408-Fixed Income HW3

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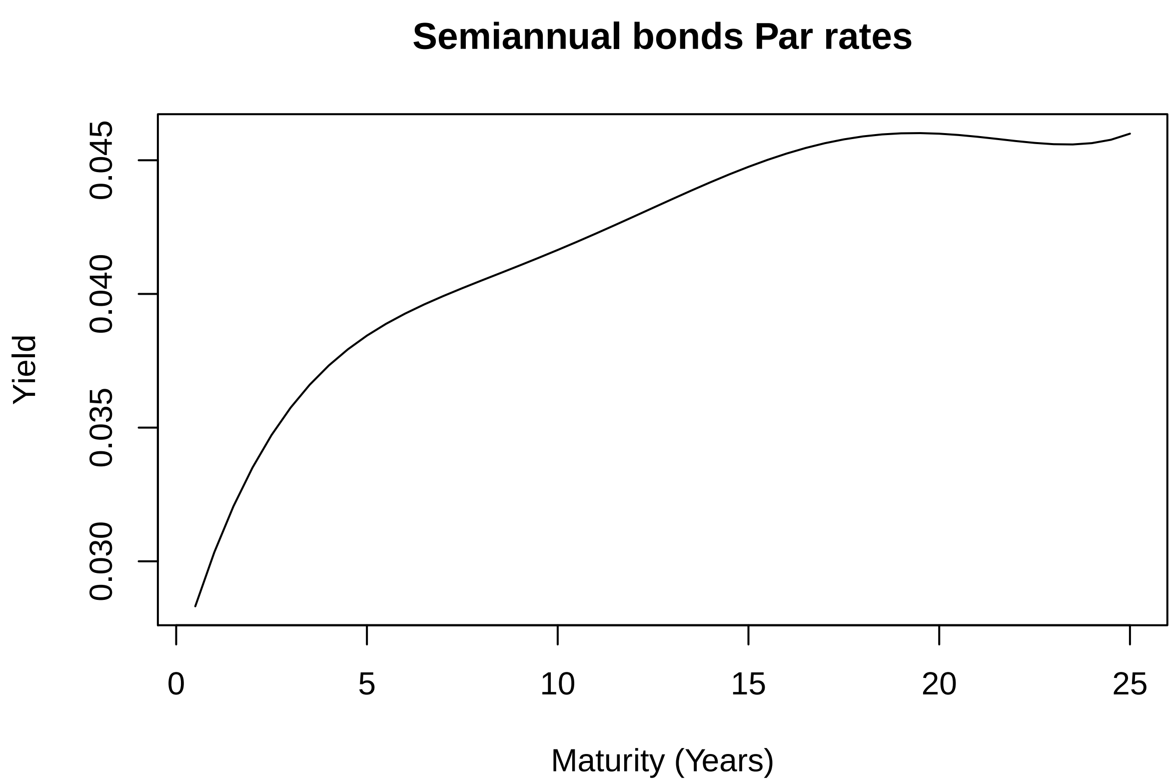
Problem 1

Use the zero-coupon curve you created in HW2 to compute the par rates for semiannual pay bonds with maturities ranging 1 year to 25 years.

```r
library(data.table)

setwd("C:/Users/Ming-Hao/Desktop/MFE/408-Fixed Income")
dt = fread(file="Homework 3 Data.csv", skip=3)
DT = dt$V2
y = 2*(DT^(-1/(2*seq(1,length(DT)))) - 1)
tList = seq(0.5, 25, 0.5)
#Q1
c = 2*(100-100*DT)/cumsum(DT)/100
plot(y=c, x=tList, xlab="Maturity (Years)", ylab="Yield", main="Semiannual bonds Par rates", type="l")
```




```r
c
```

