117.50

(3) $f(x,y) = y^2 - (x^2+x^4)y + x^6$

$\frac{\partial f}{\partial x} = -y(2x+4x^3) + 6x^5 = 0$ $\quad \frac{\partial f}{\partial y} = 2y - (x^2+x^4) = 0$

$\Rightarrow \begin{cases} x = \pm 1 \\ y = 1 \end{cases} \quad \begin{cases} x = \pm \frac{\sqrt{2}}{2} \\ y = \frac{3}{8} \end{cases}$

$\frac{\partial^2 f}{\partial x^2} = 30x^4 - y(2+12x^2) \quad \frac{\partial^2 f}{\partial y^2} = 2$

$\frac{\partial^2 f}{\partial x \partial y} = -2x - 4x^3$

对 $\begin{cases} x=1 \\ y=1 \end{cases}$ $H = \begin{vmatrix} 16 & -6 \\ -6 & 2 \end{vmatrix}$ $\therefore f(1,1)$ ~~极小=0~~ 不是.

对 $\begin{cases} x=-1 \\ y=1 \end{cases}$ $H = \begin{vmatrix} 16 & 6 \\ 6 & 2 \end{vmatrix}$ $\therefore f(-1,1)$ 不是.

对 $\begin{cases} x=\frac{\sqrt{2}}{2} \\ y=\frac{3}{8} \end{cases}$ $H = \begin{vmatrix} \frac{9}{2} & -2\sqrt{2} \\ -2\sqrt{2} & 2 \end{vmatrix} = 1$ $\therefore f(\frac{\sqrt{2}}{2}, \frac{3}{8})$ 是极小 $= -\frac{1}{64}$

对 $\begin{cases} x=-\frac{\sqrt{2}}{2} \\ y=\frac{3}{8} \end{cases}$ $H = \begin{vmatrix} \frac{9}{2} & 2\sqrt{2} \\ 2\sqrt{2} & 2 \end{vmatrix}$ 是 $\therefore f(-\frac{\sqrt{2}}{2}, \frac{3}{8})$ 是极小 $= -\frac{1}{64}$

(4) $\cancel{\frac{\partial f}{\partial y}}$ $\frac{\partial f}{\partial x}=y-\frac{50}{x^2}$ $\frac{\partial f}{\partial y}=x-\frac{20}{y^2}$

$\Rightarrow \begin{cases} x=5 \\ y=2 \end{cases}$ $\frac{\partial^2 f}{\partial x^2}=\frac{100}{x^3}$ $\frac{\partial^2 f}{\partial y^2}=\frac{40}{y^3}$ $\frac{\partial^2 f}{\partial x\partial y}=1$

$\therefore H=\begin{vmatrix} \frac{4}{5} & 1 \\ 1 & 5 \end{vmatrix}>0$ $\therefore f(5,2)_{\cancel{\min}}=30$

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(2) $\frac{\partial f}{\partial x}=\cos x-\cos(x+y)\cancel{(1+y)}=0$ $\frac{\partial f}{\partial y}=\cos y-\cos(x+y)\cancel{x}=0$

$\Rightarrow \begin{cases} x=0 \\ y=0 \end{cases}$ $\cancel{\begin{cases} x=\pi \\ y=\pi \end{cases}}$ $\cancel{\begin{cases} x=\pi \\ y=0 \end{cases}}$ $\cancel{\begin{cases} x=0 \\ y=\pi \end{cases}}$ $\begin{cases} x=\frac{2}{3}\pi \\ y=\frac{2}{3}\pi. \end{cases}$

$\frac{\partial^2 f}{\partial x^2}=-\sin x+\sin(x+y)\cancel{(1+y)}^2=0$ $\frac{\partial^2 f}{\partial y^2}=-\sin y+\cancel{\sin(x+y)})\sin(x+y)=0$

$\frac{\partial^2 f}{\partial x\partial y}=A$ $\therefore H=-A^2<0$ $\therefore$ 在D内点无

在∂D上.

$x=0$时. $f(0,y)=0$. $y=0$时 $f(x,0)=0$ $\therefore f(x,y)_{min}=0$

在 $x+y=2\pi$ 上 $f(x,y)=\sin x-\sin x-0=0$

在 $(\frac{2}{3}\pi,\frac{2}{3}\pi)$时 $H=\begin{vmatrix} -\sqrt{3} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\sqrt{3} \end{vmatrix}>0$ $\therefore f(\frac{2}{3}\pi,\frac{2}{3}\pi)=\frac{3}{2}\sqrt{3}$ 最大.

$\therefore f(x,y)_{max}=\frac{3}{2}\sqrt{3}$, $f(x,y)_{min}=0$.

117.54

(2) $2x^2+2y^2+z^2+8xz-z+8=0$ 求极值

$\therefore\ 4x\,dx+4y\,dy+2z\,dz+8x\,dz+8z\,dx-dz=0$

$\Rightarrow dz=\frac{1}{2z+8x-1}\left((-4x-8z)dx-4y\,dy\right)$

$\therefore\ \frac{\partial z}{\partial x}=-\frac{4x+8z}{2z+8x-1}=0\ ,\ \frac{\partial z}{\partial y}=-\frac{4y}{2z+8x-1}=0$

$\Rightarrow \begin{cases}x=-2\\y=0\\z=1\end{cases}$ 或 $\begin{cases}x=\frac{16}{7}\\y=0\\z=-\frac{8}{7}\end{cases}$ $\quad \frac{\partial^2 z}{\partial x^2}=-\frac{(4+8\frac{\partial z}{\partial x})(2z+8x-1)-(2\frac{\partial z}{\partial x}+8)(4x+8z)}{(2z+8x-1)^2}$

$\frac{\partial^2 z}{\partial x\partial y}=-\frac{8\frac{\partial z}{\partial y}(2z+8x-1)-(4x+8z)(2\frac{\partial z}{\partial y})}{(2z+8x-1)^2}\quad \frac{\partial^2 z}{\partial y^2}=-\frac{4(2z+8x-1)-4y(2\frac{\partial z}{\partial y})}{(2z+8x-1)^2}$

对于 $\begin{cases}x=-2\\y=0\\z=1\end{cases}$ $H=\begin{vmatrix}-\frac{4}{15} & 0\\ 0 & -\frac{4}{15}\end{vmatrix}$ $\therefore$ $\cancel{f_{1}\ z_{max}}$

$z_{极大}=z(-2,0)=1$

对于 $\begin{cases}x=\frac{16}{7}\\y=0\\z=-\frac{8}{7}\end{cases}$ $H=\begin{vmatrix}-\frac{4}{2z+8x-1} & 0\\ 0 & -\frac{4}{2z+8x-1}\end{vmatrix}$ $\therefore\ z_{极小}=z(\frac{16}{7},0)=-\frac{8}{7}$.