

一、

(1)和(2)不能推断出(3)，因为有知者中也有爱虚荣的人，故教授中可能也有爱虚荣的人。

二、

3.

(1)

1.  $\neg A \rightarrow (A \rightarrow B)$  PC中Th6
2.  $(A \rightarrow B) \rightarrow \exists V(A \rightarrow B)$  FC中Th2
3.  $\neg A \rightarrow \exists V(A \rightarrow B)$  1,2 PC中Th8
4.  $B \rightarrow (A \rightarrow B)$  PC中公理1
5.  $\neg(A \rightarrow B) \rightarrow \neg B$  PC中定理13及Tmp
6.  $\forall V(\neg(A \rightarrow B) \rightarrow \neg B)$  FC推广Th4
7.  $\forall V \neg(A \rightarrow B) \rightarrow \forall V \neg B$  FC中公理5
8.  $\neg \exists V B \rightarrow \exists V(A \rightarrow B)$  PC中定理13及Tmp
9.  $(A \rightarrow \exists V B) \rightarrow \exists V(A \rightarrow B)$  3,8 PC中Th18.

(2)

1.  $A, \exists V(A \rightarrow B), A \rightarrow B \vdash A \rightarrow B$  (E)
2.  $A, A \rightarrow B \vdash A$  (E)
3.  $A, A \rightarrow B \vdash B$  1,2 ( $\rightarrow$ -)
4.  $A \vdash (A \rightarrow B) \rightarrow B$  3 ( $\rightarrow$ +) )
5.  $A \vdash \neg B \rightarrow \neg(A \rightarrow B)$  PC中Th13及Tmp
6.  $A \vdash \forall V(\neg B \rightarrow \neg(A \rightarrow B))$  FC中Th5
7.  $A \vdash \forall V \neg B \rightarrow \forall V \neg(A \rightarrow B)$  FC中Th4及Tmp
8.  $A \vdash \exists V(A \rightarrow B) \rightarrow \exists V B$  PC中Th13及Tmp
9.  $\vdash A \rightarrow (\exists V(A \rightarrow B) \rightarrow \exists V B)$  8 ( $\rightarrow$ +) )
10.  $\vdash \exists V(A \rightarrow B) \rightarrow (A \rightarrow \exists V B)$  9 PC中Th2

- (3)
1.  $\forall V \neg(B \rightarrow A); \forall V \neg(B \rightarrow A) \vdash \forall V \neg(B \rightarrow A) \quad (E)$
  2.  $\forall V B \rightarrow A; \forall V \neg(B \rightarrow A) \vdash \forall V \neg(B \rightarrow A) \rightarrow \neg(B \rightarrow A) \quad \frac{FC \text{中Th13}}{\text{两次} +}$
  3.  $\forall V B \rightarrow A; \forall V \neg(B \rightarrow A) \vdash \neg(B \rightarrow A) \quad 1, 2 \text{Imp}$
  4.  $\forall V B \rightarrow A; \forall V \neg(B \rightarrow A) \vdash B$
  5.  $\forall V B \rightarrow A; \forall V \neg(B \rightarrow A) \vdash \neg A$
  6.  $\forall V B \rightarrow A; \forall V \neg(B \rightarrow A) \vdash \forall V B \quad 4, FC \text{中Th5}$
  7.  $\forall V B \rightarrow A; \forall V \neg(B \rightarrow A) \vdash \forall V \neg A \quad 5, FC \text{中Th5}$
  8.  $\forall V B \rightarrow A; \forall V \neg(B \rightarrow A) \vdash \forall V B \rightarrow A \quad (E)$
  9.  $\forall V B \rightarrow A; \forall V \neg(B \rightarrow A) \vdash A \quad 6, 8 \text{Imp}$
  10.  $\forall V B \rightarrow A \vdash \exists V (B \rightarrow A) \quad 5, 9 \text{Th8}$
  11.  $\vdash (\forall V B \rightarrow A) \rightarrow \exists V (B \rightarrow A) \quad \frac{FC \text{中Th6}}{\text{两次} +}$
- (4)
1.  $\exists V (B \rightarrow A); \forall V B; B \rightarrow A \vdash \forall V B \quad (E)$
  2.  $\exists V (B \rightarrow A); \forall V B; B \rightarrow A \vdash \forall V B \rightarrow B \quad FC \text{中Th1及三次} +$
  3.  $\exists V (B \rightarrow A); \forall V B; B \rightarrow A \vdash B \quad 1, 2 \text{Imp}$
  4.  $\exists V (B \rightarrow A); \forall V B; B \rightarrow A \vdash B \rightarrow A \quad (E)$
  5.  $\exists V (B \rightarrow A); \forall V B; B \rightarrow A \vdash A \quad 3, 4 \text{Imp}$
  6.  $\exists V (B \rightarrow A); \forall V B; B \rightarrow A \vdash \exists V (B \rightarrow A) \quad (E)$
  7.  $\forall V B; \exists V (B \rightarrow A) \vdash A \quad 5, 6 \text{FC中Th10}$
  8.  $\exists V (B \rightarrow A) \vdash \forall V B \rightarrow A \quad 8 \text{演绎定理}$
  9.  $\vdash \exists V (B \rightarrow A) \rightarrow (\forall V B \rightarrow A) \quad 8 \text{演绎定理}$

