P144.4

(1) $I \overset{e^{ix}=z}{=} \int_C \frac{1}{5+\frac{z+z^{-1}}{2}\cdot 3}\,\frac{1}{iz}\,dz = \int_C \frac{2}{i}\cdot\frac{1}{3z+1}\cdot\frac{1}{z+3}\,dz$

$C=\{|z|=1\}$，~~$z=0$~~，$z=-\frac{1}{3}$，$z=-3$ 为一阶极点.

$\therefore\ I = 2\pi i\,(\cancel{\mathrm{Res}[f(z),0]}+\mathrm{Res}[f(z),-\frac{1}{3}])$

$= \cancel{-\frac{19}{12}\pi}\ \cancel{-\frac{9}{4}\pi}\ \cancel{-\frac{9}{2}\pi}\ \frac{1}{2}\pi$

(3) $\int_0^{\frac{\pi}{2}} \frac{1}{2+\cos 2x}dx = \frac{1}{2}\int_0^{\pi}\frac{1}{2+\cos x}dx = \frac{1}{4}\int_0^{2\pi}\frac{1}{2+\cos x}dx$

$\overset{e^{ix}=z}{=} \frac{1}{4}\int_C \frac{1}{2+\frac{z+z^{-1}}{2}}\cdot\frac{1}{iz}dz = \frac{1}{2i}\int_C\frac{1}{z^2+4z+1}dz$

$= \pi\ \mathrm{Res}[f(z),\sqrt{3}-2] = \frac{\pi}{2\sqrt{3}}$

P144.5

(2) $I=\frac{1}{2}\int_{-\infty}^{+\infty}\frac{x^2}{(x^2+1)(x^2+4)}dx$ $x=\pm i$，$x=\pm 2i$ 为一阶极点.

$= \pi i\,(\mathrm{Res}[f(x),i]+\mathrm{Res}[f(x),2i])$

$= \frac{\pi}{6}$

(4) $\int_{-\infty}^{+\infty}\frac{dx}{(x^2+1)^n}$，$x=\pm i$ 为 n 阶极点.

$= 2\pi i\ \mathrm{Res}[f(x),i]$

$= 2\pi i\cdot\frac{1}{(n-1)!}\left(\frac{1}{(x^2+1)^n}\cdot(x-i)^n\right)^{(n-1)}\Big|_{x=i}$

$= 2\pi i\cdot\frac{i}{2^{2n-1}}\,\frac{(2n-2)!}{((n-1)!)^2} = \frac{\pi}{2^{2n-2}}\,\frac{(2n-2)!}{((n-1)!)^2}$

P144.6

(1) 设 $f(x)=\frac{e^{ix}}{(x^2+1)(x^2+9)}$，$\int_{-\infty}^{+\infty}f(x)dx = \cdots$ 1阶极点：$x=\pm i$，$x=\pm 3i$

$\therefore \int_{-\infty}^{+\infty}f(x)dx = 2\pi i\,(\mathrm{Res}[f(x),i]+\mathrm{Res}[f(x),3i])$.

$= \frac{\pi}{24}(3e^{-1}-e^{-3})$

$\therefore$ 原式 $= \mathrm{Re}(I) = \frac{\pi}{24}(3e^{-1}-e^{-3})$.

(2). $f(x)=\dfrac{xe^{2x}}{x^2+4x+20}$, $x=-2\pm4i$ 为一阶极点.

$\int_{-\infty}^{+\infty} f(x)\,dx = 2\pi i\,\mathrm{Res}[f(x), -2+4i] = -\frac{\pi}{2}(2i-1)e^{-2i-4}$

$\therefore$ 原式 $=\mathrm{Im}[I]=\frac{\pi}{2}e^{-4}(2\cos 2+\sin 2)$.

(3). $f(x)=\dfrac{xe^{ibx}}{x^4+a^4}$ $x=ae^{(-\frac{\pi}{4}+\frac{\pi}{2}k)i}$ 为一阶极点.

$\therefore \int_{-\infty}^{+\infty} f(x)\,dx = 2\pi i[\mathrm{Res}[f(x), ae^{\frac{\pi}{4}i}]+\mathrm{Res}[f(x), ae^{\frac{3}{4}\pi i}]$

$= 2\pi i\cdot\dfrac{-4i^2\sin\frac{\sqrt{2}}{2}ab}{8a^2e^{\frac{\sqrt{2}}{2}ab}} = i\frac{\pi}{a^2}e^{-\frac{\sqrt{2}}{2}ab}\sin\frac{\sqrt{2}}{2}ab.$

原式 $=\mathrm{Im}(I)$

$=\frac{\pi}{a^2}e^{-\frac{ab}{\sqrt{2}}}\sin\frac{ab}{\sqrt{2}}$.

(4).

原式 $=\frac{1}{2}\int_{-\infty}^{+\infty}\dfrac{x\sin ax}{(x^2+b^2)^2}dx$

$f(x)=\dfrac{xe^{iax}}{(x^2+b^2)^2}$. $x=\pm bi$ 为二阶极点.

$\int_{-\infty}^{+\infty} f(x)\,dx = 2\pi i\,\mathrm{Res}[f(x), bi]$

$=\dfrac{\pi a b}{2e^{ab}}\cdot i$

$\therefore$ 原式 $=\frac{1}{2}\mathrm{Im}(I)=\dfrac{\pi a}{4e^{ab}\cdot b}$

P144. B.1

令 $f(z)=\dfrac{h(z)}{(z-z_0)^m}$ $h(z_0)\neq0$, $h(z)$ 在 z_0 解析.

$\therefore g(z)\dfrac{f'(z)}{f(z)}=\dfrac{g(z)}{(z-z_0)}\left(\dfrac{h'(z)(z-z_0)}{h(z)}-m\right)$.

① $g(z_0)=0$

$\therefore z_0$ 为可去点. $\mathrm{Res}[g(z), \frac{f'(z)}{f(z)}]=0=-mg(z_0)$

② $g(z_0)\neq0$ $\therefore z_0$ 为一阶极点.

$\mathrm{Res}[g(z), \frac{f'(z)}{f(z)}]=g(z)\cdot\left(\dfrac{h'(z)(z-z_0)}{h(z)}-m\right)\Big|_{z=z_0}=-mg(z_0)$