

Hints for Crop Harvesting

- Fluffy's tractor can only hold 1 bushel of corn at a time.
- Clearly, any good strategy involves starting at the farmhouse (vertex 0), going to a harvest-able cornfield (vertex f_i), then returning to vertex 0.
- How much time do we need to pick up the i -th bushel?
 - The roads are undirected. Any path $x \rightarrow \dots \rightarrow y$ is also valid in the reversed direction.
 - If we know the shortest distance for $x \rightarrow \dots \rightarrow y$,
 - * What does that tell us about the shortest paths from y to x ?
 - * What is the shortest distance for $x \rightarrow \dots \rightarrow y \rightarrow \dots \rightarrow x$?
- In what order should we take the corn bushels?
 - If we try picking the first, second, third, ..., bushels, in chronological order, our choices become more and more restricted.
 - Instead, if we start from the last bushel and go backwards, bushels 'un-expire', and we never lose choices.
- The important parts for each corn bushel:
 - **Round-trip time:** The shortest time/distance for $0 \rightarrow \dots \rightarrow f_i \rightarrow \dots \rightarrow 0$.
 - **Last-departure time:** The final possible departure time (from the farmhouse), to collect it before expiry.
 - **Last-return time:** If taken at the last-departure time, we will return to the farmhouse at the this time.
- What is the **very last** bushel we should take?
 - The last bushel can be taken **as late as possible**.
 - Let's say we took some bushel X .
 - Before taking that bushel, we must be ready at the farmhouse this last-departure time (or earlier). This creates a **deadline**.
 - Which bushel should we pick? Would we prefer the deadline earlier or later? Which would give us (potentially) more options?
- Now, let's say we want to return to the farmhouse before time T (having planned to take some bushels later).
- Which bushel I should take right before this? What are the ones in consideration?

1. Bushels with last-return time $> T$.
If we take these bushels, we can return exactly at time T .
When should we depart?
 2. Bushels with last-return time $\leq T$.
We cannot take these bushels and return at time T , as they would expire.
When should we depart?
- Within each criterion, which is the ‘best’ one to take?
Our next deadline would be the “best”-departure time.
Would we prefer the next deadline to be earlier or later?
 - At current deadline T , the next step’s best-departure time would be:
 - For criterion (2), we can leave no later than the last departure time.
 - * Does this value depend on T ? If so, how?
 - For criterion (1):
 - * For criterion (1), we may have multiple possible bushels.
 - * We aren’t taking them last-minute, but we do want to return by T . When should we depart?
 - * Which is the ‘best’ amongst criterion (1)?
 - * Does this value depend on T ? If so, how?
 - How do we update, as the current deadline T decreases?
 - We have different ‘orderings’ for the criterion. How do we track both?
 - Can we store criterion (1) and (2) in the same data structure?
 - * When T changes, how does the “best” for (1) and (2) change?
 - * Does it change the same way, for both criteria?
 - If the best of criterion (1) is X , and best of criterion (2) is Y , how should we choose between these two?
 - Bushel Z might be best under criterion (1) and picked, then later it might be best under criterion (2) (or vice versa).
We obviously cannot take Z twice. How should we track this?
 - Even more confusingly, for some current time T , bushel Z might be the best under both criteria.
 - Initially, if we do not take anything, we have no deadlines to meet.
(Effectively, our deadline is at infinity.)
 - Conversely, when should we stop taking?