

## Case Study

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# The regional urban solid waste management system: A modelling approach

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**Abstract:** The Urban Solid Waste Management System (USWMS) is intrinsically complex, because it involves different connected problems and must achieve objectives which are often in conflict. It is difficult to evaluate the various alternatives in planning and managing it. It is thus useful to use mathematical models to provide a tool describing and evaluating the USWMS. This work concerns the development of a location-allocation model for planning USWMS and some heuristic techniques for solving it. The results of the model are the number and the location of waste disposal plants, specifying the technology adopted, the amount of waste processed and the service basin of each plant. Particular care is given to the case of the Italian region Lombardy: the regional USWMS law is in fact a useful stimulus for a constructive comparison between the actual system and possible alternatives.

**Keywords:** Environment; Location; Multi-criteria analysis

## 1. Introduction

Here we refer to a well known scheme (Hudson and Gross, 1974; Colonna and McLaren, 1976) for interpreting the Urban Solid Waste Management System (USWMS) according to which the system is structured into the four phases of *collection*, *transportation*, *processing* and *disposal*. This work considers the last three phases. Many instances nowadays advocate for selective waste collection. However here we consider the amount of waste remaining after selective collection as a whole.

For the processing phase, we consider the technologies of *incineration*, *composting* and *recycling*, the most commonly used ones; *sanitary landfills* are considered in the disposal phase. Overall, therefore, the technological choices refer to four different types of plants.

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A graph is the tool used to describe the geographical situation in which the location model is to be inserted. The nodes of the graph represent the zones into which the region is divided, and the arcs represent the transport network: the case study of the Lombardy region presented here considers just the road network (in general different transport networks can be used). The problems of identifying the zones into which the region is to be divided and the transport network, are decisive in describing the USWMS itself: zoning algorithms can be used for this purpose (Garfinkel and Nemhauser, 1972).

This problem is, as a matter of fact, multicriterial. Thus our model is characterised by a multi-objective function, with the following components: *economic cost*, *waste of resources*, *environmental impact*, associated with the system.

The *economic cost* is calculated by taking into account plant investment and management expenses, as well as transport costs. To compare the different items, the discounted cash flow method has been used, referring to a 15 years time horizon. The difficulties in determining this component, connected with the reliability of the cost parameters used, have been solved using Italian case studies (Andreottola and Frangipane, 1988).

The *waste of resources* is measured by the amount of refuse which is disposed in the sanitary landfills. We thus wish to minimise the amount of waste from which no further recovery is possible. This item measures, in a certain sense, the system's degree of 'closure', identifying the fraction of waste leaving the system (we feel it is necessary to keep this objective separate from the others, as an independent criterion in the evaluation of the system).

The *environmental impact* component tries to quantify the impact produced by the presence of the disposal system. In this case the greatest difficulties are connected with the need to compare heterogeneous factors (pollution of air and water, soil impoverishment, negative impact on the landscape, public opposition, etc.) for which it is difficult to find measurement indexes. On the basis of existing studies and current legislation (CEQ, 1978; CEC, 1985), the environmental impact depends mainly on location: the factors taken into account are technology and capacity, geographical and habitat characteristics of the zones (the latter depend in turn on the technology used). Starting with these factors, we calculate an index which quantifies the environmental impact for each possible type of plant and location. This value enables a comparison of impacts associated with different solutions, but has no meaning in absolute terms: for convenience, an index, conventionally called 'ecological unit', has been defined for this component of the objective function.

The model is also characterised by several constraints. In each zone there may be at most one process plant. This assumption must be motivated by the need not to penalise excessively some zones for the benefit of others. Total satisfaction of the demand is guaranteed, so that all the waste produced is disposed. A maximum plant capacity is established, dependent on the technology adopted.<sup>1</sup> The capacity of composting and recycling plants is linked to the potential demand of the local compost market (compost's low commercial value does not suggest a transport over large distances).

Transport between zones 'too far apart' is not permitted.<sup>2</sup> In quantifying a threshold value, reference is made to the amount of waste to be transported as well as to the transport distance.

Finally, a simplifying assumption is introduced to deal with the problem effectively. A hierarchy is established. First of all, we consider transport from zones to plants for the first treatment (*direct phase*); once the plants for this phase are located, transport from process plants to sanitary landfills (*indirect phase*) is considered. This way of proceeding is also justified by the fact that often the mean lifetime of sanitary landfills is less than that of the process plants (incinerator, recycling, composting): thus the decision-maker must relocate the landfills, while the other plants remain unchanged.

The paper is organized as follows. In Section 2, the formulation of the model is described. In Section 3, heuristic techniques to solve the model are developed. Then, in Section 4, a numerical example is provided. In Section 5 the Lombardy region test case is reported and, finally, in Section 6, a brief conclusion on the use of this kind of tools is provided.

<sup>1</sup> Introduction of a minimum capacity constraint does not seem necessary, because of the advantages of scale economies.

<sup>2</sup> This limitation is justified by the fact that we wish to avoid waste travelling backwards and forwards in the region.

## 2. Formulation of the model

Let  $n = |N|$  be the number of network nodes.

These are the indexes used in this paper from now on:

$i = 1, \dots, n$ : Indicates the location of the service users.

$j = 1, \dots, n$ : Indicates the location of the process plants.

$k = 1, \dots, n$ : Indicates the location of the sanitary landfills.

$h = 1, 2, 3$ : Indicates the process technology (incinerator = 1, recycling = 2, composting = 3).

Here are the variables of the problem:

$x_{ij}^h$ : Amount of waste that travels from user in  $i$  to process plant in  $j$  of typology  $h$ .

$y_{jk}^h$ : Amount of waste that travels from process plant in  $j$  of typology  $h$  to sanitary landfill in  $k$ .

$s_{ik}$ : Amount of waste that travels directly from user in  $i$  to sanitary landfill in  $k$ .

$z_j^h$ : Boolean variable indicating the presence of a process plant of typology  $h$  in node  $j$ .

$v_k$ : Boolean variable indicating the presence of a sanitary landfill in node  $k$ .

According to these variables, the following definitions are provided:

$r_j^h$ : Amount of waste treated by the process plant in  $j$  of typology  $h$ .

$t_k$ : Amount of waste disposed in the sanitary landfill in  $k$ .

Thus, we are able to describe completely the model.

Input data:

$d_i$ : Service demand of the user in  $i$  in weight units (tons);  $i = 1, \dots, n$ .

$c_{ab}$ : Travel cost<sup>3</sup> for unit of waste from node  $a$  to node  $b$ ;  $a, b = 1, \dots, n$ .

$\rho^h$ : Fraction (%) of waste treated by a process plant of typology  $h$  that requires disposal in a sanitary landfill;  $h = 1, 2, 3$ .

