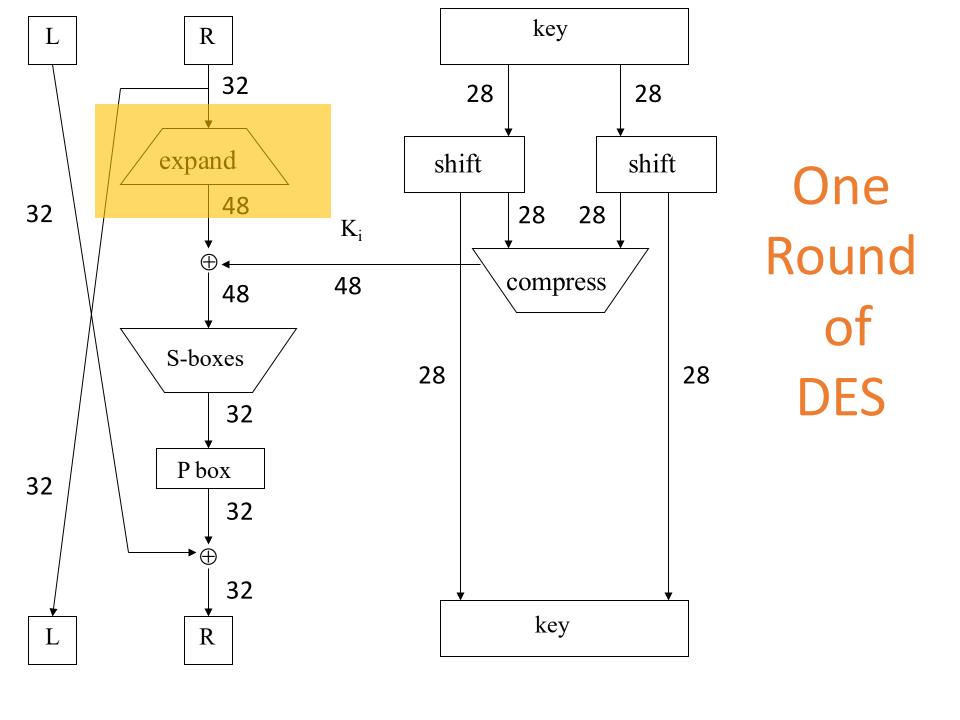


### Data Encryption Standard

- DES developed in 1970's
- An implementation of Feistel Cipher
- DES was U.S. government standard
  - NSA "secured" the algorithm.
  - But it has been cracked in 1998
- Many successors
  - Triple DES; AES; G-DES; ...

#### DES

- DES is a Feistel cipher with...
  - 64 bit block length
  - 56 bit key length
  - 16 rounds
  - 48 bits of key used each round (subkey)
- Round function is simple (for block cipher)
- Security depends heavily on "S-boxes"
  - Each S-box maps 6 bits to 4 bits