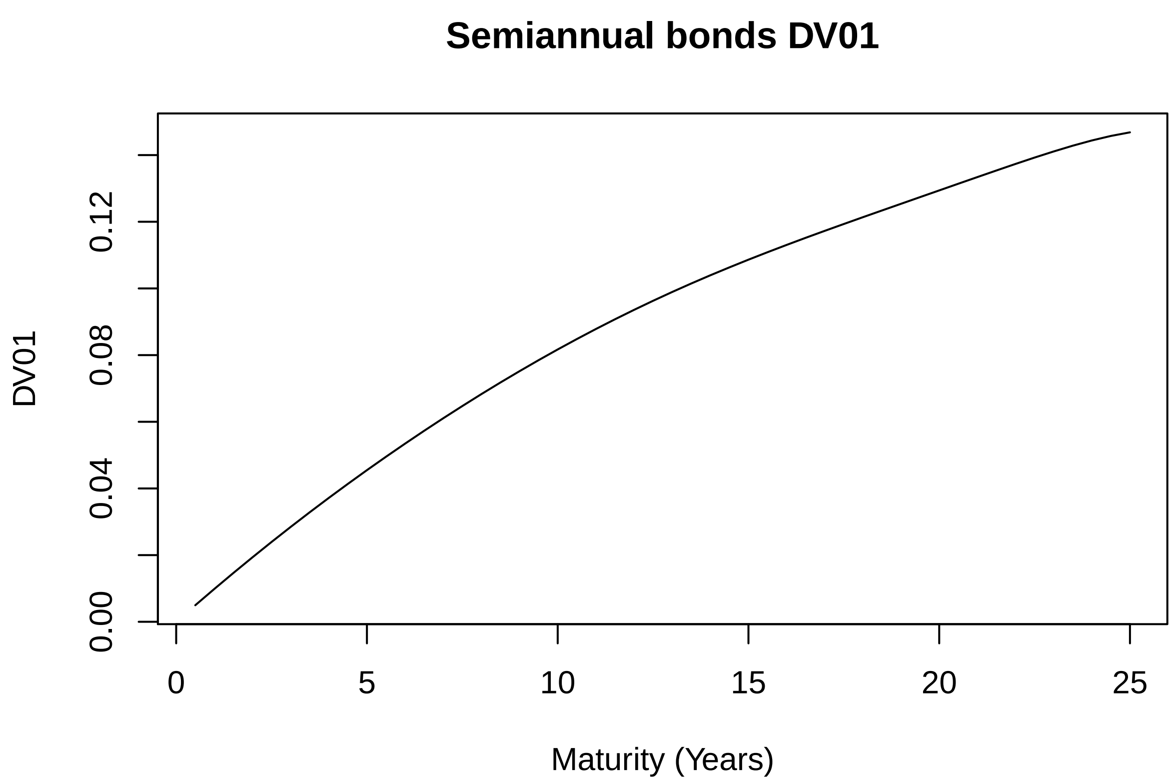
```
##  [1] 0.02831630 0.03033883 0.03205475 0.03350416 0.03472378 0.03574708
##  [7] 0.03660444 0.03732326 0.03792809 0.03844080 0.03888067 0.03926456
```

```
## [13] 0.03960700 0.03992039 0.04021506 0.04049946 0.04078027 0.04106253
## [19] 0.04134977 0.04164419 0.04194673 0.04225724 0.04257460 0.04289686
## [25] 0.04322140 0.04354500 0.04386403 0.04417457 0.04447253 0.04475380
## [31] 0.04501438 0.04525049 0.04545875 0.04563628 0.04578084 0.04589097
## [37] 0.04596612 0.04600677 0.04601461 0.04599261 0.04594521 0.04587841
## [43] 0.04579994 0.04571937 0.04564825 0.04560024 0.04559127 0.04563963
## [49] 0.04576614 0.04599427
```

Problem 2

For each of these bonds, compute their DV01.

```r
#Q2
DV01 = vector()
for(i in 1:50) {
    coupon = 100*c[i]
    ii = head(seq(1, i), -1)
    Price1 = sum(coupon/2*DT[ii]) + (100+coupon/2)*DT[i]
    y2 = 2*(DT^(-1/(2*seq(1,length(DT)))) - 1) + 0.0001/2
    DT2 = 1/(1+y2/2)^(2*seq(1,length(DT)))
    Price2 = sum(coupon/2*(DT2[ii])) + (100+coupon/2)*(DT2[i])
    DV01[i] = - (Price2 - Price1)
}

plot(y=DV01, x=tList, xlab="Maturity (Years)", ylab="DV01", main="Semiannual bonds DV01", type="l")
```



