P85. 4

(1) $(\forall x)(P(x) \to (\exists y)Q(x,y))$

$\Leftrightarrow (\forall x)(\neg P(x) \vee (\exists y)Q(x,y)) \Leftrightarrow (\forall x)(\exists y)(\neg P(x) \vee Q(x,y))$

(3) $(\exists x)P(x,y) \leftrightarrow (\forall z)Q(z) \Leftrightarrow (\forall x)\neg P(x,y) \wedge (\exists z)\neg Q(z) \vee (\exists x)P(x,y) \wedge (\forall z)Q(z)$

$\Leftrightarrow (\forall x)(\exists z)(\neg P(x,y) \wedge \neg Q(z)) \vee (\exists x)P(x,y) \wedge (\forall z)Q(z).$

$\Leftrightarrow (\forall x)(\exists z)(\forall a)(\exists b)(\neg P(x,y) \wedge \neg Q(z) \vee P(b,y) \wedge Q(a))$

(5) $(\forall x)(P(x) \to (\forall y)((P(y) \to (Q(x) \to Q(y))) \vee (\forall z)P(z)))$

$\Leftrightarrow (\forall x)(\neg P(x) \vee (\forall y)(\neg P(y) \vee (\neg Q(x) \vee Q(y))) \vee (\forall z)P(z)).$

$\Leftrightarrow (\forall x)(\forall y)(\forall z)(\neg P(x) \vee \neg P(y) \vee \neg Q(x) \vee Q(y) \vee P(z).)$

(9) $(\forall x)(P(x) \to (\exists y)Q(x,y)) \vee (\forall z)R(z)$

$\Leftrightarrow (\forall x)(\neg P(x) \vee (\exists y)Q(x,y)) \vee (\forall z)P(z)$

$\Leftrightarrow (\forall x)(\exists y)(\neg P(x) \vee Q(x,y)) \vee (\forall z)P(z)$

$\Leftrightarrow (\forall x)(\forall z)(\neg P(x) \vee Q(x, f(x)) \vee R(z))$

P88. 5

(1) $G = (\forall x)(P(x) \vee Q(x)) \wedge (\forall x)(\neg Q(x) \vee \neg R(x)) \wedge (\forall x)(\neg P(x) \wedge R(x))$

$G^* = (\forall x)(P(x) \vee Q(x)) \wedge (\forall x)(\neg Q(x) \vee \neg R(x)) \wedge (\forall x)(\neg P(x) \wedge R(x))$

$\Rightarrow S = \{P(x) \vee Q(x), \neg Q(x) \vee \neg R(x), \neg P(x), R(x)\}.$

(1) ~~$\neg P(x) \wedge R(x)$~~ $\neg P(x)$ (7) ϕ (5)(6)归结,

(2) $P(x) \vee Q(x)$

(3) $Q(x)$ (1)(2)归结.

(4) $\neg Q(x) \vee \neg R(x)$

(5) $\neg R(x)$ (3)(4)归结.

(6) $R(x)$

P86 5

② $(\forall x)(\neg P(x)\to Q(x)) \land (\forall x)\neg Q(x) \Rightarrow P(a)$.

(1) $(\forall x)(\neg P(x)\to Q(x))$ 前提

(2) $\neg P(x)\to Q(x)$ (1) $\forall -$

(3) $(\forall x)\neg Q(x)$ 前提

(4) $\neg Q(x)$ (3) $\forall -$

~~(5) $P(x)$ (2)(4)分离.~~

(5) $Q(x)\lor P(x)$ (2)置换.

(6) $P(x)$ (4)(5)分离.

(7) $(\forall x)P(x)$ (6) $\forall +$.

(8) $P(a)$ (7)代入 $x=a$.

③ $(\forall x)(P(x)\lor Q(x))\land(\forall x)(Q(x)\to\neg R(x))\land(\forall x)R(x)\Rightarrow(\forall x)P(x)$.

$G=(\forall x)(P(x)\lor Q(x))\land(\forall x)(Q(x)\to\neg R(x))\land(\forall x)R(x)\land(\exists x)\neg P(x)$.

$\therefore S=\{P(x)\lor Q(x), \neg Q(x)\lor\neg R(x), R(x), \neg P(a)\}$.

(1) $P(x)\lor Q(x)$

(2) $\neg P(a)$

(3) $Q(a)$ (1)(2)归结.

(4) $\neg Q(x)\lor\neg R(x)$

(5) $\neg R(a)$ (3)(4)归结

(6) $R(x)$

(7) $\emptyset$ (5)(6)归结.

