172.17

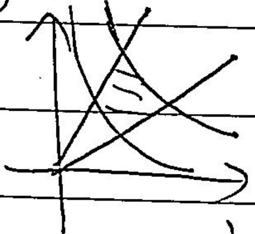
(1) $x=\frac{1}{3}r\cos\theta$, $y=\frac{1}{2}r\sin\theta$.

$D_{r\theta}=\{(r,\theta)\mid 0\le\theta\le 2\pi,\ 0\le r\le 1\}$ $\quad \frac{\partial(x,y)}{\partial(r,\theta)}=\begin{pmatrix}\frac{1}{3}\cos\theta & \frac{1}{2}\sin\theta\\ -\frac{1}{3}r\sin\theta & \frac{1}{2}r\cos\theta\end{pmatrix}$

$\therefore |J|=\frac{1}{6}r$

$\int_0^{2\pi}d\theta\int_0^1(\sin r^2)\frac{r}{6}dr=\frac{\pi}{24}\cdot(-\cos r^2)\Big|_0^1=(1-\cos 1)\frac{\pi}{24}$.

(2).



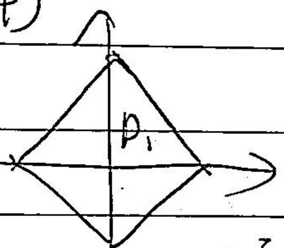
$\begin{cases}xy=u\\ x=v\end{cases}\Rightarrow D_{uv}=\{(u,v)\mid 1\le u\le 2,\ \sqrt{u}\le v\le 2\sqrt{u}\}$.

$\begin{cases}x=v\\ y=\frac{u}{v}\end{cases}$ $\therefore |J|=\frac{1}{v}$ $\therefore \int_1^2du\int_{\sqrt{u}}^{2\sqrt{u}}\frac{u^2}{v}dv=\frac{7}{3}\ln 2$

$\cancel{=\int_1^2u^{\frac{5}{2}}du=\frac{2}{7}u^{\frac{7}{2}}\Big|_1^2}$

$\cancel{=\frac{2}{7}(8\sqrt{2}-1)}$

(4)



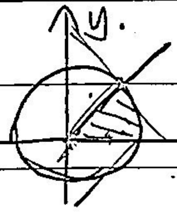
原式 $=\iint_{D}e^{x+y}dxdy$. $\quad x+y=u \quad \cancel{x-y=v}\quad |J|=\frac{1}{2}$

$\therefore D_{uv}=\{(u,v)\mid -1\le u\le 1,\ -1\le v\le 1\ \cancel{0\le v\le u}\}$.

$\cancel{\therefore 原式=4\int_0^1du\int_0^u e^u dv}$.

172.18. $\quad$ 原式 $=\int_{-1}^1du\int_{-1}^1e^udv\cdot\frac{1}{2}=e-e^{-1}$

(1) $\begin{cases}x=\frac{u+v}{2}\\ y=\frac{u-v}{2}\end{cases}\Rightarrow D_{uv}=\{(u,v)\mid$



$u\in[0,\sqrt{2}]$. $\quad 0\le\frac{u-v}{2}\le\frac{\sqrt{2}}{2}\Rightarrow$

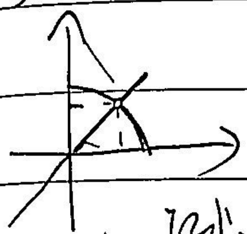
$\cancel{v\in[-1,0]}\ v\in[0,1]$, $\quad |J|=\frac{1}{2}$

$0\le v\le u\ (u\le 1)\quad 0\le v\le\sqrt{2-u^2}\ (1\le u\le\sqrt{2})$.

$\therefore \frac{1}{2}\int_0^1du\int_0^u uve^{u^2}dv+\frac{1}{2}\int_1^{\sqrt{2}}du\int_0^{\sqrt{2-u^2}}uve^{u^2}dv$

$=\cancel{\frac{1}{4}+\frac{1}{4}e^2-\frac{1}{2}e}\ \frac{1}{8}+\frac{1}{8}e^2-\frac{1}{4}e$

(2).



