(2) $\ln y = \ln x + \ln\sin x + \ln\ln x$

$\therefore \frac{y'}{y} = \frac{1}{x} + \frac{\cos x}{\sin x} + \frac{1}{x\ln x} \Rightarrow y' = \sin x\ln x + x\cos x\ln x + \sin x$

(4) $y = -\frac{2\sqrt{t}}{1-t}$ $\therefore y' = -\frac{\frac{1-t}{\sqrt{t}} + 2\sqrt{t}}{(1-t)^2} = \frac{t-1-2t}{\sqrt{t}(1-t)^2} = -\frac{1+t}{\sqrt{t}(1-t)^2}$

(6) $y = 2^x x\sin x + 2^x\cos x$

$\therefore y' = 2^x x\sin x\ln 2 + \cancel{2^x\sin x} + 2^x x\cos x + 2^x\cos x\ln 2 - \cancel{2^x\sin x}$

(8) $y = 1 - \frac{2e^{-x}}{e^x + e^{-x}} = 1 - \frac{2}{e^{2x}+1}$

$\therefore y' = \frac{4e^{2x}}{(e^{2x}+1)^2}$

(10) $y = 1 - \frac{2\ln x}{1+\ln x}$ $\therefore y' = -\frac{\frac{2}{x}(1+\ln x) - \frac{2}{x}\ln x}{(1+\ln x)^2} = -\frac{2}{x(1+\ln x)^2}$

144.20

(1) $y = \sec x - 2\cos x = \frac{1}{\cos x} - 2\cos x \Rightarrow y' = \frac{\sin x}{\cos^2 x} + 2\sin x$

$\therefore y'|_{x=\frac{\pi}{3}} = 3\sqrt{3}$

$\cancel{(3)\ f(x) = 1 - \frac{2\sqrt{x}}{1+\sqrt{x}}}$, $\cancel{f'(x) \xlongequal{u=\sqrt{x}} f'(u^2) = 1 - 2}$

$\cancel{\xlongequal{u=\sqrt{x}} f(u^2) = 1 - \frac{2u}{1+u} \Rightarrow (f(u^2))' = 2u f'(u^2) = -\frac{4u}{(1+u)^2}}$

$\cancel{\therefore f(9) \xlongequal{u=3} f'(u^2) = -\frac{3}{4}}$

(3) $f'(x) = -\frac{1}{\sqrt{x}(1+\sqrt{x})^2}$ $\therefore f'(9) = -\frac{1}{48}$

(5) $\frac{dy}{d\theta}\Big|_{\theta=\frac{\pi}{2}} = y'|_{\theta=\frac{\pi}{2}}$

$y' = \frac{\theta^2}{(\cos\theta + \theta\sin\theta)^2}$ $\therefore y'|_{\theta=\frac{\pi}{2}} = 1$

145.22

(2) $y = (x^2+x+2)^{\frac{2}{3}}$ $\therefore y' = \frac{2}{3}(x^2+x+2)^{-\frac{1}{3}}(2x+1)$

(4) $y' = \frac{\sqrt{1-x} + (1+x)\frac{1}{2\sqrt{1-x}}}{(\sqrt{1-x})^2} = \frac{2-2x+1+x}{2(\sqrt{1-x})^3} = \frac{3-x}{2(\sqrt{1-x})^3}$

(6) $y' = 2\cos 2x - 2x\sin x^2$

(8) $y' = n\cos x\sin^{n-1}x\cos nx - n\sin nx\sin^n x$

$= n\sin^{n-1}x\cos x\cos nx - n\sin^{n-1}x\sin nx\sin x$

$= n\sin^{n-1}x\cos(n+1)x$

145. 22

(10) $y=\cos^2\frac{1-\sqrt{x}}{1+\sqrt{x}}$ $\therefore$ $y'=2\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\cdot\frac{1}{\sqrt{x}(1+\sqrt{x})^2}=\sin\frac{2-2\sqrt{x}}{1+\sqrt{x}}\cdot\frac{1}{\sqrt{x}(1+\sqrt{x})^2}$

(13) $y=2^{\tan\frac{1}{x^2}}$ $\therefore$ $y'=-2^{\tan\frac{1}{x^2}+1}\ln 2\cdot\frac{1}{\cos^2\frac{1}{x^2}}\frac{1}{x^3}$

