

一、

(1)和(2)不能推断出(3)，因为有知者中也有爱虚荣的人，故教授中可能也有爱虚荣的人。

二、

3.

(1)

1. $\neg A \rightarrow (A \rightarrow B)$ PC中Th6
2. $(A \rightarrow B) \rightarrow \exists V(A \rightarrow B)$ FC中Th2
3. $\neg A \rightarrow \exists V(A \rightarrow B)$ 1,2 PC中Th8
4. $B \rightarrow (A \rightarrow B)$ PC中公理1
5. $\neg(A \rightarrow B) \rightarrow \neg B$ PC中定理13及Tmp
6. $\forall V(\neg(A \rightarrow B) \rightarrow \neg B)$ FC推广Th4
7. $\forall V \neg(A \rightarrow B) \rightarrow \forall V \neg B$ FC中公理5
8. $\neg \exists V B \rightarrow \exists V(A \rightarrow B)$ PC中定理13及Tmp
9. $(A \rightarrow \exists V B) \rightarrow \exists V(A \rightarrow B)$ 3,8 PC中Th18.

(2)

1. $A, \exists V(A \rightarrow B), A \rightarrow B \vdash A \rightarrow B$ (E)
2. $A, A \rightarrow B \vdash A$ (E)
3. $A, A \rightarrow B \vdash B$ 1,2 (\rightarrow -)
4. $A \vdash (A \rightarrow B) \rightarrow B$ 3 (\rightarrow +)
5. $A \vdash \neg B \rightarrow \neg(A \rightarrow B)$ PC中Th13及Tmp
6. $A \vdash \forall V(\neg B \rightarrow \neg(A \rightarrow B))$ FC中Th5
7. $A \vdash \forall V \neg B \rightarrow \forall V \neg(A \rightarrow B)$ FC中Th公理4及Tmp
8. $A \vdash \exists V(A \rightarrow B) \rightarrow \exists V B$ PC中Th13及Tmp
9. $\vdash A \rightarrow (\exists V(A \rightarrow B) \rightarrow \exists V B)$ 8 (\rightarrow +)
10. $\vdash \exists V(A \rightarrow B) \rightarrow (A \rightarrow \exists V B)$ 9 PC中Th2

- (3)
1. $\forall V \neg B \rightarrow A; \forall V \neg (B \rightarrow A) \vdash \forall V \neg (B \rightarrow A) \quad (E)$
 2. $\forall V B \rightarrow A; \forall V \neg (B \rightarrow A) \vdash \forall V \neg (B \rightarrow A) \rightarrow \neg(B \rightarrow A) \quad \text{FC中Th1及两次} \vdash$
 3. $\forall V B \rightarrow A; \forall V \neg (B \rightarrow A) \vdash \neg(B \rightarrow A) \quad 1, 2 \text{ Imp}$
 4. $\forall V B \rightarrow A; \forall V \neg (B \rightarrow A) \vdash B$
 5. $\forall V B \rightarrow A; \forall V \neg (B \rightarrow A) \vdash \neg A$
 6. $\forall V B \rightarrow A; \forall V \neg (B \rightarrow A) \vdash \forall V B \quad 4, \text{FC中Th5}$
 7. $\forall V B \rightarrow A; \forall V \neg (B \rightarrow A) \vdash \forall V \neg A \quad 5, \text{FC中Th5}$
 8. $\forall V B \rightarrow A; \forall V \neg (B \rightarrow A) \vdash \forall V B \rightarrow A \quad (E)$
 9. $\forall V B \rightarrow A; \forall V \neg (B \rightarrow A) \vdash A \quad 6, 8 \text{ Imp}$
 10. $\forall V B \rightarrow A \vdash \exists V (B \rightarrow A) \quad 5, 9 \text{ Th8}$
 11. $\vdash (\forall V B \rightarrow A) \rightarrow \exists V (B \rightarrow A) \quad \text{FC中Th6}$
- (4)
1. $\exists V (B \rightarrow A); \forall V B; B \rightarrow A \vdash \forall V B \quad (E)$
 2. $\exists V (B \rightarrow A); \forall V B; B \rightarrow A \vdash \forall V B \rightarrow B \quad \text{FC中Th1及三次} \vdash$
 3. $\exists V (B \rightarrow A); \forall V B; B \rightarrow A \vdash B \quad 1, 2 \text{ Imp}$
 4. $\exists V (B \rightarrow A); \forall V B; B \rightarrow A \vdash B \rightarrow A \quad (E)$
 5. $\exists V (B \rightarrow A); \forall V B; B \rightarrow A \vdash A \quad 3, 4 \text{ Imp}$
 6. $\exists V (B \rightarrow A); \forall V B; B \rightarrow A \vdash \exists V (B \rightarrow A) \quad (E)$
 7. $\forall V B; \exists V (B \rightarrow A) \vdash A \quad 5, 6 \text{ FC中Th10}$
 8. $\exists V (B \rightarrow A) \vdash \forall V B \rightarrow A \quad \text{演绎定理}$
 9. $\vdash \exists V (B \rightarrow A) \rightarrow (\forall V B \rightarrow A) \quad \text{演绎定理}$