④ ~~P(x): x是学生; Q(x): x是本科生, R(x): x是研究生~~; S(x): x是农村生

$\therefore(\forall x)(P(x)\to(Q(x)\lor R(x)))\land(\exists x)(P(x)\to S(x))\land\neg R(John)\land S(John)$ ~~$(S(x)\land P(x))$~~

$\Rightarrow P(John)\to Q(a)$

(1) $(\forall x)(P(x)\to(Q(x)\lor R(x)))$ 前提

~~(2) $P(x)\to(Q(x)\lor R(x))$ (1) $\forall -$~~

(3) $(\exists x)(P(x)\to S(x))$ ~~$(S(x)\land P(x))$~~ 前提

(4) $\neg P(x)\lor(Q(x)\lor R(x))$ (2)置换.

~~(5) $S(John)\land P(John)$ (3) $\exists -$~~

(5) $\exists x(\neg P(x)\lor S(x))$ (3)置换

(6) $\neg P(a)\lor$ ~~$S(a)$ (5)E-~~ $\exists$ John=a

(7)

④ $P(x)$：x是学生；$Q(x)$：x是本科生；$R(x)$：x是研究生；$S(x)$：x是高材生．

$\therefore (\forall x)(P(x)\to(Q(x)\bar{\vee}R(x)))\wedge(\exists x)(P(x)\to S(x))\wedge\neg R(a)\wedge S(a)$

~~$(A=(\forall x)(P(x)\to(Q(x)$~~ $\vdash P(a)\to Q(a)$

~~$(A=(\forall x)$~~

(1) $(\forall x)(P(x)\to(Q(x)\bar{\vee}R(x)))$ 前提．

(2) $P(x)\to(Q(x)\bar{\vee}R(x))$ (1) $\forall-$

(3) $\neg P(x)\vee(\neg Q(x)\wedge R(x))\vee(\neg R(x)\wedge Q(x))$ (2)置换．

(4) $(\exists x)(P(x)\to S(x))$ 前提．

(5) $(\exists x)(\neg P(x)\vee S(x))$ (4)置换．

(6) $\neg P(a)\vee(\neg Q(a)\wedge R(a))\vee(\neg R(a)\wedge Q(a))$ (3)代入．

(7) $\neg R(a)$ 前提．

(8) $\neg P(a)\vee Q(a)$ (6)(7)分离．

(9) $P(a)\to Q(a)$ (8)置换．

①

$(\forall x)(P(x)\vee Q(x)),\ (\forall x)(\neg Q(x)\vee\neg R(x)) \vdash (\exists x)(\cancel{P(x)\vee\neg R(x)}\ R(x)\to P(x))$.

(1) $(\forall x)(P(x)\vee Q(x))$ 前提.

(2) $(\forall x)(\neg Q(x)\vee\neg R(x))$ 前提.

(3) $\neg P(x)\to Q(x)$ (1) $\forall-$

(4) $Q(x)\to\neg R(x)$ (2) $\forall-$

(5) $\neg P(x)\to\neg R(x)$ (3)(4)三段论.

(6) $P(x)\vee\neg R(x)$ (5)置换.

(7) $R(x)\to P(x)$ (6)置换.

(8) $(\exists x)(R(x)\to P(x))$ $\exists+$

③

$(\forall x)(P(x)\vee Q(x)),\ (\forall x)(Q(x)\to\neg R(x)),\ (\forall x)R(x) \vdash (\forall x)P(x)$

(1) $(\forall x)(P(x)\vee Q(x))$ 前提

(2) $(\forall x)(Q(x)\to\neg R(x))$ 前提

(3) $(\forall x)R(x)$ 前提.

(4) $\neg Q(x)\vee\neg R(x)$ (2) $\forall-$

(5) $R(x)$ (3) $\forall-$

(6) $\neg Q(x)$ (4)(5)分离

(8) $P(x)\vee Q(x)$ (1) $\forall-$

(9) $P(x)$ (6)(8)分离.

(10) $(\forall x)P(x)$ (9) $\forall+$

② $(\forall x)(\neg P(x)\to Q(x))\wedge(\forall x)\neg Q(x)\vdash P(a)$.

$G=(\forall x)(\neg P(x)\to Q(x))\wedge(\forall x)\neg Q(x)\wedge\neg P(a)$.

$\quad=(\forall x)(P(x)\vee Q(x))\wedge(\forall x)\neg Q(x)\wedge\neg P(a)$

$\Rightarrow S=\{P(x)\vee Q(x),\ \neg Q(x),\ \neg P(a)\}$.

(1) $P(x)\vee Q(x)$

(2) $\neg Q(x)$

(3) $P(x)$ (1)(2)归结

(4) $P(a)$ (3)代入

(5) $\cancel{\emptyset}\ \neg P(a)$

(6) $\emptyset$ (4),(5)归结