# Speech Processing and Understanding

CSC401 Assignment 3

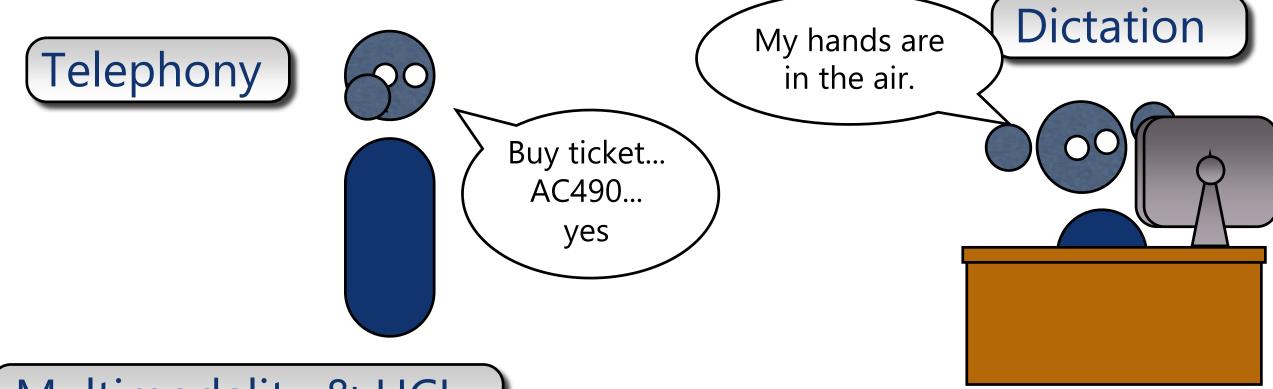


## Agenda

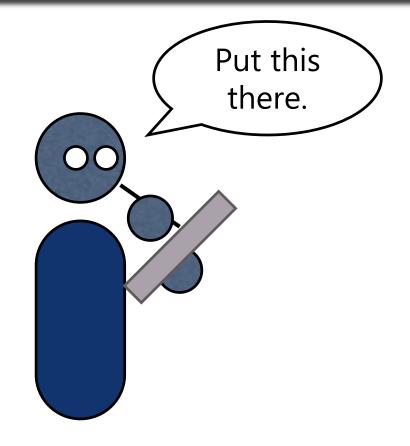
- Background
  - Speech technology, in general
  - Acoustic phonetics
- Assignment 3
  - Speaker Recognition: Gaussian mixture models
  - Speech Recognition: Word-error rates with Levenshtein distance.



# Applications of Speech Technology



#### Multimodality & HCI

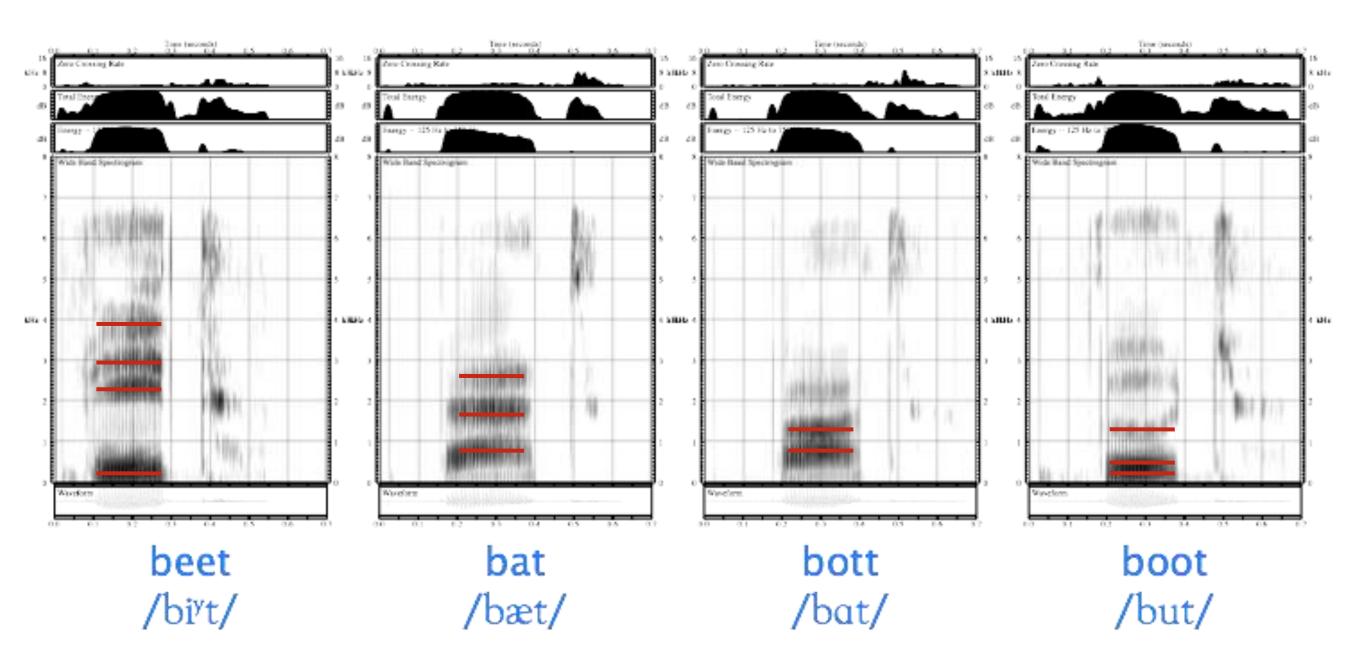


#### Emerging...

- Data mining/indexing.
- Assistive technology.
- Conversation.



