

# Optimal Distributed Power Generation for Thermal and Electrical Scheduling in a Microgrid

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**Abstract**—This paper illustrates stochastic distributed algorithms that optimally share the electrical and thermal power generation task among the several Distributed Energy Resources (DERs) within a microgrid. We borrow certain concepts from communication network theory, namely Additive-Increase-Multiplicative-Decrease (AIMD) algorithms, which are known to be convenient in terms of communication requirements and network efficiency. We adapt them to minimise a cost utility function of interest in the framework of smart grids. We then implement the AIMD utility optimisation strategies in a realistic power network simulation in Matlab-OpenDSS environment, and we show the performance of the proposed algorithms in achieving thermal and electrical power balancing<sup>1</sup>.

## I. INTRODUCTION

Many researchers in the energy field are currently revisiting the electricity smart grid paradigm into that of a *smart energy system* [1]-[2]. According to this new perspective, electricity smart grids should not be treated as an isolated system, but should be regarded only as a part of an overall smart energy system. In particular, electricity smart grids should be coordinated with other form of carriers than simply electricity, including heat, gas and bio-fuels. Also, the synergy between the electrical and the gas grid should be fully exploited to improve efficiency of both electrical and thermal power generation [3]. A Combined Heat & Power plant currently represents the most common evidence of the synergy between the electrical and gas grid, as it is a gas-fuelled plant that both provides thermal and electrical power.

The single most important aspect of an energy system architecture remains the requirement for continuously perfect balance, i.e., balancing the (electrical and thermal) energy demand and the energy supply [4]. This task is further complicated in the case that a considerable fraction of power is generated from renewable sources (e.g., solar and wind plants). The continuous balancing task requires the Energy Management System (EMS) to continuously exchange information in quasi real-time with the Distributed Energy Resources (DERs) to monitor the quantity of power that

can be delivered by each single DER, as due to the weather conditions this quantity can not be perfectly predicted in advance. Typically, the communication overhead is avoided by scheduling all power flows one day ahead [6]. In fact, the afternoon prior to the physical energy delivery, the hourly prices for the following day are already known, and at the same time, quite accurate weather and load forecasts are also available. Therefore, it is possible to schedule thermal and electrical power flows in advance, and use storage systems to accommodate for the deviations between expected patterns of energy demand and availability [6]. Although such strategies are easy to implement, and also reflect the common practices in many Large Scale VPPs (LSVPPs) [7], still they do not embed the smart grid philosophy and ultimate objective, i.e., that the smart grid should provide a “heightening of the situational awareness of the grid and an allowing of fast-acting changes in power production and power routing, thus altering the stream of electrical supply and demand on a moment-by-moment basis” [4]. Also, one-day ahead scheduling does not exploit real-time information (e.g., actual power generated from renewable sources), and poses critical stability issues in case of unexpected power supply or power consumption.

The main objective of this paper is to address the last issue and show that it is possible to implement algorithms that manage to establish the optimal thermal and electrical scheduling in quasi real-time, drastically reducing the communication requirements to one bit of communication from the EMS to the DERs every time electrical and thermal energy balancing is achieved. In particular, without the necessity of the DERs to communicate the deliverable power to the EMS, or of the EMS to communicate the required power to the DERs, and also without communication requirements between the DERs. This paper extends previous work of the authors [5] to further take into account thermal power flows. This new feature requires a modification of the algorithms designed in [5], as it is now necessary to satisfy two constraints at the same time (the electrical and the thermal one), which are tightly coupled by the presence of the CHP plants. Also, the case study presented in this paper

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is a more realistic representation of a microgrid.

This paper is organised as follows: the next section illustrates the mathematical formulation of the power generation problem, in terms of definition of utility functions of interest and the optimisation problem, and the design of distributed algorithms to solve it. Section III illustrates the performance of the algorithms in a case study of interest, and in Section IV we conclude the paper, summarise our findings and outline our current lines of research.

