FUNCTIONAL DEPENDENCIES

CS 564- Spring 2025

ACKs: *Dan Suciu, Jignesh Patel, AnHai Doan*

WHAT IS THIS LECTURE ABOUT?

Database Design Theory:

- Functional Dependencies
- Armstrong's rules
- The Closure Algorithm
- Keys and Superkeys

HOW TO BUILD A DB APPLICATION

- Pick an application
- Figure out what to model (ER model)
  - Output: ER diagram
- Transform the ER diagram to a relational schema
- Refine the relational schema (normalization)
- Now ready to implement the schema and load the data!

DB DESIGN THEORY

- Helps us identify the "bad" schemas and improve them
  1. express constraints on the data: functional dependencies (**FDs**)
  2. use the FDs to decompose the relations

- The process, called normalization, obtains a schema in a "normal form" that guarantees certain properties
  – examples of normal forms: **BCNF**, **3NF**, ...

MOTIVATING EXAMPLE

SSN	name	age	phoneNumber
934729837	Paris	24	608-374-8422
934729837	Paris	24	603-534-8399
123123645	John	30	608-321-1163
384475687	Arun	20	206-473-8221

- What is the primary key?
  - (SSN, PhoneNumber)
- What is the problem with this schema?
  - Age and name are stored redundantly

MOTIVATING EXAMPLE

SSN	name	age	phoneNumber
934729837	Paris	24	608-374-8422
934729837	Paris	24	603-534-8399
123123645	John	30	608-321-1163
384475687	Arun	20	206-473-8221

Problems:

- redundant storage
- update: change the age of Paris?
- insert: what if a person has no phone number?
- delete: what if Arun deletes his phone number?

SOLUTION: DECOMPOSITION

SSN	name	age	phoneNumber
934729837	Paris	24	608-374-8422
934729837	Paris	24	603-534-8399
123123645	John	30	608-321-1163
384475687	Arun	20	206-473-8221

SSN	name	age
934729837	Paris	24
123123645	John	30
384475687	Arun	20

SSN	phoneNumber
934729837	608-374-8422
934729837	603-534-8399
123123645	608-321-1163
384475687	206-473-8221

Functional Dependencies

FD: DEFINITION

- **Functional dependencies** (FDs) are a form of constraint
- they generalize the concept of keys

If two tuples agree on the attributes

$$A = A_1, A_2, \dots, A_n$$

then they must agree on the attributes

$$B = B_1, B_2, \dots, B_m$$

Formally:

$$A_1, A_2, \dots, A_n \longrightarrow B_1, B_2, \dots, B_m$$

We then say that A **functionally determines** B

FD: EXAMPLE 1

SSN	name	age	phoneNumber
934729837	Paris	24	608-374-8422
934729837	Paris	24	603-534-8399
123123645	John	30	608-321-1163
384475687	Arun	20	206-473-8221

- $SSN \longrightarrow name, age$
- $SSN, age \longrightarrow name$

FD: EXAMPLE 2

studentID	semester	courseNo	section	instructor
124434	4	CS 564	1	Paris
546364	4	CS 564	2	Arun
999492	6	CS 764	1	Anhai
183349	6	CS 784	1	Jeff

- $courseNo, section \longrightarrow instructor$
- $studentID \longrightarrow semester$

SPLITTING AN FD

- Consider the FD: $A, B \longrightarrow C, D$

- The attributes on the right are independently determined by A, B so we can split the FD into:
  - $A, B \longrightarrow C$ and $A, B \longrightarrow D$

- We can not do the same with attributes on the left!
  - writing $A \longrightarrow C, D$ and $B \longrightarrow C, D$ does not express the same constraint!

TRIVIAL FDS

- Not all FDs are informative:
  - $A \longrightarrow A$ holds for any relation
  - $A, B, C \longrightarrow C$ also holds for any relation

- An FD $X \longrightarrow A$ is called **trivial** if the attribute A belongs in the attribute set X
  - a trivial FD always holds!

HOW TO IDENTIFY FDS

- An FD is domain knowledge:
  - an inherent property of the application & data
  - not something we can infer from a set of tuples
- Given a table with a set of tuples
  - we can confirm that a FD **seems** to be valid
  - to infer that a FD is **definitely** invalid
  - we can **never** prove that a FD is valid

EXAMPLE 3

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Black	Toys	99
Gizmo	Stationary	Green	Office-supplies	59

Q1: Is $name \longrightarrow department$ an FD?

- not possible!

Q2: Is $name, category \longrightarrow department$ an FD ?

- we don't know!

WHY FDS?