## Formants in sonorants

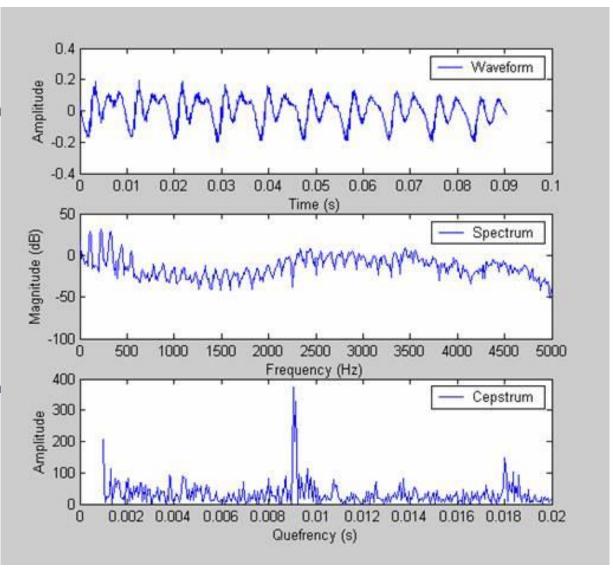


However, formants are insufficient features for use in speech recognition generally...



## Mel-frequency cepstral coefficients

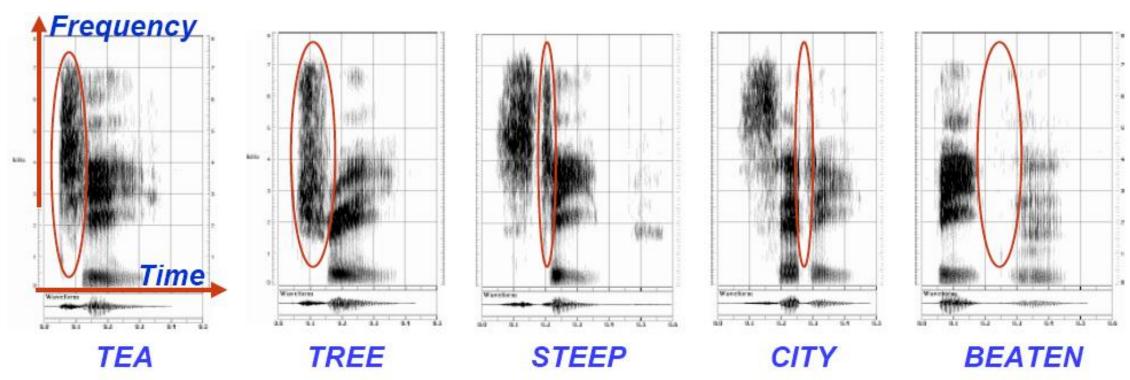
- In real speech data, the spectrogram is often transformed to a representation that more closely represents human auditory response and is more amenable to accurate classification.
- MFCCs are 'spectra of spectra'.
   They are the discrete cosine transform of the logarithms of the nonlinearly Mel-scaled powers of the Fourier transform of windows of the original waveform.





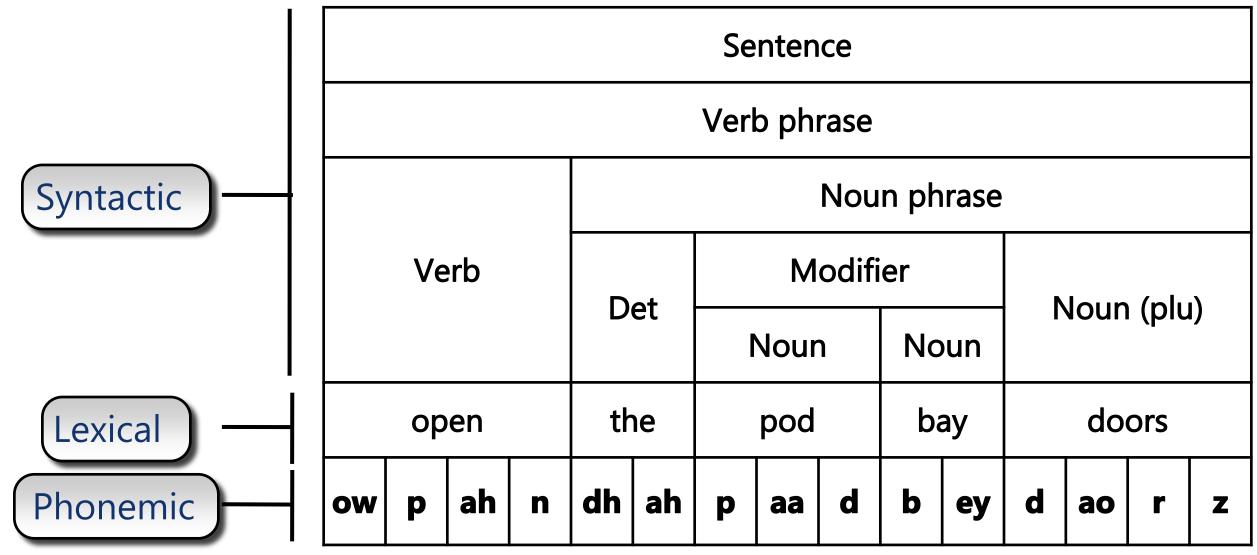
## Challenges in speech data

- Co-articulation and dropped phonemes.
- (Intra-and-Inter-) Speaker variability.
- No word boundaries.
- Slurring, disfluency (e.g., 'um').
- Signal Noise.
- Highly dimensional.