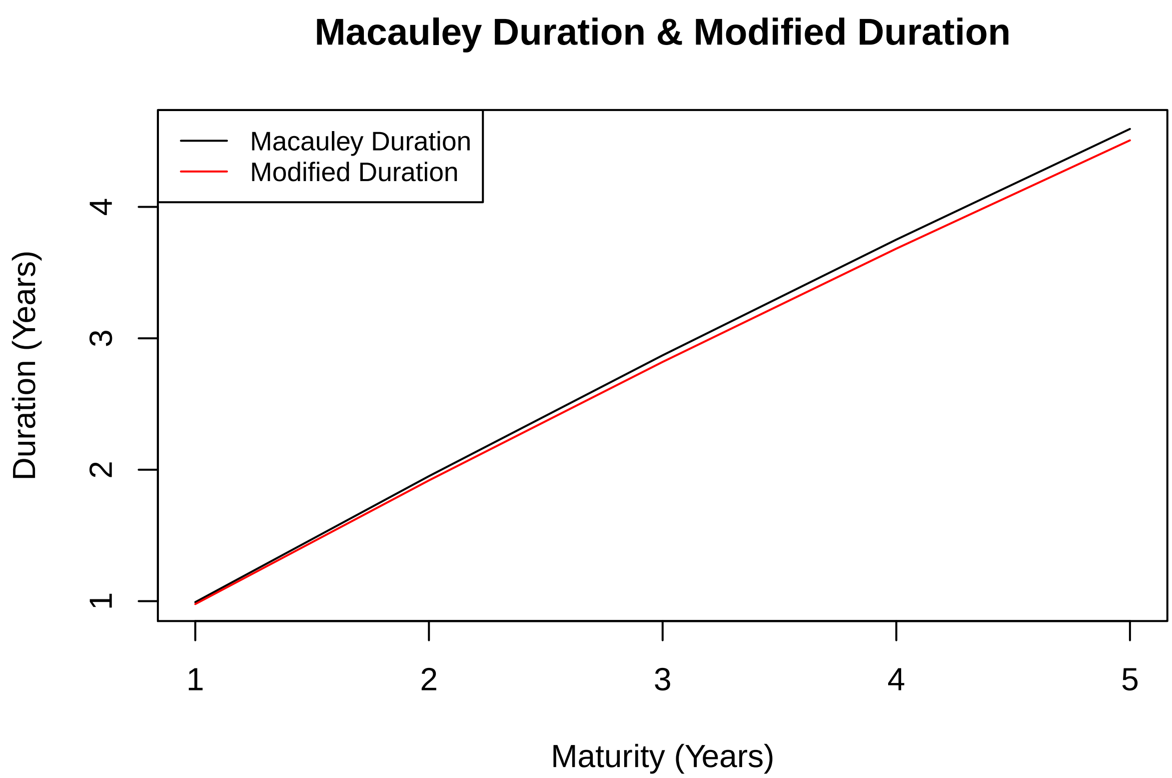
```
DV01
```

```
##  [1] 0.004964791 0.009850169 0.014646029 0.019346295 0.023947924
##  [6] 0.028450065 0.032853343 0.037159275 0.041369802 0.045486926
## [11] 0.049512442 0.053447757 0.057293785 0.061050899 0.064718948
## [16] 0.068297313 0.071785002 0.075180770 0.078483268 0.081691197
## [21] 0.084803472 0.087819387 0.090738768 0.093562118 0.096290743
## [26] 0.098926854 0.101473636 0.103935295 0.106317053 0.108625118
## [31] 0.110866594 0.113049358 0.115181873 0.117272962 0.119331512
## [36] 0.121366133 0.123384752 0.125394142 0.127399396 0.129403327
## [41] 0.131405807 0.133403033 0.135386725 0.137343253 0.139252686
## [46] 0.141087775 0.142812861 0.144382707 0.145741271 0.146820417
```