### **DES Expansion Permutation**

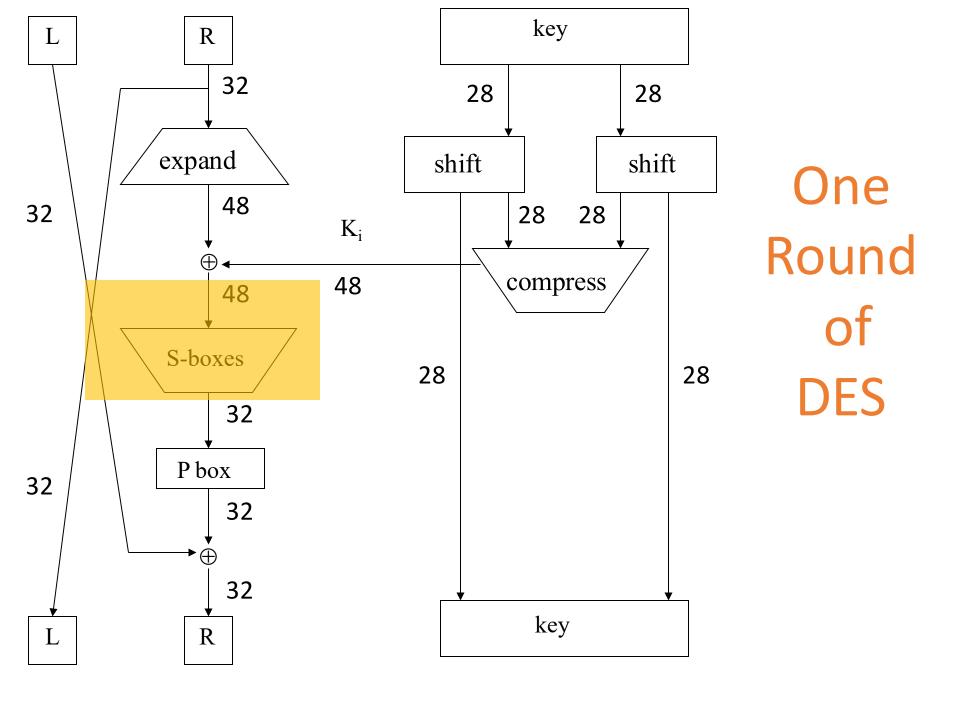
Input 32 bits

```
    0
    1
    2
    3
    4
    5
    6
    7
    8
    9
    10
    11
    12
    13
    14
    15

    16
    17
    18
    19
    20
    21
    22
    23
    24
    25
    26
    27
    28
    29
    30
    31
```

Output 48 bits

Output contains eight 6-bit (8 \* 6 = 48 bits) pieces

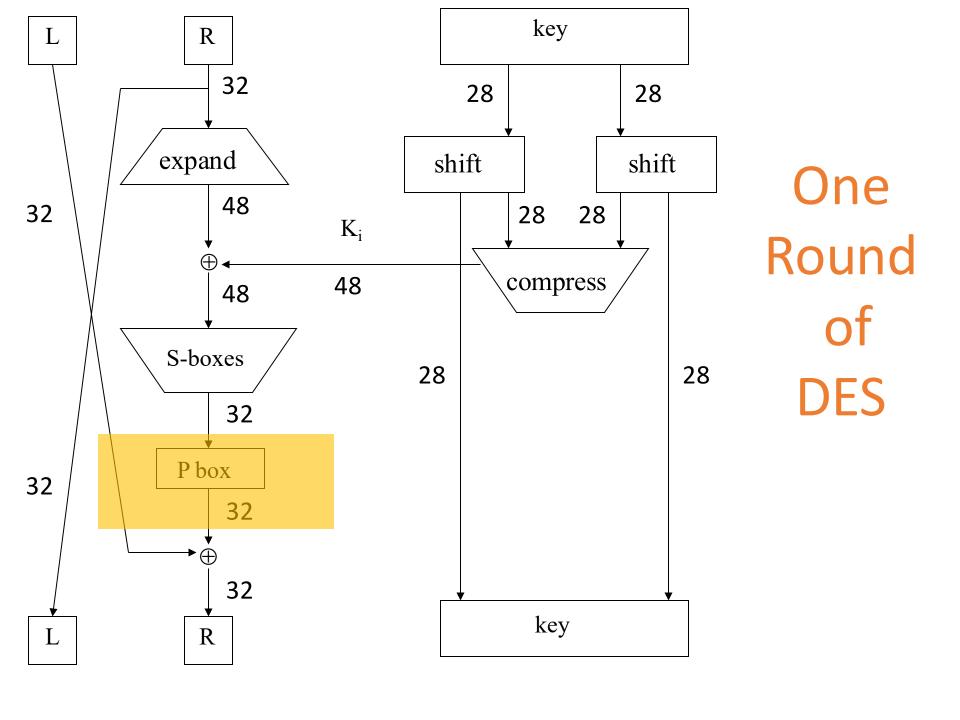


#### **DES S-box**

- 8 "substitution boxes" or S-boxes
  - In DES S-boxes are carefully chosen to resist cryptanalysis.
  - Thus, that is where the security comes from.
- Each S-box maps 6 bits to 4 bits

