Fair Charging Strategies for EVs Connected to a Low-Voltage Distribution Network

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***Abstract*—The development of smart grid technologies and appropriate charging strategies are key to accommodating large numbers of Electric Vehicles (EV) charging on the grid. In this paper a general framework is presented for formulating the EV charging optimization problem and three different charging strategies are investigated and compared from the perspective of charging fairness while taking into account power system constraints. Two strategies are based on distributed algorithms, namely, Additive Increase and Multiplicative Decrease (AIMD), and Distributed Price-Feedback (DPF), while the third is an ideal centralized solution used to benchmark performance. The algorithms are evaluated using a simulation of a typical residential low voltage distribution network with 50% EV penetration.**

***Index Terms*—smart grid, demand side management, electric vehicle charging, distributed algorithms**

I. Introduction

In recent years, a combination of government policy, aimed at promoting energy sustainability, and rapid advances by the automotive industry, have meant electric vehicles (EVs) are increasingly being prioritized as a means of reducing pollution, combating climate change, and improving energy security. The Irish Government, for example, has set a target of 10% penetration of EVs in Ireland by 2020 [1].

In the near future, as the number of EVs plugging into the grid increases in residential areas, it is likely that coincident uncontrolled charging would overload local distribution networks and substantially increase peak power requirements [2]-[3]. There has been much research carried out in recent years with the aim of optimizing EV charging while minimizing the impact of this charging on the grid [3]-[12].

In this paper, a general framework is presented for formulating the EV charging optimization problem. Within this framework, both instantaneous and temporal optimization objectives can be defined. Two promising instantaneous distributed charging strategies which seek to achieve a fair distribution of available power are then introduced, namely, Additive Increase and Multiplicative Decrease (AIMD) [9] and Distributed Price-Feedback (DPF) [12]. The basic algorithms are modified to incorporate power system constraints and a heuristic Time-Of-Use (TOU) price adjustment mechanism that enables temporal objectives to be simultaneously addressed. The resulting algorithms are benchmarked against an ideal centralized solution to the fair EV charging problem.

The remainder of the paper is organised as follows. Section II presents the mathematical framework for EV charging, while Section III introduces the instantaneous optimization methods considered in the paper. Then, in Section IV, simulation results are presented and discussed for a residential low voltage distribution network with 50% EV penetration. Finally, conclusions are presented in Section V.

II. Problem Formulation

In this section, a general description of the mathematical framework used to define the EV charging optimization problem is given.

A scenario in which a number of houses incorporating EVs are connected to a distribution power network is shown in Fig.1. In this network S is defined as the number of distribution transformers. These transformers are then connected to the Medium-Voltage (MV) substation bus, SubBus. The Substation bus is powered by a transformer called TR0, which connects to an external bulk power system.

A number of simplifications are made in the system model representation. The load power consumption in the network is discretized into M discrete time slots (indexed 1,2…,M), each of length ΔT. The loads are artificially classified as non-EV loads, and EV loads, in the Low-Voltage (LV) areas. The number of houses in all LV areas is given by N, and $N(k)$ denotes the number of active charge points at time k, distributed over all LV areas. The index set of all houses connected to the transformer $TR(r)$ is given by $\mathfrak{T}_{\mathrm{H}}^{r}$, and similarly the index set of all EVs connected to transformer $TR(r)$ is given by $\mathfrak{T}_{\mathrm{C}}^{r}$.