$Q_j^h$ : Maximum capacity<sup>4</sup> for process plant in  $j$  of typology  $h$ ;  $j = 1, \dots, n$ ;  $h = 1, 2, 3$ .

$Q_k$ : Maximum capacity for sanitary landfill in  $k$ ;  $k = 1, \dots, n$ .

$\gamma^h, \delta^h$ : Fixed and variable (per unit) costs used in the economic cost function for process plant of typology  $h$ ;  $h = 1, 2, 3$ .

$\bar{\gamma}, \bar{\delta}$ : Fixed and variable (per unit) costs used in the economic cost function for sanitary landfill;

$\eta_j^h, \mu_j^h$ : Fixed and variable (per unit) costs used in the environmental impact function for process plant in  $j$  of typology  $h$ ;  $j = 1, \dots, n$ ;  $h = 1, 2, 3$ .

$\bar{\eta}_k, \bar{\mu}_k$ : Fixed and variable (per unit) costs used in the environmental impact function for sanitary landfill in  $k$ ;  $k = 1, \dots, n$ .

MultiObjective function:

$$\min F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

where:

$$F_1 = \left[ \sum_h \sum_j (\delta^h z_j^h + \gamma^h r_j^h) + \sum_k (\bar{\delta} v_k + \bar{\gamma} t_k) \right] + \left[ \sum_h \sum_i \sum_j (c_{ij}^h x_{ij}^h) + \sum_i \sum_k (c_{ik} s_{ik}) + \sum_h \sum_j \sum_k (c_{jk}^h y_{jk}^h) \right], \quad (1.a)$$

$$F_2 = \sum_k t_k, \quad (1.b)$$

$$F_3 = \sum_h \sum_j (\eta_j^h z_j^h + \mu_j^h r_j^h) + \sum_k (\bar{\eta}_k v_k + \bar{\mu}_k t_k). \quad (1.c)$$

<sup>3</sup> Transport costs over a threshold value are fixed to infinite, to avoid these links.

<sup>4</sup> In our model the dependence of maximum capacity on the site of the plant is meaningful only for recycling and composting technology, but, as a general approach, this dependency is introduced for all the technologies considered.

Constraints:

$$\sum_h \sum_j x_{ij}^h + \sum_k s_{ik} = d_i, \quad i = 1, \dots, n, \quad (2)$$

$$r_j^h \leq Q_j^h z_j^h, \quad j = 1, \dots, n, \quad h = 1, 2, 3, \quad (3)$$

$$t_k \leq Q_k v_k, \quad k = 1, \dots, n, \quad (4)$$

$$\sum_i \rho^h x_{ij}^h = \sum_k y_{jk}^h, \quad j = 1, \dots, n, \quad h = 1, 2, 3, \quad (5)$$

$$\sum_h z_j^h \leq 1, \quad j = 1, \dots, n. \quad (6)$$

Variable conditions:

$$x_{ij}^h \geq 0, \quad i, j = 1, \dots, n, \quad h = 1, 2, 3, \quad (7.a)$$

$$y_{jk}^h \geq 0, \quad j, k = 1, \dots, n, \quad h = 1, 2, 3, \quad (7.b)$$

$$s_{ik} \geq 0, \quad i, k = 1, \dots, n, \quad (7.c)$$

$$z_j^h = 0, 1, \quad j = 1, \dots, n, \quad h = 1, 2, 3, \quad (7.d)$$

$$v_k = 0, 1, \quad k = 1, \dots, n. \quad (7.e)$$

Definitions:

$$r_j^h \equiv \sum_i x_{ij}^h \quad j = 1, \dots, n, \quad h = 1, 2, 3, \quad (8)$$

$$t_k \equiv \sum_i s_{ik} + \sum_h \sum_j y_{jk}^h, \quad k = 1, \dots, n. \quad (9)$$

Component  $F_1$  (see (1.a)) of the objective function describes the overall costs; the first part is connected with investment and management expenses (compared with the discounted cash flow to calculate the costs in the time unit), the second one describes the transport costs. The function forms imply a fixed cost (parameters  $\delta^h$  and  $\bar{\delta}$ ) and a variable one that grows linearly with the amount of waste processed (parameters  $\gamma^h$  and  $\bar{\gamma}$ ). Component  $F_2$  (see (1.b)) determines the waste of resources within the system in the time unit. Component  $F_3$  concerns the environmental impact: the function forms are the same of those described for (1.a).

Constraint (2) requires that all the waste produced is correctly disposed. Constraint (3) fixes the maximum capacity for a process plant of typology  $h$ . Constraint (4) does the same for sanitary landfills. Constraint (5) is a balance equation: all the waste produced by the process plants has to be finally disposed in a landfill. Constraint (6) implies no more than one process plant for each node.

Variables  $x$ ,  $y$  and  $s$  in (7.a), (7.b), (7.c) are defined as nonnegative. Variables  $z$  and  $v$  in (7.d) and (7.e) are defined as boolean.

Definition (8) indicates the amount of waste treated in a process plant. Similarly, definition (9) indicates the amount of waste disposed in a sanitary landfill.

The heuristic procedure described in the next section is aimed at finding out a subset of approximate Pareto solutions using a weight method. A scalar objective function is computed in the following manner:

$$F = w_1 F_1 + w_2 F_2 + w_3 F_3$$

where  $w_1$  is the weight for the function  $F_1$ . The method used is based on the assumption that every efficient solution can be found as an optimal solution by applying certain nonnegative weights in a linear function (Geoffrion et al., 1972). A subset of Pareto solutions can be generated by repeatedly solving the problem for various combinations of nonnegative weights: thus, as is well known, it only holds for convex problems. The *reference point* method (Wierzbicki, 1980) is used to help the decision-maker in identifying the final solution.

### 3. Solution algorithms

In this section we first describe some algorithms that constitute the solution procedure, namely **Dominant, Connect, TechnicalChoice Add, Drop, Substitute**. Then we illustrate the complete procedure, and finally we provide the two algorithms that allow respectively the computation of the new weights in the **iterative weight method**, and the choice of the final solution in the **reference point method**.

**Dominant.** This routine establishes the best covering of the network, without considering capacity constraints. First of all we give a few definitions. For each user node  $i$ , let  $A_i$  be the set of potential plants  $j$  with existing arcs  $c_{ij}$  (i.e. nodes  $i, j$  are not 'too far apart'). Conversely, for each plant  $j$ , let  $B_j$  be the set of users with existing arcs  $c_{ij}$ . Let  $S$  be the set of users already assigned to a plant: at the beginning of this algorithm,  $S$  is the empty set. For each potential plant  $j$ , let  $E_j$  be a set defined as follows:

$$E_j = (B_j - (B_j \cap S)).$$

Then  $E_j$  is the set of users not yet assigned that plant  $j$  can serve.