1. keys are special cases of FDs
2. more integrity constraints for the application
3. having FDs will help us detect that a schema has redundancies and tell us how to normalize it

MORE ON FDS

- If the following FDs hold:
  - $A \longrightarrow B$
  - $B \longrightarrow C$

  then the following FD is also true:
  - $A \longrightarrow C$
- We can find more FDs like that using what we call **Armstrong's Axioms**

ARMSTRONG'S AXIOMS: 1

Reflexivity

For any subset $X \subseteq \{A_1, \ldots, A_n\}$:

$$A_1, A_2, \ldots, A_n \longrightarrow X$$

- Examples
  - $A, B \longrightarrow B$
  - $A, B, C \longrightarrow A, B$
  - $A, B, C \longrightarrow A, B, C$

Armstrong's axioms: 2

Augmentation

For any attribute sets X, Y, Z :

$$\text{if } X \longrightarrow Y \quad \text{then} \quad X, Z \longrightarrow Y, Z$$

- Examples
  - $A \longrightarrow B$ implies $A, C \longrightarrow B, C$
  - $A, B \longrightarrow C$ implies $A, B, C \longrightarrow C$

ARMSTRONG'S AXIOMS: 3

<u>Transitivity</u>
For any attribute sets X, Y, Z :
if $X \longrightarrow Y$ and $Y \longrightarrow Z$ then $X \longrightarrow Z$

- Examples
  - $A \longrightarrow B$ and $B \longrightarrow C$ imply $A \longrightarrow C$
  - $A \longrightarrow C, D$ and $C, D \longrightarrow E$ imply $A \longrightarrow E$

APPLYING ARMSTRONG'S AXIOMS

Product(name, category, color, department, price)

1. $name \longrightarrow color$
2. $category \longrightarrow department$
3. $color, category \longrightarrow price$

- Infer: $name, category \longrightarrow price$
  1. We apply the augmentation axiom to (1) to obtain (4) $name, category \longrightarrow color, category$
  2. We apply the transitivity axiom to (4), (3) to obtain $name, category \longrightarrow price$

APPLYING ARMSTRONG'S AXIOMS

Product(name, category, color, department, price)

1. $name \longrightarrow color$
2. $category \longrightarrow department$
3. $color, category \longrightarrow price$

- Infer: $name, category \longrightarrow color$
  1. We apply the reflexivity axiom to obtain
     (5) $name, category \longrightarrow name$
  2. We apply the transitivity axiom to (5), (1) to obtain
     $name, category \longrightarrow color$

FD CLOSURE

<u>FD Closure</u>
If F is a set of FDs, the closure F^+ is the set of all FDs logically implied by F

Armstrong's axioms are:

- **<u>sound</u>**: any FD generated by an axiom belongs in F^+
- **<u>complete</u>**: repeated application of the axioms will generate all FDs in F^+

CLOSURE OF ATTRIBUTE SETS

Attribute Closure

If X is an attribute set, the closure X^+ is the set of all attributes B such that:

$$X \longrightarrow B$$

In other words, X^+ includes all attributes that are functionally determined from X

WHY IS CLOSURE NEEDED?

1. Does $X \longrightarrow Y$ hold?
   – we can check if Y $\subseteq X^+$
2. To compute the closure F^+ of FDs
   – for each subset of attributes X, compute X^+
   – for each subset of attributes Y $\subseteq X^+$, output the FD $X \longrightarrow Y$

EXAMPLE

Product(name, category, color, department, price)

- $name \longrightarrow color$
- $category \longrightarrow department$
- $color, category \longrightarrow price$

Attribute Closure:

- $\{name\}^+ = \{name, color\}$
- $\{name, category\}^+ =$
  $\{name, color, category, department, price\}$

THE CLOSURE ALGORITHM

- Let $X = \{A_1, A_2, \dots, A_n\}$
- **UNTIL** X doesn't change **REPEAT**:

  IF $B_1, B_2, \dots, B_m \longrightarrow C$ is an FD **AND**

  $B_1, B_2, \dots, B_m$ are all in X

  THEN add C to X
- Output X

EXAMPLE

R(A, B, C, D, E, F)

- $A, B \longrightarrow C$
- $A, D \longrightarrow E$
- $B \longrightarrow D$
- $A, F \longrightarrow B$

Compute the attribute closures:

- $\{A, B\}^+ = \{A, B, C, D, E\}$
- $\{A, F\}^+ = \{A, F, B, D, E, C\}$

KEYS & SUPERKEYS

superkey: a set of attributes $A_1, A_2, \ldots, A_n$ such that for any other attribute B in the relation:

$$A_1, A_2, \ldots, A_n \longrightarrow B$$

key (or candidate key): a minimal superkey

- none of its subsets functionally determines all attributes of the relation

If a relation has multiple candidate keys, we specify one to be the **primary key**

COMPUTING KEYS & SUPERKEYS

- Compute X^+ for all sets of attributes X
- If $X^+ = all\ attributes$, then X is a superkey
- If no subset of X is a superkey, then X is also a key

EXAMPLE

Product(name, category, price, color)

- $name \longrightarrow color$
- $color, category \longrightarrow price$

Superkeys:

- $\{name, category\}, \{name, category, price\}$
  $\{name, category, color\}, \{name, category, price, color\}$

Keys:

- $\{name, category\}$

HOW MANY KEYS?

Q: Is it possible to have many keys in a relation **R** ?

YES!! Take relation **R**(A, B, C) with FDs

- $A, B \longrightarrow C$
- $A, C \longrightarrow B$

RECAP

- FDs and (super)keys
- Reasoning with FDs:
  - given a set of FDs, infer all implied FDs
  - given a set of attributes X, infer all attributes that are functionally determined by X
- Next, we will look at how to use them to detect that a table is "bad"