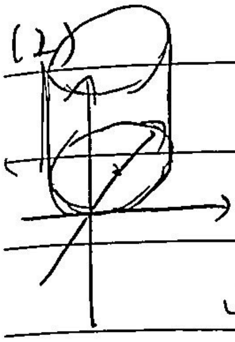
174.26

(2)



原式 $= \int_0^2 dz \iint_{D_z} d\sigma$.

$D_z = \{(x,y) \mid x^2+(y-1)^2 \le 1\}$ 关于 x 奇. 以关于 yoz

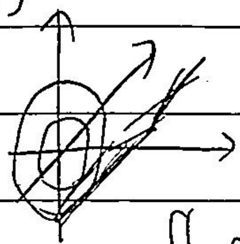
~~令 $x = r\cos\theta,\ y = r\sin\theta$~~ ∴ $=0$

∴ ~~$D_{r\theta} = \{(r,\theta) \mid 0\le\theta\le 2\pi,\ 0\le r\le 2\sin\theta\}$ $J = \frac{\partial(x,y)}{\partial(r,\theta)} = r$~~

∴ ~~原式 $= \int_0^2 dz \int_0^{2\pi} d\theta \int_0^{2\sin\theta} r^4\cos\theta\, dr$~~

~~$= \frac{64}{15}$~~

(3)



~~$\int_0^{\dots} dx \iint_{D_z} d\sigma \quad \int_0^4 dx \iint_{D_{z_1}} d\sigma - \int_0^4 dx \iint_{D_{z_2}} d\sigma$.~~

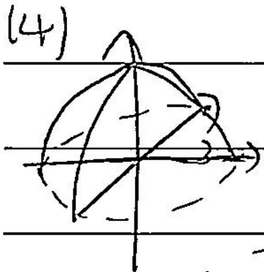
~~$D_{z_1} = \{(y,z) \mid x-2\le z\le 2,\ \dots\ z^2+y^2\le 4\}$.~~

∴ ~~$D_{r\theta_1} = \{(r,\theta) \mid 0\le\theta\le 2\pi,$~~ r

$\iint_D d\sigma \int_0^{z+2} y\, dx$. ~~$D = \{(y,z)$~~ $D_{r\theta} = \{(r,\theta) \mid 0\le\theta\le 2\pi,\ 1\le r\le 2\}$.

$= 0$. f关于 y 奇, 以关于 xoz 对称.

(4)



$\int_0^9 dz \iint_{D_z} d\sigma$.

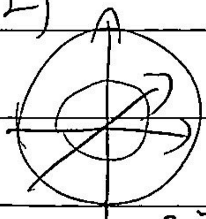
$D_z = \{(x,y) \mid x^2+y^2 \le 9-z\}$.

∴ $D_{r\theta} = \{(r,\theta) \mid 0\le\theta\le 2\pi,\ 0\le r\le\sqrt{9-z}\}$.

∴ 原式 $= \frac{324}{5}\pi$

174.27

(1)



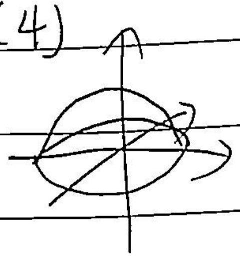
$D_{\rho\varphi\theta} = \{(\rho,\varphi,\theta) \mid 1\le\rho\le 2,\ 0\le\varphi\le\frac{\pi}{2},\ 0\le\theta\le\frac{\pi}{2}\}$.

~~$f(x,y,z) = -f(-x,y,z)$ 以关于 yoz 对称~~ ∴ 原式 $=0$

$\int_0^{\frac{\pi}{2}} d\theta \int_0^{\frac{\pi}{2}} d\varphi \int_1^2 \rho\sin\varphi\cos\theta\, e^{\rho^4} \rho^2 \sin\varphi\, d\rho$

$= \frac{1}{16}(e^{16}-e)\pi$

(4) $D_{\rho\varphi\theta}=\{(\rho,\varphi,\theta)\mid 0\le\rho\le1,\ 0\le\varphi\le\frac{\pi}{2},\ 0\le\theta\le2\pi\}$.

