

1.(1)设 2 是素数为命题  $p$ , 2 是偶数为命题  $q$ , 则该语句可以写成  $p \wedge q$

(2)设一个整数是奇数为命题  $p$ , 一个整数不能被 2 整除为命题  $q$ , 则该语句可写成  $p \leftrightarrow q$

(3)设大学里的学生是本科生为  $p$ , 大学里的学生是研究生为  $q$ , 则该语句可写成  $\neg p \leftrightarrow q$

(4)设车速超过每小时 100 公里是命题  $p$ , 接到超速罚单为命题  $q$ , 则该语句可以写成  $q \rightarrow p$

2.(1)成立。设任意一种赋值  $v$  使得  $A^v = 1$ , 则  $(B \rightarrow A)^v = \neg B^v \vee A^v = 1$ , 故成立。

(3)成立。设任意一种赋值  $v$  使得  $\neg A^v \vee (B \rightarrow C)^v = \neg A^v \vee (\neg B^v \vee C^v) = 1$ , 则

$$\begin{aligned}(\neg(A \rightarrow B) \vee (A \rightarrow C))^v &= (A^v \wedge \neg B^v) \vee (\neg A^v \vee C^v) = (\neg A^v \vee A^v) \wedge (\neg A^v \wedge \neg B^v) \vee C^v \\&= \neg A^v \vee (\neg B^v \vee C^v) = 1\end{aligned}$$

故成立。

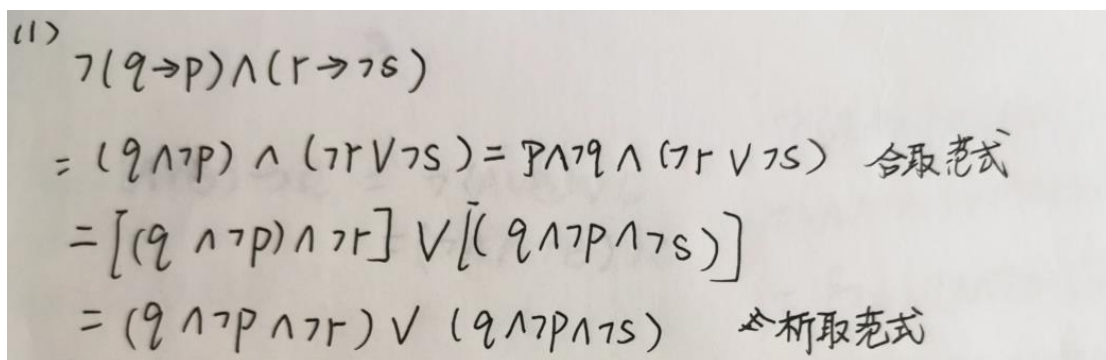
(5) 成立。设  $v$  为任意一种赋值则有:

$$((A \vee B) \rightarrow C)^v = (\neg A^v \wedge \neg B^v) \vee C^v$$

$$\begin{aligned}((A \rightarrow C) \wedge (B \rightarrow C))^v &= (\neg A^v \vee C^v) \wedge (\neg B^v \vee C^v) = (\neg A^v \wedge \neg B^v) \vee (\neg B^v \wedge C^v) \vee (C^v \wedge \neg A^v) \vee C^v \\&= (\neg A^v \wedge \neg B^v) \vee C^v \text{ (使用吸收率)}\end{aligned}$$

故成立。

3.


$$\begin{aligned}(1) \quad & \neg(q \rightarrow p) \wedge (r \rightarrow \neg s) \\&= (q \wedge \neg p) \wedge (\neg r \vee \neg s) = \neg p \wedge q \wedge (\neg r \vee \neg s) \quad \text{合取范式} \\&= [(q \wedge \neg p) \wedge \neg r] \vee [(q \wedge \neg p) \wedge \neg s] \\&= (q \wedge \neg p \wedge \neg r) \vee (q \wedge \neg p \wedge \neg s) \quad \text{析取范式}\end{aligned}$$

$$\begin{aligned}
 (2) \quad & \neg(P \vee Q) \leftrightarrow (P \wedge Q) \\
 & = [\neg(P \vee Q) \wedge (P \wedge Q)] \vee [(P \vee Q) \wedge \neg(P \wedge Q)] \\
 & = [\neg P \wedge \neg Q \wedge P \wedge Q] \vee [(P \vee Q) \wedge (\neg P \vee \neg Q)] \\
 & = (P \vee Q) \wedge (\neg P \vee \neg Q) \quad \text{合取范式} \\
 & = [\neg P \wedge (P \vee Q)] \vee [\neg Q \wedge (P \vee Q)] \\
 & = (\neg P \wedge Q) \vee (\neg Q \wedge P) \quad \text{析取范式}
 \end{aligned}$$

4.

