234.21

(3) $\frac{\partial P}{\partial y}=1+e^{-x}\cos y \quad \frac{\partial Q}{\partial x}=1+e^{-x}\cos y$ 且 $P, Q \in C'(D)$.

$\therefore$ 无关 $\quad \frac{\partial u}{\partial x}=y+e^{-x}\sin y \Rightarrow u=xy-e^{-x}\sin y+C_1(y)$.

$\frac{\partial u}{\partial y}=x-e^{-x}\cos y+C_1'(y)=x-e^{-x}\cos y \Rightarrow C_1'(y)=0 \Rightarrow C_1(y)=C$

$\therefore u=xy-e^{-x}\sin y+C \quad \therefore I=u(1,\frac{\pi}{2})-u(0,0)=\frac{\pi}{2}-e^{-1}$

(4) $\frac{\partial P}{\partial y}=\frac{1-x^2y^2}{(1+x^2y^2)^2}, \frac{\partial Q}{\partial x}=\frac{1-x^2y^2}{(1+x^2y^2)^2}$ 又且 $P, Q \in C'(D)$

$\therefore$ 无关 $\quad \frac{\partial u}{\partial x}=\frac{y}{1+x^2y^2} \Rightarrow u=\arctan xy+C_1(y)$.

$\frac{\partial u}{\partial y}=\frac{x}{1+x^2y^2}+C_1'(y)=\frac{x}{1+x^2y^2} \Rightarrow C_1(y)=C$

$\therefore u=\arctan xy+C \quad \therefore I=u(1,1)-u(0,0)=\frac{\pi}{4}$

234.22

(2) $\frac{\partial P}{\partial y}=-\frac{2y}{x^2}\cos\frac{y}{x}+\frac{y^2}{x^3}\sin\frac{y}{x}, \quad \frac{\partial Q}{\partial x}=-\frac{y}{x^2}\cos\frac{y}{x}-\frac{y}{x^2}\cos\frac{y}{x}+\frac{y^2}{x^3}\sin\frac{y}{x}$.

$\therefore \frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}, \quad P, Q \in C'(D) \therefore$ 存在.

$\frac{\partial u}{\partial x}=1-\frac{y^2}{x^2}\cos\frac{y}{x} \Rightarrow u=x+y\sin\frac{y}{x}+C_1(y)$. $\frac{\partial u}{\partial y}=\sin\frac{y}{x}+\frac{y}{x}\cos\frac{y}{x}+C_1'(y)$

$\therefore \sin\frac{y}{x}+\frac{y}{x}\cos\frac{y}{x}+C_1'(y)=\sin\frac{y}{x}+\frac{y}{x}\cos\frac{y}{x} \Rightarrow C_1'(y)=0$

$\therefore u(x,y)=x+y\sin\frac{y}{x}+C$

(4) $\frac{\partial P}{\partial y}=\frac{y^2-2xy-x^2}{(x^2+y^2)^2}. \quad \frac{\partial Q}{\partial x}=\frac{y^2-2xy-x^2}{(x^2+y^2)^2}$ 且 $P, Q \in C'(D) \therefore$ 存在.

$\frac{\partial u}{\partial x}=\frac{x-y}{x^2+y^2} \Rightarrow u=\frac{1}{2}\ln(x^2+y^2)-\arctan\frac{x}{y}+C_1(y)$.

$\frac{\partial u}{\partial y}=\frac{y}{x^2+y^2}+\frac{x}{x^2+y^2}+C_1'(y)=\frac{x+y}{x^2+y^2} \Rightarrow C_1(y)=C$

$\therefore u=\frac{1}{2}\ln(x^2+y^2)-\arctan\frac{x}{y}+C$

234.23

(2). $\frac{\partial(f(x)y)}{\partial y}=-\frac{\partial(f(x)x)}{\partial x} \Rightarrow xf'(x)+2f(x)=0 \Rightarrow f(x)=\frac{C}{x^2}$

$f(1)=1 \Rightarrow f(x)=\frac{1}{x^2}$

(3). $\frac{\partial P}{\partial y}=e^x f(x)-\frac{1}{x}$，$\frac{\partial Q}{\partial x}=-\frac{f'(x)}{f(x)}$ ~~∴ $\iint_D \frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}=0$，$\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$~~

∂ $P\in C'(D)$，但 Q 不一定，~~……~~ 当 $Q\in C'(D)$ 时.

~~$(1,0)\to(1,y)\to(x,y)\to(x,0)\to(1,0)$~~ $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}\Rightarrow f(x)=\frac{x}{(x-1)e^x+2}$.

234.24

$2xy$ ~~…~~，$Q\in C'(D)$ $\frac{\partial(2xy)}{\partial y}=2x=\frac{\partial Q}{\partial x}\Rightarrow Q=x^2+C_1(y)$.