### Phonemes

- Words are formed by **phonemes** (aka 'phones'), e.g., 'pod' = /p aa d/
- Words have different pronunciations. and in practice we can never be certain of which phones were uttered, nor their start/stop points.



## Phonetic alphabets

- International Phonetic Association (IPA)
  - Can represent sounds in all languages
  - Contains non-ASCII characters
- ARPAbet
  - One of the earliest attempts at encoding English for early speech recognition.
- TIMIT/CMU
  - Very popular among modern databases for speech recognition.



## Example phonetic alphabets

IPA	CMU	TIMIT	Example	IPA symbol name
[a]	AA	aa	father, hot	script a
[æ]	AE	ae	h <u>a</u> d	digraph
[e]	AH0	ax	sof <u>a</u>	schwa (common in unstressed syllables)
[ \( \) ]	AH1	ah	b <u>u</u> t	turned v
[3:]	AO	ao	c <u>aug</u> ht	open o – Note, many speakers of Am. Eng. do not distinguish between [o:] and [α]. If your "caught" and "cot" sound the same, you do not.
[8]	EH	eh	h <u>ea</u> d	epsilon
[I]	IH	ih	h <u>i</u> d	small capital I
[i:]	IY	iy	h <u>ee</u> d	lowercase i
[ប]	UH	uh	h <u>oo</u> d, b <u>oo</u> k	upsilon
[u:]	UW	uw	b <u>oo</u> t	lowercase u
[aɪ]	AY	ay	h <u>i</u> de	
[aʊ]	AW	aw	h <u>ow</u>	
[eɪ]	EY	ey	tod <u>a</u> y	
[ 00 ]	OW	ow	h <u>oe</u> d	
[pi]	OY	oy	joy, ahoy	
[&]	ER0	axr	h <u>er</u> self	schwar (schwa changed by following r)
[3,]	ER1	er	b <u>ir</u> d	reverse epsilon right hook

IPA	CMU	TIMIT	Example	IPA symbol name
[ŋ]	NG	ng	si <u>ng</u> so <u>ng</u>	eng or angma
[ [ ] ]	SH	<u>sh</u>	sheet, wish	esh or long s
[t[]	CH	<u>ch</u>	<u>ch</u> eese	
[j]	Y	У	<u>y</u> ellow	lowercase j
[3]	ZJ	zh	vi <u>s</u> ion	long z or yogh
[dʒ]	JH	jh	ju <u>dg</u> e	
[ð]	DH	dh	thee, this	eth

- The other consonants are transcribed as you would expect
  - i.e., p, b, m, t, d, n, k, g, s,z, f, v, w, h



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## Assignment 3

- Two parts:
  - Speaker identification: Determine which of 30 speakers an unknown test sample of speech comes from, given Gaussian mixture models you will train for each speaker.
  - Speech recognition: Compute word-error rates for speech recognition systems using Levenshtein distance.

## Speaker Data

- 32 speakers (e.g., S-3C, S-5A).
- Each speaker has up to 12 training utterances.
  - e.g., /u/csc401/A3/data/S-3C/0.wav
- Each utterance has 3 files:
  - \*.wav : The original wave file.
  - \*.mfcc.npy: The MFCC features in NumPy format
  - \* .txt: Sentence-level transcription.



## Speaker Data (cont.)

- All you need to know: A speech utterance is an Nxd matrix
  - Each row represents the features of a d-dimensional point in time.
  - There are N rows in a sequence of N frames.
  - The data is in numpy arrays \* .mfcc.npy
  - To read the files: np.load('1.mfcc.npy')

		1	d		
	1	X <sub>1</sub> [1]	X <sub>1</sub> [2]	•••	X <sub>1</sub> [d]
time	frames	X <sub>2</sub> [1]	X <sub>2</sub> [2]	•••	X <sub>2</sub> [d]
tir	fra	•••	•••	•••	•••
•	N	X <sub>N</sub> [1]	X <sub>N</sub> [2]	•••	X <sub>N</sub> [d]

## Speaker Data (cont.)

- You are given human transcriptions in transcripts.txt
- You are also given Kaldi and Google transcriptions in transcripts.\*.txt.
- Ignore any symbols that are not words.

