3.1

(a) $x(n)=\{\underset{-5}{3},\underset{-4}{0},\underset{-3}{0},\underset{-2}{0},\underset{-1}{0},\underset{\uparrow 0}{6},\underset{1}{1},\underset{2}{-4}\}$

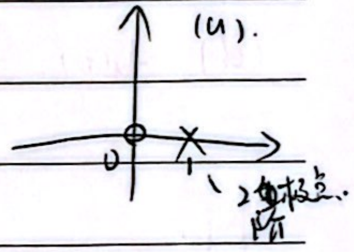
$X(z)=\sum_{n=-\infty}^{+\infty}x(n)z^{-n}=3z^{5}+6+z^{-1}-4z^{-2}$, ROC: ~~$0<|z|<+\infty$~~ $|z|\neq 0$

3.2.

(1) $x(n)=(1+n)u(n)$, $X(z)=\sum_{n=-\infty}^{+\infty}x(n)z^{-n}=\sum_{n=0}^{+\infty}(1+n)z^{-n}=\sum_{n=0}^{+\infty}z^{-n}+\sum_{n=0}^{+\infty}nz^{-n}$

$nx(n)\longleftrightarrow -z\frac{dX(z)}{dz}$, $u(n)\longleftrightarrow\frac{1}{1-z^{-1}}$, $nu(n)\longleftrightarrow -z\cdot\left(-\frac{1}{(z-1)^2}\right)=\frac{z}{(z-1)^2}$

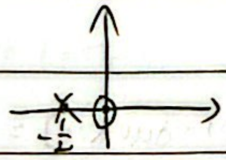
$\therefore$ ~~$x(n)\longleftrightarrow$~~ $X(z)=\frac{1}{1-z^{-1}}+\frac{z}{(z-1)^2}$. ROC: $|z|>1$



(2) $x(n)=(-1)^n 2^{-n}u(n)=\left(-\frac{1}{2}\right)^n u(n)$.

$X(z)=\sum_{n=0}^{+\infty}\left(-\frac{1}{2}\right)^n z^{-n}=\sum_{n=0}^{+\infty}\left(-\frac{1}{2z}\right)^n=\frac{1}{1+\frac{1}{2z}}=\frac{2z}{2z+1}$.

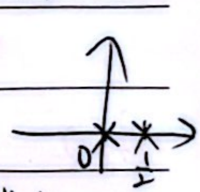
ROC: $|z|>\frac{1}{2}$



(3) $x(n)=\left(\frac{1}{2}\right)^n u(n)-\left(\frac{1}{2}\right)^n u(n-10)=\left(\frac{1}{2}\right)^n u(n)-\left(\frac{1}{2}\right)^{n-10}\left(\frac{1}{2}\right)^{10}u(n-10)$.

$Z\left[\left(\frac{1}{2}\right)^n u(n)\right]=\frac{2z}{2z-1}$, $Z\left[\left(\frac{1}{2}\right)^{n-10}u(n-10)\right]=z^{-10}\frac{2z}{2z-1}$.

$\therefore Z[x(n)]=\frac{2z}{2z-1}-\left(\frac{1}{2}\right)^{10}z^{-10}\frac{2z}{2z-1}$ ROC: $|z|>\frac{1}{2}$



$\lim_{z\to 0}X(z)=\frac{1}{10}\left(\frac{1}{2}\right)^9 z^{-9}$. 0是9阶极点 0点: $z_n=\frac{1}{2}e^{j\frac{\pi}{5}k}$ ~~$e^{j\frac{\pi}{5}k}\cdot\frac{1}{2}$~~

$k\in Z$. 让 $|z|=\frac{1}{2}$ 十等分了.

3.14.