It is possible to define the 'degree of coverage' of a plant with respect to a set  $S$  as the cardinality of set  $E_j$ . Thus, a plant with higher degree covers the users not yet assigned better than a plant of lower degree. Finally, let  $P$  be the set of selected plants. As for  $S$ , at the beginning  $P$  is the empty set.

**Dominant** proceeds as follows:

Step 1. Set  $S = \emptyset$  and  $P = \emptyset$  and compute sets  $A_i, B_j$ .

Step 2. Identify the user node  $i^* \notin S$ , with the lowest  $|A_{i^*}|$ .

Step 3. Compute sets  $E_j$  for all  $j \in A_{i^*}$ , then determine the plant  $j^*$  with the highest degree.

Step 4. Put  $P = P \cup \{j^*\}$  and  $S = S \cup E_{j^*}$ .

Step 5. If  $S = N$ , stop; otherwise go to Step 2.

In Figure 1 a graph with 5 nodes is described (only arcs with transport cost  $c_{ab} \leq 3$  are considered). It is thus possible to see **Dominant** working on this graph as an example.

Step 1.  $S = \emptyset, P = \emptyset$ ;

$A_1 = \{1, 2, 3\}, A_2 = \{1, 2, 3\}, A_3 = \{3, 4\}, A_4 = \{1, 4, 5\}, A_5 = \{1, 5\}$ ;

$B_1 = \{1, 2, 4, 5\}, B_2 = \{1, 2\}, B_3 = \{1, 2, 3\}, B_4 = \{3, 4\}, B_5 = \{4, 5\}$ .

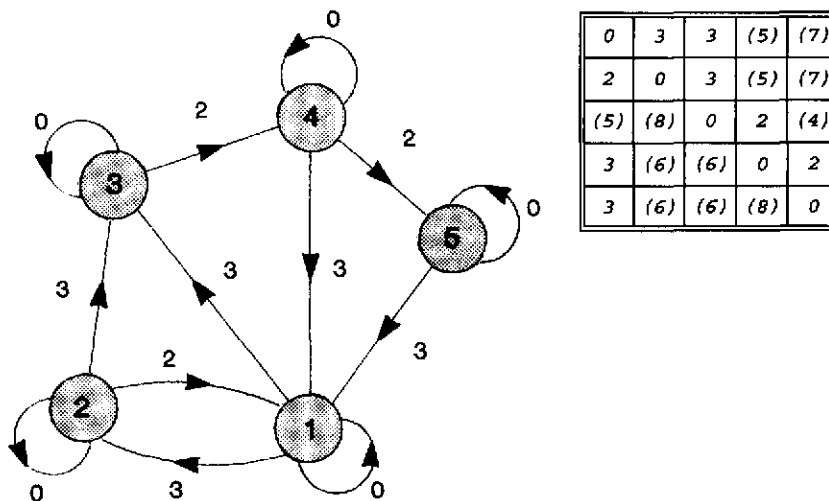


Figure 1. A graph and its related cost matrix

Step 2.  $i = 5$ , as  $|A_5| = 2$ ;  $A_5 = \{1, 5\}$ .

Step 3.  $E_1 = \{1, 2, 4, 5\}$ ,  $E_5 = \{4, 5\}$ ;  $j = 1$ , as  $|E_1| = 4$ .

Step 4.  $S = \{1, 2, 4, 5\}$ ,  $P = \{1\}$ .

Step 5.  $S = N$ , go to Step 2.

Step 2.  $i^* = 3$ , as  $|A_3| = 2$  (3 is the only user not in  $S$ );  $A_3 = \{3, 4\}$ .

Step 3.  $E_3 = \{3\}$ ,  $E_4 = \{3\}$ ;  $j = 3$ , as  $|E_3| = 1$ .

Step 4.  $S = \{1, 2, 3, 4, 5\}$ ,  $P = \{1, 3\}$ .

Step 5.  $S = N$ , stop.

**Connect.** This algorithm heuristically assigns the users to the nearest plant taking into account the capacity constraint of the largest plant and defines the service basins  $U_j$  for all  $j \in P$ .<sup>5</sup> Capacity constraints can determine situations where a user is not assigned to the plant that is in the same node. If no feasible solution is found for set  $P$ , more plants are added (see **Add** algorithm). With reference to Figure 1, let  $P = \{1, 2, 3\}$ ,  $d_i = 2$  for all  $i$ ,  $Q_j^h = 4$  for all  $h, j$  and  $Q_k = 4$  for all  $k$ . Thus users 4 and 5 must be assigned to plant in 1 and then  $U_1 = \{4, 5\}$ ,  $U_2 = \{1, 2\}$  and  $U_3 = \{3\}$ .

**TechnicalChoice.** This routine chooses the typology of the plants located in the previous phase. It explores each plant  $j \in P$  starting with the one with higher number of ending arcs considering only the subgraph with node set  $P$ . With reference at Fig. 1, let  $P = \{1, 2, 3\}$ . For nodes 3, 1 and 2 the arcs number equals 3, 2 and 2 respectively: thus that is the order with which they are explored.

Then **TechnicalChoice** computes the three parts of the objective function for all the feasible technologies: the best typology is then chosen.

This algorithm is used only during the direct phase for both processing and disposal plants (during the indirect phase only sanitary landfills are considered and thus no typology choice is required). Moreover, for the processing technologies, a forecast of the increase of the objective function values, due to the indirect phase, is calculated: this forecast is obtained by adding economic and environmental costs due to the transport and disposal of the residual waste in the nearest existing landfill (if no landfill is available, we suppose its construction in the same node).

**Add, Substitute, Drop.** All these algorithms are described together because they are very similar. They are used to change the set  $P$  of selected plants, starting from a current solution: whenever a better solution is found, it becomes the current one. **Add&Drop** (Kuehn and Hamburger, 1963) is a well known algorithm to find new solutions. **Add** changes the set  $P$  adding a new plant. **Substitute** modifies  $P$  dropping a plant and adding a new one. **Drop** changes  $P$  dropping one of the selected plants.

It is easy to see that, starting from a set  $P$ , more solutions are reachable using **Substitute** instead of **Add** or **Drop**. As an example, with reference to Figure 1, if  $P = \{1, 2, 3\}$ , there are these new sets:

**Add:**  $P_1 = \{1, 2, 3, 4\}$ ,  $P_2 = \{1, 2, 3, 5\}$ .

**Substitute:**  $P_1 = \{2, 3, 4\}$ ,  $P_2 = \{2, 3, 5\}$ ,  $P_3 = \{1, 3, 4\}$ ,  
 $P_4 = \{1, 3, 5\}$ ,  $P_5 = \{1, 2, 4\}$ ,  $P_6 = \{1, 2, 5\}$ .

