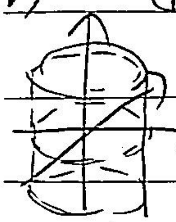
233.17

(6) $(P,Q,R)=\left(\frac{x}{x^2+y^2+z^2},\ 0,\ \frac{z^2}{x^2+y^2+z^2}\right)$



$\cancel{\frac{\partial P}{\partial x}=\frac{y^2+z^2-2x^2}{(x^2+y^2+z^2)^2},\ \frac{\partial R}{\partial z}=\frac{2z(x^2+y^2)}{(x^2+y^2+z^2)^2}}$.

$\cancel{\therefore \text{原式}=\iiint_\Omega\left(\frac{\partial P}{\partial x}+\frac{\partial R}{\partial z}\right)dV=}$

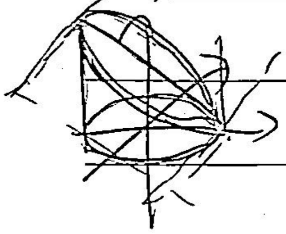
~~侧面~~ $x^2+y^2=R^2\quad z\in[-R,R]\quad \cancel{\iint_{\Sigma_1}\vec F\cdot d\vec S=(P,Q,R)(0,0,}$

$\iint_{\Sigma_{上下}}(P,Q,R)\cdot d\vec S=\iint_{D_{xy}}\frac{R^2}{x^2+y^2+R^2}d\sigma-\iint_{D_{xy}}\frac{R^2}{x^2+y^2+R^2}d\sigma=0$

$\iint_{\Sigma_{侧}}(P,Q,R)d\vec S=\iint_{D_{yz}}(P,Q,R)(1,-x_y,0)+(P,Q,R)(-1,x_y,0)\,d\sigma$

$=\frac{R}{2}\pi^2$

(7) $(P,Q,R)=(0,-y,z+1)$



$\text{原式}=\cancel{\iint_{D_{yz}}}\iint_{D_{xz}}-y\,d\sigma+\iint_{D_{xz}}-y\,d\sigma$

$=\iint_{D_{xz}}-\sqrt{4-x^2}+\iint_{D_{xz}}-\sqrt{4-x^2}=-2\iint_{D_{xz}}\sqrt{4-x^2}\,d\sigma.$

$\because \cancel{D_{xy}}\ D_{xz}:\{-2\le x\le 2,\ 0\le z\le 2-x\}.$

$\therefore \text{原式}=-8\pi$

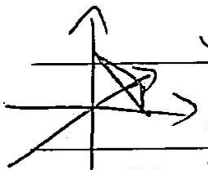
(8) $\because \vec r=(u\cos v,\ u\sin v,\ v)\quad \vec r_u=(\cos v,\sin v,0)\quad \vec r_v=(-u\sin v,u\cos v,1)$

$\therefore \vec r_u\times\vec r_v=(\sin v,-\cos v,u)\ \therefore \text{原式}=\iint_{D_{uv}}(y,x,z^2)(\sin v,-\cos v,u)\,d\sigma$

$=\iint_{D_{uv}}u(\sin^2 v-\cos v+v^2)\,d\sigma=\frac{1}{6}\pi^3$

233.29

(1) $\vec F=(x^2,y^2,z^2)\ \therefore \frac{\partial P}{\partial x},\frac{\partial Q}{\partial y},\frac{\partial R}{\partial z}\in C^1(\Omega)$.



$\therefore \text{原式}=\iiint_\Omega(2x+2y+2z)\,dV=6\iiint_\Omega x\,dV=6\iint_{D_{xy}}d\sigma\int_0^{1-x-y}x\,dz.$

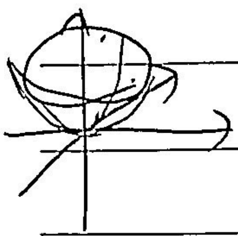
$0\le z\le 1-x-y,\ D_{xy}:\ 0\le x\le 1,\ 0\le y\le 1-x\quad=\frac{1}{4}$

(3) $\frac{\partial P}{\partial x},\frac{\partial Q}{\partial y},\frac{\partial R}{\partial z}\in C^1(\Omega)$. $\therefore \text{原式}=-\iiint_\Omega(3x^2+3y^2+3z^2)\,dV$