## II. DECENTRALISED OPTIMISATION METHODS

### A. Basic AIMD algorithm

In the Internet, one way of achieving a distributed resource allocation is that sources increase transmission rate until a packet loss signal is received, as this indicates that a congestion event has occurred. Upon detecting congestion, the sources instantaneously decrease the transmission rate [8]. Such algorithms are called AIMD (Additive Increase Multiplicative Decrease) algorithms [9]. We now adapt the algorithms to the power generation case. In this case we have that the power sources increase their generation until a congestion notification signal is received, which means that the overall generated power equals the required power, at which point the power sources suddenly decrease the power generation. We observe that the original congestion control in the Internet can be easily adapted to the power generation case as there are many similarities: the quantities of interest are positive (bandwidth/power); locally bounded (local maximum transmission rate/maximum power that the power source can generate); available capacity/power required by users varies over time in an unpredictable fashion; and the underlying system (Internet/Smart grid) exhibits similar large-scale characteristics. The general algorithm in the power generation case can be described as in the following scheme:

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#### Algorithm II.1: BASIC AIMD ALGORITHM ( $p_i$ )

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 $p_i(0) = p_{i0}$ 
repeat
   $t = t + 1$ 
  if  $\sum_{i=1}^n p_i(t) < d(t)$ 
     $p_i(t+1) = \min[p_i(t) + \alpha_i, \bar{p}_i(t)], \forall i = 1, \dots, n$  (AI)
  else
     $p_i(t+1) = \max[\beta_i p_i(t), \underline{p}_i(t)], \forall i = 1, \dots, n$  (MD)
until end of simulation

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In Algorithm II.1, the parameter  $\alpha_i$  is the positive additive parameter associated with the additive increase phase of the algorithm (AI) (expressed in  $kW$  or  $MW$ ), and  $0 < \beta_i \leq 1$  is the multiplicative parameter used in the decrease phase (MD). The quantities  $p_i(t)$  represent the power generated by the  $i$ 'th of  $n$  DERs at time  $t$ , and they are increased

until they equal the demanded power  $d(t)$ . The quantities  $\underline{p}_i(t)$  and  $\bar{p}_i(t)$  denote the minimum and maximum bound on the power respectively. In fact, the values of  $p_i$  cannot take arbitrary values due to: (i) the limited sizes of the DERs; (ii) the availability of renewables (sun/wind); (iii) the power networks constraints; and (iv) due to the fact that DERs can modulate their power generation without exceeding a nominal ramp reference. Note also that such bounds are time varying.

### B. AIMD application to power generation

In principle, there are many ways in which the AIMD algorithms can share the power generation task among the available power plants. In this paper, we are interested in the solution that minimises a utility function of interest, namely, the cost of producing the total power. Note that this solution makes sense as within a microgrid, power plants cooperate to produce the required power with minimum cost, and are not in economic competition among themselves. In this paper we adopt quadratic cost functions, as typical in the microgrid literature (see for instance references [11], [10] and [16]):

$$f_i(p_i) = a_i \cdot p_i^2 + b_i \cdot p_i + c_i, \quad (1)$$

where  $f_i(p_i)$  is the hourly cost of power generation in Currency Unit (C.U.) per hour of the  $i$ 'th DER;  $p_i$  is its generated power in  $MW$ ; and  $a_i$ ,  $b_i$  and  $c_i$  are coefficients of appropriate measurement unit that depend on the technology of the power plant (e.g., fuel cost, efficiency, etc). In particular,  $b_i$  includes operation and maintenance (O&M) costs, and fuel and carbon costs, which are usually expressed in  $\text{€}/MWh$  (or in  $\text{\$/MWh}$ ). The coefficient  $c_i$  takes into account the expenses that are incurred even if no energy is produced at all.