**Drop:**  $P_1 = \{2, 3\}$ ,  $P_2 = \{1, 3\}$ ,  $P_3 = \{1, 2\}$ .

Moreover, only **Add** and **Drop** modify the cardinality of set  $P$ . In Figure 2 we have a lattice describing all the possible  $P$  sets for our example. Neat lines connect sets that can be reached using **Add** or **Drop** once starting from  $P = \{1, 2, 3\}$ . Dotted lines connect sets that can be reached using **Substitute** once starting from the same set  $P = \{1, 2, 3\}$ . In the lattice, **Add** corresponds to downwards vertical displacements, **Substitute** to horizontal and **Drop** to upwards vertical movements. Our procedure uses the three algorithms in this order: **Add**, **Substitute** and **Drop**.

As these algorithms are performed until no improvement is achieved, in the worst case, for **Substitute**, all the solutions with the same cardinality of  $P$  are evaluated. In the real case tested (125 nodes, see Section 3) we had no improvements after few iterations of **Substitute**.

As stated at the beginning of this section, now we describe the whole procedure, using the flow-chart of Figure 3.

<sup>5</sup> If the resulting amount of waste overcomes the capacity constraints of some typologies, these ones are no more considered in the **TechnicalChoice** algorithm.

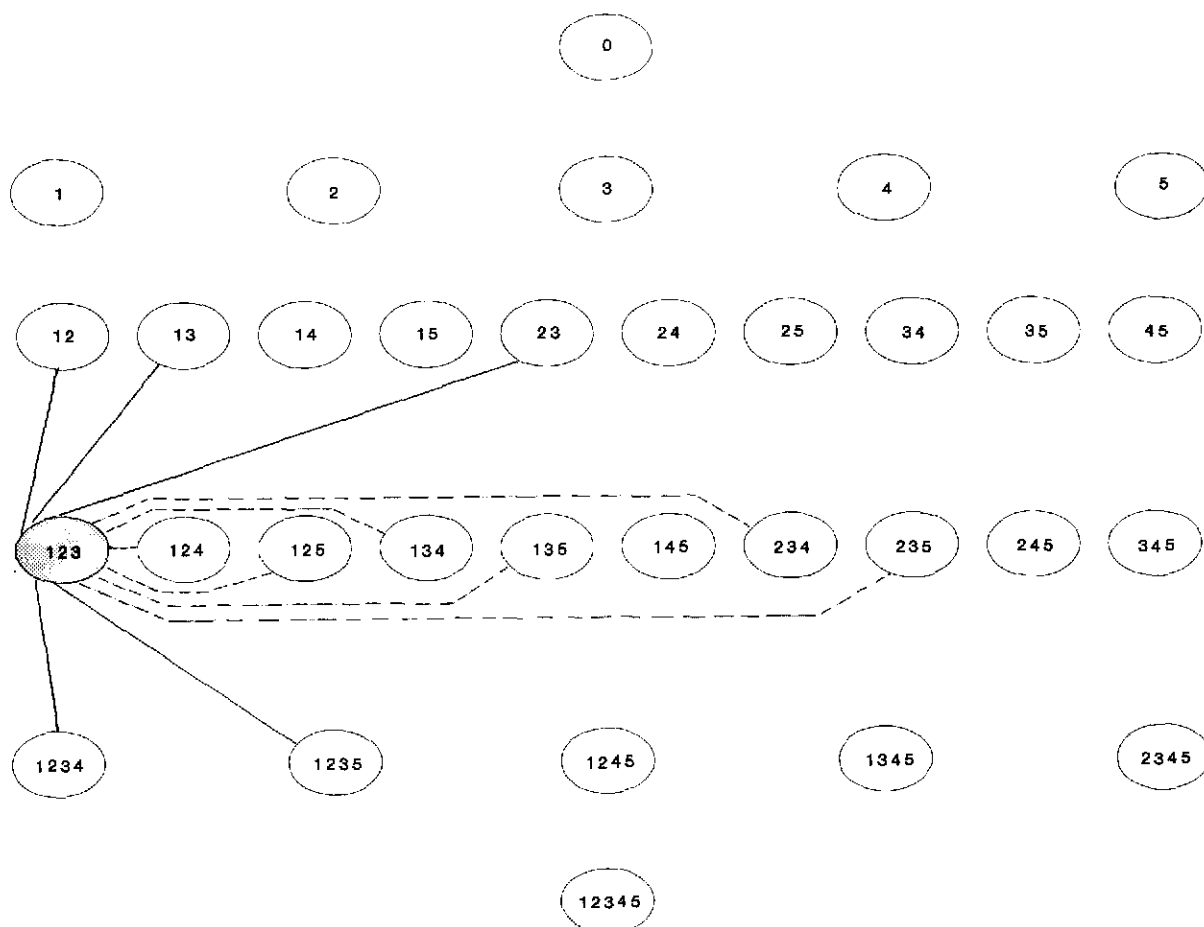


Figure 2. The solution lattice with a few links

A few comments are useful for a better comprehension of this flow-chart. In fact, to avoid a very detailed and formally correct but perhaps confusing representation, an 'ad hoc' notation has been used.

The main cycle is due to weight recomputation. For each set of weights a new Pareto solution is found out.

The routines separated by commas in the same block are executed in cascade (for instance, the block **DOMINANT, CONNECT, TEC\_CHOICE** performs the three routines one after the other).

In both direct and indirect phases **Add**, **Substitute** and **Drop** are used in cascade. Thus the cardinality of  $P$  starts growing, then remains unchanged and finally can diminish. It has to be said that the initial solution, computed by **Dominant**, should find a minimal number of plants for the graph: this is the reason why these three algorithms are used in that order.

After each step of **Add**, **Substitute** and **Drop** algorithms in both phases, **TEST 1** is checked: when a better solution is found, it becomes the new one, and the current algorithm starts again. Otherwise new  $P$  sets are tested until no other change is possible. In other words, each algorithm stops only when all possible changes bring no improvement.

In the indirect phase, as said before, the **TechnicalChoice** routine is not used. At the end of the indirect phase, the **NEW WEIGHT COMPUTATION** is performed and then **TEST 2** is checked: if the new weights are less distant from the previous ones than a predefined threshold value (as explained in the following), the main cycle exists and finally the *reference point* method is started.

