P19. 1

(2) $\frac{i}{-1+2-3i} = \frac{i}{1-3i} = \frac{i-3}{10}$ ∴ $\mathrm{Re}\,z = -\frac{3}{10}$, $\mathrm{Im}\,z = \frac{1}{10}$

$|z| = \frac{\sqrt{10}}{10}$, $\arg z = \arctan\frac{-1}{3} + \pi$ $\quad \bar{z} = \frac{-3-i}{10}$

(4) $1+i = \sqrt{2}e^{i\frac{\pi}{4}}$, $1-i = \sqrt{2}e^{i(-\frac{\pi}{4})}$

∴ $z = 2^{50}e^{i25\pi} + 2^{50}e^{i(-25\pi)} = -2^{51}$ ∴ $\cancel{\mathrm{Im}\,z = -2^{51}}$

$\mathrm{Re}\,z = -2^{51}$ $\mathrm{Im}\,z = 0$, $|z| = 2^{51}$ $\arg z = \pi$. $\bar{z} = -2^{51}$.

(6) $(1+\sqrt{3}i)\frac{1}{2} = e^{i\frac{\pi}{3}}$ ∴ $z = e^{i\frac{5}{3}\pi} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

∴ $\mathrm{Re}\,z = \frac{1}{2}$, $\mathrm{Im}\,z = -\frac{\sqrt{3}}{2}$, $|z| = 1$. $\arg z = -\frac{\pi}{3}$, $\bar{z} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$.

P20. 2

(1) $(2-3i)(-2+i) = -4+6i+2i+3 = -1+8i$

$r = \sqrt{1+64} = \sqrt{65}$ $\Rightarrow z = \sqrt{65}(\cos\theta + i\sin\theta)$ $\theta = \arccos -\frac{1}{\sqrt{65}}$

$z = \sqrt{65}\, e^{i(\arctan -8 + \pi)}$.

(4) $\frac{1-i\tan\theta}{1+i\tan\theta} = \frac{1-\tan^2\theta - 2i\tan\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta - 2i\cos\theta\sin\theta$.

$= (\cos\theta - i\sin\theta)^2$

令 $\cos\theta - i\sin\theta = z$ ∴ $z = e^{i(-\theta)}$

$z^2 = e^{i(-2\theta)} = Y$ ∴ $Y = \cos2\theta - i\sin2\theta$

P20. 3

(2) $\sqrt{3} - i = 2e^{i(-\frac{\pi}{6})}$ ∴原式 $= 2^{12}e^{i(-2\pi)} = 2^{12}(\cos2\pi - i\sin2\pi) = 2^{12}$

(4) $1^{\frac{1}{5}} = \{1, e^{i\frac{2\pi}{5}}, e^{i\frac{4\pi}{5}}, e^{i\frac{6\pi}{5}}, e^{i\frac{8\pi}{5}}\}$.

(8) $\omega = \rho e^{i\varphi}$, $\omega^6 = 64 \Rightarrow \rho^6 e^{i6\varphi} = 64e^{i(0+2\pi k)}$.

∴ $\rho^6 = 64$, $6\varphi = 2\pi k \Rightarrow \rho = \pm 2$, $\varphi = \frac{\pi}{3}k$.

∴ $\omega = \pm 2e^{i\frac{\pi}{3}k}$, $k \in \mathbb{Z}$. θ只取主值，有

$\omega = \pm 2$, $1 \pm \sqrt{3}i$, $-1 \pm \sqrt{3}i$

P20. 2(4)

(1) 令 $z=x+iy$

$\therefore\ x^2+2xyi-y^2-3[x-y+(x+y)i]+5i=0$

$\Rightarrow \begin{cases} x^2-y^2-3x+3y=0 \\ 2xy-3(x+y)+5=0 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=2 \end{cases}$ 或 $\begin{cases} x=2 \\ y=1 \end{cases}$

$\therefore$ ~~$z=\sqrt{5}+\sqrt{5}i$~~ $z=1+2i\ /\ 2+i$

P20. 6

(2). $x=a\cos t,\ y=b\sin t\quad \therefore \frac{x^2}{a^2}+\frac{y^2}{b^2}=1\quad \therefore$ 椭圆.

(3). $x=t,\ y=\frac{1}{t}\quad \therefore\ xy=1$ 双曲线

P20. 7

(2) $z=x+iy \Rightarrow z^2=x^2-y^2+2xyi$

$\therefore\ x^2-y^2=a^2$ 双曲线 $(a\neq0)$. $(x-y)(x+y)=0.\ (a=0)$ 直线

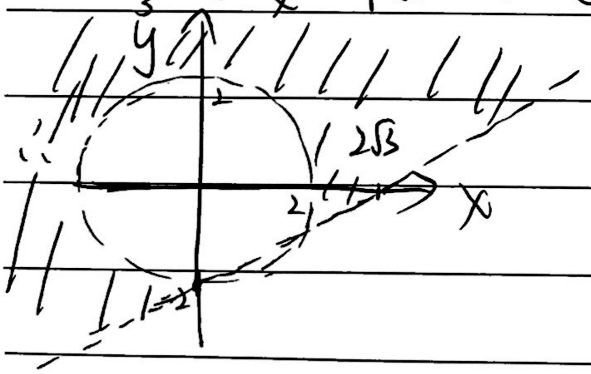
(3) $z=x+iy \Rightarrow \left|\frac{(x-a)+iy}{(1-ax)-ay}\right| = \frac{|\ \cdots\ |}{|\ \cdots\ |} = \frac{(x-a)^2+y^2}{(1-ax)^2+a^2y^2}=1$

$\Rightarrow x^2+y^2=1$ 圆.