Therefore, the objective of the AIMD algorithm is to compute the optimal instantaneous  $p_i^{el}(t)$  and  $p_i^{th}(t)$  (i.e., electrical and thermal power generated by the  $i$ 'th DER) that minimise the total cost of power generation, while satisfying the power demand by the users.

$$\begin{aligned} \min \sum_{i=1}^n f_i^{el}(p_i^{el}(t)) + f_i^{th}(p_i^{th}(t)) \\ \left\{ \begin{array}{l} \sum_{i=1}^n p_i^{el}(t) = d^{el}(t) \\ \sum_{i=1}^n p_i^{th}(t) = d^{th}(t) \end{array} \right. \end{aligned} \quad (2)$$

In the optimisation problem, the quantities denoted by  $^{el}$  and  $^{th}$  refer to electrical and thermal power flows respectively. The formulation of problem (2) is a general one, as we are assuming that each DER could in principle generate both thermal and electrical power. Also, we are assuming that the cost functions for producing thermal or electrical power could be different. This general formulation allows us to consider indirect thermal power production, e.g., a PV plant could be connected to a boiler and be altogether used to provide thermal power. However, in this paper we consider quadratic utility functions (1) for both the electrical and the thermal case.

**Remark:** We also make the assumption that the feasibility

region of problem (2) is not empty. In practice, this corresponds to assuming that at any time of the day the collectivity of power plants has the ability to satisfy both the thermal and electrical power demand by the users. In the case that this assumption does not hold, then the EMS must take some alternative scheduling decisions, such as: buy power from the outer grid, or use power stored in some storage systems, or disconnect some of the loads. However, as this possibility is not investigated here, we simply assume that power supply is always greater than the power demand.

One of the main contributions of this paper is the integration of thermal balancing issues in the optimal power generation problem. For this purpose, we need to suitably tailor the AIMD algorithms in order to achieve a second balancing objective, related to the thermal power flows. Thus, we design a new algorithm which alternatively performs an AIMD step on the electrical power, and one on the thermal power; also, as the two steps are strongly related (e.g., the electrical power production of a CHP is coupled with its thermal production), we give priority to one of the two power balancing loops, namely, the thermal loop. The motivation to do so is that most power plants have the ability to generate electrical power, while usually only a subset of them can also directly provide thermal power.

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**Algorithm II.2:** FULL AIMD ALGORITHM ( $p_i^{th}(t), p_i^{el}(t)$ )

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$$p_i^{th}(0) = p_{i0}^{th}, p_i^{el}(0) = p_{i0}^{el},$$

**repeat**

$$t = t + 1$$

**if**  $\sum_{i=1}^n p_i^{th}(t) < d^{th}(t)$

$$p_i^{th}(t+1) = \min [p_i^{th}(t) + \alpha_i^{th}, \bar{p}_i^{th}(t)], \forall i = 1, \dots, n \quad (\text{AI})$$

**else**

$$p_i^{th}(t+1) = \max [\beta_i^{th} p_i^{th}(t), \underline{p}_i^{th}(t)], \forall i = 1, \dots, n \quad (\text{MD})$$

**endif**

**if**  $\sum_{i=1}^n p_i^{el}(t) < d^{el}(t)$

$$p_i^{el}(t+1) = \min [p_i^{el}(t) + \alpha_i^{el}, \bar{p}_i^{el}(t)], \forall i = 1, \dots, m \quad (\text{AI})$$

**else**

$$p_i^{el}(t+1) = \max [\beta_i^{el} p_i^{el}(t), \underline{p}_i^{el}(t)], \forall i = 1, \dots, m \quad (\text{MD})$$

**endif**

**until** end of simulation

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Note that in Algorithm II.2, the electrical AIMD step is performed by a subset of  $m < n$  DERs; in particular, these are the  $m$  DERs that are not involved in the thermal power balancing loop. This strategy has been used to deal with the case that the thermal AIMD algorithm requires some DERs to increase their thermal power generation (i.e., because the thermal demand is greater than the supply), while the electrical AIMD algorithm requires some DERs to decrease their electrical power generation (i.e., because the electrical

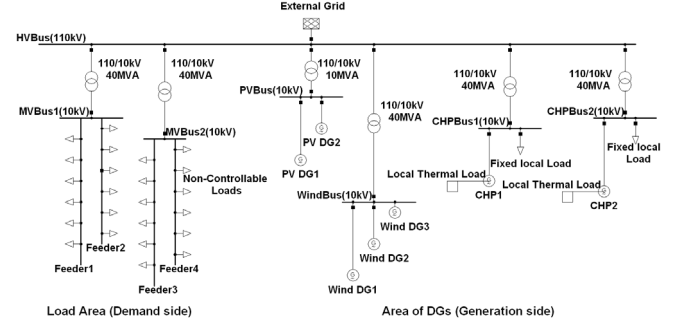


Fig. 1. Schematic topology of the tested microgrid network.