三、

(1) 先证 $\forall x(A \rightarrow B) \vdash A \rightarrow \forall x B$

由演绎定理只需证 $\forall x(A \rightarrow B); A \vdash \forall x B$

1. $\forall x(A \rightarrow B) \rightarrow (A \rightarrow B)$ FC中Th1
2. $\forall x(A \rightarrow B)$ 假设
3. $A \rightarrow B$ 2.1 Imp
4. A 假设
5. B 4.3 Imp
6. $B \Rightarrow \forall x B$ 5用推广定理5
- ~~7. $\forall x B$ 5.6 Imp~~

再证 $A \rightarrow \forall x B \vdash \forall x(A \rightarrow B)$

1. $\forall x B \rightarrow B$ FC中Th1
2. $A \rightarrow \forall x B$ 假设
3. $A \rightarrow B$ 2.1 三段论
4. (~~$\forall x(A \rightarrow B)$~~) $\forall x(A \rightarrow B)$ 3用FC中Th5

综上 $\forall x(A \rightarrow B) \vdash A \rightarrow \forall x B$

(2) 先证 $\forall x(A \rightarrow B) \vdash \exists x A \rightarrow B$

1. $\forall x(A \rightarrow B); \neg B \vdash \forall x(A \rightarrow B)$ (E)
2. $\forall x(A \rightarrow B); \neg B \vdash \forall x(A \rightarrow B) \rightarrow (A \rightarrow B)$ FC中Th1及两次(+)
3. $\forall x(A \rightarrow B); \neg B \vdash A \rightarrow B$ 1,2 Tmp
4. $\forall x(A \rightarrow B); \neg B \vdash (A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$ PC中Th13及两次(+)
5. $\forall x(A \rightarrow B); \neg B \vdash \neg B \rightarrow \neg A$ 3,4 Tmp
6. $\forall x(A \rightarrow B); \neg B \vdash \neg B$ (E)
7. $\forall x(A \rightarrow B); \neg B \vdash \neg A$ 6,5 Tmp
8. $\forall x(A \rightarrow B); \neg B \vdash \forall x \neg A$ 推广定理5
9. $\forall x(A \rightarrow B) \vdash \neg B \rightarrow \forall x \neg A$ 演绎定理
10. $\forall x(A \rightarrow B) \vdash (\neg B \rightarrow \forall x \neg A) \rightarrow (\neg \forall x \neg A \rightarrow B)$ PC中Th14及一次(+)
11. $\forall x(A \rightarrow B) \vdash \neg \forall x \neg A \rightarrow B$ 即 $\forall x(A \rightarrow B) \vdash \exists x A \rightarrow B$ 9,10 Tmp

再证 $\exists x A \rightarrow B \vdash \forall x(A \rightarrow B)$

1. $\exists x A \rightarrow B; A \vdash A$ (E)
2. $\exists x A \rightarrow B; A \vdash A \rightarrow \exists x A$ FC中Th1及两次(+)
3. $\exists x A \rightarrow B; A \vdash \exists x A$ 1,2 Tmp
4. $\exists x A \rightarrow B; A \vdash \exists x A \rightarrow B$ (E)
5. $\exists x A \rightarrow B; A \vdash B$ 3,4 Tmp
6. $\exists x A \rightarrow B \vdash A \rightarrow B$ 演绎定理
7. $\exists x A \rightarrow B \vdash \forall x(A \rightarrow B)$ 推广定理5.

得证

(13) 先证 $\forall x(A \wedge B) \vdash \forall x A \wedge \forall x B$

1. $\forall x(A \wedge B) \vdash \forall x(A \wedge B)$ (E)
2. $\forall x(A \wedge B) \vdash \forall x(A \wedge B) \rightarrow (A \wedge B)$ FC中Th1及(+)
3. $\forall x(A \wedge B) \vdash A \wedge B$ 1,2 \wedge mp
4. $\forall x(A \wedge B) \vdash A$ 3(\wedge -)
5. $\forall x(A \wedge B) \vdash \forall x A$ 4推广定理5
6. $\forall x(A \wedge B) \vdash B$ 3(\wedge -)
7. $\forall x(A \wedge B) \vdash \forall x B$ 6用推广定理5
8. $\forall x(A \wedge B) \vdash \forall x A \wedge \forall x B$ 5,7 (\wedge +)