1111 1100 1000 0010 0100 1001 0001 0111 0101 1011 0011 1110 1010 0000 0110 1101

non-linear transformation, provided in the form of a lookup table



### DES P-box

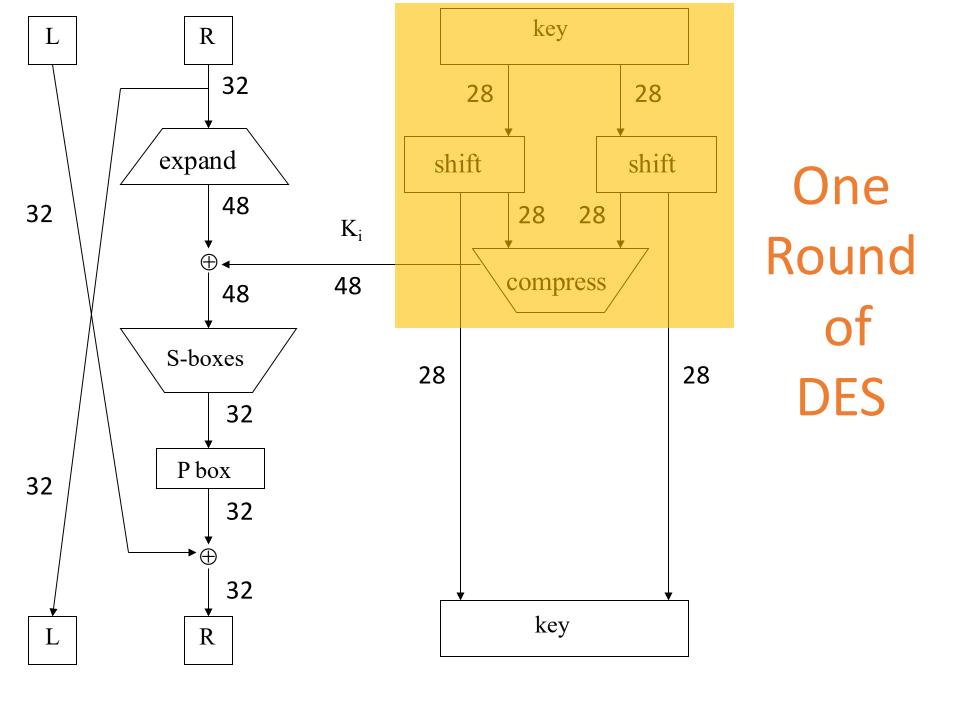
• Basically we further permutate the output of S-box.

#### Input 32 bits

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
```

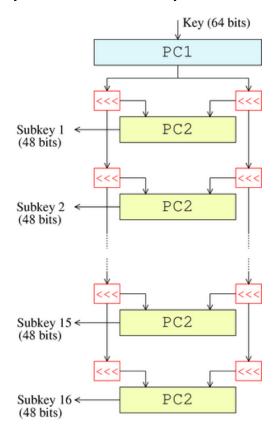
#### Output 32 bits