(15) $\ln y=\cos 2x+\sqrt{1-x}$ $\therefore$ $\frac{y'}{y}=-2\sin 2x-\frac{1}{2\sqrt{1-x}}$

$\therefore$ $y'=-e^{\cos 2x+\sqrt{1-x}}\left(2\sin 2x+\frac{1}{2\sqrt{1-x}}\right)$

(17) $e^y=\ln(\ln x)$ $\therefore$ $y'e^y=\frac{1}{x\ln(x)}$

$\therefore$ $y'=\frac{1}{x\ln x\cdot\ln(\ln(x))}$

(19) $e^y=\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)$ $\therefore$ $y'e^y=\frac{1}{2\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)}$

$\therefore$ $y'=\frac{1}{2\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)}=\frac{1}{2\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)}=\frac{1}{\sin\left(x+\frac{\pi}{2}\right)}=\frac{1}{\cos x}$

(22) $y'=\frac{1}{2}\sqrt{x^2+a^2}+\frac{x^2}{2\sqrt{x^2+a^2}}+\frac{a^2}{2}\frac{1+\frac{x}{\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}}$

$=\frac{1}{2}\sqrt{x^2+a^2}+\frac{x^2}{2\sqrt{x^2+a^2}}+\frac{a^2}{2}\frac{x+\sqrt{x^2+a^2}}{x+\sqrt{x^2+a^2}}\frac{1}{\sqrt{x^2+a^2}}$

$=\frac{1}{2}\sqrt{x^2+a^2}+\frac{x^2}{2\sqrt{x^2+a^2}}+\frac{a^2}{2\sqrt{x^2+a^2}}=\sqrt{x^2+a^2}$

146.23

(1) $f(x)=\begin{cases} 2xe^{-x^2}-2x^3e^{-x^2} & |x|\le 1 \\ 0 & |x|>1 \end{cases}$ $\quad f'_{1^{\pm}}(x) = f'_{-1^{\pm}}(x) = 0$

146.24.

(2) $\ln y = \frac{1}{2}\ln x + \frac{1}{2}\ln\sin x + \frac{1}{4}\ln(1-e^x)$

$\therefore \frac{y'}{y} = \frac{1}{2x} + \frac{\cos x}{2\sin x} - \frac{e^x}{4(1-e^x)}$

$\therefore y' = \sqrt{x\sin x\sqrt{1-e^x}}\left(\frac{1}{2x} + \frac{\cos x}{2\sin x} - \frac{e^x}{4(1-e^x)}\right)$

(4) $\ln y = x\ln\frac{x}{1+x}$

$\therefore \frac{y'}{y} = \ln\frac{x}{1+x} + x\frac{1+x}{x}\cdot\frac{1+x-x}{(1+x)^2} = \ln\frac{x}{1+x} + \frac{1}{1+x}$

$\therefore y' = \cancel{x}\left(\frac{x}{1+x}\right)^x\left(\ln\frac{x}{1+x} + \frac{1}{1+x}\right)$

146.25

(2) $y = (x-1)^2(x+1)^2|x+1| = \begin{cases} (x-1)^2(x+1)^3 & x\ge -1 \\ -(x-1)^2(x+1)^3 & x<-1 \end{cases}$

$\therefore \ln y = 2\ln(x-1) + 3\ln(x+1) \quad (x\ge -1).$

$\therefore \frac{y'}{y} = \frac{2}{x-1} + \frac{3}{x+1} \Rightarrow y' = (x-1)^2(x+1)^3\left(\frac{5x-1}{x^2-1}\right) = (x-1)(x+1)^2(5x-1) \quad x\ge -1$

$\ln y = 2\ln(x-1) + 3\ln(x+1)$

$\therefore \frac{y'}{y} = \frac{2}{x-1} + \frac{3}{x+1} \Rightarrow y' = -(x-1)(x+1)^2(\overset{5x-1}{\cancel{5-x}}) \quad x<-1$

