Factorial Distribution

A **factorial distribution** happens when a set of variables are independent events. In other words, the variables don't interact at all; Given two events x and y, the probability of x doesn't change when you factor in y. Therefore, the probability of x, given that y has happened —P(x|y)— will be the same as P(x).

The factorial distribution can be written in many ways (Hinton, 2013; Olshausen, 2004):

- p(x,y) = p(x)p(y)
- p(x,y,z) = p(x)p(y)p(z)
- $p(x_1, x_2, x_3, x_4) = p(x_1)\ P(x_2)\ p(x_3)\ p(x_4)$

In the case of a probability vector, the meaning is exactly the same. That is, a probability vector from a factorial distribution is the product of probabilities of the vector's individual terms.

You may notice a lack of the factorial symbol (!) in any of the definitions. That's because the distribution is named because successive frequencies are factorial quantities, rather than the terms being factorials themselves.

Defining a Factorial Distribution

For a factorial distribution, P(x,y) = P(x)P(y). We can generalize this for more than two variables (Olshausen, 2004) and write:

$P(x_1, x_2,\ldots,x_n) = P(x_1) \cdot P(x_2 \cdot \ldots\ldots \cdot P(x_n)$.

This expression can also be written more concisely as:

$P(x_1, x_2,\ldots,x_n)= \Pi_i P(x_i)$.

Factorial Distribution Examples

We like to work with factorial distributions because their statistics are easy to compute. In some fields such as neurology, situations best represented by complicated, intractable probability distributions are approximated by factorial distributions in order to take advantage of this ease of manipulation.

One example of an often-encountered factorial distribution is the p-generalized normal distribution, represented by the equation

$$g\left(\sum_{i=1}^{n} |x_i|^p\right) = \frac{p^n}{\left(2\Gamma\left(\frac{1}{p}\right)(2\sigma^2)^{\frac{1}{p}}\right)^n} e^{-\frac{\sum_{i=1}^{n} |x_i|^p}{2\sigma^2}}$$

I won't go into the meaning of that formula here; if you'd like to go deeper, feel free to read up on it here. But note that when p = 2, this is exactly the normal distribution. So the normal distribution is also factorial.