Dot Product

Algebraic definition [edit]

The dot product of two vectors **a** = [a_1, a_2, …, a_n] and **b** = [b_1, b_2, …, b_n] is defined as:[1]

$$\mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^{n} a_i b_i = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

where Σ denotes summation and n is the dimension of the vector space. For instance, in three-dimensional space, the dot product of vectors [1, 3, −5] and [4, −2, −1] is:

$$\begin{aligned} [1, 3, -5] \cdot [4, -2, -1] &= (1 \times 4) + (3 \times -2) + (-5 \times -1) \\ &= 4 - 6 + 5 \\ &= 3 \end{aligned}$$

If vectors are identified with row matrices, the dot product can also be written as a matrix product

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a}\mathbf{b}^{\top},$$

where $\mathbf{b}^{\top}$ denotes the transpose of $\mathbf{b}$.

Expressing the above example in this way, a 1 × 3 matrix (row vector) is multiplied by a 3 × 1 matrix (column vector) to get a 1 × 1 matrix that is identified with its unique entry:

$$\begin{bmatrix} 1 & 3 & -5 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \\ -1 \end{bmatrix} = 3.$$

Hadamard Product

Definition [edit]

For two matrices A and B of the same dimension $m \times n$, the Hadamard product $A \circ B$ (or $A \odot B$[3][4][5]) is a matrix of the same dimension as the operands, with elements given by

$$(A \circ B)_{ij} = (A \odot B)_{ij} = (A)_{ij}(B)_{ij}.$$

For matrices of different dimensions ($m \times n$ and $p \times q$, where $m \neq p$ or $n \neq q$) the Hadamard product is undefined.

Example [edit]

For example, the Hadamard product for a 3 × 3 matrix $\mathbf{A}$ with a 3 × 3 matrix $\mathbf{B}$ is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \circ \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} a_{11}\, b_{11} & a_{12}\, b_{12} & a_{13}\, b_{13} \\ a_{21}\, b_{21} & a_{22}\, b_{22} & a_{23}\, b_{23} \\ a_{31}\, b_{31} & a_{32}\, b_{32} & a_{33}\, b_{33} \end{bmatrix}.$$