(4) $\frac{1}{|x|}\le 1 \Rightarrow 1\le |x|$

$\therefore y = \begin{cases} \arccos\frac{1}{x} & x\ge 1 \\ \arccos -\frac{1}{x} & x\le -1 \end{cases}$ $\quad \therefore y' = \begin{cases} \frac{-1}{\sqrt{1-\frac{1}{x^2}}}\cdot -\frac{1}{x^2} = \frac{1}{x\sqrt{x^2-1}} & x\ge 1 \\ \frac{-1}{\sqrt{1-\frac{1}{x^2}}}\cdot\frac{1}{x^2} = \frac{-1}{x\sqrt{x^2-1}} & x\le -1 \end{cases}$

$\therefore y = \frac{1}{x\sqrt{1-x^2}} \quad |x|>1$

141.26

(3) $y' = f'(\sin^2 x)\cdot 2\sin x\cos x - f'(\cos^2 x)\, 2\cos x\sin x$

$= f'(\sin^2 x)\sin 2x - f'(\cos^2 x)\sin 2x = \sin 2x\,(f'(\sin^2 x) - f'(\cos^2 x))$

(5) $y' = \cancel{\arctan}\ \dfrac{\frac{\varphi'(x)\psi(x) - \varphi(x)\psi'(x)}{\psi^2(x)}}{1+\left(\frac{\varphi(x)}{\psi(x)}\right)^2} = \dfrac{\varphi'(x)\psi(x) - \psi'(x)\varphi(x)}{\psi^2(x) + \varphi^2(x)}$

146.27 (2)

$\cancel{y = x + \tfrac{1}{2}\left(\sqrt{x^2+1} + \frac{x^2}{\sqrt{x^2+1}}\right) + \frac{\quad}{\sqrt{\ }}}$ $y' = x + \sqrt{x^2+1}$

$\cancel{2y = xy' + \ln y' \Rightarrow 2\frac{y}{y'} = x + \frac{\ln y'}{y'}}$

$\cancel{e^{2y} = e^{xy' + y'}\ y' e^{xy'}\ 2 \therefore e^{2y} = e^{x^2} e^{x\sqrt{x^2+1}}(x + \sqrt{x^2+1})}$

$\therefore 2y = x^2 + x\sqrt{x^2+1} + \cancel{\ln}\ \ln x + \sqrt{x^2+1}$.

$xy' + \ln y' = x^2 + x\sqrt{x^2+1} + \ln x + \sqrt{x^2+1} = 2y$.

146.28

(2) $3x^2 + 3y^2y' - 3a(y + xy') = 0 \Rightarrow x^2 + y^2y' - ay - axy' = 0$

$\Rightarrow ay - x^2 = y'(y^2 - ax) \Rightarrow y' = \dfrac{ay - x^2}{y^2 - ax}$

(4) $y\sin x + y\cos x + (1-y')\sin(x-y) = 0 \Rightarrow y' = \dfrac{-y\cos x - \sin(x-y)}{\sin x - \sin(x-y)}$

146.29

(1) $\cos xy\cdot(y + xy') + \dfrac{y'-1}{y-x} = 1$ 又 $x=0$ 时 $y=1$

$\therefore y'|_{x=0} = 1$

(3) $e^{2x+y}(2+y') + \sin(xy)(y + xy') = 0$. $x=0$ 时. $y=1$

$\therefore y'|_{x=0} = -2$

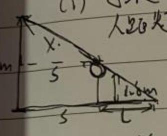
147.37

(1) $\begin{cases} x=(\cos\theta+\sin\theta)\cos\theta \\ y=(\cos\theta+\sin\theta)\sin\theta \end{cases} \Rightarrow \frac{dy}{dx}=-\tan(2\theta-\frac{\pi}{4}) \quad \therefore \frac{dy}{dx}\Big|_{\theta=\frac{\pi}{4}}=-1$

$\because$ 过 $(1,1)$ $\therefore$ $l_{切}: y=-x+2 \quad l_{法}: y=x$

147.39

(1) 影长 L ， 人距灯 S



$\frac{L}{1.8}=\frac{S+L}{6} \Rightarrow L(\frac{1}{1.8}-\frac{1}{6})=\frac{S}{6}$

$\therefore \frac{dL}{dt}(\frac{4.2}{10.8})=\frac{1}{6}\frac{dS}{dt}$

$\Rightarrow V_L=\frac{3}{7}V_人=24\ m/min=0.4\ m/s$

(2) $x^2=S^2+(4.2)^2$

$\therefore 2x\cdot\frac{dx}{dt}=2S\frac{dS}{dt} \Rightarrow V_{头}=\frac{S}{x}V_人=\frac{5}{\sqrt{25+(4.2)^2}}\times 56\ m/min$

