Test 2 (20%)

Mathematics 4 (SCIFF 021)

Spring Semester 2021/2022

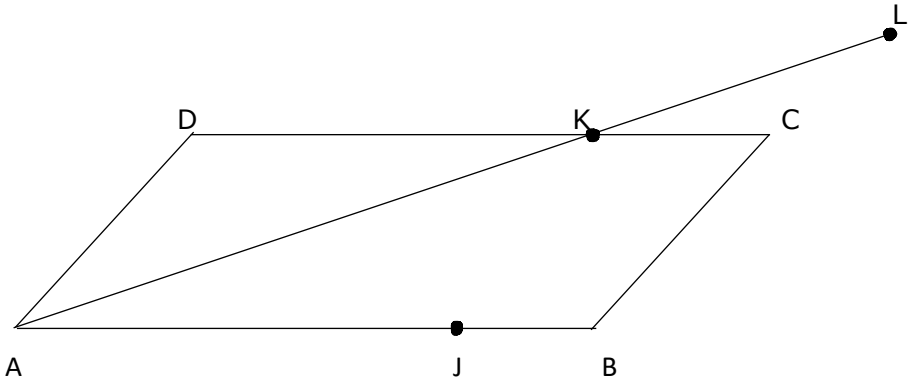
Answer all the questions. **[Total: 40m]**

1. a) Given that $z = -4 + 6i$. Calculate arg z, giving your answer in degree. [2 m]

   b) The complex number w is given by $w = \frac{A}{2-i}$, where A is a positive constant. Given that $|w| = \sqrt{20}$.
   i) find w in the form $a + ib$, where a and b are constant, [5 m]
   ii) calculate arg $\left(\frac{w}{z}\right)$. [4 m]

   c) Given that 2 and $5+2i$ are the roots of the equation $x^3 - 12x^2 + cx + d = 0$. Find the
   i) values of c and d, [7 m]
   ii) other complex root [5 m]

2. Diagram 1 shows a parallelogram ABCD. Point J lies on the straight line AB and point K lies on the straight line DC. The straight line AK is extended to a point L such that 3AK = 2AL. It is given that:
   AJ : JB = 3 : 1, DK : KC = 3 : 1, $\overrightarrow{AB} = 8p, \overrightarrow{AD} = q$



   a) Express $\overrightarrow{AK}$ and $\overrightarrow{JC}$ in terms of p and q. [3 m]
   b) Hence, show that the points J, C and L are collinear. [4 m]

   c) It is given that $p = 3i$ and $q = 2i + 5j$.

      i) Express $\overrightarrow{JC}$ in terms of i and j. [2 m]
      ii) Find the unit vector in the direction of $\overrightarrow{JC}$. [3 m]
      iii) Find the area of the triangle ADK. [5 m]