Gradient descent in general has often been regarded as slow or unreliable. In the past, the application of gradient descent to non-convex optimization problems was regarded as foolhardy or unprincipled. Today, we know that the machine learning models described in part II work very well when trained with gradient descent. The optimization algorithm may not be guaranteed to arrive at even a local minimum in a reasonable amount of time, but it often finds a very low value of the cost function quickly enough to be useful.

Stochastic gradient descent has many important uses outside the context of deep learning. It is the main way to train large linear models on very large datasets. For a fixed model size, the cost per SGD update does not depend on the training set size m. In practice, we often use a larger model as the training set size increases, but we are not forced to do so. The number of updates required to reach convergence usually increases with training set size. However, as m approaches infinity, the model will eventually converge to its best possible test error before SGD has sampled every example in the training set. Increasing m further will not extend the amount of training time needed to reach the model's best possible test error. From this point of view, one can argue that the asymptotic cost of training a model with SGD is $O(1)$ as a function of m.

Prior to the advent of deep learning, the main way to learn nonlinear models was to use the kernel trick in combination with a linear model. Many kernel learning algorithms require constructing an $m \times m$ matrix $G_{i,j} = k(\boldsymbol{x}^{(i)}, \boldsymbol{x}^{(j)})$. Constructing this matrix has computational cost $O(m^2)$, which is clearly undesirable for datasets with billions of examples. In academia, starting in 2006, deep learning was initially interesting because it was able to generalize to new examples better than competing algorithms when trained on medium-sized datasets with tens of thousands of examples. Soon after, deep learning garnered additional interest in industry, because it provided a scalable way of training nonlinear models on large datasets.

Stochastic gradient descent and many enhancements to it are described further in chapter 8.

5.10 Building a Machine Learning Algorithm

Nearly all deep learning algorithms can be described as particular instances of a fairly simple recipe: combine a specification of a dataset, a cost function, an optimization procedure and a model.

For example, the linear regression algorithm combines a dataset consisting of

$\boldsymbol{X}$ and $\boldsymbol{y}$, the cost function

$$J(\boldsymbol{w}, b) = -\mathbb{E}_{\mathbf{x},\mathrm{y}\sim\hat{p}_{\mathrm{data}}} \log p_{\mathrm{model}}(y \mid \boldsymbol{x}), \tag{5.100}$$

the model specification $p_{\mathrm{model}}(y \mid \boldsymbol{x}) = \mathcal{N}(y; \boldsymbol{x}^\top \boldsymbol{w} + b, 1)$, and, in most cases, the optimization algorithm defined by solving for where the gradient of the cost is zero using the normal equations.

By realizing that we can replace any of these components mostly independently from the others, we can obtain a very wide variety of algorithms.

The cost function typically includes at least one term that causes the learning process to perform statistical estimation. The most common cost function is the negative log-likelihood, so that minimizing the cost function causes maximum likelihood estimation.

The cost function may also include additional terms, such as regularization terms. For example, we can add weight decay to the linear regression cost function to obtain

$$J(\boldsymbol{w}, b) = \lambda ||\boldsymbol{w}||_2^2 - \mathbb{E}_{\mathbf{x},\mathrm{y}\sim\hat{p}_{\mathrm{data}}} \log p_{\mathrm{model}}(y \mid \boldsymbol{x}). \tag{5.101}$$

This still allows closed-form optimization.

If we change the model to be nonlinear, then most cost functions can no longer be optimized in closed form. This requires us to choose an iterative numerical optimization procedure, such as gradient descent.

The recipe for constructing a learning algorithm by combining models, costs, and optimization algorithms supports both supervised and unsupervised learning. The linear regression example shows how to support supervised learning. Unsupervised learning can be supported by defining a dataset that contains only $\boldsymbol{X}$ and providing an appropriate unsupervised cost and model. For example, we can obtain the first PCA vector by specifying that our loss function is

$$J(\boldsymbol{w}) = \mathbb{E}_{\mathbf{x}\sim\hat{p}_{\mathrm{data}}} ||\boldsymbol{x} - r(\boldsymbol{x}; \boldsymbol{w})||_2^2 \tag{5.102}$$

while our model is defined to have $\boldsymbol{w}$ with norm one and reconstruction function $r(\boldsymbol{x}) = \boldsymbol{w}^\top \boldsymbol{x} \boldsymbol{w}$.

In some cases, the cost function may be a function that we cannot actually evaluate, for computational reasons. In these cases, we can still approximately minimize it using iterative numerical optimization so long as we have some way of approximating its gradients.

Most machine learning algorithms make use of this recipe, though it may not immediately be obvious. If a machine learning algorithm seems especially unique or

hand-designed, it can usually be understood as using a special-case optimizer. Some models such as decision trees or k-means require special-case optimizers because their cost functions have flat regions that make them inappropriate for minimization by gradient-based optimizers. Recognizing that most machine learning algorithms can be described using this recipe helps to see the different algorithms as part of a taxonomy of methods for doing related tasks that work for similar reasons, rather than as a long list of algorithms that each have separate justifications.

5.11 Challenges Motivating Deep Learning

The simple machine learning algorithms described in this chapter work very well on a wide variety of important problems. However, they have not succeeded in solving the central problems in AI, such as recognizing speech or recognizing objects.

The development of deep learning was motivated in part by the failure of traditional algorithms to generalize well on such AI tasks.

This section is about how the challenge of generalizing to new examples becomes exponentially more difficult when working with high-dimensional data, and how the mechanisms used to achieve generalization in traditional machine learning are insufficient to learn complicated functions in high-dimensional spaces. Such spaces also often impose high computational costs. Deep learning was designed to overcome these and other obstacles.

5.11.1 The Curse of Dimensionality

Many machine learning problems become exceedingly difficult when the number of dimensions in the data is high. This phenomenon is known as the **curse of dimensionality**. Of particular concern is that the number of possible distinct configurations of a set of variables increases exponentially as the number of variables increases.