$\approx 42.9\ m/min$

147.41

(2) $\ln y=\ln x+\ln\arctan\sqrt{x}$

$\therefore \frac{y'}{y}=\frac{1}{x}+\frac{\frac{1}{2\sqrt{x}}}{\arctan\sqrt{x}}\cdot\frac{1}{1+x}$

$\therefore dy=(\arctan\sqrt{x}+\frac{\sqrt{x}}{2}\frac{1}{(1+x)})\,dx$

(3) ~~$y^2=x+\sqrt{x+\sqrt{x}}$~~ ~~$y^4=$~~

~~$y^2-x=\sqrt{x+\sqrt{x}} \quad \therefore y^4-2xy^2+x^2=x+\sqrt{x}$~~

$\therefore y'=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\cdot\left[1+\frac{1}{2\sqrt{x+\sqrt{x}}}\cdot(1+\frac{1}{2\sqrt{x}})\right]$

$\therefore dy=\frac{4\sqrt{x}\sqrt{x+\sqrt{x}}+2\sqrt{x}+1}{8\sqrt{x+\sqrt{x+\sqrt{x}}}\sqrt{x+\sqrt{x}}\sqrt{x}}\,dx$

147.43 (1)

即 $y'=\frac{3x^2-12x^5-9x^8}{3x^2}=1-4x^3-3x^6$

146.30

(1) $y + y'x + \frac{y'}{y} = 0 \Rightarrow y' = \frac{-y}{x+\frac{1}{y}} \Rightarrow f'(x) = \frac{-f(x)}{x+\frac{1}{f(x)}}$

(2) $g'(x) = f'(\ln x)\frac{1}{x}e^{f(x)} + f'(x)e^{f(x)}f(\ln x)$

$\therefore g'(1) = f'(0)e^{f(1)} + f'(1)e^{f(1)}f(0)$

$f(1) = 1,\ f'(1) = -\frac{1}{2},\ f(0) = e,\ f'(0) = -e^2$

$\therefore g'(1) = -e^3 - \frac{1}{2}e^2$

147.34

(1) $\frac{dy}{dx} = \frac{(1-\arctan t)'}{(\ln(1+t^2))'} = -\frac{1}{2t} = 0 - \frac{1}{2}$

(2) $\frac{dy}{dx} = \tan t \quad \therefore \left.\frac{dy}{dx}\right|_{t=\frac{\pi}{4}} = 1,\ \left.\frac{dy}{dx}\right|_{x=-\frac{\pi}{4}} = -1$

147.35.(2)

$\frac{dy}{dx} = \frac{-2\sin 2t}{\cos t} \qquad \left.\frac{dy}{dx}\right|_{t=\frac{\pi}{6}} = -2$

$(\frac{1}{2}, \frac{1}{2}) \quad \therefore$ 切: $y = -2(x-\frac{1}{2}) + \frac{1}{2}$ 法: $y = \frac{1}{2}(x-\frac{1}{2}) + \frac{1}{2}$

147.36(2).

$\cancel{\frac{dy}{dx}} \ \frac{dy}{dx} = t.$

$\therefore 3 + 2\ln t = yy'$

$2xy'^2 + 1 = \frac{1+\ln t}{t^2} \cdot t^2 \cdot 2 + 1 = 3 + 2\ln t = yy'$

147.42

(1) $dy = \frac{1}{2}\frac{2u\,du\,v^2 + 2v\,dv\,u^2}{u^2+v^2}$

(2) $dy = \dfrac{\frac{1}{u^2}(dv\,u - du\,v)}{1+\frac{v^2}{u^2}} = \dfrac{u\,dv - v\,du}{u^2+v^2}$

148.49

~~$l = R\alpha$~~

~~$\therefore \alpha = \frac{1}{R} l \qquad \delta\alpha = \frac{1}{R}\delta_l = \frac{1}{2000}$ rad~~

$\frac{l}{2} = R \cdot \sin\frac{\alpha}{2} \Rightarrow \frac{L}{2R} = \sin\frac{\alpha}{2} \Rightarrow \alpha = 2\arcsin\frac{L}{2R}$