Generating Pareto solutions by means of systematic variation of weights may lead to a set of solutions which is far too big for practical purposes. Thus, there is a need for a more clever strategy to generate a

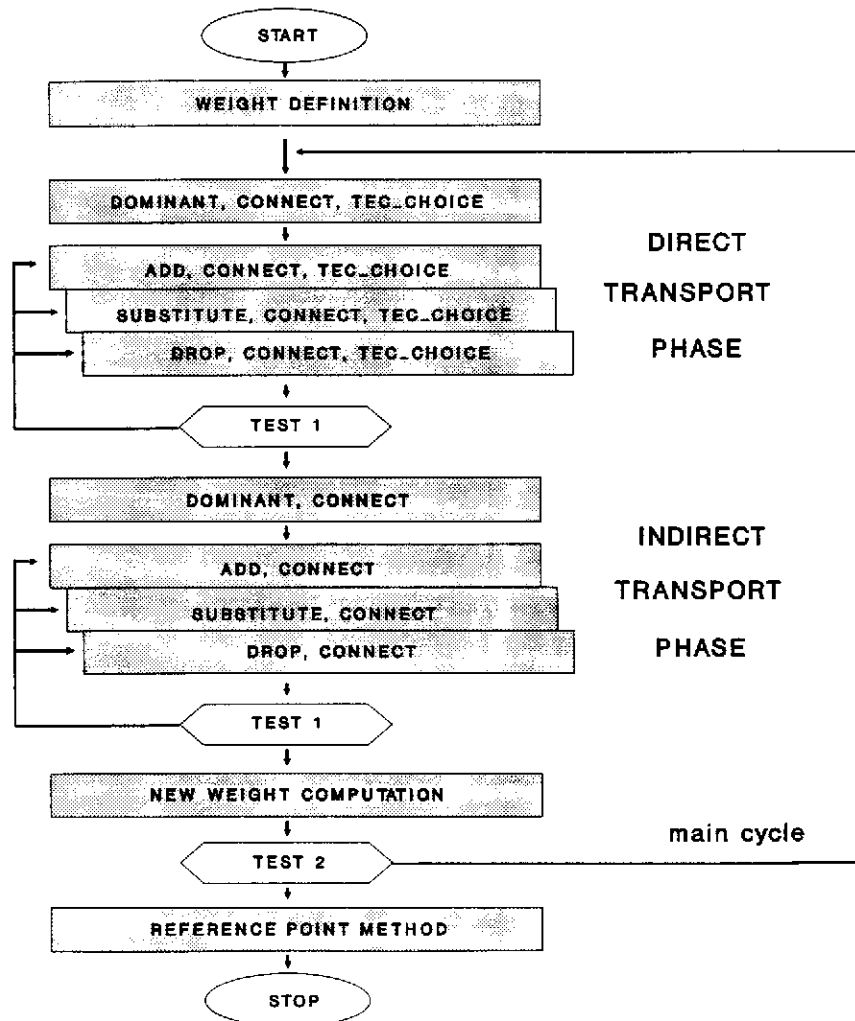


Figure 3. Flow chart of the solution procedure

limited number of efficient solutions which gives a fair representation of the whole set. There are many ways of generating this representative subset. (Steuer, 1977). IN our case the selection procedure is based on an heuristic method given in the following.

At a certain iteration a weight vector  $w$  is provided. When the iteration is over, the best solution, with  $F = F_l^0$ , is identified. During the weight iteration minimum ( $F_l^n$ ) and maximum ( $F_l^*$ ) values (for all solutions explored during the current weight cycle) for each component of the objective function are stored.

Given a value  $e > 1$ , the new weight vector is computed as follows:

$$w'_i = (e - w_i) * (1 + (F_l^0 - F_l^n) / (F_l^* - F_l^n)). \quad (10)$$

Weights  $w'_i$  are then normalized and used in the new iteration. The greater  $e$ , the smaller the oscillations of  $w_i$ -values. The second term of the formula gives more importance to the weight  $w_i$  whose component of the objective function  $F_l^0$  is more distant from the best found value  $F_l^n$ , relative to the observed range of  $F$ -values.



The weight iteration stops when the new weights  $w'_i$  are less distant from the previous ones  $w_i$  than a predefined value  $\psi$ , in formula:

$$\sum_i |w_i - w'_i| \leq \psi. \quad (11)$$

The *reference point* method (Wierzbicki, 1979; Lewandowski and Grauer, 1982) helps the decision-maker in choosing the final solution in a multicriteria problem. The decision-maker fixes an ideal solution (reference point), indicating this point in the objective function space: the method determines the closest <sup>6</sup> real solution (efficient point) and iterates until he is satisfied of the identified solution.

#### 4. A numerical example

In this section we show an example of application of our USWMS model to a simple case using the graph in Figure 1 and considering as process plants incinerators ( $h = 1$ ) and recycling plants ( $h = 2$ ) only. Moreover the objective functions used are  $F_1$  (economic cost) and  $F_2$  (waste of resources) only. There are 5 nodes in the graph,  $N = \{1, 2, 3, 4, 5\}$ , each of them with a service demand  $d_i = 2$ . We indicate only the arcs with a transport cost  $c_{ab} \leq 3$ ; the capacities are  $Q_j^1 = 4.5$ ,  $Q_j^2 = 2.5$  for all  $j$ ,  $\bar{Q}_k = 4.5$  for all  $k$ ; the fixed costs are respectively  $\delta^1 = 30$ ,  $\delta^2 = 20$ ,  $\bar{\delta} = 25$ ,  $\gamma^1 = 1$ ,  $\gamma^2 = 3$ ,  $\bar{\gamma} = 2.5$ ; the fractions of waste treated by a process plant of typology  $h$  that require disposal in a sanitary landfill are respectively  $\rho^1 = 10\%$  and  $\rho^2 = 25\%$ . In the following, the economic costs are described as the sum of two terms: plant costs  $C_p$  (the first term of (1.a)) and transport costs  $C_t$  (the second term of (1.a)). The method works as follows:

**Weights choice.** We set initially  $w_1 = 1$  and  $w_2 = 0$ , deciding to give importance to the economic cost  $F_1$  only

**Direct transport phase.** The method uses first of all the **Dominant** algorithm to obtain a graph coverage with a low number of plants: it finds an initial solution  $P = \{1, 3\}$  with two plants (see Section 3, algorithm **Dominant**). The algorithm **Connect** allocates the users to the open plants taking into account plant capacities. As it is impossible to satisfy all the users, even using the greatest capacity (4.5), other plants must be added to the ones already chosen.

The method continues using the algorithm **Add**; then a new solution is obtained with three plants  $P = \{1, 2, 3\}$ . The algorithm **Connect** determines the three service basins,  $U_1 = \{4, 5\}$ <sup>7</sup>,  $U_2 = \{1, 2\}$ ,  $U_3 = \{3\}$ . The algorithm **TechnicalChoice** sets the typology of plants; it explores the plants starting with node 3 (see Section 3, algorithm **TechnicalChoice**). In this first elaboration all the three plants set up are landfills. The plant costs are  $C_p(1) = 25 + 4 \cdot 2.5 = 35$ ,  $C_p(2) = 25 + 4 \cdot 2.5 = 35$ ,  $C_p(3) = 25 + 2 \cdot 2.5 = 30$  and the transport costs are  $C_t = 2 \cdot 3 + 0 + 0 + 2 \cdot 3 + 2 \cdot 3 = 18$ . The solution found,  $P = \{1, 2, 3\}$ , has a total cost of 118 and an amount of waste disposed in landfills of 10. Having  $w_1 = 1$  and  $w_2 = 0$  the global objective function results  $F = 118$ .