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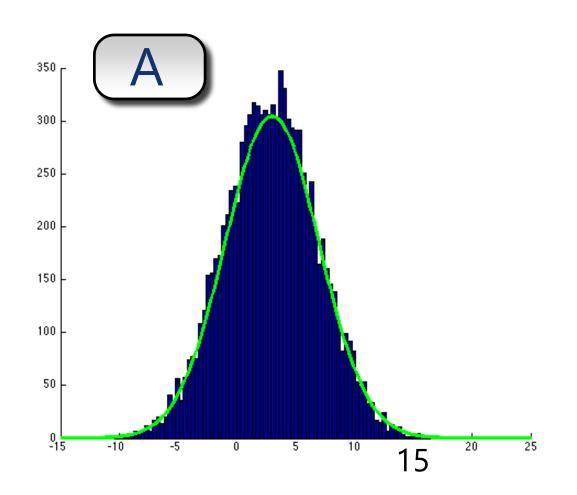
## Speaker Recognition

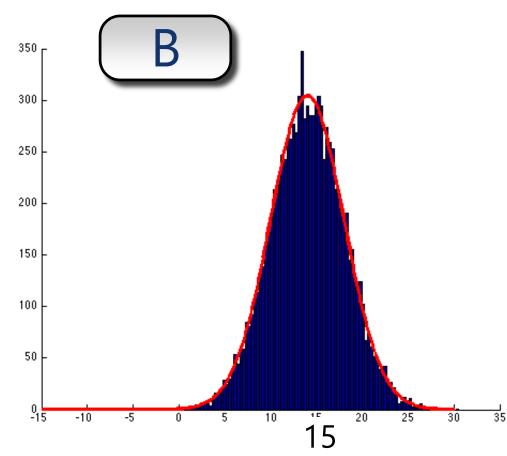
- The data is randomly split into training and testing utterances. We don't know which speaker produced which test utterance.
- Every speaker occupies a characteristic part of the acoustic space.
- We want to learn a probability distribution for each speaker that describes their acoustic behaviour.
  - Use those distributions to identify the speaker-dependent features of some unknown sample of speech data.



## Some background: fitting to data

- Given a set of observations X of some random variable, we wish to know how X was generated.
- Here, we assume that the data was sampled from a Gaussian Distribution (validated by data).
- Given a new data point (x=15), It is more likely that x was generated by B.



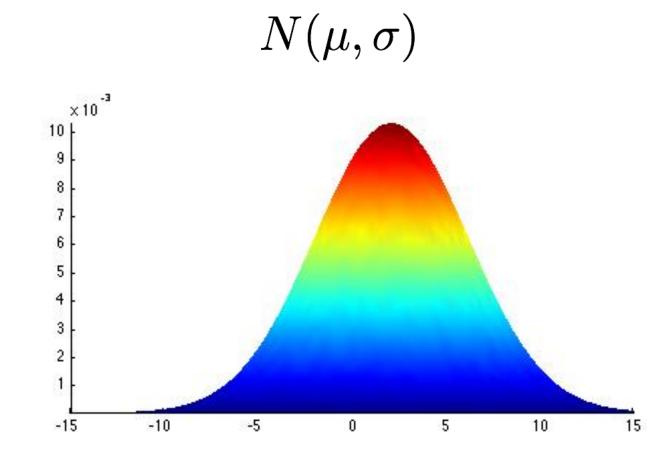


## Finding parameters: 1D Gaussians

#### Often called Normal distributions

$$p(x) = \frac{\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}{\sqrt{2\pi}\sigma}$$

$$\mu = E(x) = \int xp(x)dx$$



$$\sigma^{2} = E((x - \mu)^{2}) = \int (x - \mu)^{2} p(x) dx$$

The parameters we can adjust to fit the data are  $\mu$  and  $\sigma^2$  :  $\theta = \langle \mu, \sigma \rangle$ 



#### Maximum likelihood estimation

- Given data:  $X = \{x_1, x_2, \dots, x_n\}$
- and Parameter set:  $\theta$
- Maximum likelihood attempts to find the parameter set that maximizes the likelihood of the data.

$$L(X, \theta) = p(X \mid \theta) = p(x_1, x_2, \dots, x_n \mid \theta) = \prod_{i=1}^{n} p(x_i \mid \theta)$$

• The likelihood function  $L(X,\theta)$  provides a surface over all possible parameterizations. In order to find the Maximum Likelihood, we set the derivative to zero:

$$\frac{\partial}{\partial \theta} L(X, \theta) = 0$$



#### MLE - 1D Gaussian

Estimate  $\hat{\mu}$ 

imate 
$$\hat{\mu}$$

$$L(X,\mu) = p(X \mid \mu) = \prod_{i=1}^{n} p(x_i \mid \mu) = \prod_{i=1}^{n} \frac{\exp\left(-\frac{(x_i - \mu)^2}{2\sigma^2}\right)}{\sqrt{2\pi}\sigma}$$

$$\log L(X,\mu) = -\frac{\sum_{i} (x_i - \mu)^2}{2\sigma^2} - n \log \sqrt{2\pi}\sigma$$

$$\frac{\partial}{\partial \mu} \log L(X, \mu) = \frac{\sum_{i} (x_i - \mu)}{\sigma^2} = 0$$

$$\hat{\mu} = \frac{\sum_{i} x_i}{n}$$

A similar approach gives the MLE estimate of  $\hat{\sigma}^2$ :