$d\alpha = \dfrac{2 \cdot \frac{1}{2R}}{\sqrt{1-\frac{L^2}{4R^2}}}\,dl \Rightarrow \delta\alpha = \cdots$

148.50 (2)

$f(x) = \arctan x$, $f(1+0.04) \approx f(1) + df(0.04) = \frac{\pi}{4} + \frac{dx}{1+x^2} = \frac{\pi}{4} + 0.02$

148.51

(2) $y' = \cos x - x\sin x$, $y'' = -\sin x - \sin x - x\cos x$

(4) $y' = 2x\arctan x + 1$, $y'' = 2\arctan x + \frac{x}{1+x^2}$

(6) $y' = f'(\varphi(x))\varphi'(x)$, $y'' = f''(\varphi(x))(\varphi'(x))^2 + \varphi''(x)f'(\varphi(x))$

148.52

(2) $y'(x) = \frac{1}{2\sqrt{x}}e^{\sqrt{x}} - \frac{1}{2\sqrt{x}}e^{-\sqrt{x}}$, $y''(x) = -\frac{1}{4}x^{-\frac{3}{2}}e^{\sqrt{x}} + \frac{1}{4x}e^{\sqrt{x}} + \frac{1}{4}x^{-\frac{3}{2}}e^{-\sqrt{x}} + \frac{1}{4x}e$

$\therefore xy'' + \frac{1}{2}y' - \frac{1}{4}y = -\frac{1}{4}x^{-\frac{1}{2}}e^{\sqrt{x}} + \frac{1}{4}e^{\sqrt{x}} + \frac{1}{4}x^{-\frac{1}{2}}e^{-\sqrt{x}} + \frac{1}{4}e^{-\sqrt{x}} + \frac{1}{4}x^{-\frac{1}{2}}e^{\sqrt{x}} - \frac{1}{4}x^{-\frac{1}{2}}e^{-\sqrt{x}}$

$-\frac{1}{4}e^{\sqrt{x}} - \frac{1}{4}e^{-\sqrt{x}} = 0$

149.52.(3)

$y' = m\cos(m\arcsin x)\dfrac{1}{\sqrt{1-x^2}}$, $y'' = m(-\sin(m\arcsin x)\cdot m\dfrac{1}{(1-x^2)} + \dfrac{x}{(1-x^2)^{\frac{3}{2}}}\cos(m\arcsin x))$

$\therefore (1-x^2)y'' - xy' + m^2y = \cancel{-m\left\{m\sin(m\arcsin x)\frac{1}{1-x^2} - \frac{2x}{1-x^2}\cos(m\arcsin x)\right\}}$

$= 0$

$\cancel{- m\cos(m\arcsin x)\frac{x}{1-x^2} + m^2\sin(m\arcsin x)}$

$= \cancel{\frac{mx}{1-x^2}\cos(m\arcsin x) - m^2\frac{x^2}{1-x^2}\sin(m\arcsin x)}$

$= \cancel{\frac{mx}{1-x^2}(\cos(m\arcsin x) - mx\sin(m\arcsin x)) = \frac{mx}{1-x^2}\cos(m\arcsin x+\varphi)}$

$\cancel{\tan\varphi = mx}$

149.53

$x = \cos t$, $dx = -\sin t\, dt$ $\cancel{d^2x =}$

$\dfrac{d^2y}{dx^2} = \cancel{\dfrac{d^2y}{dt^2}\dfrac{dt^2}{dx^2}\, d\left(\dfrac{dy}{dx}\right)/d(x) = \dfrac{d(\frac{dy}{dx})}{-\sin t\, dt}}$ $y = f(x) = f(\cos t)$.

$\cancel{\dfrac{dy}{dx} = \dfrac{dy}{-\sin t\, dt}}$ $\cancel{d\left(\dfrac{dy}{-\sin t\, dt}\right) = \dfrac{d^2y}{-\cos t\, dt - \sin t}\, dx}$

$\dfrac{d^2y}{dx^2} = \dfrac{d(\frac{dy}{dx})}{dx} = \dfrac{d(\frac{dy}{dx})}{dt}\dfrac{dt}{dx} = \dfrac{d(\frac{dy}{dt}\frac{dt}{dx})}{dt}$ $\dfrac{dt}{dx} = \dfrac{1}{\sin^2 t}\dfrac{d^2y}{dt^2} - \dfrac{\cos t}{\sin^2 t}\dfrac{dy}{dt}$