In this paper, $h_j(k)$ denotes the non-EV power consumption for the j th house at time k, and $c_i(k)$ denotes the charge rate of the i th active EV charge point at time k, for all $k \in \{1,2,3 \ldots M\}$. The price signal at sample time k, $E(k)$, can represent either TOU or real-time signal pricing. In the model used here, it is assumed that each house can only have a

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maximum of one EV. The charge rate vector is given by $\mathbf{c}(k)^{\mathrm{T}} := [c_1(k), c_2(k) \dots c_N(k)]$ for all $k \in \{1,2,3 \dots M\}$. A charge rate matrix is also given by $\mathbf{C} := [\mathbf{c}(1), \mathbf{c}(2) \dots \mathbf{c}(M)]$. The plug-in time and plug-out time of the ith EV are given by a_i and b_i respectively. Therefore, the ith EV must be charged within $[a_i, b_i]$. Due to the battery specifications, each EV may have a different battery size (kWh), and this parameter is denoted as B_i for the ith vehicle. The State-of-Charge (SOC) for the ith EV at time k, $SOC_i(k)$, within $[a_i, b_i]$ is given by:

$$SOC_i(k) = SOC_i(a_i) + \sum_{t=a_i}^{k-1} c_i(t)\, \Delta T / B_i \tag{1}$$

Here $SOC_i(a_i)$ is the initial SOC for the ith EV when it plugs in. The maximum achievable SOC for the ith EV is given by:

$$SOC_{max}^i = min(1, SOC_i(a_i) + \frac{c_{max}^i (b_i - a_i)\Delta T}{B_i}) \tag{2}$$

where c_{max}^i is the maximum charge rate of the ith EV charge point.

There are 3 types of constraints associated here with the EV charging problem: plug-in constraints, power systems constraints, and optimization constraints.

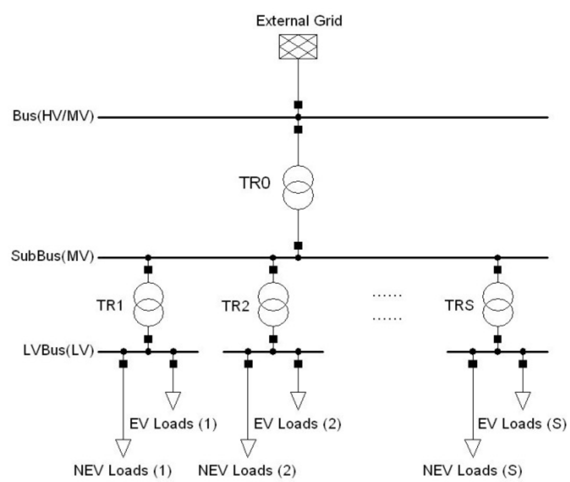

Fig.1: Schematic diagram of the distribution network. (**HV**: High Voltage; **MV**: Medium Voltage; **LV**: Low Voltage; **SubBus**: Substation Bus; **TR**: Transformer; **EV**: Electrical Vehicle; **NEV**: Non-Electric Vehicle)

➢ Plug-in constraints

These define the constraints imposed by the EV charge point and vehicle battery on the feasible charge rates which can be drawn from the charging socket by each EV, according to the specifications for the battery specified by the manufacturers. The minimum and maximum charge rates for the i th vehicle are denoted by c_{min}^i and c_{max}^i, respectively. Considering the charging rate over the course of the full M time slots and noting that charging can only take place when the EV is plugged in and not already fully charged, gives the following plug-in constraints:

If: $a_i \leq k \leq b_i$

$$c_{min}^i \leq c_i(k) \leq c_{max}^i \tag{3}$$

If: $k \leq a_i$ or $k \geq b_i$ or $SOC_i(k) = 100\%$

$$c_i(k) = 0 \tag{4}$$

➢ Power system constraints

In this paper, the power system performance in the model is investigated, with the aim of mitigating the impact of EVs on the grid from the perspective of transformer loading levels and voltage profiles. The total loading conditions for each transformer $P_{TR}^r(k)$, $r \in \{0,1,2 \dots S\}$ at each time slot k is inspected, and the voltage for each connected node $v_j(k)$ is used to evaluate the power system performance at each time instant. We define the maximum rating for transformer r as $\mathrm{TR}_{\mathrm{max}}^r$, and define the lower bound of the voltage value that can be tolerated by the distribution network as V_{min}. Therefore, the power system constraints can be set as follows:

$$P_{TR}^r(k) = \sum_{j \in \mathfrak{T}_H^r} h_j(k) + \sum_{i \in \mathfrak{T}_C^r} c_i(k)\, , r \in \{1,2,3 \dots S\} \tag{5}$$

$$P_{TR}^0(k) = \sum_{r=1}^{S} P_{TR}^r(k) \tag{6}$$

$$P_{TR}^r(k) \leq TR_{max}^r, r \in \{0, 1,2,3 \dots S\} \tag{7}$$

$$v_j(k) \geq V_{min}, j \in \{1,2,3 \dots N\} \tag{8}$$

➢ Optimization constraints

Optimization constraints refer to constraints that may arise as part of the formulation of the EV optimization problem. For example, if the objective is defined as minimizing the cost of charging EVs, i.e.

$$min_{\mathbf{C}}\{\sum_{k=1}^{M} \sum_{i=1}^{N} c_i(k). \Delta T. E(k)\} \tag{9}$$

an additional constraint needs to be added to avoid the trivial solution, $c_i(k) = 0, \forall i, k$, such as

$$SOC_i(M) = SOC_{max}^i, \forall i. \tag{10}$$

Having established the model parameters and three types of constraints listed above, cost functions need to be defined to capture the EV charging optimization objective. The possible cost functions can be divided into two groups, those which seek to optimize temporally $J(\mathbf{C})$, and those which optimize instantaneously $J(\mathbf{c}(k))$, i.e. only consider the current time instant. Temporal cost functions usually consider long-term benefits such as the total cost of charging to the consumer, or peak-shaving and valley filling for the utility company, whereas instantaneous cost functions objectives address issues such as fairness of the distribution of the available power for charging [3,6].

In the case of temporal optimization, which is essentially a scheduling problem, the optimum solution requires a priori knowledge of the all the relevant parameters over the optimization horizon, $k = 1,2,3 \dots M$, so that the full $\mathbf{C}$ matrix can be determined. In reality, base load, plug-in, plug-out and SOC patterns for each user can only be approximated, leading to sub-optimal solutions. In addition, these problems are computationally very challenging and do not scale well.

Instantaneous cost functions are defined in terms of the information available at the current time instant, allowing for much better defined optimization problems. In particular, they are much more amenable to decentralized implementation. However, they cannot address temporal objectives such as peak-shaving.

III. Decentralized Optimization Methods

A. Distributed AIMD algorithm

Stüdli et al. proposed a novel fair EV charging strategy based on the decentralized AIMD method [9]. Subsequently Liu et al. [5], [10] developed an AIMD implementation which attempts to benefit both utilities and customers by maximizing the utilization of low-price electricity during off-peak times as well as fairly distributing the available power to connected EVs. In order to do this power system constraints on voltages and load balance are incorporated into the AIMD implementation so that all EVs can share the maximum amount of available power fairly while ensuring that the distribution network continues to operate within acceptable limits. In addition, the available power signal is modulated in response to a varying electricity price [13] so as to affect a shift in EV loads away from periods of high demand, thereby reducing peak-power capacity requirements, and ultimately minimizing the cost to the customers. The pseudo code below summarized the main functionality of the proposed algorithm. For a more detailed description please refer to [5], [9] and [10].

Define 0.5< ϕ_i <1, 0< $\beta^{(2)} < \beta^{(1)} < 1$
while *battery not charged* **do**
  if *capacity congestion event*
  then generate uniform random number*, 0<p<1*
    if $p < \phi_i$ **then**
      $c_i(k+1) = \beta^{(1)}.c_i(k)$
    else
      $c_i(k+1) = \beta^{(2)}.c_i(k)$
    end if

  else
    $c_i(k+1) = min(c_i(k) + \alpha.\Delta T, c^i_{max})$
  end if
  if $v_j(k) < V_{min}$ **then** *transmit voltage event message*
  end if
end while