$u\in[0,1]$ $\quad y^2\le x^2\le 1-y^2$, $0\le y\le \frac{\sqrt{2}}{2}$

$v\in[0,\frac{1}{2}]$. $\Rightarrow 0\le v\le \frac{1}{2}u$. $\quad |J|=$

$\therefore$ 原式 $=\int_0^1 du\int_0^{\frac{1}{2}u} \cancel{\sqrt{u^2-4v^2}\,e^{u+2v}}\,(x^2-y^2)e^{u+2v}\frac{1}{2(x^2-y^2)}dv$

$=\frac{1}{8}+\frac{1}{8}e^2-\frac{1}{4}e$.

173.20

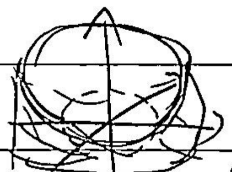
(3) $(x^2+y^2-1)^2\le 1-x^2-y^2 \Rightarrow (x^2+y^2)\in[0,3]$.

$D_{r\theta}=\{(r,\theta)\mid 0\le\theta\le 2\pi, 0\le r\le 1\}$.

$V=\iint_D (1+\sqrt{1-x^2-y^2}-x^2-y^2)d\sigma = \int_0^{2\pi}d\theta\int_0^{1}(1+\sqrt{1-r^2}-r^2)\,rdr$

$=\pi\frac{7}{6}=\frac{7}{6}\pi$.

(4) $D=\{(x,y)\mid 1\le x^2+y^2\le 2\}$.

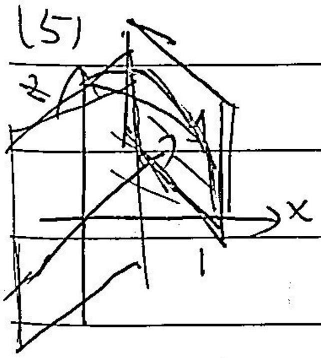


$2\times\pi\times\sqrt{2}^2$.

$D_{r\theta}=\{(r,\theta)\mid 0\le\theta\le 2\pi, 1\le r\le\sqrt{2}\}$.

$\Omega'=\int_0^{2\pi}d\theta\int_1^{\sqrt{2}}\sqrt{r^2-1}\,rdr=\frac{2}{3}\pi$. $\quad\Omega=2\Omega'=\frac{4}{3}\pi$; $\Omega_0=4\pi-\frac{4}{3}\pi=\frac{8}{3}\pi$.

(5)



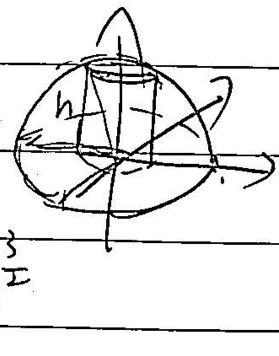
$D=\{(x,y)\mid 0\le x\le 1, 0\le y\le 1-x\}$.

$\int_0^1 dx\int_0^{1-x}(6-(x^2+y^2))dy$

$=\int_0^1(\frac{4}{3}x^3-2x^2-5x+\frac{17}{3})dx$

$=\frac{17}{6}$

173.22



(1) 先求半个. $D=\{(r,\theta)\mid 0\le\theta\le 2\pi, 0\le r\le r_0\}$.

$\Omega=\int_0^{2\pi}d\theta\int_0^{r_0}\sqrt{R^2-r^2}\,rdr=\frac{2\pi}{3}R^3-\frac{2\pi}{3}(R^2-r^2)^{\frac{3}{2}}$

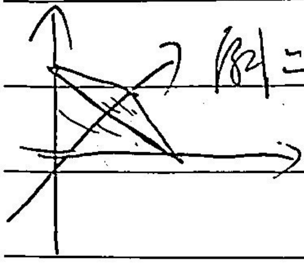
$\therefore \Omega_{剩}=\frac{4}{3}\pi R^3-2\Omega=\frac{4\pi}{3}(R^2-r^2)^{\frac{3}{2}}$

$h^2+r^2=R^2 \Rightarrow \Omega_{剩}=\frac{4}{3}\pi h^3\frac{1}{8}=\frac{\pi}{6}h^3$.