$\therefore$ 原式 $\Rightarrow \dfrac{d^2y}{dt^2} + y = 0$

149.56

$\cancel{\dfrac{d(\frac{dt}{dy})}{dy}}$ $\cancel{dt}$ $\cancel{dt = \dfrac{1}{dy}}$

$\cancel{dy = (1+\tfrac{1}{x})dx}$, $\cancel{f'(y) = \dfrac{1}{(f'(x))'} = \dfrac{1}{(x+\ln x)'} = \dfrac{1}{1+\frac{1}{x}} = \dfrac{x}{x+1}}$

$f^{(2)}(y) = (f'(y))'$

$f'(y) = \dfrac{1}{(f^{-1}(x))'} = \dfrac{1}{(x+\ln x)'} = \dfrac{1}{1+\frac{1}{x}} = \dfrac{x}{x+1}$

$\dfrac{d^2t}{dy^2} = \dfrac{d(\frac{dt}{dy})}{dx}\cdot\dfrac{dx}{dy} = \left(\dfrac{x}{x+1}\right)'\cdot\dfrac{x}{x+1} = \dfrac{x}{(x+1)^3}$.

149.57

(2) $y' = \frac{1+y'}{\cos^2(x+y)} \Rightarrow y' = \frac{1}{\cos^2(x+y)-1}$

$y'' = \frac{1}{(\cos^2(x+y)-1)^2} 2\cos(x+y)\sin(x+y)(1+y')$

$= \frac{\sin(2x+2y)\cos^2(x+y)}{[\cos^2(x+y)-1]^3}$

(3) $\frac{\frac{y-y'x}{y^2}}{1+\frac{x^2}{y^2}} = \frac{y-y'x}{y^2+x^2} = \frac{\frac{2x+2yy'}{2\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}}$

$\Rightarrow y-y'x = x+yy' \Rightarrow y' = \frac{y-x}{y+x}$

$y'' = \frac{(y'-1)(y+x)-(y'+x)(y-x)}{(y+x)^2} = \frac{\cancel{\frac{2x(y-x)}{y+x}}+x^2-y-yx}{(y+x)^2}$

$= -\frac{2(x^2+y^2)}{(x+y)^3}$

149.58

$y' = e^{xy} + x(y+xy')e^{xy} = e^{xy}(xy+x^2y'+1)$

$x=0$ 时, $y=1$ $\therefore$ ~~$y'|_{x=0}=$~~

$y' = \frac{xye^{xy}+e^{xy}}{1-x^2e^{xy}}$ $\therefore y'|_{x=0} = 1$

$y'' = (xy'+y)e^{xy}(xy+x^2y'+1) + e^{xy}(y+xy'+2xy'+y'x^2)$

$\therefore y''|_{x=0} = 2$

149.62 (2)

$y^{(100)} = \sum_{k=0}^{100} C_{100}^{k} [(x+1)^2]^{(k)} (e^{2x})^{(100-k)}$

$= (x+1)^2 2^{100} e^{2x} + 100(2x+2)2^{99}e^{2x} + 100\times 99\times 2^{98} e^{2x}$

149.63

(1) $y = 1-\frac{2x}{1+x}$ $\therefore y' = -\frac{2}{(1+x)^2}$

$\therefore y^{(n)} = -2(-2)(-3)\cdots(\overset{-1-n}{-2-n+1})\frac{1}{(1+x)^{n+1}}$

$= -2\frac{(-1)^{n-1}n!}{(1+x)^{n+1}} = (-1)^n \frac{2\cdot n!}{(1+x)^{n+1}}$

(3) $y' = 2\sin x\cos x = \sin 2x$.

$\therefore y^{(n)} = 2^{n-1}\sin(2x + \frac{n-1}{2}\pi)$.