demand is lower than the supply). As the quantity of electrical power provided by a DER is usually related to the provided thermal power, such a conflicting situation is not simple to handle. A possible way to solve it is to appropriately choose the values of  $\alpha_i^{th}$ ,  $\alpha_i^{el}$ ,  $\beta_i^{th}$  and  $\beta_i^{el}$  to give more importance to the thermal balancing loop; alternatively, which is the solution implemented here in Algorithm II.2, we assume that a subset of DERs is used only to accommodate the residual electrical power request.

**Comment:** As proved in [5], the optimisation problem (2) can be easily solved by appropriately choosing the AIMD parameters; in particular, the optimal choice is

$$\alpha_i^{th,el} = \frac{\alpha_\lambda^{th,el}}{2a_i^{th,el}}, \quad (3)$$

$$\beta_i^{th,el}(t) = \beta_\lambda^{th,el} + b_i^{th,el} \cdot \frac{\beta_\lambda^{th,el} - 1}{2a_i^{th,el} p_i^{th,el}(t)}$$

where  $th, el$  means that the same equation can either refer to the thermal loop parameters, or to the electrical loop parameters. Parameters  $a_i^{th}$ ,  $a_i^{el}$ ,  $b_i^{th}$  and  $b_i^{el}$  are the utility function parameters, as from Equation (1).

Also,  $0 < \beta_\lambda^{th,el} < 1$  and  $\alpha_\lambda^{th,el}$  are two arbitrary parameters that can be used to adjust the AIMD speed of convergence properties [5].

### III. MICROGRID SIMULATIONS AND RESULTS

#### A. Simulation set-up

To evaluate the performance of our proposed double prioritised AIMD utility optimisation algorithms in a real microgrid environment, we tested our algorithms on a specific tested power network with the dedicated power system simulation software OpenDSS [12]. In this network, we assume that several DERs are installed, which have sufficient capacity to participate in the electricity market and serve both thermal and electrical demand in the load area. The topology of the tested network is illustrated in Figure 1. In our simulation, the base voltage of the High Voltage (HV) network was set to 110kV (1.0pu) at the source-end of the external grid. Two HV/MV transformers were applied to step down the voltage from

TABLE I  
PARAMETERS OF THE UTILITY FUNCTIONS

Plant	$a_i^{el}$	$b_i^{el}$	$c_i^{el}$	$a_i^{th}$	$b_i^{th}$	$c_i^{th}$
Wind Plant 1	0.0021	20.59	3.438	-	-	-
Wind Plant 2	0.0021	19.54	3.440	-	-	-
Wind Plant 3	0.0020	25.33	3.418	-	-	-
Solar PV 1	0.0042	23.73	3.428	-	-	-
Solar PV 2	0.0042	15.88	3.435	-	-	-
CHP 1	0.0064	39.22	4.011	0.0064	39.22	4.011
CHP 2	0.0062	33.84	4.024	0.0064	39.22	4.011