$=-\int_0^{2\pi}d\theta\int_0^{\pi}d\varphi\int_0^R 3\rho^2\rho^2\sin\varphi\,d\rho$

$=-\frac{12}{5}R^5\pi$

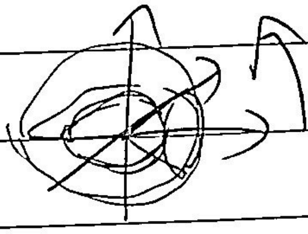
(5) $\vec{F}=(2x+z,\ 0,\ z)$, $\frac{\partial P}{\partial x},\frac{\partial Q}{\partial y},\frac{\partial R}{\partial z}\in C^1(\Omega)$ 再又取补面内侧为Σ_0.



$\iint_{\Sigma\cup\Sigma_0} d\vec{S} = -\iiint_{\Omega} 3\,dV = -\frac{3}{2}\pi$

原式 $= -\frac{3}{2}\pi + \iint_{\Sigma_0外} d\vec{S} = -\frac{3}{2}\pi + \iint_{D_{xy}} 1\,d\sigma$

$= -\frac{\pi}{2}$



转一转不变.

(7) $\vec{F}=(x^3,\ \frac{1}{z}f(\frac{y}{z})+y^3,\ \frac{1}{y}f(\frac{y}{z})+z^3)$

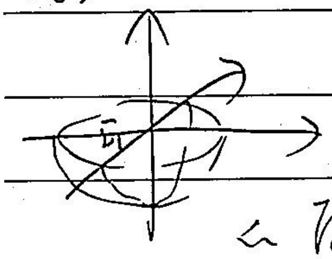
$\frac{\partial P}{\partial x},\frac{\partial Q}{\partial y},\frac{\partial R}{\partial z}\in C^1(\Omega)$

∴原式 $= 3\iiint_{\Omega}(x^2+y^2+z^2)\,dV = 3\int_0^{2\pi}d\theta\int_0^{\pi}d\varphi\int_1^2\rho^2\rho^2\sin\varphi\,d\rho$

$- 3\int_0^{2\pi}d\theta\int_0^{\frac{\pi}{4}}d\varphi\int_1^2\rho^2\rho^2\sin\varphi\,d\rho$

$= \frac{93}{5}(2\pi-\sqrt{2}\pi)$

(8).



$x^2+y^2+z^2=R^2 \Rightarrow \vec{F}=(x,\ 0,\ \frac{1}{R}(z+R)^2)$.

$\frac{\partial P}{\partial x},\frac{\partial Q}{\partial y},\frac{\partial R}{\partial z}\in C^1(\Omega)$ 补全,

∴原式 $= \iiint_{\Omega}(1+\frac{2}{R}(z+R))\,dV + \iint_{\Sigma_1下侧}(x,0,\frac{1}{R}(z+R)^2)\cdot(0,0,-1)\,d$

$= \frac{\pi}{2}R^3$

235.30

$r^2=x^2+y^2+z^2$, $\oiint_{\Sigma}\frac{1}{r^2}\cos(\hat{r,n})\,dS = \oiint_{\Sigma}\frac{\vec{r}}{r^3}\,d\vec{S}$

则 $\vec{F}=(\frac{x}{(x^2+y^2+z^2)^{3/2}},\ \frac{y}{\cdots},\ \frac{z}{\cdots})$ $\frac{\partial P}{\partial x}=\frac{x^2+y^2+z^2-3x^2}{(x^2+y^2+z^2)^{5/2}}$ ∴ $\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}=0$

当Σ不包含$(0,0,0)$时 直接Gauss. ∴原式$=0$

当Σ包含$(0,0,0)$时 挖去 $x^2+y^2+z^2=\varepsilon^2$, $\lim_{\varepsilon\to 0}$, 取内侧.

则原式 $= \oiint_{\Sigma+\Sigma_0} d\sigma + \oiint_{\Sigma_0外} d\sigma = 0 + \frac{1}{\varepsilon^3}\iiint_{\Omega} 1+1+1\,dV$

$= \frac{3}{\varepsilon^3}\cdot\frac{4}{3}\pi\varepsilon^3 = 4\pi$