(5) $y = \frac{1}{(x-1)(x-2)} = \frac{1}{x-2} - \frac{1}{x-1}$

$\therefore y^{(n)} = \frac{(-1)^n n!}{(x-2)^{n+1}} - \frac{(-1)^n n!}{(x-1)^{n+1}}$

(7) $y = (x^2+2x+2)e^{-x}$

$y^{(n)} = (-1)^n(x^2+2x+2)e^{-x} + n(-1)^{n-1}(2x+2)e^{-x} + n(n-1)(-1)^{n-2}e^{-x}$

1~~4~~30.64

$f(x) = (x-a)^n\varphi(x)$. ~~$f'(x) = n(x-a)^{n-1}\varphi(x)$. $f''(x) = n(n-1)(x-a)^{n-2}\varphi$~~

$f^{(n)}(x) = \sum_{k=0}^{n} C_n^k (x-a)^{n(k)}\varphi^{(n-k)}(x) = (x-a)^n\varphi^{(n)}(x) + n^2(x-a)^{n-1}\varphi^{(n-1)}(x)$.

$+ \cdots + n!\varphi(x)$.

$\therefore f^{(n)}(a) = \varphi(a)\cdot n!$

130.65

$f'(x) = \frac{1}{1+x^2}$ $f''(x) = \frac{-2x}{(1+x^2)^2} = \frac{1}{(1+x^2)}\frac{-2x}{1+x^2} = \frac{-2x}{1+x^2}f'(x)$.

(1) $\therefore (1+x^2)f''(x) + 2xf'(x) = 0$

$\therefore [(1+x^2)f''(x) + 2xf'(x)]^n = 0$

$\therefore (1+x^2)f^{(n+2)}(x) + 2nxf^{(n+1)}(x) + n(n-1)f^{n}(x) + 2xf^{(n+1)}(x) + 2nf^{(n)}(x) = 0$

$\therefore (1+x^2)f^{(n+2)}(x) + (2nx+2x)f^{(n+1)}(x) + (n^2+n)f^{n}(x) = 0$.

$n \to n-1$ 得证.

(2) $f'(0) = 1$, $f''(0) = 0$

$(1+x^2)f^{(n+2)}(x) + 2x(n+1)f^{(n+1)}(x) + n(n+1)f^{n}(x) = 0$

$\therefore f^{(n+2)}(0) + n(n+1)f^{(n)}(0) = 0$.

$\therefore f^{(n)}(0) = \begin{cases} 0 & n\text{为偶} \\ (-1)^{k-1}(2k-2)! & n = 2k-1. \end{cases}$

~~150. 66.~~

$y' = \cancel{-\frac{1}{2}(1-x^2)^{-\frac{3}{2}}\arcsin x + \frac{1}{1-x^2}}$

$y'' = \cancel{-\frac{1}{2}\left(3x(1-x^2)^{-\frac{5}{2}}\arcsin x + (1-x^2)^{-2}\right)} + \frac{2x}{(1-x^2)^2}$

$= \cancel{-\frac{3}{2}x(1-x^2)^{-\frac{5}{2}}\arcsin x + \frac{2x-\frac{1}{2}}{(1-x^2)^2}}.$

$\therefore \cancel{(1-x^2)^2 y'' = 3(1-x^2)y' + 2x - \frac{7}{2}}$

$\therefore \cancel{((1-x^2)^2 y'')^{(n)} = (3(1-x^2)y' + 2x - \frac{7}{2})^{(n)}}$

$\therefore \cancel{(1-x^2)^2 y^{(n+2)} - 4xn(1-x^2)y^{(n+1)}} -$

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$y' = x(1-x^2)^{-\frac{3}{2}}\arcsin x + (1-x^2)^{-1}.$

$y'' = 3x^2(1-x^2)^{-\frac{5}{2}}\arcsin x + x(1-x^2)^{-2} + (1-x^2)^{-\frac{3}{2}}\arcsin x. + 2x(1-x^2)^{-2}$

$= \frac{3x}{1-x^2}y' + \frac{1}{1-x^2}y.$

$\therefore (1-x^2)y'' = 3xy' + y.$

$\therefore (1-x^2)y^{(n+1)} - (2n+1)xy^{(n)} - n^2y^{(n-1)} = 0.$

(2). 当 $x=0$ 时. $y^{(n+1)} = n^2 y^{(n-1)}$

$y'|_{x=0} = 1, \quad y''|_{x=0} = 0.$ $\quad y^{(n+2)} = (n+1)^2 y^{(n)}$

$\therefore y^{(n)}(0) = \begin{cases} 0 & n=2k \\ \cancel{[(2k-1)!!]^2} \; [(2k+2)!!]^2 \; 4^k(k!)^2 & n=2k+1. \end{cases}$