$$\hat{\sigma}^2 = \frac{\sum_i (x_i - \hat{\mu})^2}{n}$$



#### Multidimensional Gaussians

 When your data is d-dimensional, the input variable is

$$\vec{x} = \langle x[1], x[2], \dots, x[d] \rangle$$

the mean vector is

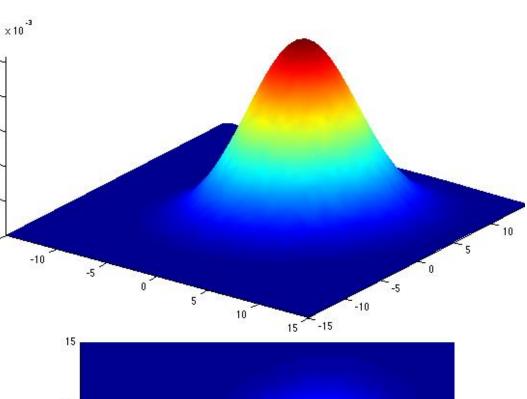
$$\vec{\mu} = E(\vec{x}) = \langle \mu[1], \mu[2], \dots, \mu[d] \rangle$$

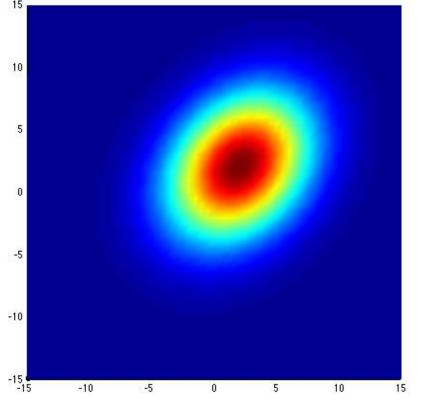
the covariance matrix is

$$\Sigma = E((\vec{x}-\vec{\mu})(\vec{x}-\vec{\mu})^T)$$
 with 
$$\Sigma[i,j] = E(x[i]x[j]) - \mu[i]\mu[j]$$

and

$$p(\vec{x}) = \frac{\exp\left(-\frac{(\vec{x} - \vec{\mu})^T \Sigma^{-1} (\vec{x} - \vec{\mu})}{2}\right)}{(2\pi)^{d/2} |\Sigma|^{1/2}}$$

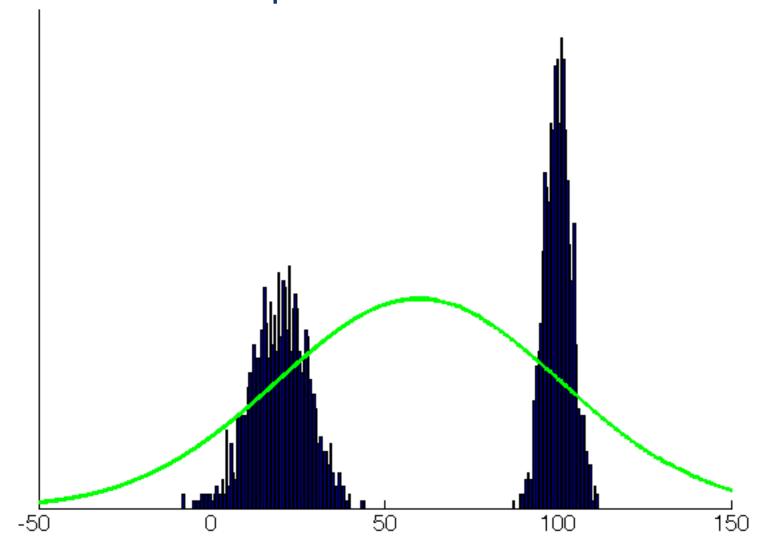






#### Non-Gaussian data

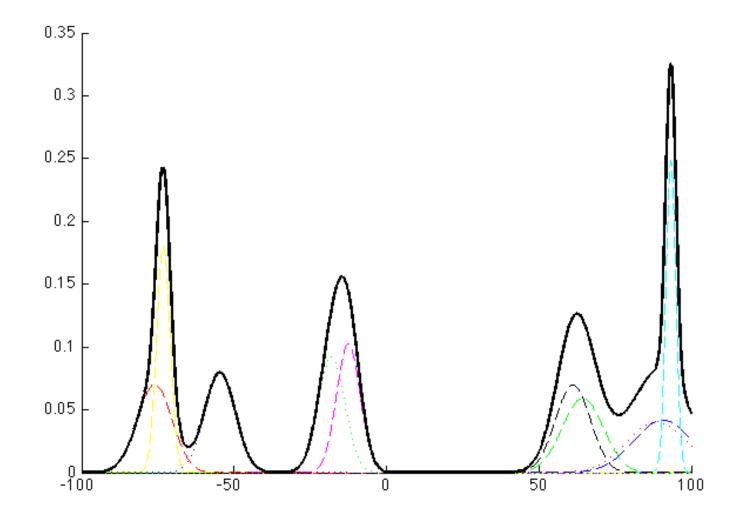
- Our speaker data does not behave unimodally.
  - i.e., we can't use just 1 Gaussian per speaker.
- E.g., observations below occur mostly bimodally, so fitting 1 Gaussian would not be representative.