$x(n)$因果. 且 $X(z)$ 为分式, 故 ROC: $|z|>r_{max}$. ROC: $|z|>2$

(a) $X(z)=\frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}}=1-\frac{2}{(z+1)(z+2)}=1-2\left(\frac{1}{z+1}-\frac{1}{z+2}\right)=1-\frac{2}{z+1}+\frac{2}{z+2}$

$\Rightarrow$ ~~$x(n)=\delta(n)-(-1)^n u(n)$~~ $=-\frac{1}{1+\frac{2}{z}}+\frac{2}{1+\frac{1}{z}}$

$\Rightarrow x(n)=2(-1)^n u(n)-(-2)^n u(n)$

$ROC: |z|>1$

(c). $X(z) = \frac{z^{-6}+z^{-7}}{1-z^{-1}} = z^{-6}\frac{1}{1-z^{-1}} + z^{-7}\frac{1}{1-z^{-1}}$, $\frac{1}{1-z^{-1}} \longleftrightarrow u(n)$

$x(n-n_0) \longleftrightarrow z^{-n_0}X(z) \Rightarrow x(n) = u(n-6)+u(n-7)$

3.16

(a) $x_1(n) = (\frac{1}{4})^n u(n-1)$, $x_2(n) = [1+(\frac{1}{2})^n]u(n)$.

$Z[x_1(n) * x_2(n)] = X_1(z)\cdot X_2(z)$. $X_1(z) = \frac{1}{4z}\frac{1}{1-\frac{1}{4}z^{-1}}$ $ROC: |z|>\frac{1}{4}$

$X_2(z) = \frac{1}{1-z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}$ $ROC: |z|>\frac{1}{2}$

$\therefore X_1(z)\cdot X_2(z) = \frac{z}{(4z-1)(z-1)} + \frac{2z}{(4z-1)(2z-1)}$ $ROC: |z|>\frac{1}{2}$.

$= -\frac{1}{3}\frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{3}\frac{1}{1-z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}$

$= -\frac{4}{3}\frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{3}\frac{1}{1-z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}$

$\Rightarrow x_1(n) * x_2(n) = [-\frac{4}{3}(\frac{1}{4})^n + \frac{1}{3} + (\frac{1}{2})^n]u(n)$

(b) $x_1(n) = u(n)$, $x_2(n) = \delta(n) + (\frac{1}{2})^n u(n)$

$X_1(z) = \frac{1}{1-z^{-1}}$ $ROC: |z|>1$, $X_2(z) = 1+\frac{1}{1-\frac{1}{2}z^{-1}}$ $|z|>\frac{1}{2}$.

$X_1(z)\cdot X_2(z) = \frac{1}{1-z^{-1}} + \frac{1}{(1-z^{-1})(1-\frac{1}{2}z^{-1})}$ $ROC: |z|>1$.

$= \frac{3}{1-z^{-1}} - \frac{1}{1-\frac{1}{2}z^{-1}}$ $\Rightarrow x_1(n) * x_2(n) = [3-(\frac{1}{2})^n]u(n)$.

3.25.

$X(z) = \frac{1}{1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}} = \frac{2}{1-z^{-1}} - \frac{1}{1-\frac{1}{2}z^{-1}}$ ROC: 未知.

① 右边信号. $ROC: |z|>1 \Rightarrow x(n) = [2-(\frac{1}{2})^n]u(n)$.

② 左边信号. $ROC: |z|<\frac{1}{2} \Rightarrow x(n) = [(\frac{1}{2})^n - 2]u(-n-1)$

③ 双边信号. $ROC: \frac{1}{2}<|z|<1 \Rightarrow x(n) = -2u(-n-1) - (\frac{1}{2})^n u(n)$

补1.

$\mathcal{X}_1(z) = \sum_{n=0}^{+\infty} x_1(n) z^{-n}$，求 $y(n) = x(n+1)$ 的单边 z 变换。

$$Y(n) = \sum_{n=0}^{+\infty} x(n+1) z^{-n} = \sum_{n=0}^{+\infty} x(n+1) z^{-(n+1)} \cdot z = z \sum_{n=1}^{+\infty} x(n) z^{-n}$$

$$= \cancel{z Y \; z \mathcal{X}_1(n))} \cdot z\left(\mathcal{X}_1(z) - x(0)\right) \qquad \text{ROC: } \text{ROC}_x - \{0\}.$$