(1) 由下列真值表:

P	Q	$P \wedge Q$	$P \rightarrow P \wedge Q$
0	0	0	1
0	0	0	1
0	1	0	1
1	0	0	0
1	1	1	1

可得主合取范式为  $(\neg P \vee Q) \wedge$

主析取范式为  $(\neg P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (P \wedge Q)$

(2) 由下列真值表:

P	Q	R	$P \wedge Q$	$P \rightarrow (P \wedge Q)$	$(P \rightarrow (P \wedge Q)) \vee R$
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	0	0	0
1	0	1	0	0	1
1	1	0	1	1	1
1	1	1	1	1	1

可得主合取范式为:  $\neg P \vee Q \vee \neg R$

主析取范式为:  $(\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R)$

$$5. (1) \quad P \vee (P \wedge Q) \leftrightarrow P$$

$$= ((P \vee (P \wedge Q)) \wedge P) \vee (\neg(P \vee (P \wedge Q)) \wedge \neg P)$$

$$= P \vee (P \wedge (P \wedge Q)) \vee (\neg(P) \wedge \neg P)$$

$$= P \vee \neg P$$

$$= P \rightarrow P$$

$$\begin{aligned}
 (2) \quad & (P \wedge Q) \wedge R \leftrightarrow (P \wedge (Q \wedge R)) \\
 & = ((P \wedge Q) \wedge R \wedge P \wedge (Q \wedge R)) \vee (\neg((P \wedge Q) \wedge R) \wedge \neg(P \wedge (Q \wedge R))) \\
 & = \cancel{P \wedge Q} (P \wedge Q \wedge R) \vee (\neg P \vee \neg Q \vee \neg R) \\
 & = \cancel{P \wedge Q} \neg(P \wedge Q \wedge R) \rightarrow (\neg P \vee \neg Q \vee \neg R) \\
 & = (\neg(P \rightarrow \neg Q) \rightarrow \neg R) \rightarrow (\neg(P \rightarrow \neg Q) \rightarrow \neg R)
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & (P \vee (Q \wedge R)) \leftrightarrow (P \vee Q) \wedge (P \vee R) \\
 & = [(P \vee (Q \wedge R)) \wedge (P \vee Q) \wedge (P \vee R)] \vee [\neg(P \vee (Q \wedge R)) \wedge \neg((P \vee Q) \wedge (P \vee R))] \\
 & = [P \vee (Q \wedge R) \wedge (P \vee R)] \vee \neg[P \vee (Q \wedge R) \wedge (P \vee R)] \\
 & = [P \vee (Q \wedge R)] \vee \neg[P \vee (Q \wedge R)] \\
 & = [P \vee (Q \wedge R)] \rightarrow [P \vee (Q \wedge R)] \\
 & = [\neg P \rightarrow \neg(Q \wedge R)] \rightarrow [\neg P \rightarrow \neg(Q \wedge R)]
 \end{aligned}$$

$$\begin{aligned}
 6. (1) \quad & \neg P \vee Q = \neg(P \wedge \neg Q) = P \uparrow (Q \uparrow Q) \\
 & = \neg \neg(\neg P \vee Q) = \neg(\neg P \downarrow Q) = ((P \downarrow P) \downarrow (P \downarrow P) \downarrow Q)
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & \neg P \vee \neg Q = \neg(P \wedge Q) = P \uparrow Q \\
 & = \neg \neg(\neg P \vee \neg Q) = \neg(\neg P \downarrow \neg Q) \\
 & = ((P \downarrow P) \downarrow (Q \downarrow Q)) \downarrow ((P \downarrow P) \downarrow (Q \downarrow Q))
 \end{aligned}$$