1.

$$\boxed{\begin{array}{l}\text{(SGD)} \\ \theta^{k+1} = \theta^{k} - \alpha \nabla J \qquad \text{where } J = (y - h(x, \theta^{k}))^2\end{array}}$$

Consider $h(x, \theta^k) = \sigma(b + w_1 x_1 + w_2 x_2)$, σ is sigmoid function

$$\Rightarrow \theta^{k+1} = \theta^{k} - \alpha \nabla J$$

$$= \theta^{k} - 2\alpha (y - h(x, \theta^{k})) \cdot \nabla(-h(x, \theta^{k}))$$

$$= \theta^{k} + 2\alpha (y - h(x, \theta^{k})) \cdot \underbrace{\nabla h(x, \theta^{k})}_{= \sigma(z) \cdot \begin{pmatrix} 1 \\ x_1 \\ x_2 \end{pmatrix}}$$

and $z := b + w_1 x_1 + w_2 x_2$

$\therefore z^0 = 4 + 5 \cdot 1 + 6 \cdot 2 = 21$

$$\therefore \theta^{1} = \begin{pmatrix} b^1 \\ w_1^1 \\ w_2^1 \end{pmatrix} = \begin{pmatrix} 4 + 2\alpha(3 - \sigma(z^0))\sigma(z^0) \cdot 1 \\ 5 + 2 \cdot \alpha \cdot (3 - \sigma(z^0))\sigma(z^0) \cdot 1 \\ 6 + 2 \cdot \alpha (3 - \sigma(z^0) \cdot \sigma(z^0) \cdot 2 \end{pmatrix} \#$$

2. (a)

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

$$\Rightarrow \frac{d}{dx}\sigma(x) = -(1+e^{-x})^{-2} \cdot (e^{-x})'$$
$$= (1+e^{-x})^{-2} \cdot e^{-x}$$
$$= \frac{e^{-x}}{(1+e^{-x})^2}$$
$$= \sigma(x) \cdot \frac{e^{-x}+1-1}{1+e^{-x}}$$
$$= \sigma(x)(1-\sigma(x))$$

$$\Rightarrow \frac{d^2}{dx^2}\sigma(x) = \sigma'(x)(1-\sigma(x)) + \sigma(x)(1-\sigma(x))'$$
$$= \sigma(x)(1-\sigma(x))^2 - \sigma(x)\cdot\sigma(x)(1-\sigma(x))$$
$$= \sigma(x)(1-\sigma(x))(1-\sigma(x)-\sigma(x))$$
$$= \sigma(x)(1-\sigma(x))(1-2\sigma(x))$$

$$\Rightarrow \frac{d^3}{dx^3}\sigma(x) = \underbrace{[\sigma(x)(1-\sigma(x))]'}_{=\frac{d^2}{dx^2}\sigma(x)}(1-2\sigma(x)) + \sigma(x)(1-\sigma(x))(1-2\sigma(x))'$$
$$= \sigma(x)(1-\sigma(x))(1-2\sigma(x))(1-2\sigma(x)) - 2\sigma(x)(1-\sigma(x))\cdot\sigma(x)(1-\sigma(x))$$
$$= \sigma(x)(1-\sigma(x))[(1-2\sigma(x))(1-2\sigma(x)) - 2\sigma(x)(1-\sigma(x))]$$
$$= \sigma(x)(1-\sigma(x))(1-4\sigma(x)+4\sigma^2(x) - 2\sigma(x) + 2\sigma(x))$$
$$= \sigma(x)(1-\sigma(x))(1-6\sigma(x)+6\sigma^2(x))$$

$\vdots$

2(b)

$$\sigma(x) = \frac{1}{1+e^{-x}} \qquad \sinh(x) = \frac{e^{x}-e^{-x}}{2} \qquad \cosh(x) = \frac{e^{x}+e^{-x}}{2}$$

$$\tanh(x) = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

where $\tanh(x) = \dfrac{(e^{x}-e^{-x})\cdot\frac{1}{e^{x}}}{(e^{x}+e^{-x})\cdot\frac{1}{e^{x}}}$

$$= \frac{1-e^{-2x}}{1+e^{-2x}}$$

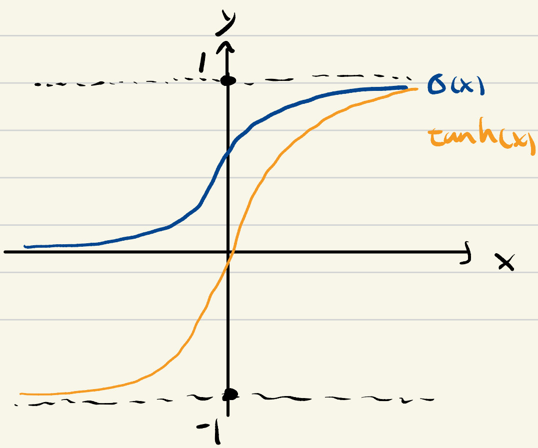
$$= \frac{-(e^{-2x}+1)+1+1}{1+e^{-2x}}$$

$$= -1 + \frac{2}{1+e^{-2x}}$$

$$= 2\cdot\sigma(2x) - 1$$

$$= 2\left(\sigma(2x) - \tfrac{1}{2}\right)$$

∴ $\tanh(x)$ 是由 $\sigma(x)$“伸缩平移”得来的



由函数图形也有感觉！