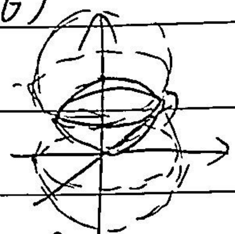


原式 $=\int_0^{2\pi}d\theta\int_0^{\frac{\pi}{2}}d\varphi\int_0^1\frac{\rho\cos\varphi\ln(1+\rho^2)}{1+\rho^2}\cdot\rho^2\sin\varphi\,d\rho$.

$=2\pi\cdot(\frac{3}{2}\ln2-1)\cdot\int_0^{\frac{\pi}{2}}\sin\varphi\cos\varphi\,d\varphi$.

$=\cancel{\frac{\pi}{2}(\frac{3}{2}\ln2-1)}\ \frac{\pi}{2}(\ln2-\frac{1}{2}-\frac{1}{2}\ln^2 2)$.

(6)

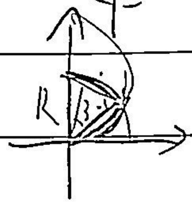


$0\le\varphi\le\frac{\pi}{3}$

$D_{\rho\varphi\theta}=\{(\rho,\varphi,\theta)\mid 0\le\theta\le2\pi,\ 0\le\varphi\le\frac{\pi}{2},\ 0\le\rho\le R$.

$0\le\rho\le2R\cos\varphi$.

$\frac{\pi}{3}\le\varphi\le\frac{\pi}{2}$.

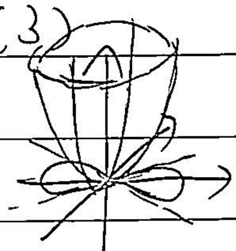


∴原式 $=2\pi\Big(\int_0^{\frac{\pi}{3}}d\varphi\int_0^R\rho^4\cos^2\varphi\sin\varphi\,d\rho+\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}d\varphi\int_0^{2R\cos\varphi}\rho^4\cos^2\varphi\sin\varphi\,d\rho\Big)$

$=\cancel{\frac{13\pi}{60}R^5}\ \frac{59}{480}R^5\pi$.

174.28

(3)

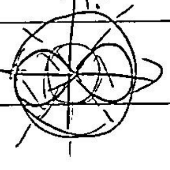


$\cancel{\int_0^{\frac{1}{2}}dz\iint_{D_z}d\sigma}$ $\iint_D d\sigma\int_0^{\frac{1}{2}(x^2+y^2)}z\,dz$.

$\cancel{D_z=\{(x,y)\mid}$

$\cancel{D_{r\theta}=\{(r,\theta)\mid\frac{\pi}{4}\le\theta\le\frac{\pi}{4}}$

∴ $D=\{(x,y)\mid(x^2+y^2)^2=x^2-y^2\}$.

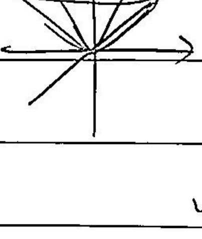


$\frac{1}{8}\iint_D 1-(x^2+y^2)^2\,d\sigma$. $D_{r\theta_1}=\{(r,\theta)\mid-\frac{\pi}{4}\le\theta\le\frac{\pi}{4},\ 0\le r\le\sqrt{\cos2\theta}\}$.

$=\frac{2}{8}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}d\theta\int_0^{\sqrt{2\cos2\theta}}r^5\,dr$.