110kV to 10kV in the load area. In addition, several 110/10kV transformers were used to step up the voltage generated from the DERs to the HV power network. We considered three wind plants, two PV plants, and two CHPs. We assume that the generation capacity of each CHP is sufficient to serve its local fixed electrical load. Besides, CHPs must have the ability to generate enough thermal energy. All of the DERs in the network were modeled as the constant P-Q generators with the same power factors equal to 1.0 to generate pure active power for the loads. We chose the capacity of each DER according to existing plants for which parameters were available from reference [18]. In particular, the capacities of the wind plants were chosen as 12.15 MW, 13.5 MW and 7.7 MW respectively. The capacities for the PV plants were selected as 1.8 MW and 2.1 MW and the capacities for the CHPs were chosen as 24 MW and 14 MW. The fixed loads located around the CHPs were taken as 10 MW and 5 MW respectively. We assumed that the CHP were the only DERs able to produce thermal power, with a heat to power ratio equal to 1.0. In particular, the microgrid was sized to satisfy both the electrical and the thermal demand at any moment during the simulation. In the simulation, we further assumed that each load had a power factor of 0.98 lagging, and load profiles were randomly chosen for a period of 24 hours, according to reference [14]. Loads are located in the microgrid area at a random distance smaller than 1 Km among each other. The maximum wind power output for each wind DER was also randomly chosen from the real onshore wind turbine data from National Renewable Energy Laboratory (NREL) [15]. The maximum solar power generation profile of each PV was computed according to a quadratic function with non-zero values from 6am to 6pm, randomly perturbed to simulate cloud disturbances, as in [16]. The real thermal power profile was referred from [17] and applied to the total thermal power demand in our simulation, proportionally to a maximum thermal peak of 18 MW during the day. The parameters of the utility functions were taken from [18] and [11] and are summarised in Table I. In particular, as the CHPs provide revenues due to thermal power generation, a heat credit is subtracted from total unit costs to establish an equivalent of the levelised costs of producing only electricity [18]. We sampled the load profiles and the maximum output of the DERs every 10 minutes, so that the values of  $\bar{p}_i^{th}(t)$ ,  $p_i^{th}(t)$ ,  $\bar{p}_i^{el}(t)$ ,  $p_i^{el}(t)$ ,  $d^{th}(t)$  and  $d^{el}(t)$  did actually change every 10 minutes.

## B. Simulation results

This section reports the simulation results obtained by implementing the proposed double prioritised AIMD utility optimization algorithm in the network described in the previous section. For better evaluation of the performance of the proposed algorithm, we compared the solution with the optimal one obtained in a centralised fashion with a full exchange of information. The centralised solution is computed every 10 minutes, assuming that the EMS is informed of the maximum power that each DER can provide (depending on wind/sun availability) and also of the power required by the users. Then we assumed that the EMS had the ability to solve instantaneously the optimisation problem and to schedule the optimal power flows to the DERs. Clearly, this solution cannot be realistically implemented, and requires a great exchange of information. As for the AIMD implementation, we settled with 1 second the time step to perform additive increase and multiplicative decrease steps. Figure 2.(a) depicts the optimal share of electrical power generated by each available DER. Figure 2.(b) shows that both a centralised full-communication and the proposed distributed algorithm manage to match supply and demand of both electrical and thermal power. Figure 2.(c) shows that both algorithms achieve the same minimum cost result. Finally, Figure 2.(d) depicts the minimum and the maximum per-unit voltage in the network, as computed from the OpenDSS environment. It should be noted that we assumed that all the DGs were allocated outside of the load centre and produced a large aggregated available power. For the sake of simplicity, we assumed that the required reactive power was taken from the external grid, and the figure only illustrates the voltage impact on the grid in case no voltage control actions are taken.

## IV. CONCLUSION

This paper extends some preliminary work of the authors in this topic [5] by further considering thermal balancing, which had been neglected in the first paper, and by constructing a test example that more realistically portrays a microgrid. The main objective of the paper was to show that distributed algorithms can successfully solve the power generation problem in an optimal fashion, greatly reducing the communication overhead of centralised algorithms, even in the presence of a second thermal constraint.

The proposed scenario is a realistic one, consisting of several DERs, some of which are based on renewable sources, and some have the ability to provide both electrical and thermal power. The scenario could be further extended by also considering the presence of storage systems, and Electric Vehicles (EVs), and this possibility is currently being explored by the authors. Another interesting line of research involves the management of reactive power. So far, we have only focused on active power generation, thus assuming that reactive power was provided by some ancillary services in the microgrid (e.g., reactive Vehicle-to-Grid (V2G) operations) or bought from the external grid. Obviously, this assumption