Problem 3

Compute the Macauley and modified for the 1,2,3,4 and 5 year bonds in problem 1 above.

```
#Q3
MC = vector()
for(i in 2:10){
    ii = seq(1,i-1)
    MC[i] = (sum( ii/2*100*c[i]/2*DT[ii] ) + i/2*(100+100*c[i]/2)*DT[i])/100
}
MD = MC/(1+c[1:10]/2)

Q3_MC = MC[seq(2,10,2)]
Q3_MD = MD[seq(2,10,2)]
names(Q3_MC) =  names(Q3_MD) = c("1 year", "2 year", "3 year", "4 year", "5 year")
plot(y=Q3_MC, x=seq(1,5), xlab="Maturity (Years)", ylab="Duration (Years)",
     main="Macauley Duration & Modified Duration", type="l")
points(y=Q3_MD, x=seq(1,5), type="l", col="red")
legend("topleft", c("Macauley Duration", "Modified Duration"), col=c("black", "red"), cex=0.8, lwd=1)
```




```
Q3_MC
```

```
##     1 year    2 year    3 year    4 year    5 year
## 0.9925212 1.9509823 2.8707737 3.7512106 4.5933551
```

```
Q3_MD
```

```
##     1 year    2 year    3 year    4 year    5 year
## 0.9776902 1.9188378 2.8203638 3.6824894 4.5067339
```

Problem 4

You have a $5,000,000 liability due in 3 years. How much do you need to invest in a 3 year zero-coupon bond to defease the liability? Use the same zero-coupon curve as in 1.

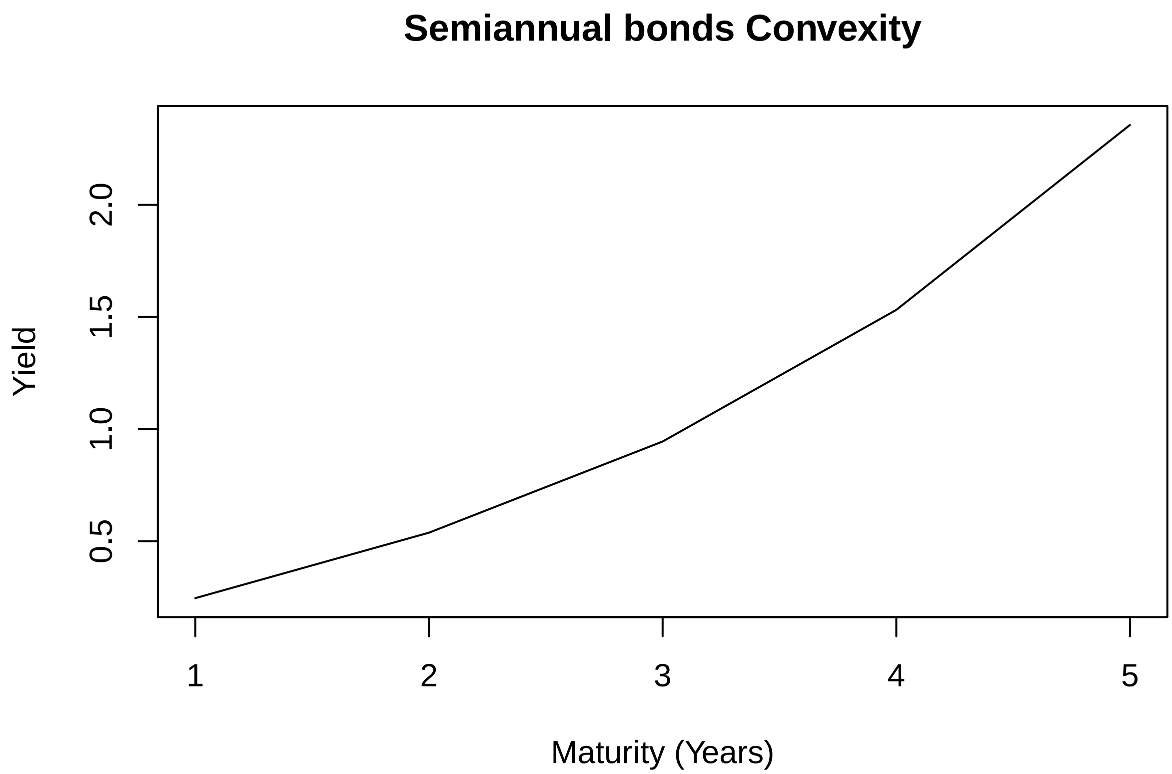
```
#Q4
L = 5e6*DT[6]
#Invest L in zero-coupon bond which gets paid $5000000 in 3rd year.
L
```

```
## [1] 4494051
```