The methods proceeds using the algorithm **Add** again and obtaining a new solution with 4 plants  $P = \{1, 2, 3, 4\}$ . The algorithm **Connect** then determines  $U_1 = \{1, 5\}$ ,  $U_2 = \{2\}$ ,  $U_3 = \{3\}$ ,  $U_4 = \{4\}$ . **TechnicalChoice** sets the typologies of plants (recycling for nodes 2 and 4, sanitary landfills for nodes 1 and 3). As the resulting total cost is 128.5, this solution is however discarded. The same applies for the next solution,  $P = \{1, 2, 3, 5\}$ , which has recycling plants in nodes 1 and 2, landfills in 3 and 5, with a total cost of 126.5. As no more changes are possible, **Add** stops.

Then the algorithm **Substitute** starts from the current solution  $P = \{1, 2, 3\}$ . It obtains first the solution  $P = \{2, 3, 5\}$  in which all the 3 plants are landfills and the total cost is 110; the solution

<sup>6</sup> It is possible to define distance in different ways; in our case it has been assumed as the Euclidean distance in a 3-dimensional space.

<sup>7</sup> The user of node 1 is not served by the plant in node 1 but by the one in node 2. For this point see the example of the **Connect** algorithm in Section 3.

$P = \{1, 3, 5\}$  is then found, again with 3 landfills and a total cost of 108. Finally the solution  $P = \{1, 4, 5\}$  with the total cost 106.75 (104 in the direct phase) and an amount of waste at landfills of 8.5 (8 in the direct phase) is found. It has 2 sanitary landfills in nodes 1 and 4, and a recycling plant in node 5.

As no better solutions are found using again **Substitute**, the method uses the algorithm **Drop**, trying to decrease the number of plants. In this simple example this is impossible due to the maximum allowed capacities. Thus the direct phase ends with the solution  $P = \{1, 4, 5\}$ .

*Indirect transport phase.* In this phase sanitary landfills found in the previous phase remain fixed (in the example on the nodes 1 and 4) and one tries to find the best allocation for the wastes coming from the process plants already set up (in the example a recycling plant in node 5 with 0.5 units of waste). Of course the algorithm **TechnicalChoice** is never used in this phase. The method uses the algorithms **Add**, **Substitute** and **Drop** to explore the adjacent solution to the current one,  $P = \{1, 4\}$ .<sup>8</sup>

No improvements to the current solution are found. Then the current solution is assumed as the best one (point A in Figure 4) for the problem, corresponding to the weights  $w_1 = 1$  and  $w_2 = 0$ .

*New weight calculation.* Using (10), with  $e = 1.33$ , we obtain the following new values for the weights:

$$w_1 = (1.33-1) * (1 + (106.75-106.75)/(128.5-106.75)) = 0.33,$$

$$w_2 = (1.33-0) * (1 + (8.5-7.0)/(10.0-7.0)) = 1.995,$$

which normalised become which normalised become  $w_1 = 0.14$ ,  $w_2 = 0.86$ .

*Direct transport phase (second iteration).* As in the first iteration, the use of algorithms **Dominant**, **Add**, **Connect** and **TechnicalChoice** produces the solution  $P = \{1, 2, 3\}$  (incinerators on nodes 1 and 2, sanitary landfill on node 3), with  $U_1 = \{4, 5\}$ ,  $U_2 = \{1, 2\}$ ,  $U_3 = \{3\}$ , and resulting plants costs and transport cost respectively of  $C_p(1) = 30 + 4 * 1.0 = 34$ ,  $C_p(2) = 30 + 4 * 1.0 = 34$ ,  $C_p(3) = 25 + 2 * 2.5 = 30$ , and  $C_t = 2 * 3 + 0 + 0 + 2 * 3 + 2 * 3 = 18$ .

To these costs must be added the expected cost of the indirect phase, in which the 8 units treated in the incinerators 1 and 2 yield 0.8 units to be sent to the landfill in 3. That generates a cost of 2.4 for the transport and a cost of 2 for the final disposal and consequently 0.8 more waste units in landfills.

Finally one obtains  $F_1 = 120.4$  and  $F_2 = 2.8$ , with  $F = 120.4 * 0.14 + 2.8 * 0.86 = 19.264$ . This solution remains the best one for the entire direct phase.

*Indirect transport phase (second iteration).* In this phase the transport occurs only from the incinerators of nodes 1 and 2 to the landfill in node 3. The solution is  $P = \{3\}$  and remains the best one for the problem (point B in Figure 4) corresponding to the weights  $w_1 = 0.14$  and  $w_2 = 0.86$ .

*New weights calculations and new iterations.* Continuing the iterative procedure with different sets of weights, the method finds 2 others solutions (points C and D). The complete set of results is given in Figure 4.

*Reference point method.* The normalised distances using as reference the Utopia Point with  $F_1 = 106.75$  and  $F_2 = 1.3$ , are shown in Table 1. In this case the final choice is solution B.

Using a different reference point, for example with  $F_1 = 115$  and  $F_2 = 5$ , we have different distances, also shown in Table 1. In this case the solution is given by point C.

## 5. A case study

We now present the application of the model to the Lombardy region case. Lombardy produces around 7000 tons per day (1981) of urban solid waste. In our model this production is partitioned into the 125 zones into which Lombardy has been divided. This zoning procedure is suggested by various considerations: the limits of the computer used (a PC IBM), both in terms of memory capacity and computer time, and the geographical situation of Lombardy. The number of zones chosen seems a good compromise between the previous requirements.

<sup>8</sup> Node 5, already belonging to the solution, is irrelevant because in the indirect phase only sanitary landfills are considered.

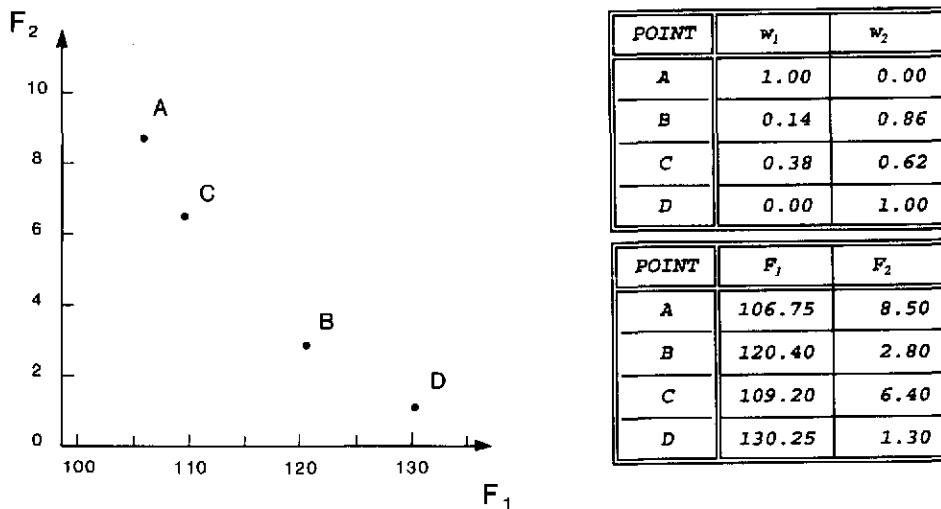


Figure 4. The solutions found

A data base has been constructed for each zone, describing the geographical, morphological and population characteristics, to calculate the environmental impact of the plants on the zone itself (see model formulation). The data base also includes the category and the measure of each arc of the main road network so that the transport costs can be calculated.