三、

(1) 先证  $\forall x(A \rightarrow B) \vdash A \rightarrow \forall x B$

由演绎定理只需证  $\forall x(A \rightarrow B); A \vdash \forall x B$

1.  $\forall x(A \rightarrow B) \rightarrow (A \rightarrow B)$  FC中Th1
2.  $\forall x(A \rightarrow B)$  假设
3.  $A \rightarrow B$  2.1 Imp
4.  $A$  假设
5.  $B$  4.3 Imp
6.  $B \Rightarrow \forall x B$  5用推广定理5
- ~~7.  $\forall x B$  5.6 Imp~~

再证  $A \rightarrow \forall x B \vdash \forall x(A \rightarrow B)$

1.  $\forall x B \rightarrow B$  FC中Th1
2.  $A \rightarrow \forall x B$  假设
3.  $A \rightarrow B$  2.1 三段论
4. ( ~~$\forall x(A \rightarrow B)$~~ )  $\forall x(A \rightarrow B)$  3用FC中Th5

综上  $\forall x(A \rightarrow B) \vdash A \rightarrow \forall x B$

(2) 先证  $\forall x(A \rightarrow B) \vdash \exists x A \rightarrow B$

1.  $\forall x(A \rightarrow B); \neg B \vdash \forall x(A \rightarrow B)$  (E)
2.  $\forall x(A \rightarrow B); \neg B \vdash \forall x(A \rightarrow B) \rightarrow (A \rightarrow B)$  FC中Th1及两次(+)
3.  $\forall x(A \rightarrow B); \neg B \vdash A \rightarrow B$  1,2 Tmp
4.  $\forall x(A \rightarrow B); \neg B \vdash (A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$  PC中Th13及两次(+)
5.  $\forall x(A \rightarrow B); \neg B \vdash \neg B \rightarrow \neg A$  3,4 Tmp
6.  $\forall x(A \rightarrow B); \neg B \vdash \neg B$  (E)
7.  $\forall x(A \rightarrow B); \neg B \vdash \neg A$  6,5 Tmp
8.  $\forall x(A \rightarrow B); \neg B \vdash \forall x \neg A$  推广定理5
9.  $\forall x(A \rightarrow B) \vdash \neg B \rightarrow \forall x \neg A$  演绎定理
10.  $\forall x(A \rightarrow B) \vdash (\neg B \rightarrow \forall x \neg A) \rightarrow (\neg \forall x \neg A \rightarrow B)$  PC中Th14及一次(+)
11.  $\forall x(A \rightarrow B) \vdash \neg \forall x \neg A \rightarrow B$  即  $\forall x(A \rightarrow B) \vdash \exists x A \rightarrow B$  9,10 Tmp

再证  $\exists x A \rightarrow B \vdash \forall x(A \rightarrow B)$

1.  $\exists x A \rightarrow B; A \vdash A$  (E)
2.  $\exists x A \rightarrow B; A \vdash A \rightarrow \exists x A$  FC中Th1及两次(+)
3.  $\exists x A \rightarrow B; A \vdash \exists x A$  1,2 Tmp
4.  $\exists x A \rightarrow B; A \vdash \exists x A \rightarrow B$  (E)
5.  $\exists x A \rightarrow B; A \vdash B$  3,4 Tmp
6.  $\exists x A \rightarrow B \vdash A \rightarrow B$  演绎定理
7.  $\exists x A \rightarrow B \vdash \forall x(A \rightarrow B)$  推广定理5.

