Modeling the Melt: Using Best-Fit Line Approximations to Measure the Decaying of Earth's Glaciers

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1 Introduction

This project will use least-squares approximation to calculate the two different functions used to estimate glacial melting. I recently watched James Balog's documentary *Chasing Ice*, a feature length film of his project to document the melting of the ice shelf over the years. In setting up 25 cameras around the world, he would periodically return to find that the footage would capture astonishing scenes - thousands of tons of ice falling into the sea every year. It is extraordinary and alarming the rate at which this occurs, and how the rate is increasing every year.

I first take data from the Environmental Protection Agency (EPA) data on average Arctic Sea Ice Mass (*Climate Change Indicators in the United States*, EPA, www.epa.gov/climate-indicators), in million square miles, as measured in March.

Year	Arctic Sea Ice Mass
1980	6.19307863
1985	6.135163307
1990	6.127441263
1995	5.891918946
2000	5.87647486
2005	5.671840716
2010	5.845586687
2015	5.548288025
2020	5.70658991

In the following models, the the independent variable x shall represent the year itself, and the dependent variable y shall be the mass of ice in million square miles. We choose x so we may interpolate discrete points and extrapolate beyond the range of 1980-2020. For sake of simplicity, the mass values will be rounded to the nearest hundredth place.

2 Best-Fit Line Model

We first compute the best-fit line for the data utilizing least-squares approximation. We do this by finding the coefficients r_0, r_1 for which the line $y = r_0 + r_1 x$ best fits our mass data. To compute these coefficients, we use the normal equation $A^T A r = A^T y$, where

$$A^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 5 & 10 & 15 & 20 & 25 & 30 & 25 & 40 \end{bmatrix}$$

$$y^T = \begin{bmatrix} 6.19 & 6.14 & 6.13 & 5.90 & 5.88 & 5.67 & 5.84 & 5.55 & 5.71 \end{bmatrix}$$

The normal equations simplify to

$$\begin{bmatrix} 53.01 \\ 1037.7 \end{bmatrix} = \begin{bmatrix} 9 & 180 \\ 180 & 5100 \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \end{bmatrix}$$

We find that $r_0 \approx 6.19$ and $r_1 \approx -0.015$. Therefore, the best-fit line equation is:

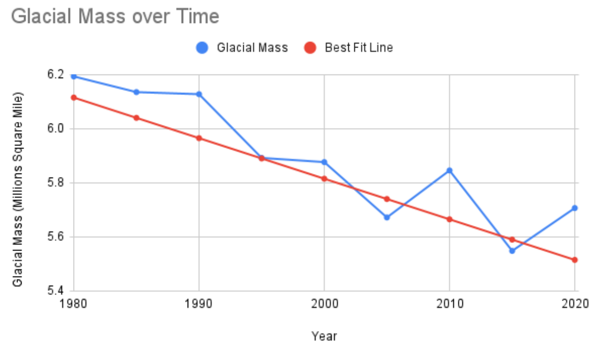
$$y = 6.19 - 0.015x$$

3 Visualizing the Data

A graph plotting the shrinking of the glacial mass and the calculated best fit line are graphed in blue and red, respectively, to the right.

The data suggests a oscillating downward trend. This seems sensible, as from year to year, the climate may cause more or less mass to melt off.

To best replicate this behavior, a sine and cosine function with the period of 10 years will be tried.



4 Downward Oscillation Model

For the second model, we consider the basis functions $f_0(x) = 1$, $f_1(x) = \sin\frac{\pi x}{6}$, and $f_2(x) = \cos\frac{\pi x}{6}$. This trigonometric function has a 5 year period. Thus, the new coefficient matrix is:

$$A^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -0.491 & -0.857 & -1.000 & -0.881 & -0.531 & -0.0431 & 0.456 & 0.836 & 0.998 \\ -0.871 & -0.514 & -0.023 & 0.474 & 0.847 & 0.999 & 0.890 & 0.548 & 0.063 \end{bmatrix}$$

Plugging into the normal equations, we find the linear system:

$$\begin{bmatrix} 52.996 \\ -9.981 \\ 13.348 \end{bmatrix} = \begin{bmatrix} 9 & -1.513 & 2.413 \\ -1.513 & 4.92 & 0.919 \\ 2.413 & 0.919 & 4.01 \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \\ r_3 \end{bmatrix}$$

We solve the system and find that $r_0 \approx 5.909$, $r_1 \approx -0.177$, and $r_2 \approx -0.186$, providing the approximation:

$$y = 5.090 - 0.177\sin\left(\frac{5\pi x}{6}\right) - 0.186\cos\left(\frac{5\pi x}{6}\right)$$

5 Analysis of Models

To analyze the accuracy of our models, we find quantifiable measures like coefficient of determination R. The average glacial mass in million square miles is $\overline{x} \approx 5.888$. Using the coefficient determination formula, we find that:

$$\text{Best fit line: } R^2 = 0.291$$

$$\text{Trigonometric Approximation: } R^2 = 0.783$$

The coefficient of determination for the best fit line is close to 0, showing that the line does not predict the values well and therefore isn't a good model for the relationship in the data. This is sensible as the data oscillates downwards, rather than trend downwards in a straight line. The best-fit oscillation model, however, works well to show a general downwards trend of the melt, that is, it downs how downward the oscillation is.

The trigonometric approximation is much closer to 1, revealing that is a better representation of the data. This point of data gives confidence that ice melting works in an oscillating trend. Around every 5 years, glacial mass is in the opposite trend than the previous 5 years.

Together, both lines show that glacial mass is both downward and oscillating. We see that every 5 years the trend of the mass changes. However, over the 40 year period, and possibly before and after the data set, the overall trend is downwards at an alarming rate. From 1980 to 2020, Almost 600,000 square miles of glacial mass has melted into the ocean. If Long Island, NY is 1,400 square miles, that melt is the equivalent of 428.5 Long Islands.

While specific calls to action is beyond the scope of this project, it is clear the present danger the Earth faces in terms of glacial melt and global warming. The trend continues downward, and there are and will be innumerable ecological health problems if bilateral action isn't taken immediately.