## Gaussian mixtures

 Gaussian mixtures are a weighted linear combination of M component gaussians.

$$\langle \Gamma_1, \dots, \Gamma_M \rangle$$
 
$$p(\vec{x}) = \sum_{j=1}^{M} p(\Gamma_j) p(\vec{x} \mid \Gamma_j)$$





## MLE for Gaussian mixtures

• For notational convenience,  $\omega_m = p(\Gamma_m), \; b_m(\vec{x_t}) = p(\vec{x_t} \mid \Gamma_m)$ 

• So 
$$p_{\Theta}(\vec{x_t}) = \sum_{m=1}^{M} \omega_m b_m(\vec{x_t}), \; \Theta = \langle \omega_m, \vec{\mu_m}, \Sigma_m \rangle, \; m = 1, \dots, M$$

$$b_m(\vec{x_t}) = \frac{\exp\left(-\frac{1}{2} \sum_{i=1}^{d} \frac{(x_t[i] - \mu_m[i])^2}{\sigma_m^2[i]}\right)}{(2\pi)^{d/2} \left(\prod_{i=1}^{d} \sigma_m^2[i]\right)^{1/2}}$$

• To find  $\hat{\Theta}$ , we solve  $abla_{\Theta} \log L(X,\Theta) = 0$  where

$$\log L(X, \Theta) = \sum_{t=1}^{N} \log p_{\Theta}(\vec{x_t}) = \sum_{t=1}^{N} \log \left( \sum_{m=1}^{M} \omega_m b_m(\vec{x_t}) \right)$$

...see Appendix for more



## MLE for Gaussian mixtures (pt. 2)

- Given  $\frac{\partial \log L(X,\Theta)}{\partial \mu_m[n]} = \sum_{t=1}^N \frac{1}{p_{\Theta}(\vec{x_t})} \left[ \frac{\partial}{\partial \mu_m[n]} \omega_m b_m(\vec{x_t}) \right]$
- Since  $\frac{\partial}{\partial \mu_m[n]} b_m(\vec{x_t}) = b_m(\vec{x_t}) \frac{x_t[n] \mu_m[n]}{\sigma_m^2[n]}$
- We obtain  $\hat{\mu_m}[n]$  by solving for  $\mu_m[n]$  in :

$$\frac{\partial \log L(X,\Theta)}{\partial \mu_m[n]} = \sum_{t=1}^N \frac{\omega_m}{p_{\Theta}(\vec{x_t})} b_m(\vec{x_t}) \frac{x_t[n] - \mu_m[n]}{\sigma_m^2[n]} = 0$$

$$p(\Gamma_m \mid \vec{x_t}, \Theta) = p(\vec{x_t} \mid \Gamma_m)$$
 and: 
$$p(\Gamma_m \mid \vec{x_t}, \Theta) = \frac{\omega_m}{p_{\Theta}(\vec{x_t})} b_m(\vec{x_t})$$
 and: 
$$\frac{\hat{\mu_m}[n]}{\sum_t p(\Gamma_m \mid \vec{x_t}, \Theta)} = \frac{\sum_t p(\Gamma_m \mid \vec{x_t}, \Theta) x_t[n]}{\sum_t p(\Gamma_m \mid \vec{x_t}, \Theta)}$$

$$\hat{\mu_m}[n] = \frac{\sum_t p(\Gamma_m \mid \vec{x_t}, \Theta) x_t[n]}{\sum_t p(\Gamma_m \mid \vec{x_t}, \Theta)}$$



## Recipe for GMM ML estimation

Do the following for each speaker individually. Use all the frames available in their respective **Training** directories

- <u>Initialize</u>: Guess  $\Theta = \langle \omega_m, \vec{\mu_m}, \Sigma_m \rangle, \ m = 1, \dots, M$  with M random vectors in the data, or by performing M-means clustering.
- Compute likelihood: Compute  $b_m(\vec{x_t})_{and} P(\Gamma_m \mid \vec{x_t}, \Theta)$ Update parameters:  $\hat{\omega_m} = \frac{1}{T} \sum_{i=1}^T p(\Gamma_m \mid \vec{x_t}, \Theta)$

$$\left| \hat{\omega_m} = \frac{1}{T} \sum_{t=1}^T p(\Gamma_m \mid \vec{x_t}, \Theta) \right|$$

$$\begin{vmatrix} \hat{\vec{\sigma}}_m^2 = \frac{\sum_t p(\Gamma_m \mid \vec{x_t}, \Theta) \vec{x_t}^2}{\sum_t p(\Gamma_m \mid \vec{x_t}, \Theta)} - \hat{\vec{\mu}}_m^2 \end{vmatrix} \hat{\vec{\mu}}_m = \frac{\sum_t p(\Gamma_m \mid \vec{x_t}, \Theta) \vec{x_t}}{\sum_t p(\Gamma_m \mid \vec{x_t}, \Theta)}$$