```
15 6 19 20 28 11 27 16 0 14 22 25 4 17 30 9
1 7 23 13 31 26 2 8 18 12 29 5 21 10 3 24
```



### **DES Subkey**

- 56 bit DES key out of 64-bit, numbered 0,1,2,...,55
  - 24 \* 2 bits of key are eventually extracted from 28 \* 2 bits.



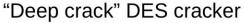
### DES: Prologue and Epilogue

- An initial permutation before round 1
- Halves are swapped after last round
- A final permutation (inverse of initial perm) applied to  $(R_{16}, L_{16})$
- None of this serves any security purpose
  - But this is how the algorithm is designed...

### Security of DES

- Security depends heavily on S-boxes
- Attacks by exhaustive key search
  - given message x and a ciphertext c such that  $DES_k(x)=c \rightarrow \text{find } k$
  - Wiener: \$1,000,000 3.5 hours; never built in public
  - 1998, the EFF DES Cracker, which was built for less than \$250,000 < 3 days
  - 1999, Distributed.Net (*idle time of your computer*), 22 hours and 15 minutes (over many machines)
  - You can assume that NSA and agencies likely can crack DES in milliseconds







### Triple DES

- Today, 56 bit DES key is too small
  - As aforementioned, exhaustive key search is feasible
- Triple DES or 3DES (with 112 bit key, why?)
  - $C = E(D(E(P,K_1),K_2),K_1)$
  - $P = D(E(D(C,K_1),K_2),K_1)$
- Why Encrypt-Decrypt-Encrypt with 2 keys?
  - Why not encrypt-encrypt-encrypt mode?
    - Backward compatible: E(D(E(P,K),K),K) = E(P,K)
  - And 112 is a lot of bits

### 3DES

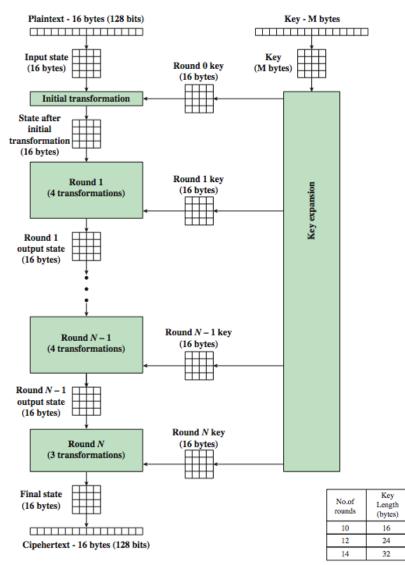
- Why not C = E(E(P,K),K) instead?
  - still just 56 bit key
- But why not  $C = E(E(P,K_1),K_2)$  instead?
  - Unfortunately, brute force search difficulty can be reduced from 2^112 to 2^57
- Meet-in-the-middle attack

### Advanced Encryption Standard

- Replacement for DES
  - The de-facto symmetric cipher since late 1990
- Not a Feistel cipher (unlike DES)
  - Variable key lengths
  - Fast implementation in hardware and software
  - Small code and memory footprint → but still too much
- We will provide a (simplified) implementation of AES after today's class.
  - Just for your reference, you are NOT required to remember its implementation.

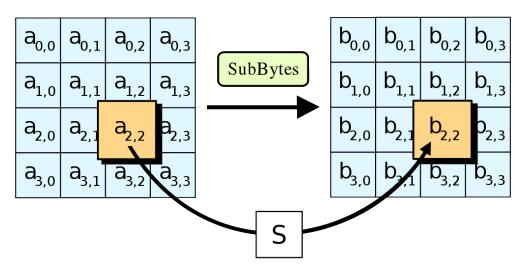
### AES: Executive Summary

- Block size: 128 bits
- Key length: 128, 192 or 256 bits
- 10 to 14 rounds (depends on key length)
- Each round uses 4 functions
  - ByteSub (S-box layer)
  - ShiftRow
  - MixColumn
  - AddRoundKey (key addition layer)



### AES ByteSub

Treat 128 bit block as 4x4 byte array



ByteSub is AES's "S-box"

No fixed point a<sub>i,j</sub> = S(a<sub>i,j</sub>)

### AES "S-box"

#### Last 4 bits of input