$\frac{\partial u}{\partial x}=2xy\Rightarrow u=x^2y+C_2(y)$ $\frac{\partial u}{\partial y}=x^2+C_2'(y)=x^2+C_1(y)$

∴ $C_2'(y)=C_1(y)$ $u(t,1)=u(1,t)\Rightarrow\int_1^t C_1(y)dy=t^2-t$.

令 $t^2-t=F(t)$ ∴ $F(t)=\int_1^t C_1(y)dy\Rightarrow F'(t)=C_1(t)=2t-1$

∴ $C_1(y)=2y-1$ ∴ $Q=x^2+2y-1$

234.25

$P,Q\in C'(D)$ ∴ $\frac{\partial P}{\partial y}=-\frac{x}{y^2}(x^2+y^2)^p+2px(x^2+y^2)^{p-1}$

~~$\frac{\partial Q}{\partial x}=-\frac{2x}{y^2}(x^2+y^2)^p-\frac{2x^2p}{y}(x^2+y^2)^{p-1}=\frac{\partial P}{\partial y}\Rightarrow$~~

$\frac{\partial Q}{\partial x}=-\frac{2x}{y^2}(x^2+y^2)^p-p\frac{2x^3}{y^2}(x^2+y^2)^{p-1}=\frac{\partial P}{\partial y}\Rightarrow P=-\frac{1}{2}$.

$P=\frac{x}{y\sqrt{x^2+y^2}}$，$Q=-\frac{x^2}{y^2\sqrt{x^2+y^2}}$ ∴ $\frac{\partial u}{\partial x}=P\Rightarrow u=\frac{1}{y}\sqrt{x^2+y^2}+C_1(y)$.

$\frac{\partial u}{\partial y}=-\frac{x^2}{y^2\sqrt{x^2+y^2}}=Q\Rightarrow C_1(y)=C$ ∴ $u=\frac{1}{y}\sqrt{x^2+y^2}+C$.

~~$C(0,2)-C(1,1)=$~~ $u(0,2)-u(1,1)=1-\sqrt{2}$

234.26

(2) $\frac{\partial P}{\partial y}=\cos xy-xy\sin xy$，$\frac{\partial Q}{\partial x}=\cos xy-xy\sin xy$ ∴ 存.

$\frac{\partial u}{\partial x}=1+y\cos xy\Rightarrow u=x+\sin xy+C_1(y)$，$\frac{\partial u}{\partial y}=x\cos xy+C_1'(y)=Q$.

∴ $u=x+\sin xy+C$ ∴ $x+\sin xy=C$

(4) $\frac{\partial P}{\partial y}=1+4xy$，$\frac{\partial Q}{\partial x}=1-4xy$ ∴ ~~$xy=0$~~ ∴ 不是，改写.

~~$\frac{\partial u}{\partial x}=y+2xy^2\Rightarrow u=xy+x^2y^2+C_1(y)$ $\frac{\partial u}{\partial y}=x+2yx^2+C_1'(y)=x-2x^2y+$~~

~~$\Rightarrow C_1(y)=-2x^2y^2+C$ ∴ $u=xy-x^2y^2+C$~~ $d(xy)+2xy^3d\frac{x}{y}$ $xy=u$，$\frac{x}{y}=v$

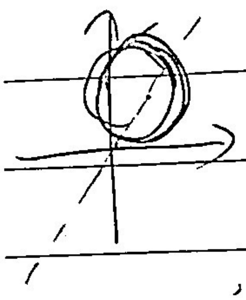
$(ydx+xdy)+2xy(ydx-xdy)=0\Rightarrow$ ∴ $2\ln\frac{x}{y}-\frac{1}{xy}=C$

~~25~~235. 27)

同理 $\oint_{\partial D^+} uv\,dy = \iint_D (v\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial x})\,d\sigma$

又∵ $u, v \in C^1(D)$ ∴ $\oint_{\partial D^+} uv\,dy = \iint_D (u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x})\,d\sigma$ ∴得证.

235. 28.



$xf(y), -\frac{y}{f(x)} \in C^1(D)$

~~∴ $\oint_C xf(y)dy - \frac{y}{f(x)}dx = \iint_D xf'(y) + \frac{yf'(x)}{f^2(x)}\,d\sigma$.~~

又∵ ~~x, y 轮换对称.~~

∴ 原式 ~~$= \iint_D xf'(x)(1-\frac{1}{f^2(x)})\,d\sigma$~~

$\oint_C xf(y)dy - \frac{y}{f(x)}dx = \iint_D f(y) + \frac{1}{f(x)}\,d\sigma$. x, y 轮换对称

∴ 原式 $\geq 2A_D = 2\pi$.