$$\log p(X \mid \hat{\Theta}_{i+1}) - \log p(X \mid \hat{\Theta}_i) < \epsilon$$

Repeat 2&3 until converges



#### Cheat sheet

$$b_m(\vec{x_t}) = p(\vec{x_t} \mid \Gamma_m)$$

$$b_m(\vec{x_t}) = \frac{\exp\left(-\frac{1}{2}\sum_{i=1}^d \frac{(x_t[i] - \mu_m[i])^2}{\sigma_m^2[i]}\right)}{(2\pi)^{d/2} \left(\prod_{i=1}^d \sigma_m^2[i]\right)^{1/2}} \text{ Probability of observing } \mathbf{x_t in the m^{th} Gaussian}$$

$$\omega_m = p(\Gamma_m)$$

Prior probability of the m<sup>th</sup> Gaussian

$$p(\Gamma_m \mid \vec{x_t}, \Theta) = \frac{\omega_m}{p_{\Theta}(\vec{x_t})} b_m(\vec{x_t})$$
 Probability of the m<sup>th</sup> Gaussian, given x<sub>t</sub>

$$p_{\Theta}(ec{x_t}) = \sum_{m=1}^{M} \omega_m b_m(ec{x_t})$$
 Probability of  $x_t$  in the GMM



## Initializing theta

$$\Theta = \langle \omega_1, \mu_1, \Sigma_1, \omega_2, \mu_2, \Sigma_2, \dots, \omega_M, \mu_M, \Sigma_M \rangle$$

- Initialize each mu<sub>m</sub> to a random vector from the data.
- Initialize Sigma<sub>m</sub> to a `random' diagonal matrix (or identity matrix).
- Initialize omega<sub>m</sub> randomly, with these constraints:

$$0 \le \omega_m \le 1$$

$$\sum_{m} \omega_{m} = 1$$

A good choice would be to set omega<sub>m</sub> to 1/M



#### Over-fitting in Gaussian Mixture Models

 Singularities in likelihood function when a component 'collapses' onto a data point:

$$\mathcal{N}(\mathbf{x}_n|\mathbf{x}_n,\sigma_j^2\mathbf{I})=rac{1}{(2\pi)^{1/2}}rac{1}{\sigma_j}$$
 then consider  $\sigma_j o 0$ 

- Likelihood function gets larger as we add more components (and hence parameters) to the model
  - not clear how to choose the number K of components

#### Solutions:

- Ensure that the variances don't get too small.
- Bayesian GMMs



### Your Task

- For each speaker, train a GMM, using the EM algorithm, assuming diagonal covariance.
- Identify the speaker of each test utterance.
- Experiment with the number of mixture elements in the models, the improvement threshold, number of possible speakers, etc.
- Comment on the results



## Practical tips for MLE of GMMs

- We assume diagonal covariance matrices. This reduces the number of parameters and can be sufficient in practice given enough components.
- Numerical Stability: Compute likelihoods in the <u>log domain</u> (especially when calculating the likelihood of a sequence of frames).

$$\log b_m(\vec{x_t}) = -\sum_{n=1}^d \frac{(\vec{x_t}[n] - \vec{\mu_m}[n])^2}{2\vec{\sigma_m}^2[n]} - \frac{d}{2}\log 2\pi - \frac{1}{2}\log \prod_{n=1}^d \vec{\sigma_m}^2[n]$$

• Here,  $\vec{x_t}$ ,  $\vec{\mu_m}$  and  $\vec{\sigma_m}^2$  are d-dimensional vectors.



## Practical tips (pt. 2)

• Efficiency: Pre-compute terms not dependent on  $ec{x_t}$ 

$$\log b_m(\vec{x_t}) = -\sum_{n=1}^d \left( \frac{1}{2} \vec{x_t} [n]^2 \vec{\sigma_m}^{-2} [n] - \vec{\mu_m} [n] \vec{x_t} [n] \vec{\sigma_m}^{-2} [n] \right)$$

$$- \left( \sum_{n=1}^d \frac{\vec{\mu_m} [n]^2}{2 \vec{\sigma_m}^2 [n]} + \frac{d}{2} \log 2\pi + \frac{1}{2} \log \prod_{n=1}^d \vec{\sigma_m}^2 [n] \right)$$



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#### Word-error rates

- If somebody said REF: how to recognize speech but an ASR system heard HYP: how to wreck a nice beach how do we measure the error that occurred?
- One measure is #CorrectWords/#HypothesisWords e.g., 2/6 above
- Another measure is (S+I+D)/#ReferenceWords
  - S: # Substitution errors (one word for another)
  - I: # Insertion errors (extra words)
  - D: # Deletion errors (words that are missing).



## Computing Levenshtein Distance

In the example

REF: how to recognize speech.

HYP: how to wreck a nice beach

How do we count each of S, I, and D?

• If "wreck" is a substitution error, what about "a" and "nice"?

## Computing Levenshtein Distance

In the example

REF: how to recognize speech.

HYP: how to wreck a nice beach

How do we count each of S, I, and D?

If "wreck" is a substitution error, what about "a" and "nice"?