```
c5 30
                                 2b fe d7
                  6f
                           01 67
                     f0 ad d4 a2 af 9c
         26 36 3f f7 cc 34 a5 e5 f1 71
04 c7 23 c3 18 96 05 9a 07 12 80 e2 eb
         1a 1b 6e 5a a0 52 3b d6 b3 29
53 d1 00 ed 20 fc b1 5b 6a cb be 39 4a 4c
|d0 ef |aa |fb 43 4d 33 85 45 f9 02
                                 7f
51 a3 40 8f 92 9d 38 f5 bc b6
                        c4 a7 7e 3d 64
           22 2a 90 88 46 ee b8 14 de
         0a 49 06 24 5c c2 d3 ac 62 91
         6d 8d d5 4e a9 6c 56 f4 ea 65
ba 78 25 2e 1c a6 b4 c6 e8 dd 74 1f 4b bd 8b 8a
70 3e b5 66 48 03 f6 0e 61 35 57 b9 86
e1 f8 98 11 69 d9 8e 94 9b 1e 87 e9 ce 55 28 df
8c a1 89 Od bf e6 42 68 41 99 2d Of
```

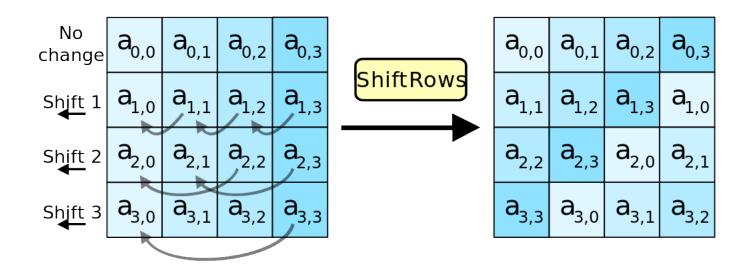
First 4 bits of input

Then, how to invert this table? (need another table)

•  $0x62 \rightarrow 0xaa \rightarrow 0xaa \rightarrow 0x62$ 

### **AES ShiftRow**

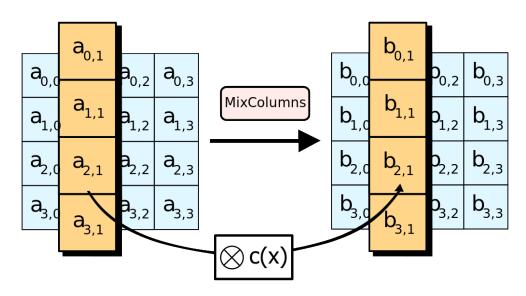
Cyclic shift rows



To avoid the columns being encrypted independently

#### **AES MixColumn**

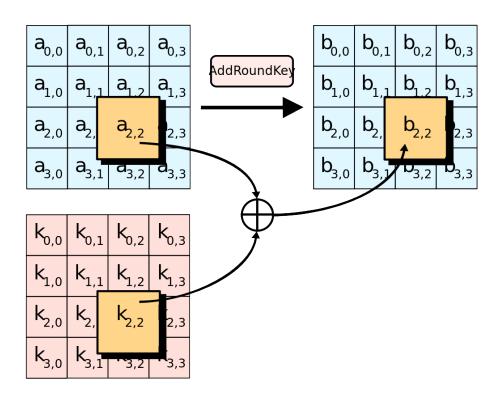
Invertible operation applied to each column



- It's a matrix multiplication.
  - Implemented as a (big) lookup table

### AES AddRoundKey

#### XOR subkey with block



#### **AES Comments**

- Can be effectively implemented in 8-bit and 32-bit CPU.
  - One key reason it is becoming so popular
    - The franchise player in block cipher
  - Specific hardware instructions.
    - AES-NI
    - AESENC AESDEC
- Some other good properties
  - Avalanche effect: one bit difference in plaintext causes a big change in cipher text.
    - Talk about that later..

# Tiny Encryption Algorithm

- 64 bit block, 128 bit key
- Assumes 32-bit arithmetic
- Number of rounds is variable (32 is considered secure)
- Uses "weak" round function, so large number of rounds required

# TEA Encryption

#### Assuming 32 rounds:

```
(K[0], K[1], K[2], K[3]) = 128 \text{ bit key}
(L,R) = plaintext (64-bit block)
delta = 0x9e3779b9  Magic number!
sum = 0
for i = 1 to 32
   sum += delta
   L += ((R << 4) + K[0]) \oplus (R + sum) \oplus ((R >> 5) + K[1])
   R += ((L << 4) + K[2]) \oplus (L + sum) \oplus ((L >> 5) + K[3])
ciphertext = (L,R)
```

# TEA Decryption

#### Assuming 32 rounds:

```
(K[0], K[1], K[2], K[3]) = 128 \text{ bit key}
(L,R) = ciphertext (64-bit block)
delta = 0x9e3779b9
sum = delta << 5
for i = 1 to 32
   R = ((L << 4) + K[2]) \oplus (L + sum) \oplus ((L >> 5) + K[3])
   L = ((R << 4) + K[0]) \oplus (R + sum) \oplus ((R >> 5) + K[1])
   sum -= delta
plaintext = (L,R)
```

#### **TEA Comments**

- "Almost" a Feistel cipher?
  - Uses + and instead of ⊕ (XOR)
  - Each step both left and right halves are used.
- Simple, easy to implement, fast, low memory requirement, etc.
  - Can you memorize the implementation like how you memorize quick sort algorithm?
    - Well, that's why it's called "tiny".
- Possibly enable attacks?
  - four different keys that all give the exact same encrypted output
  - $2^{128} \rightarrow 2^{126}$
  - X-box attacks.



# Overview of Linear Cryptanalysis

# Linear Cryptanalysis

- Non-linear functions (i.e., s-box) are the primary contribution of security of block ciphers.
- To attack, we approximate the nonlinearity with linear equations
- How well can we do this?

# Vulnerable S-box Linear Analysis

□ Input  $x_0x_1x_2$  where  $x_0$  is row and  $x_1x_2$  is column

Output	$y_0y_1$
--------	----------

	column			
row	00	01	10	11
0	10	01	11	00
1	00	10	01	11

# Linear Analysis

□ For example,

$$y_1 = x_1$$
 with prob. 3/4

□ And  $y_0 = x_0 \oplus x_2 \oplus 1$  with prob. 1

□ And  $y_0 \oplus y_1 = x_1 \oplus x_2$  with prob. 3/4

	column				
row	00	01	10	11	
0	10	01	11	00	
1	00	10	01	11	

# Linear Cryptanalysis of DES

- DES is linear except for S-boxes
- How well can we approximate S-boxes with linear functions?
- DES S-boxes designed so there are no good linear approximations to any one output bit
- But there are linear combinations of output bits that can be approximated by linear combinations of input bits
  - Still not very practical, requires a huge amount of plaintext

# Block Cipher Modes

# Multiple Blocks

- How to encrypt multiple blocks?
  - What's the block size of AES/DES/TEA?
  - How large it is?
- Do we need a new key for each block?
  - If so, as impractical as a one-time pad!
- So, what can we do?
  - Encrypt each block individually?
  - Make encryption depends on previous block?
    - Then "chain" them together?
    - Partial block? Not considered in this course

### Modes of Operation

- Many modes we discuss 3 most popular
- Electronic Codebook (ECB) mode
  - Encrypt each block independently
  - Most obvious approach, but a bad idea
- Cipher Block Chaining (CBC) mode
  - Chain the blocks together
  - More secure than ECB, a little bit extra work
- Counter (CTR) mode
  - Block ciphers acts like a stream cipher
  - Popular for random access

#### ECB Mode

- Notation: C = E(P, K)
- Given plaintext  $P_0, P_1, ..., P_m, ...$
- Most obvious way to use a block cipher:

# Encrypt Decrypt $C_0 = E(P_0, K)$ $P_0 = D(C_0, K)$ $C_1 = E(P_1, K)$ $P_1 = D(C_1, K)$ $C_2 = E(P_2, K)$ ... $P_2 = D(C_2, K)$ ...

- For fixed key K, this is "electronic" version of a codebook cipher (without additive)
  - With a different codebook for each key

#### ECB Cut and Paste

Suppose plaintext is

Eva. pings Bob. Ted. pings Tom.

Assuming 64-bit blocks and 8-bit ASCII:

```
P_0= "Eva. pin", P_1= "gs Bob. ", P_2= "Ted. pin", P_3= "gs Tom. "
```

- Ciphertext:  $C_0$ ,  $C_1$ ,  $C_2$ ,  $C_3$
- Attacker <u>cuts and pastes</u>: C<sub>0</sub>, C<sub>3</sub>, C<sub>2</sub>, C<sub>1</sub>
- Decrypts as

Eva. pings Tom. Ted. pings Bob.

# Principles of CIA



#### Confidentiality:

- No one is supposed to read what data or information is sent unless he is authorized.
   Integrity:
- The ability to ensure that data is an accurate and unchanged representation of the original secure information.

#### Availability:

the information concerned is readily accessible to the authorised viewer at all times

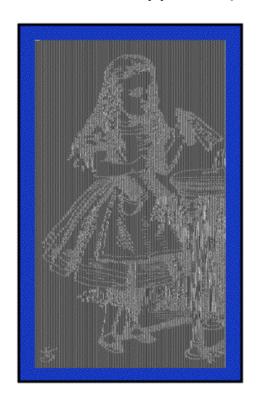
#### ECB Weakness

- Suppose  $P_i = P_j$
- Then  $C_i = C_j$  and attacker knows  $P_i = P_j$
- This gives attacker some information, even if he does not know  $P_i$  or  $P_i$
- Attacker might know P<sub>i</sub>
- Is this a serious issue?
  - Any information can be leaked?

#### Alice Hates ECB Mode

Alice's uncompressed image, and ECB encrypted (TEA)





- Why does this happen?
- Same plaintext yields same ciphertext!

#### CBC Mode

- Blocks are "chained" together
- A random initialization vector, or IV, is required to initialize CBC mode
- IV is random, but not secret

#### **Encryption**

$$C_0 = E(IV \oplus P_0, K),$$
  
 $C_1 = E(C_0 \oplus P_1, K),$   
 $C_2 = E(C_1 \oplus P_2, K),...$ 

#### **Decryption**

$$P_0 = IV \oplus D(C_0, K),$$

$$P_1 = C_0 \oplus D(C_1, K),$$

$$P_2 = C_1 \oplus D(C_2, K),...$$

Analogous to classic codebook with additive

#### CBC Mode

- Identical plaintext blocks yield different ciphertext blocks — this is very good!
- But what about errors in transmission?
  - If C<sub>1</sub> is garbled to, say, G then

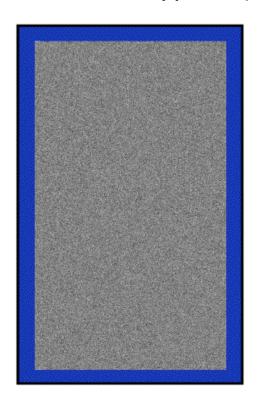
$$P_1 \neq C_0 \oplus D(G, K), P_2 \neq G \oplus D(C_2, K)$$

- But  $P_3 = C_2 \oplus D(C_3, K), P_4 = C_3 \oplus D(C_4, K), ...$
- "G" can only influence limited decryptions!
- Cut and paste is still possible
  - Talk more on "integrity" later today.

### Alice Likes CBC Mode

Alice's uncompressed image, Alice CBC encrypted (TEA)





- Why does this happen?
- Same plaintext yields different ciphertext!

# Counter Mode (CTR)

- CTR is popular for random access
- Use block cipher like a stream cipher

#### **Encryption**

$$C_0 = P_0 \oplus E(IV, K),$$
  
 $C_1 = P_1 \oplus E(IV+1, K),$   
 $C_2 = P_2 \oplus E(IV+2, K),...$ 

#### **Decryption**

$$P_0 = C_0 \oplus \mathbf{E}(IV, K),$$

$$P_1 = C_1 \oplus \mathbf{E}(IV+1, K),$$

$$P_2 = C_2 \oplus \mathbf{E}(IV+2, K),...$$

# Integrity

### Data Integrity

- Integrity detect unauthorized writing (i.e., detect unauthorized mod of data)
- Example: Inter-bank fund transfers
  - Confidentiality may be nice, integrity is critical
- Encryption provides confidentiality (prevents unauthorized disclosure)
- Encryption alone does not provide integrity
  - One-time pad, ECB cut-and-paste, etc., etc.

#### MAC

- Message Authentication Code (MAC)
  - Used for data integrity
  - Integrity not the same as confidentiality
- MAC is computed as CBC residue
  - That is, compute CBC encryption, saving only final ciphertext block, the MAC
  - The MAC serves as a cryptographic checksum for data