B. Distributed Price-Feedback algorithm

The original DPF algorithm proposed in [12] is modified to fit the framework considered in this paper as follows. At the beginning of the EV charging process, each EV owner provides a willingness to pay (WTP) parameter, $\omega_i > 0$, that is used to determine their charge rate. In general, the larger ω_i the more charge a vehicle can expect to receive, and in the absence of power system or plug-in constraints available power is distributed proportionally to ω_i. The real price signal *E(k)* in our algorithm is then defined as:

$$E(k) = f\left(\sum_{i=1}^{N(k)} c_i(k)\right) \tag{11}$$

where $f(x)$ is given by

$$f(x) = a\left(\frac{x}{P_c(k)}\right)^k \ . \tag{12}$$

Here a and k are constants and $P_c(k)$ is the maximum market power capacity for charging at time slot k for all active EV charging points, i.e.

$$P_c(k) = min(P_{av}(k), \sum_{i=1}^{N(k)} c^i_{max}) \tag{13}$$

and

$$P_{av}(k) = TR^0_{max} - \sum_{j=1}^{N} h_j(k) - \Delta(k) \ . \tag{14}$$

$\Delta(k)$ is introduced as a utility regulatory factor to regulate the total available charging power in order to increase the long-term benefits (e.g. valley filling). Each EV i is associated with a utility function $u_i(x)$ with respect to the charging demand x at each charging time slot. $u_i(x)$ is a non-decreasing function to achieve proportional fair pricing defined in [12] as:

$$u_i(x) = \omega_i logx \ . \tag{15}$$

Hence, our objective is that each EV chooses $c_i(k)$ so as to maximize:

$$u_i\big(c_i(k)\big) - c_i(k)E(k). \tag{16}$$

According to [12], user i adapts its charge rate using the following equation to maximize the above utility function:

$$c_i(k+1) = c_i(k) + \gamma_i(\omega_i - c_i(k)E(k)) \tag{17}$$

Here γ_i is a parameter that controls the rate of convergence of the algorithm. However, in order to optimize the objective function in (15) while satisfying power system constraints defined in Section II, the algorithm is altered as follows.

Define 0.5< ϕ_i <1, 0< $\beta^{(2)} < \beta^{(1)} < 1$
while *battery not charged* **do**
  if *capacity congestion event*
  then generate uniform random number*, 0<p<1*
    if $p < \phi_i$ **then**
      $c_i(k+1) = \beta^{(1)}.c_i(k)$
    else
      $c_i(k+1) = \beta^{(2)}.c_i(k)$
    end if

  else
    calculate $E(k)$ using (11)
    $c_i(k+1) = min(c_i(k) + \gamma_i\big(\omega_i - c_i(k)E(k)\big), c^i_{max})$
  end if
  if $v_j(k) < V_{min}$ **then** *transmit voltage event message*
  end if
end while

C. Ideal centralized solution

In this section, we introduce an ideal centralized solution based on a hierarchical structure. At each time slot *k*, the *N*(*k*) EVs connected to the grid send charging requests of $c^i_{\max}$ to their local transformers. Each local transformer calculates the charge rate for the EVs in its area taking account of the current local capacity $\mathrm{TR}^r_{\max} - \mathrm{P}^r_{TR}(k)$, $i \in \mathfrak{X}^r_{\mathrm{C}}$ and forwards the requested power requirements to the main substation. If the total amount of requested power exceeds the available power the main substation TR^0 allocates the available power to each substation in proportion to the requested values. Each substation then updates their EV charge rates accordingly and broadcasts the information to the charge points.

Voltage constraint violations can be handled by reducing the charge rates for all EVs connected to the affected transformer by a multiplicative factor, in a similar manner to the approach used in the AIMD and DPF algorithms. Since perfect knowledge is assumed for optimization, this solution is taken as an ideal reference for comparison to other methods. This ideal solution ensures fairness in each local area, such that each EV is allocated the same charge rate and this is the highest charge rate possible taking into account the local power system constraints.