173、25.

(2)

$D=\{(x,y) \mid 0\le x\le 1,\ 0\le y\le 1-x\}$.

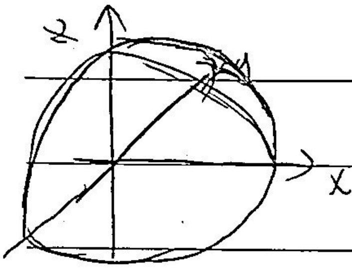


原 $=\iint\limits_D d\sigma \int_0^{1-x-y} e^{x+y+z}\,dz$

$=\int_0^1 dx\int_0^{1-x}dy\int_0^{1-x-y}e^{x+y+z}\,dz$

$=(e^x-\frac{1}{2}ex^2)\Big|_0^1=\frac{e}{2}-1$.

(4)



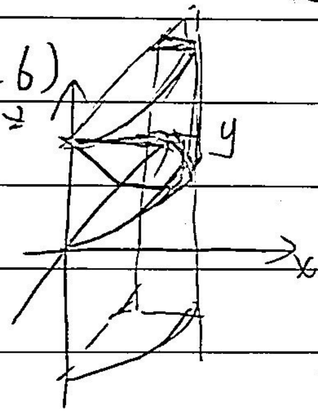
设 $\Omega=\{(x,y,z) \mid -\sqrt{1-x^2-z^2}\le y\le\sqrt{1-x^2-z^2},\ (x,z)\in D\}$.

$D=\{(x,z) \mid 0\le x^2+z^2\le 1,\ x\ge 0,\ z\ge 0\}$.

$f(x,y,z)=-f(x,-y,z)$. ~~y在Ω上对称~~

∴ ∵ Ω 关于 xOz 对称 ∴ 原 $=0$

(6)



$\Omega=\{(x,y,z) \mid 0\le x\le\sqrt{y},\ (y,z)\in D\}$.

$D=\{(y,z) \mid 0\le y\le\frac{\pi}{2},\ 0\le z\le\frac{\pi}{2}-y\}$.

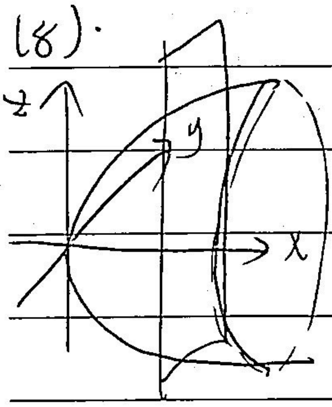
$\iint\limits_D d\sigma\int_0^{\sqrt{y}} x\sin(y+z)\,dx$

$=\int_0^{\frac{\pi}{2}}dy\int_0^{\frac{\pi}{2}-y}\frac{y}{2}\sin(y+z)\,dz$

$=\int_0^{\frac{\pi}{2}}\frac{y}{2}\cos y\,dy=\frac{y}{2}\sin y\Big|_0^{\frac{\pi}{2}}-\frac{1}{2}\int_0^{\frac{\pi}{2}}\sin y\,dy$

$=\frac{\pi}{4}-\frac{1}{2}$

(8).



$\int_0^4 dx\iint\limits_{D_x}d\sigma$. $D_x=\{(y,z) \mid 0\le y^2+z^2\le\frac{x}{4}\}$.

$D_{r\theta}=\{(r,\theta) \mid 0\le\theta\le 2\pi,\ 0\le r\le\frac{\sqrt{x}}{2}\}$.

$\int_0^4 dx\int_0^{2\pi}d\theta\int_0^{\frac{\sqrt{x}}{2}} x r\,dr$

$=\frac{16\pi}{3}$.