    - Any tempering of data will be detected.

# MAC Computation

• MAC computation (assuming N blocks)

$$C_0 = E(IV \oplus P_0, K),$$

$$C_1 = E(C_0 \oplus P_1, K),$$

$$C_2 = E(C_1 \oplus P_2, K), \dots$$

$$C_{N-1} = E(C_{N-2} \oplus P_{N-1}, K) = MAC$$

- Send IV,  $P_0$ ,  $P_1$ , ...,  $P_{N-1}$  and MAC
- $\bullet$  Receiver does same computation and verifies that result agrees with MAC
- Both sender and receiver must know K

#### Does a MAC work?

- Suppose Alice has 4 plaintext blocks
- Alice computes

$$C_0 = E(IV \oplus P_0, K), C_1 = E(C_0 \oplus P_1, K),$$
  
 $C_2 = E(C_1 \oplus P_2, K), C_3 = E(C_2 \oplus P_3, K) = MAC$ 

- Alice sends IV, P<sub>0</sub>, P<sub>1</sub>, P<sub>2</sub>, P<sub>3</sub> and MAC to Bob
- ullet Suppose the attacker changes  $P_1$  to X
- Bob computes

$$C_0 = E(IV \oplus P_0, K), C_1 = E(C_0 \oplus X, K),$$
  
 $C_2 = E(C_1 \oplus P_2, K), C_3 = E(C_2 \oplus P_3, K) = MAC \neq MAC$ 

- It works since error <u>propagates</u> into **MAC**
- The attacker can't make **MAC** == **MAC** without K

# Confidentiality and Integrity

- Encrypt with one key, MAC with another key
- Why not use the same key?
  - Send last encrypted block (MAC) twice?
  - This cannot add any security!

# Confidentiality and Integrity

- Using different keys to encrypt and compute MAC works, even if keys are related
  - But, twice as much work as encryption alone
  - Can do a little better about 1.5 "encryptions"
- Confidentiality *and* integrity with same work as one encryption is a still research topic...
  - Authenticated encryption (AE)
  - Optional reading materials on this topic.

- Quantum computers use weird properties of quantum mechanics
- These use "quantum bits" or "qubits"
  - Bits are either 0 or 1, but...
  - Qubits both 0 and 1? Or range 0 to 1?
- With N bits, in one of 2<sup>N</sup> states
- With N qubits, 2<sup>N</sup> quantum amplitudes
  - Which are continuous, not discrete, values, so potential for vastly more computing power

- Quantum computers are challenging to build and to program
- Special algorithms needed to take advantage of qubits
  - Need to use reversible logic operations
  - Results generally only probabalistic
- Today, quantum computers are small
  - IBM claims to have one with 433 qubits
  - Engineering challenges are immense

- Assuming big quantum computers are built, are they a threat to symmetric ciphers?
- Best quantum algorithm for exhaustive search is due to Grover (1996)
- Grover's algorithm is square root faster
- For n bit symmetric key...
  - Conventional computer: Work factor 2<sup>n-1</sup>
  - Grover's algorithm: Work factor about 2<sup>n/2</sup>

- Assuming big quantum computers are built, are they a threat to symmetric ciphers?
- Not really a serious threat
- If we double the length of the key, work factor for Grover's algorithm is same
- For AES, most common to use 128-bit key
  - But, we can use 256-bit keys
  - Just a little slower with 256 than 128-bit key

# Uses for Symmetric Crypto

- Confidentiality
  - Transmitting data over insecure channel
  - Secure storage on insecure media
- Integrity (MAC)
- Authentication protocols (later...)
- Hash function