It should be remembered that in May 1988 the Lombardy region planned its USWMS (Lombardy Regional Council, 1988). Thus we already know the locations chosen, the plant technologies adopted and the service basin defined for each plant: our work proposes a comparison with the regional choices.

Some simulations have been performed starting by the following initial conditions:

1. *ex-novo scenario*, with no already-existing plant (this condition is not realistic, but may describe 'the best possible solution').
2. *present scenario*, corresponding to the actual situation in Lombardy with the regional law of 1988 (this is the simulation of greatest interest for the decision-maker);
3. *planned scenario*, corresponding to the choices of location suggested by the regional law (this is the reference situation).

It has to be said that in *present* and *planned scenarios* installation costs for already existing plants are not calculated. Thus, solutions with different scenarios are hardly comparable.

A package was prepared, called PURPLE<sup>9</sup> (Planning Urban Refuse Plants: Location Exercise). We briefly describe some of its characteristics.

- *Initialisation phase*. For each zone it is possible to indicate the presence of a plant (specifying process technology and capacity) and predefined location choices. It is also possible to exclude landfills in the direct phase and to close already existing (but not very useful) plants. A graphic aid is provided to illustrate the initial setting (see Figure 5).

- *Execution phase*. This phase is the core of the package, enabling execution of the algorithms presented. Weight vector and threshold values discriminating existing arcs can be modified. The execution takes place using just one set of selected weights or through an automatic weight assignment procedure. Finally, the value of the multiobjective function for a completely defined solution (without performing any optimisation) can be calculated: this is the case of the *planned scenario*.

- *Display phase*. This part examines the adopted solutions. In particular, it shows the plants selected, describing their technology and capacity, the service basins and the transport flows inside the network (see Figure 6).

<sup>9</sup> PURPLE is a program developed in TurboPascal. It works on PC IBM computers.

Table 1  
The reference point method

Points	Reference point					
	$F_1 = 106.75, F_2 = 1.3^a$			$F_1 = 115, F_2 = 5$		
	$\Delta_{F1}$	$\Delta_{F2}$	$\Delta$	$\Delta_{F1}$	$\Delta_{F2}$	$\Delta$
A	0.00	1.00	1.00	0.35	0.49	0.60
B	0.58	0.21	0.62	0.23	0.30	0.38
C	0.10	0.71	0.72	0.25	0.19	0.31
D	1.00	0.00	1.00	0.65	0.51	0.83

<sup>a</sup> Utopia point.

– *Research phase in the Pareto set.* This part uses the reference point technique to look for the final solution, also displaying the values for the various solutions of the Pareto set.

– *Parameter calibration Phase.* The values of all the parameters used in calculating the economic cost of the USWMS can be changed: in particular, the resources value (considered in the economic coefficients  $\gamma^h$  and  $\bar{\gamma}$  and subject to quite considerable fluctuations) can be changed.

As mentioned above, three different scenarios were examined. Table 2 gives the characteristics of the best solutions found for each scenario: to identify the best solution the reference point was set to the utopia point. For each solution, the values of the objective function are reported, together with the number of plants and amount of waste processed for each typology. Obviously, the amount of waste