Problem 5

Using the data in question 1, compute the convexities of the 1,2,3,4 and 5 year bonds.

```
#Q5
Convexity = vector()
for(i in 1:10){
    ii = head(seq(1,i),-1)
    Convexity[i] = (sum( ii*(ii+1)*100*c[i]/2*DT[ii] ) + i/2*(100+100*c[i]/2)*DT[i])/(1+c[i]/2)^2/2^2/1
}
Q5 = Convexity[seq(2,10,2)]
names(Q5) = c("1 year", "2 year", "3 year", "4 year", "5 year")
plot(y=Q5, x=seq(1,5), xlab="Maturity (Years)", ylab="Yield", main="Semiannual bonds Convexity", type="
```

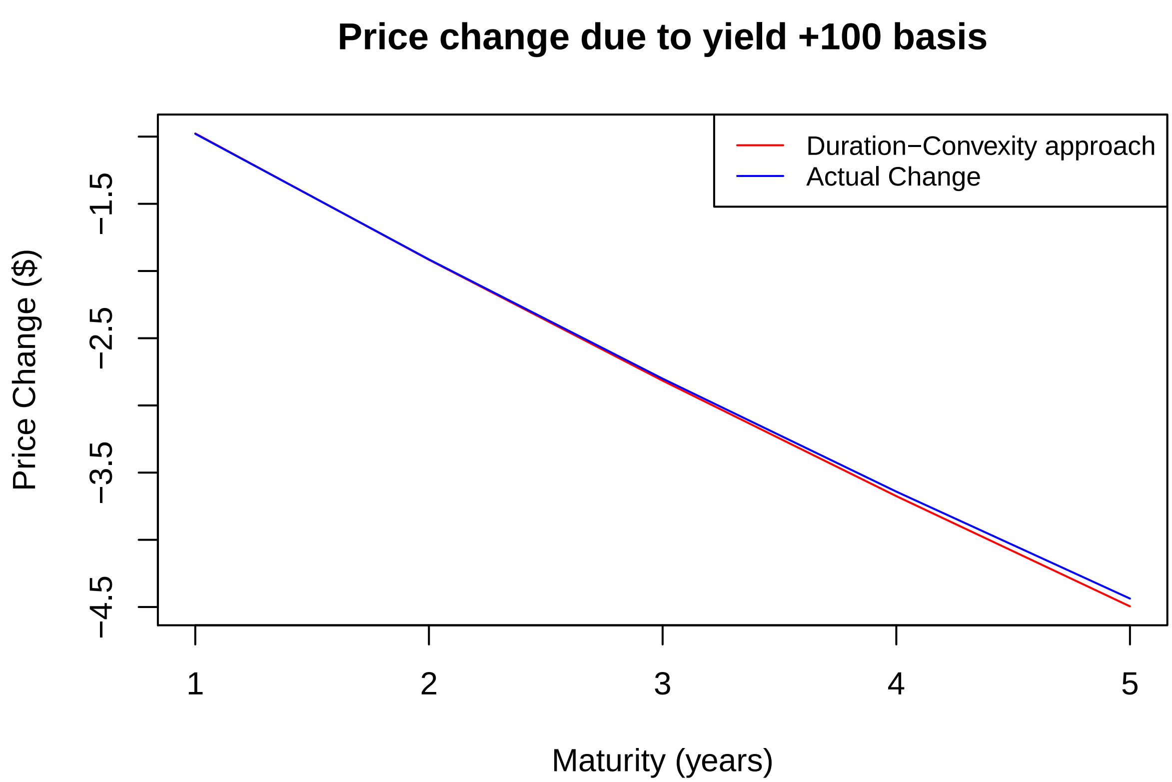


Problem 6

Use the computed dollar durations and convexities for the 1,2,3,4, and 5 year bonds, compute the price change of a 100 basis point upward and downeard parallel shift in the zero-curve. Compare the price changes with actual price change obtained by recomputing the price of the bond from shifted spot curve.

```
#Q6
#dP = -P*MD*dr + 0.5*P*Convexity*dr^2
dp_upward = -100*Q3_MD*100/10000 + 0.5*100*Q5*(100/10000)^2
dp_downward = -100*Q3_MD*(-100)/10000 + 0.5*100*Q5*(-100/10000)^2
dp_upward_actual = dp_downward_actual = dp1 = dp2 = vector()
for(i in 1:10) {
    coupon = 100*c[i]
    ii = head(seq(1, i), -1)
    y2 = 2*(DT^(-1/(2*seq(1,length(DT)))) - 1) + 0.01/2
```

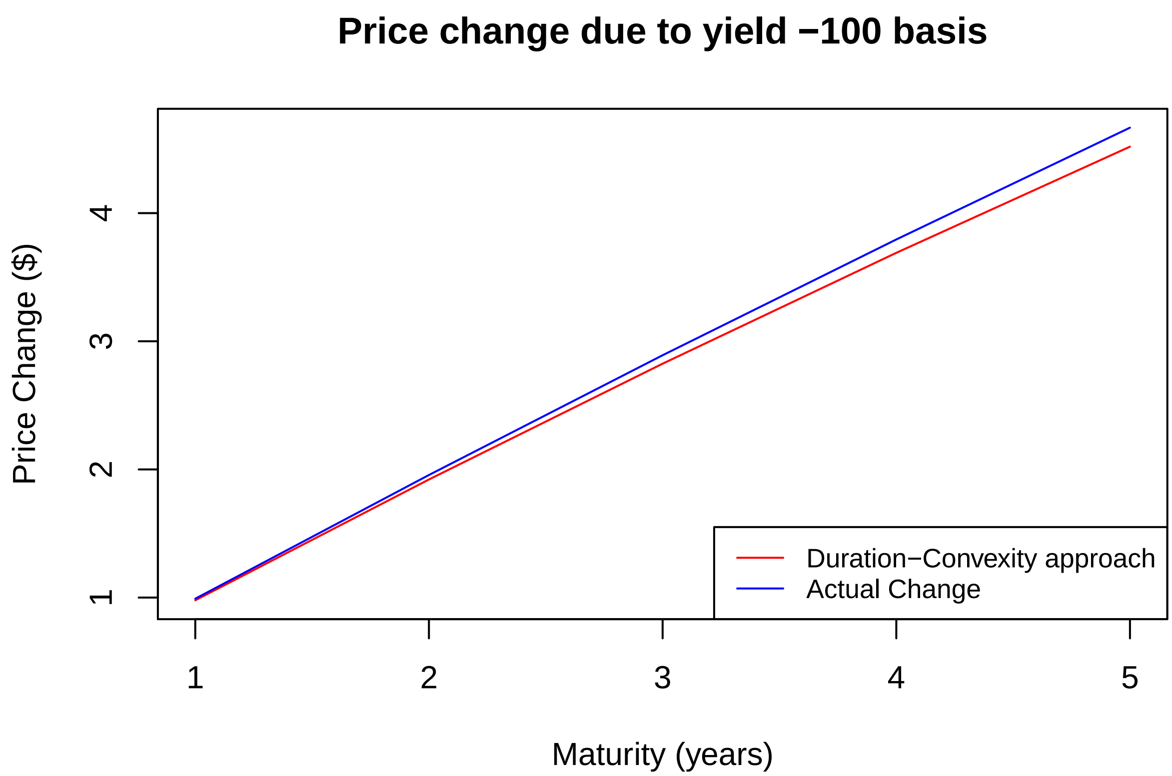
```
    DT2 = 1/(1+y2/2)^(2*seq(1,length(DT)))
    y3 = 2*(DT^(-1/(2*seq(1,length(DT)))) - 1) - 0.01/2
    DT3 = 1/(1+y3/2)^(2*seq(1,length(DT)))
    Price2 = sum(coupon/2*(DT2[ii])) + (100+coupon/2)*(DT2[i])
    Price3 = sum(coupon/2*(DT3[ii])) + (100+coupon/2)*(DT3[i])
    dp1[i] = Price2 - 100
    dp2[i] = Price3 - 100
}
dp_upward_actual = dp1[seq(2,10,2)]
dp_downward_actual = dp2[seq(2,10,2)]

ylim = c(min(dp_upward, dp_upward_actual),
         max(dp_upward, dp_upward_actual))