IV. Simulations and Results

For our algorithm implementations, a three day simulation was run with ΔT set to 5 minutes (M = 288). The simulation consists of a typical residential low voltage distribution network with S=3 and N=160 (distributed evenly across phases) and 50% EV penetration, and was implemented using OpenDSS and Matlab as described in [5]. The rating of the main substation was 400kVA and the rating for each local transformer was 150kVA. The minimum voltage that can be tolerated on the grid was set to 0.9pu. The non-EV household load profiles for each house was generated by randomly selecting winter load profiles from a customer smart meter electricity trial dataset provided by the Commission for Energy Regulation (CER) in Ireland [14].

For each EV the minimum and maximum charge rates were set as 0 and 3.7kW, respectively. Battery capacities were fixed at 20 kWh and the initial SOCs for each EV selected randomly in the interval 5kWh to 15kWh. Plug-in times were normally distributed around 6 pm with a standard deviation of one hour. These values are in accordance with the travel pattern assumptions made in [9]. For comparison purposes the same plug-in times were used with each method considered. It was assumed that all EVs charge overnight and that once an EV is plugged in it will only physically plug-out at the scheduled plug-out time. This was taken as 6am the next day. In the price-feedback algorithm the WTP parameter was chosen to be 1.5, 2.5 or 3.5. These values can be interpreted as slow, normal and fast charging. For each method the utility regulatory factor $\Delta(\mathrm{k})$ in (14) was set up according to the TOU pricing defined in [5] as a price-adjusted available power heuristic to motivate load shifting from peak to off peak times.

Summary results are presented in Table I for the performance of each charging strategy when implemented with (P) and without (NP) the price-adjusted available power heuristic. In the case of the Ideal algorithm results are presented for two implementations; one that includes a voltage limit violation correction mechanism and one that does not. Power flow and voltage profile analyses are also presented in Fig.2 and Fig. 3, respectively. The results show that both AIMD and DPF provide a good approximation to the ideal solution, and as such are competitive alternatives to the ideal solution given the substantially reduced communication overhead associated with their distributed implementation. DPF has the advantage that by using its price-feedback approach it is able to provide users with charge rates that reflect their WTP. However, these are not necessarily proportional to their WTP due to the impact of local power system and charging infrastructure constraints. In our simulations the average charge rates obtained by slow, normal and fast charging consumers were 1, 1.4 and 1.6 kW respectively (plots are not shown due to space constraints). In contrast, AIMD does not distinguish between customers, but in return requires the least communication overhead.

TABLE I. Comparison Talble for 50% EV penetration

Charging Strategy	*Minimum voltage(p.u.)*	*Maximum Load (%)*	*Average Cost (cents/kWh)*	*Duration of voltage dips(min.)*
NoEV	0.91	94	-	0
Unctrl	0.86	151	19.70	305
AIMD(P)	0.90	96	11.18	0
DPF(P)	0.90	100	11.37	0
Ideal(P)	0.90	100	10.98	0
IdealN(P)	0.89	100	10.99	5
AIMD(NP)	0.90	102	12.84	0
DPF(NP)	0.90	105	12.28	0
Ideal(NP)	0.89	100	14.47	5
IdealN(NP)	0.89	100	14.60	25

NoEV*: Grid without EVs;* ***Unctrl****: Uncontrolled charging;* ***AIMD****: AIMD Smart charging;* ***DPF****: Smart charging based on price-feedback;* ***Ideal****: Ideal Centralized charging solution.* ***IdealN****: Ideal Centralized charging solution without voltage control;* ***Maximum Load****: Maximum loading as a percentage of the sub-station rating (400kVA).* ***Duration of voltage dips:*** *Total time in minutes over the 3 days that the voltage drops below 0.9pu;* ***Average cost:*** *Average charging cost per kWh in cents.* ***P****: With price-adjusted available power;* ***NP****: Without price-adjusted available power;*

It is also clear from Table I that the simple price-adjusted available power heuristic achieves the desired effect of shifting the peak load to off-peak times. This is reflected in the lower charging costs to the consumer when price-adjustment is in effect and indirectly in the lower maximum loads observed with AIMD(P) and DPF(P), relative to AIMD(NP) and DPF(NP).