$=\frac{1}{56}$

(4)



$\int_0^1dz\iint_{D_z}d\sigma$. $D_{\rho\varphi\theta}=\{(\rho,\varphi,\theta)\mid 0\le\theta\le2\pi$

$\cancel{D_z=\{(x,y)\mid\frac{z^2}{3}\le x^2+y^2\le z^2\}}$ $\frac{\pi}{6}\le\varphi\le\frac{\pi}{4}$

$\cancel{∴D_{r\theta}=\{(r,\theta)\mid}$ $0\le\rho\le\frac{1}{\cos\varphi}\}$

∴原式 $=\cancel{\frac{3\sqrt{2}}{135}}\ \cancel{(\frac{54\sqrt{2}-16\sqrt{3}}{135}+\ln\frac{3}{2})}\pi$.

$(\frac{9\sqrt{2}}{27}-\frac{4\sqrt{3}}{27}+\ln\frac{3}{2})\pi$

(5) 令 $\begin{cases} x = a\rho\sin\varphi\cos\theta, \\ y = b\rho\sin\varphi\sin\theta \\ z = c\rho\cos\varphi. \end{cases}$ $\therefore |J| = abc\rho^2\sin\varphi$

$\therefore$ 原式 $= \iiint_{\Omega'} \sqrt{1-\rho^3}\, dV$, $\Omega' = \{(\rho,\varphi,\theta) \mid 0 \le \rho \le 1, 0 \le \theta \le 2\pi, 0 \le \varphi \le \pi\}$.

$= \cancel{(\int_0^{2\pi} d\theta)(\int_0^{\pi} d\varphi)(\int_0^1 \sqrt{1-\rho^3}\, abc\rho^2\sin\varphi\, d\rho)}$

$= \cancel{\frac{8}{9}\pi abc}$

(6) $x = u+a$, $y = v+b$, $z = w+c$ $\therefore |J| = 1$

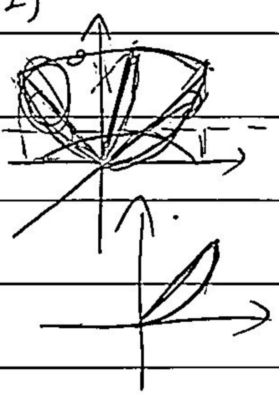
$\iiint_{\Omega'} u^2+v^2+w^2+2au+2bv+2cw+a^2+b^2+c^2\, dV$, $\Omega' = \{u^2+v^2+w^2 \le R\}$.

$\therefore \int_0^{2\pi} d\theta \int_0^{\pi} d\varphi \int_0^R \cancel{\rho^2+\ldots}\ \rho^2\sin\varphi(\rho^3+\rho^2(2a\sin\varphi\cos\theta+2b\sin\varphi\sin\theta+2c\cos\varphi)+(a^2+b^2+c^2)\rho)\, d\rho$

$= \cancel{\frac{1}{2}\pi^2 R^4 + R^2\pi^2(a^2+b^2+c^2)}\ \frac{4}{5}\pi R^5 + \frac{4}{3}\pi R^3(a^2+b^2+c^2)$

175. 29

(2)



$\cancel{\iiint_{\Omega} xyz\, dV}$ ~~以关于~~

$\Omega = \{(\rho,\varphi,\theta) \mid 0 \le \theta \le \frac{\pi}{2}, \frac{\pi}{4} \le \varphi \le \frac{\pi}{2}, 0 \le \rho \le \frac{\cos\varphi}{\sin^2\varphi}\}$.

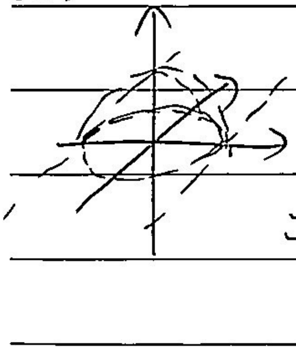
$\therefore \cancel{\int_0^{\frac{\pi}{2}} d\theta \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_0^{\frac{\cos\varphi}{\sin^2\varphi}}}$

$\int_0^1 dz \iint_{D_z} d\sigma$. $D_{r\theta} = \{(r,\theta) \mid 0 \le \theta \le \frac{\pi}{2}, z \le r \le \sqrt{z}\}$.