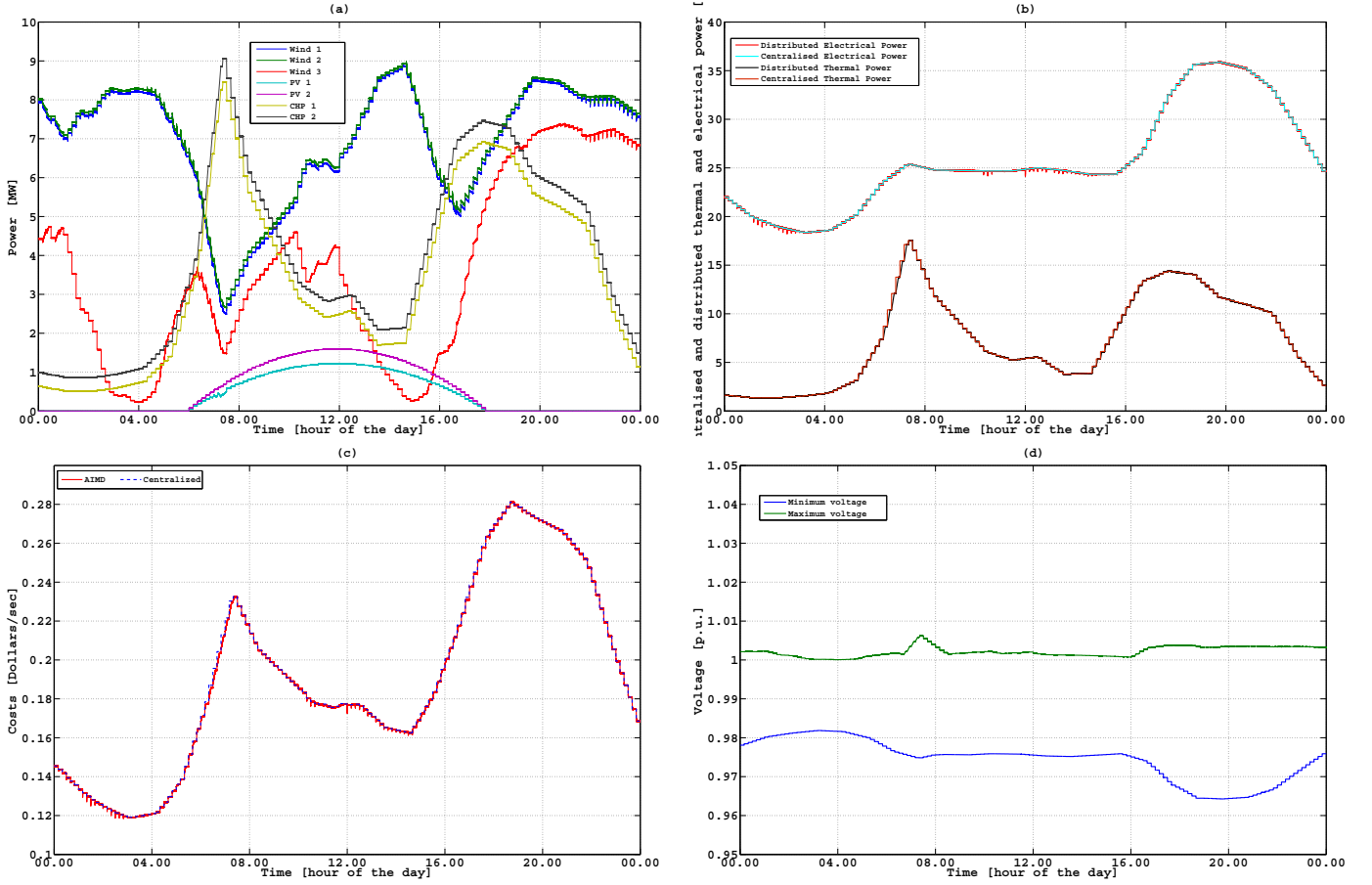


Fig. 2. Figure (a) gives the detail of how much electrical power is generated by every single DER; Figure (b) shows that both the distributed and the centralised solution manage to satisfy the required thermal and electrical power constraints, while achieving the same minimum value of the cost function (c); Figure (d) shows the minimum and the maximum voltage in the network obtained with the two algorithms.

should be removed, and treated as a further balancing event (i.e., reactive power balancing).

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