V. Conclusions

In this paper, a mathematical framework for formulating EV charging problems has been presented that incorporates both power system and charging infrastructure constraints and caters for both instantaneous and temporal optimization objectives. Within this framework two decentralized EV charging algorithms based on Stüdli et al [9] (AIMD) and Fan [12] (DPF) have been introduced and compared with an ideal centralized charging strategy which distributes available power equally between customers, taking into account power system constraints. In addition to addressing power system constraints, the modified AIMD and DPF implementations also include a utility regulatory factor that allows temporal objectives to be indirectly achieved by artificially adjusting the available power for charging. Specifically, a TOU price-adjusted available power heuristic is proposed as a means of shifting load to off-peak times.

An evaluation of the performance of the proposed distributed EV charging strategies on a simulated low voltage distribution network serving a residential area with 50% EV penetration confirms the efficacy of each method at managing the EV charging requirements while meeting power system constraints. In particular, the price-adjusted available power heuristic is shown to be effective at shifting load to off-peak times, reducing the strain on the power system infrastructure and reducing the cost to the consumer. The heuristic integrates seamlessly with each method and comes at no additional communication overhead of significance. As such the deployment of this heuristic with either AIMD or DPF has the potential to provide significant benefits to utility companies by reducing the necessity for new infrastructure as EVs rollout.

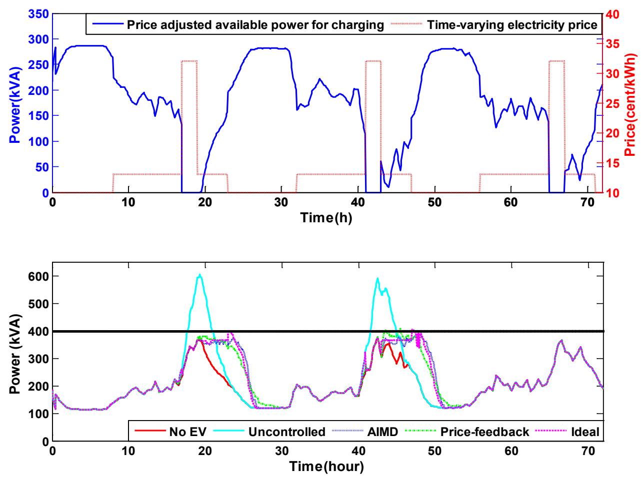

Fig. 2: A comparison of the power flow achieved with different EV charging strategies relative to uncontrolled charging and no-charging: (a) TOU pricing signal and resulting price adjusted available power; (b) Power flow at substation (TR0) over a 72 hour period incorporating two charging cycles.

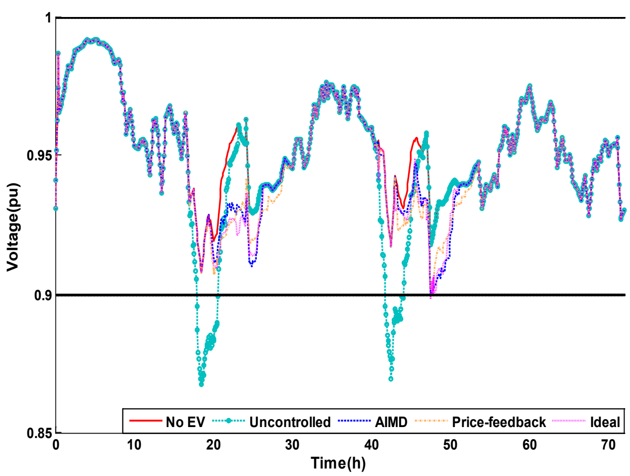

Fig. 3: Minimum voltage levels achieved with the different charging strategies compared to uncontrolled charging and no-charging scenarios

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