plot(y=dp_upward, x=seq(1,5), main="Price change due to yield +100 basis",
     xlab="Maturity (years)", ylab="Price Change ($)", type="l", col="red", ylim=ylim)
points(y=dp_upward_actual, x=seq(1,5), type="l", col="blue")
legend("topright", c("Duration-Convexity approach", "Actual Change"), col=c("red", "blue"), cex=0.8, lwd
```




```
Q6_up = matrix(nrow=2, ncol=length(dp_upward), c(dp_upward, dp_upward_actual), byrow=T)
colnames(Q6_up) = c("1 year", "2 year", "3 year", "4 year", "5 year")
rownames(Q6_up) = c("Duration-Convexity", "Actual Change")
Q6_up
```

```
##                       1 year    2 year    3 year    4 year    5 year
## Duration-Convexity -0.9764591 -1.916148 -2.815641 -3.674831 -4.494953
## Actual Change      -0.9790157 -1.913631 -2.800872 -3.641351 -4.437133
```

```
ylim = c(min(dp_downward, dp_downward_actual),
         max(dp_downward, dp_downward_actual))
plot(y=dp_downward, x=seq(1,5), main="Price change due to yield -100 basis",
     xlab="Maturity (years)", ylab="Price Change ($)", type="l", col="red", ylim=ylim)
points(y=dp_downward_actual, x=seq(1,5), type="l", col="blue")
legend("bottomright", c("Duration-Convexity approach", "Actual Change"), col=c("red", "blue"), cex=0.8,
```




```
Q6_down = matrix(nrow=2, ncol=length(dp_downward), c(dp_downward, dp_downward_actual), byrow=T)
colnames(Q6_down) = c("1 year", "2 year", "3 year", "4 year", "5 year")
rownames(Q6_down) = c("Duration-Convexity", "Actual Change")
Q6_down
```

```
##                       1 year   2 year   3 year   4 year   5 year
## Duration-Convexity 0.9789213 1.921528 2.825087 3.690148 4.518515
## Actual Change      0.9912002 1.956407 2.891077 3.794279 4.666650
```