得证

(13) 先证  $\forall x(A \wedge B) \vdash \forall x A \wedge \forall x B$

1.  $\forall x(A \wedge B) \vdash \forall x(A \wedge B)$  (E)
2.  $\forall x(A \wedge B) \vdash \forall x(A \wedge B) \rightarrow (A \wedge B)$  FC中Th1及(+)
3.  $\forall x(A \wedge B) \vdash A \wedge B$  1,2  $\Gamma_{mp}$
4.  $\forall x(A \wedge B) \vdash A$  3( $\wedge$ -)
5.  $\forall x(A \wedge B) \vdash \forall x A$  4推广定理5
6.  $\forall x(A \wedge B) \vdash B$  3( $\wedge$ -)
7.  $\forall x(A \wedge B) \vdash \forall x B$  6用推广定理5
8.  $\forall x(A \wedge B) \vdash \forall x A \wedge \forall x B$  5,7 ( $\wedge$ +)

再证  $\forall x A \wedge \forall x B \vdash \forall x(A \wedge B)$

1.  $\forall x A \wedge \forall x B \vdash \forall x A \wedge \forall x B$  (E)
2.  $\forall x A \wedge \forall x B \vdash \forall x A$  1( $\wedge$ -)
3.  $\forall x A \wedge \forall x B \vdash \forall x A \rightarrow A$  FC中Th1及(+)
4.  $\forall x A \wedge \forall x B \vdash A$  2,3  $\Gamma_{mp}$
5.  $\forall x A \wedge \forall x B \vdash \forall x B$  1( $\wedge$ -)
6.  $\forall x A \wedge \forall x B \vdash \forall x B \rightarrow B$  FC中Th1及(+)
7.  $\forall x A \wedge \forall x B \vdash B$  5,6  $\Gamma_{mp}$
8.  $\forall x A \wedge \forall x B \vdash A \wedge B$  4,7 ( $\wedge$ +)
9.  $\forall x A \wedge \forall x B \vdash \forall x(A \wedge B)$  8用推广定理5

得证



(4) 先证  $\exists x(A \vee B) \vdash \exists x A \vee \exists x B$

1.  $\exists x(A \vee B); A \vee B; A \vdash A$  (e)
2.  $\exists x(A \vee B); A \vee B; A \vdash A \rightarrow \exists x A$  FC中Th2及三次(+)
3.  $\exists x(A \vee B); A \vee B; A \vdash \exists x A$  1,2 ( $\rightarrow$ -)
4.  $\exists x(A \vee B); A \vee B; A \vdash \exists x A \vee \exists x B$  3用(V+)
5.  $\exists x(A \vee B); A \vee B; B \vdash B$  (e)
6.  $\exists x(A \vee B); A \vee B; B \vdash B \rightarrow \exists x B$  FC中Th2及三次(+)
7.  $\exists x(A \vee B); A \vee B; B \vdash \exists x B$  5,6 ( $\rightarrow$ -)
8.  $\exists x(A \vee B); A \vee B; B \vdash \exists x A \vee \exists x B$  7用(V+)
9.  $\exists x(A \vee B); A \vee B \vdash A \vee B$  (e)
10.  $\exists x(A \vee B); A \vee B \vdash \exists x A \vee \exists x B$  4,8,9用(V-)
11.  $\exists x(A \vee B) \vdash \exists x(A \vee B)$  (e)
12.  $\exists x(A \vee B) \vdash \exists x A \vee \exists x B$  10,11用FC中Th10

再证  $\exists x A \vee \exists x B \vdash \exists x(A \vee B)$

1.  $\exists x A \vee \exists x B \vdash \exists x A \vee \exists x B$  (e)
- ~~2.  $\exists x A \vee \exists x B \vdash \exists x A$  (e)~~
2.  $\exists x A \vee \exists x B; \exists x A; A \vdash A$  (e)
3.  $\exists x A \vee \exists x B; \exists x A; A \vdash A \vee B$  (V+)
4.  $\exists x A \vee \exists x B; \exists x A; A \vdash \exists x(A \vee B)$  FC中Th2及T<sub>imp</sub>
5.  $\exists x A \vee \exists x B; \exists x A \vdash \exists x A$  (e)
6.  $\exists x A \vee \exists x B; \exists x A \vdash \exists x(A \vee B)$  4,5用FC中Th10
7.  $\exists x A \vee \exists x B; \exists x B; B \vdash B$  (e)
8.  $\exists x A \vee \exists x B; \exists x B; B \vdash A \vee B$  7(V+)
9.  $\exists x A \vee \exists x B; \exists x B; B \vdash \exists x(A \vee B)$  FC中Th2及T<sub>imp</sub>
10.  $\exists x A \vee \exists x B; \exists x B \vdash \exists x B$
11.  $\exists x A \vee \exists x B; \exists x B \vdash \exists x(A \vee B)$  9,10用FC中Th10
12.  $\exists x A \vee \exists x B \vdash \exists x(A \vee B)$  1,6,11用(V+)

得证