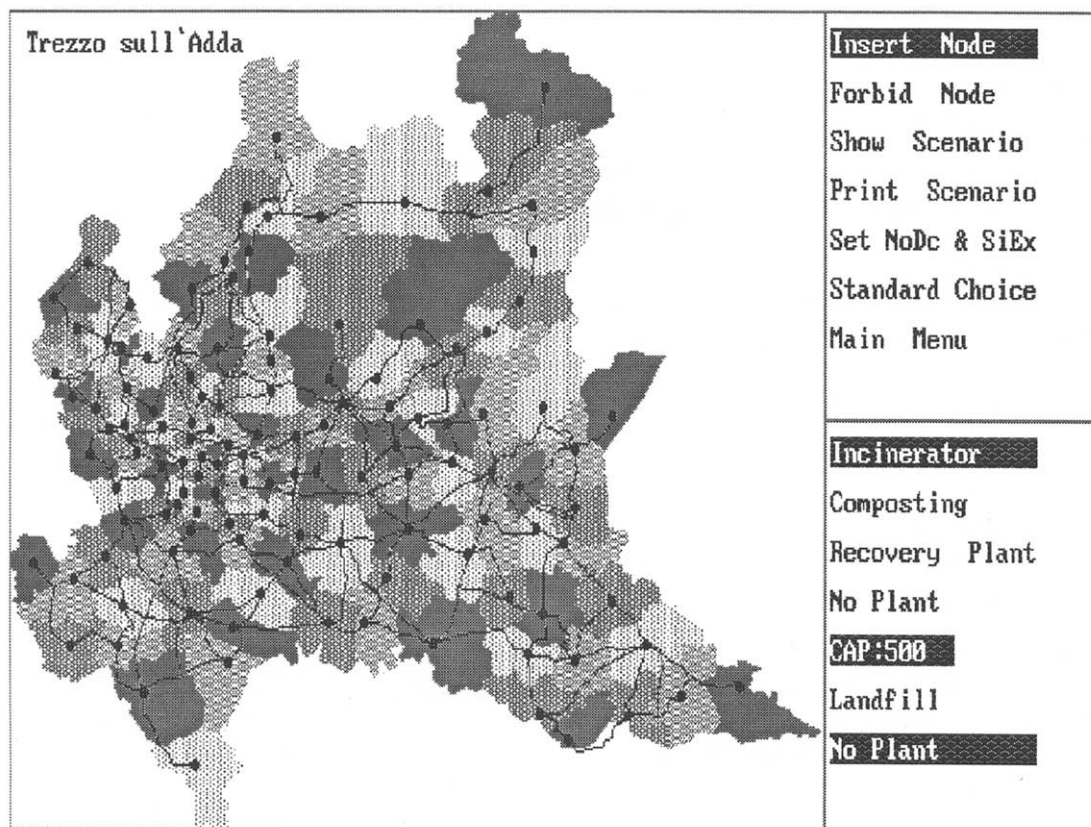


Figure 5. PURPLE package: the initialization phase

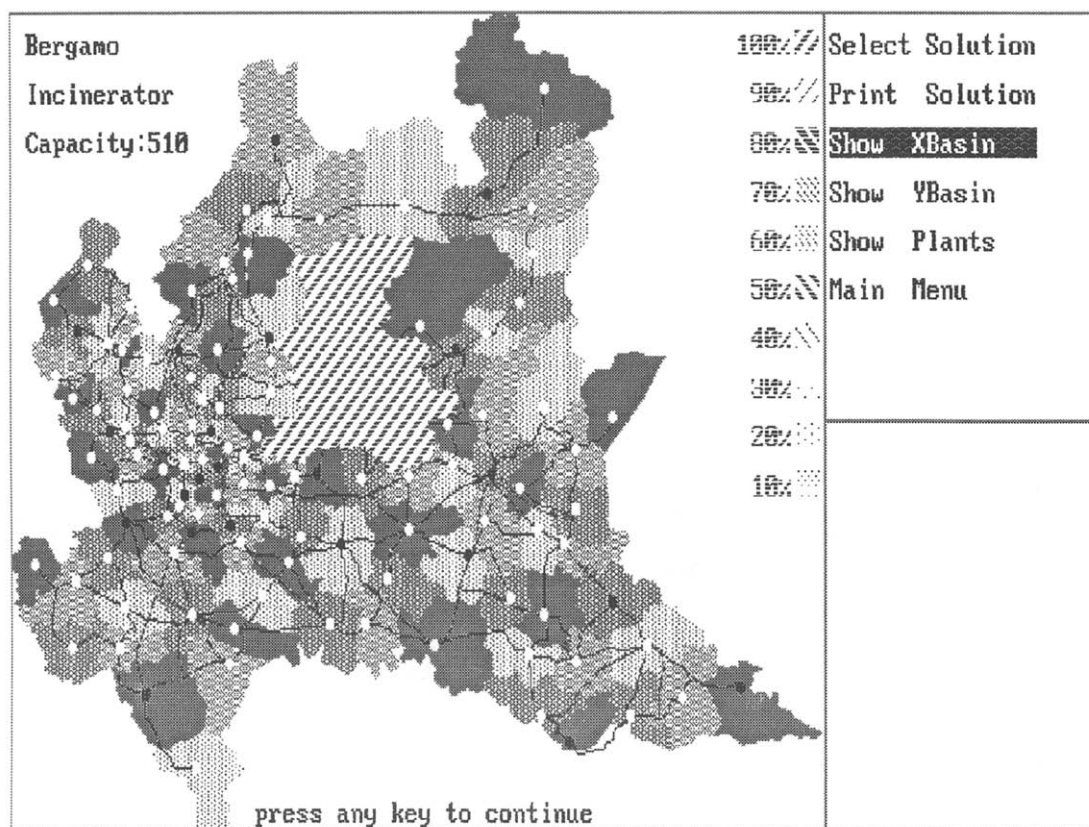


Figure 6. PURPLE package: the display phase

disposed in the sanitary landfills equals the value of the second component of the objective function, the waste of resources.

It has to be remembered that the planned scenario describes a predefined solution and so no optimisation is performed.

We have performed several simulations (not reported here) for exploring the Lombardy case study. The analysis of the results makes it possible to introduce some comments. A first one is that there is a considerable advantage, mainly from the economic point of view, in working with medium-large plans:

Table 2  
Best solutions for each scenario

	Ex-novo scenario	Present scenario	Planned scenario
$F_1$	674	595	705
$F_2$	1876	1894	2155
$F_3$	8680	8952	12462
Incinerator	9 (6048)	10 (6192)	15 (4680)
Composting	0 (0)	2 (72)	6 (792)
Recycling	5 (1152)	5 (936)	10 (2088)
Landfill	14 (1876)	9 (1894)	24 (2155)

this leads us to indicate (for the Lombardy region) a suitable number of process plants, about 15–20 units and a related number of landfills, between 10–12 units.

A second remark is technological. A close correlation (due to the coefficients quoted in the Italian literature and used here) was noted between plant size and technological choices. In particular, incinerators are best for medium–large capacities, recycling plants for medium–small capacities, composting plants for very small capacities. There are no indications against the use of sanitary landfills in large capacity intervals: a considerable economic advantage is thus obtained whenever these plants are used in the direct phase (the only disadvantage of this strategy is that waste inside the system increases enormously: the adoption of the second component of the objective function is still more justifiable).

There are also considerations which are more closely linked to the situation in Lombardy. In particular it was found that technological choices can often be correlated with geographical and population characteristics: incinerator is practically the unique technology adopted in the Milan–Como–Varese area and it is also strongly suggested for the large urban agglomerations of Bergamo and Brescia (about 70% of the region's plants are concentrated in these zones). Recycling plants are mainly used in the remaining zones, apart from particular situations where the small sizes required suggest the use of composting, particularly in the provinces of Sondrio and Cremona.

The costs of the system vary between 450 and 700 thousand million liras in a time horizon of 15 years. This sum is mainly for investment costs (the large variation is justified by the fact that situations with and without existing plants are considered). The management costs of these systems can even be negative. In fact, incineration, in large plants enables profits in management because of the energy production and thus if this type of process is used on a large scale, the profits deriving from it may cover the management costs of other plants. Transport costs are not very high (about 10% of the management costs): it should be remembered that this item does not consider the costs of the collection phase, which are much greater.

In our whole set of simulations, the waste of resources varies between 1850 and 300 tons per year. This variation depends mainly on whether sanitary landfills are used or not in the direct phase. If they are, the percentage of refuse wasted can increase considerably, although considerable economic advantages (and small variations of environmental impact) are obtained.

Finally, the environment impact varies between 8500 and 12500 'ecological units': it should be remembered that, in its determination, the choice of location plays an important role. Solutions with greatest impact generally have plants located in places that are unsuitable from the point of view of environmental safeguarding.

It can be seen that some of these general indications are partially disregarded by regional planning, which thus is judged unfavourably, on the basis of the indicators provided by the model. It is difficult to compare effectively the regional planning and the model results (because the model is based on some simplifying assumptions and because the regional plan overestimates the production of waste), but two main comments can be made:

- the planned number of process plants could be reduced, with significant economies of scale;
- plant capacities could be more correlated to the process technologies adopted.

## 6. Concluding remarks

We have presented a set of algorithms and a package which help to structure a location–allocation multicriteria problem in a suitable way for modelling an urban solid waste management system. The resulting procedure could help to solve this kind of problems, and, in our test case, it is applied to the Lombardy regional management of urban waste disposal.

The package PURPLE (Caruso and Paruccini, 1989), which implements the procedure on a personal computer, was actually used in the Joint Research Centre of Ispra to deal with different regional siting problems, with useful remarks.

Presently, we are trying to modify the heuristics in order to study applications on toxic waste management.

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