再证 $\forall x A \wedge \forall x B \vdash \forall x(A \wedge B)$

1. $\forall x A \wedge \forall x B \vdash \forall x A \wedge \forall x B$ (E)
2. $\forall x A \wedge \forall x B \vdash \forall x A$ 1(\wedge -)
3. $\forall x A \wedge \forall x B \vdash \forall x A \rightarrow A$ FC中Th1及(+)
4. $\forall x A \wedge \forall x B \vdash A$ 2,3 \wedge mp
5. $\forall x A \wedge \forall x B \vdash \forall x B$ 1(\wedge -)
6. $\forall x A \wedge \forall x B \vdash \forall x B \rightarrow B$ FC中Th1及(+)
7. $\forall x A \wedge \forall x B \vdash B$ 5,6 \wedge mp
8. $\forall x A \wedge \forall x B \vdash A \wedge B$ 4,7 (\wedge +)
9. $\forall x A \wedge \forall x B \vdash \forall x(A \wedge B)$ 8用推广定理5

得证

(4) 先证 $\exists x(A \vee B) \vdash \exists x A \vee \exists x B$

只需证 $\exists x(\neg A \rightarrow B) \vdash \forall x \neg A \rightarrow \exists x B$

1. $\exists x(\neg A \rightarrow B); \forall x \neg A; \neg A \rightarrow B \vdash \forall x \neg A$ (E)
 2. $\exists x(\neg A \rightarrow B); \forall x \neg A; \neg A \rightarrow B \vdash \forall x \neg A \rightarrow \neg A$ FC 中 Th1 及三次 (+)
 3. $\exists x(\neg A \rightarrow B); \forall x \neg A; \neg A \rightarrow B \vdash \neg A$ 1, 2 \rightarrow Imp
 4. $\exists x(\neg A \rightarrow B); \forall x \neg A; \neg A \rightarrow B \vdash \neg A \rightarrow B$ (E)
 5. $\exists x(\neg A \rightarrow B); \forall x \neg A; \neg A \rightarrow B \vdash B$ 3, 4 \rightarrow Imp
 6. $\exists x(\neg A \rightarrow B); \forall x \neg A; \neg A \rightarrow B \vdash B \rightarrow \exists x B$ FC 中 Th1 及三次 (+)
 7. $\exists x(\neg A \rightarrow B); \forall x \neg A; \neg A \rightarrow B \vdash \exists x B$ 5, 6 \rightarrow Imp
 8. $\exists x(\neg A \rightarrow B); \forall x \neg A \vdash \exists x(\neg A \rightarrow B)$ (E)
 9. $\exists x(\neg A \rightarrow B); \forall x \neg A \vdash \exists x B$ 7, 8 FC 中 Th10
 10. $\exists x(\neg A \rightarrow B) \vdash \forall x \neg A \rightarrow \exists x B$ \rightarrow 演绎定理
- 即 $\exists x(A \vee B) \vdash \exists x A \vee \exists x B$

再证 $\exists x A \vee \exists x B \vdash \exists x(A \vee B)$

1. $\exists x A \vee \exists x B \vdash \exists x A \vee \exists x B$ (E)
2. ~~$\exists x A \vee \exists x B \vdash \exists x A$~~ (E)
2. $\exists x A \vee \exists x B; \exists x A; A \vdash A$ (E)
3. $\exists x A \vee \exists x B; \exists x A; A \vdash A \vee B$ (V+)
4. $\exists x A \vee \exists x B; \exists x A; A \vdash \exists x(A \vee B)$ FC 中 Th2 及 \rightarrow Imp
5. $\exists x A \vee \exists x B; \exists x A \vdash \exists x A$ (E)
6. $\exists x A \vee \exists x B; \exists x A \vdash \exists x(A \vee B)$ 4, 5 用 FC 中 Th10
7. $\exists x A \vee \exists x B; \exists x B; B \vdash B$ (E)
8. $\exists x A \vee \exists x B; \exists x B; B \vdash A \vee B$ \vee (V+)
9. $\exists x A \vee \exists x B; \exists x B; B \vdash \exists x(A \vee B)$ FC 中 Th2 及 \rightarrow Imp
10. $\exists x A \vee \exists x B; \exists x B \vdash \exists x B$
11. $\exists x A \vee \exists x B; \exists x B \vdash \exists x(A \vee B)$ 9, 10 用 FC 中 Th10
12. $\exists x A \vee \exists x B \vdash \exists x(A \vee B)$ 1, 6, 11 用 (V+)

得证