#### Levenshtein distance:

```
Initialize R[0,0] = 0, and R[i,j] = \infty for all i=0 or j=0 for i=1...n (#ReferenceWords)
    for j=1...m (#Hypothesis words)
    R[i,j] = \min(R[i-1,j] + 1 \text{ (deletion)}
    R[i-1,j-1] (only if words match)
    R[i-1,j-1]+1 (only if words differ)
    R[i,j-1]+1 ) (insertion)

Return 100*R(n,m)/n
```

		how	to	wreck	а	nice	beach
	0	∞	∞	∞	∞	∞	∞
how	∞	• 0 –	<b>→ 1</b> −	→ 2 <b>—</b>	→ 3 <b>—</b>	+ 4 -	<b>→</b> 5
to	∞						
recognize	∞						
speech	∞						

		how	to	wreck	а	nice	beach
	0	∞	∞	∞	∞	∞	∞
how	∞	• 0 -	<b>→ 1</b> −	→ 2 <b>—</b>	→ 3 <b>—</b>	+ 4 -	<b>→</b> 5
to	∞	1	` 0 –	<b>→ 1</b> —	→ 2 <b>—</b>	→ 3 <b>–</b>	<b>→</b> 4
recognize	∞						
speech	∞						

		how	to	wreck	а	nice	beach
	0	∞	<b>∞</b>	∞	∞	<b>∞</b>	∞
how	∞	• 0 -	<b>→ 1</b> –	→ 2 <b>—</b>	→ 3 <b>—</b>	<b>→ 4</b> —	<b>→</b> 5
to	∞	1	0 -	→ 1 <del>_</del>	→ 2 <del>_</del>	→ 3 <del>-</del>	<b>→</b> 4
recognize	∞	2	1	1 —	→ 2 <b>—</b>	<b>→</b> 3 –	<del>\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ </del>
speech	∞						

		how	to	wreck	а	nice	beach
	0	∞	∞	<b>∞</b>	∞	8	8
how	∞	· 0 -	<b>→ 1</b> −	<b>→</b> 2 −	<b>→</b> 3 <i>—</i>	<b>→ 4</b> —	<b>→</b> 5
to	∞	1	0 -	→ 1 <del>-</del>	→ 2 <del>_</del>	→ 3 <del>-</del>	<b>→</b> 4
recognize	∞	2	1	1 -	<del>2</del> 2 <del>-</del>	× 3 <del>-</del>	<b>→</b> 4
speech	∞	3	2	2	2 —	   3   × ↑	<b>→</b> 4

Word-error rate is 4/4 = 100%

2 substitutions, 2 insertions



# Appendices



## Multidimensional Gaussians, pt. 2

If the i<sup>th</sup> and j<sup>th</sup> dimensions are statistically independent,

$$E(x[i]x[j]) = E(x[i])E(x[j])$$

and

$$\Sigma[i,j] = 0$$

• If all dimensions are statistically independent,  $\Sigma[i,j] = 0, \ \forall i \neq j$  and the covariance matrix becomes diagonal, which means

$$p(\vec{x}) = \prod_{i=1}^{d} p(x[i])$$

where

$$p(x[i]) \sim N(\mu[i], \Sigma[i, i])$$
  
$$\Sigma[i, i] = \sigma^{2}[i]$$



## MLE example - dD Gaussians

• The MLE estimates for parameters  $\Theta=\langle \theta_1,\theta_2,\dots,\theta_d\rangle$  given i.i.d. training data  $X=\langle \vec{x_1},\dots,\vec{x_n}\rangle$  are obtained by maximizing the joint likelihood

$$L(X,\Theta) = p(X \mid \Theta) = p(\vec{x_1}, \dots, \vec{x_n} \mid \Theta) = \prod_{i=1}^{n} p(\vec{x_i} \mid \Theta)$$

• To do so, we solve  $\nabla_{\Theta}L(X,\Theta)=0$  where

$$\nabla_{\Theta} = \left\langle \frac{\partial}{\partial \theta_1}, \dots, \frac{\partial}{\partial \theta_d} \right\rangle$$

Giving

$$\hat{\vec{\mu}} = \frac{\sum_{t=1}^{n} \vec{x_t}}{n} \qquad \hat{\Sigma} = \frac{\sum_{t=1}^{n} \left( \vec{x_t} - \hat{\vec{\mu}} \right) \left( \vec{x_t} - \hat{\vec{\mu}} \right)^T}{n}$$



## MLE for Gaussian mixtures (pt1.5)

- Given  $\log L(X,\Theta) = \sum_{t=0}^{\infty} \log p_{\Theta}(\vec{x_t})$  and  $p_{\Theta}(\vec{x_t}) = \sum_{t=0}^{\infty} \omega_m b_m(\vec{x_t})$ 
  - lacktriangle Obtain an ML estimate,  $\mu_m$ , of the mean vector by maximizing  $\log L(X, \vec{\mu_m})$  w.r.t.  $\mu_m[n]$

$$\frac{\partial \log L(X,\Theta)}{\partial \mu_m[n]} = \sum_{t=1}^N \frac{\partial}{\partial \mu_m[n]} \log p_{\Theta}(\vec{x_t}) = \sum_{t=1}^N \frac{1}{p_{\Theta}(\vec{x_t})} \left[ \frac{\partial}{\partial \mu_m[n]} \omega_m b_m(\vec{x_t}) \right]$$

• Why? d of sum = sum of d d rule for loge

d wrt  $\mu_{m}$ is 0 for all other mixtures in the sum in  $p_{\Theta}(\vec{x_{t}})$ 