P20. 8

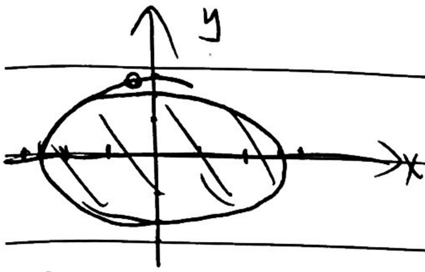
(4) $z=x+iy.\quad \arg(x+(y+2)i)=\arctan\frac{y+2}{x}$

$\therefore \frac{\sqrt{3}}{3}<\frac{y+2}{x}<+\infty \Rightarrow y+2>\frac{\sqrt{3}}{3}x$, 又 $\because x^2+y^2>4$



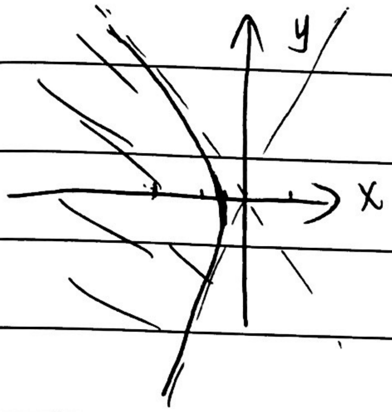
单连通

(8). $c=2$, $a=\frac{5}{2}$, $b=\frac{3}{2}$ 的椭圆.



单连通, 无边界. $\frac{4x^2}{25}+\frac{4y^2}{9}<1$.

(9). $c=2$, $a=\frac{1}{2}$, $b=\frac{\sqrt{15}}{2}$ 的双曲线的一支.



单连通, 无边界.

$4x^2-\frac{4}{15}y^2>1$; $(x<0)$.

P21. 4

~~$z_1=x_1+iy_1$, $z_2=x_2+iy_2$. , $x_1, x_2>0$~~

∴ ~~$z_1z_2=x_1x_2-y_1y_2+(x_1y_2+x_2y_1)i$~~

∴ ~~$\arg(z_1z_2)=$~~ $z_1=r_1e^{i\theta_1}$, $z_2=r_2e^{i\theta_2}$ $\theta_1, \theta_2\in(-\frac{\pi}{2},\frac{\pi}{2})$.

∴ $z_1z_2=r_1r_2e^{i(\theta_1+\theta_2)}$ $\arg(z_1z_2)=\theta_1+\theta_2$ // ∵ $(\theta_1+\theta_2)\in(-\pi,\pi)$.

$=\arg(z_1)+\arg(z_2)$.

同理 $\frac{z_1}{z_2}=\frac{r_1}{r_2}e^{i(\theta_1-\theta_2)}$ $(\theta_1-\theta_2)\in(-\pi,\pi)$ ∴ — — —

P53.1

(1) $\lim\limits_{z\to 0}\frac{\operatorname{Re}\bar{z}}{z^2} = \lim\limits_{z\to 0}\frac{\operatorname{Re}z}{z^2} = \lim\limits_{r\to 0}\frac{r\cos\theta}{r^2} = \lim\limits_{r\to 0}\frac{\cos\theta}{r}$ ∴不存在.

(2) $\lim\limits_{z\to i}\frac{z-i}{z(1+z^2)} = \lim \frac{z-i}{z(z-i)(z+i)} = \lim\limits_{z\to i}\frac{1}{z(z+i)} = \frac{1}{i(2i)} = -\frac{1}{2}$

P53.2

(1) $x=ky$ ∴ $\lim\limits_{z\to 0}f(z) = \lim\limits_{\substack{x=ky\\ y\to 0}}\frac{ky^2}{(k^2+1)y^2} = \frac{k}{k^2+1} \neq 0$ ∴不连续.

P21.12.

(1) $w_1 = -i$ $\quad w_2 = (1+i)^3 = \sqrt{2}^3 e^{\frac{3}{4}\pi i} = 2\sqrt{2}(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i) = -2+2i$

$w_3 = (\sqrt{3}+i)^3 = 2^3 e^{i\pi} = 8(-1) = -8$

(2) $z = re^{i\theta}$ $\quad \theta\in(2\pi k, \frac{\pi}{3}+2\pi k)$ $k\in\mathbb{Z}$. $r\in(0,+\infty)$

∴ $w = r^3 e^{3i\theta} = r' e^{i\theta'}$. $\theta'\in(6\pi k, \pi+6\pi k)$ $k\in\mathbb{Z}$.

∴ $0 < \arg w < \pi$.