We can observe that the Duration-Convexity approximation will deviate with the actual price change as the maturity goes longer. Since the Duration-Convexity is the first and second Taylor expansion of the bond price to the par rate, there must be some error. And the longer the maturity is a bond, the larger the effect will be to the price of the bond, then the error between approximating calculation and the real value will increase.

Yahoo Junk Bond Case

Our goal is to construct a bond portfolio consisting of 10-year Treasury Bond and 1-year Treasury Bill that has same DV01 of the Yahoo Junk Bond.

```r
c_Bond = 0.05
c_Bill = 0
c_Yahoo = 0.11

yield_Bond = 0.05 #par rate
yield_Bill = 0.0075
yield_Yahoo = 0.08

Price_Bond = 100
Price_Bill = 100/(1+yield_Bill)
Price_Yahoo = sum(c_Yahoo*100/2/(1+yield_Yahoo/2)^seq(1,13)) + (100+c_Yahoo*100/2)/(1+yield_Yahoo/2)^14

Case = matrix(nrow=3, ncol=3, c(c_Bond, c_Bill, c_Yahoo, yield_Bond, yield_Bill,
                                yield_Yahoo, Price_Bond, Price_Bill, Price_Yahoo), byrow=T)
colnames(Case) = c("Bond", "Bill", "Yahoo")
rownames(Case) = c("Coupon", "Yield", "Price")
Case
```

```
##         Bond     Bill    Yahoo
## Coupon 5e-02  0.00000   0.1100
## Yield  5e-02  0.00750   0.0800
## Price  1e+02 99.25558 115.8447
```

After calculation, we get DV01 for Bond, Bill and Yahoo Junk Bond, then we use DV01 to calculate the composition of portfolio. For both cases of +/-100 basis point of the yield, we recalculate the portfolio value and the capital gain of the portfolio. We attach the result in a word file.