$\therefore$ 原式 $= \int_0^1 dz \int_0^{\frac{\pi}{2}} d\theta \int_z^{\sqrt{z}} z\sin\theta\cos\theta\, r^3\, dr$

$= \frac{1}{96}$

(3)



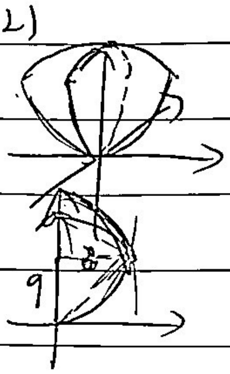
$D_{\rho\varphi\theta} = \{(\rho,\varphi,\theta) \mid 0 \le \theta \le 2\pi, 0 \le \varphi \le \frac{\pi}{2}, 0 \le \rho \le 3\}$.

$\int_0^{2\pi} d\theta \int_0^{\frac{\pi}{2}} d\varphi \int_0^3 \rho^2\cos\varphi\, \rho^2\sin\varphi\, d\rho$.

$= \frac{243}{5}\pi$

175. 30

(2)



$\cancel{D_{\rho\varphi\theta}=\{(\rho,\varphi,\theta)\mid 0\le\theta\le 2\pi,\ 0\le\varphi\le\frac{\pi}{2},\ \rho}$

$\Omega = 2\Omega_1$.

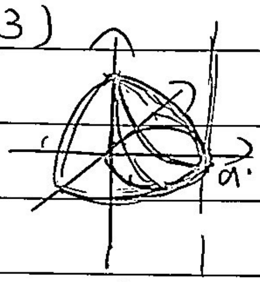
$\Omega_1=\{(x,y,z)\mid 9\ge z\ge \cancel{x^2}+y^2\}$,

$\therefore V = 2\iiint_{\Omega_1} dV = 2\int_0^9 dz\iint_{D_z} d\sigma$

$D_{r\theta}=\{(r,\theta)\mid 0\le\theta\le 2\pi,\ 0\le r\le\sqrt{z}\}$.

$\therefore V = 81\pi$.

(3)



$\cancel{D_{\rho\varphi\theta}=\{(\rho,\varphi,\theta)\mid -\frac{\pi}{2}\le\theta\le\frac{\pi}{2},\ 0\le\varphi\le\pi,\ \rho}$

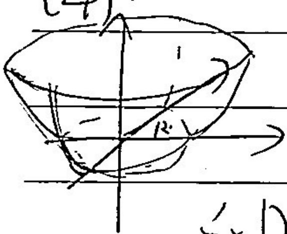
$\cancel{\int_0^a dz\iint_{D_z} d\sigma}$ $\iint_D d\sigma\int_0^{\sqrt{a^2-x^2-y^2}} dz$.

$\cancel{D_{r\theta}=\{(r,\theta)\mid\}}$ σ

$\cancel{x=r\cos\theta}$ $D_{r\theta}=\{(r,\theta)\mid -\pi\le\theta\le\pi,\ 0\le r\le a\cos\theta\}$.

$\therefore V' = 2\int_{0}^{\frac{\pi}{2}} d\theta\int_0^{a\cos\theta}\sqrt{a^2-r^2}\, r\, dr = \frac{2a^3}{3}(\pi-\frac{4}{3})$.

(4).



$\int_0^H dz\iint_{D_z} d\sigma$

$D_z=\{(x,y)\mid 0\le x^2+y^2\le z^2+R^2\}$.

$\therefore D_{r\theta}=\{(r,\theta)\mid 0\le\theta\le 2\pi,\ 0\le r\le\sqrt{z^2+R^2}\}$.

$\int_0^H dz\int_0^{2\pi} d\theta\int_0^{\sqrt{z^2+R^2}} r\, dr = \pi(\frac{1}{3}H^3+HR^2)$.