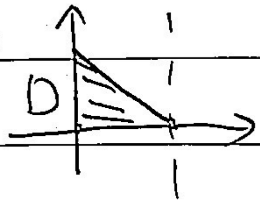
171.13

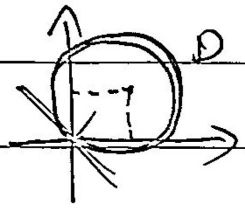
(3)



$\theta \in [0, \frac{\pi}{2}]$, $\quad r \in [0, \frac{1}{\sin\theta+\cos\theta}]$.

原式 $= \int_0^{\frac{\pi}{2}} d\theta \int_0^{\frac{\sqrt{2}}{2\sin(\theta+\frac{\pi}{4})}} f(r\cos\theta, r\sin\theta)\, r\, dr$

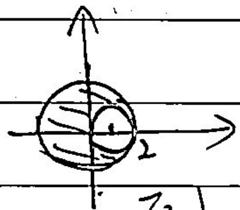
(4)



$\theta \in [-\frac{\pi}{4}, \frac{3}{4}\pi]$, $\quad r \in [0, 2(\cos\theta+2\sin\theta))$

$\therefore$ 原式 $= \int_{-\frac{\pi}{4}}^{\frac{3}{4}\pi} d\theta \int_0^{2\sqrt{2}\sin(\theta+\frac{\pi}{4})} f(r\cos\theta, r\sin\theta)\, r\, dr$

(5)



$\theta \in [0, 2\pi]$. $\quad r \in [2\cos\theta, 2]$

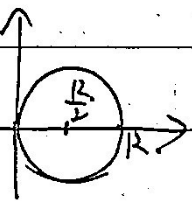
$\cancel{D_{\theta r} = \cdots}$

原式 $= \cancel{\int_0^{2\pi} d\theta \int_{2\cos\theta}^{2} f(r\cos\theta, r\sin\theta)\, r\, dr}$

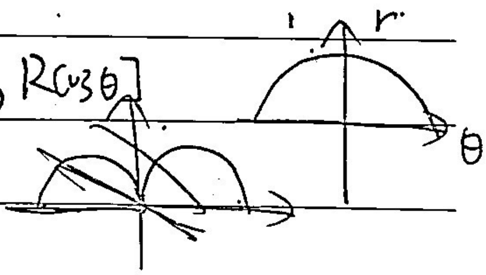
$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_{2\cos\theta}^{2} f(r\cos\theta, r\sin\theta)\, r\, dr + \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} d\theta \int_0^2 f(r\cos\theta, r\sin\theta)\, r\, dr.$

171.14

(1)



$\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ $\quad r \in [0, R\cos\theta]$



$\therefore$ 原式 $= \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^{R\cos\theta} \sqrt{R^2-r^2}\, r\, dr$

$= -\frac{1}{3} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} R^3(|\sin^3\theta|-1)\, d\theta$

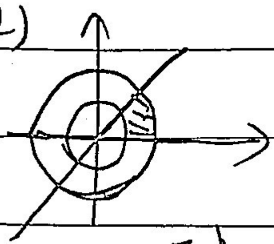
$= \frac{R^3}{3} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1\, d\theta = \frac{\pi}{3} R^3 \quad \times$

$\cancel{2\int_0^{\frac{\pi}{2}} R^3(\sin^3\theta-1)\, d\theta = \frac{R^3}{3}(\pi-\frac{4}{3})}$

~~为什么这里不用对称结果不一样~~ / 马虎了开根忘记

171.14

(2)

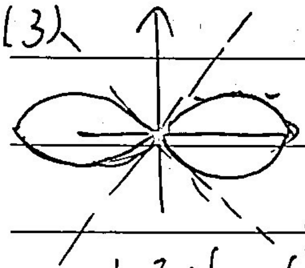


$\theta \in [0, \frac{\pi}{4}]$, $r \in [1, 2]$

$\therefore$ 原式 $= \int_0^{\frac{\pi}{4}} d\theta \int_1^2 \arctan(\tan\theta) r\, dr$

$= \int_0^{\frac{\pi}{4}} \frac{3}{2}\theta\, d\theta = \frac{3}{4}\theta^2 \Big|_0^{\frac{\pi}{4}} = \frac{3\pi^2}{64}$

(3)



$\theta \in [-\frac{\pi}{4}, \frac{\pi}{4}] \cup [\frac{3\pi}{4}, \frac{5}{4}\pi]$

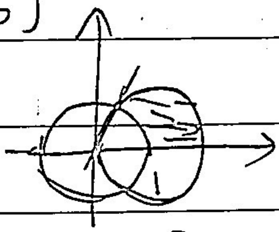
$r \in [0, a\sqrt{\cos 2\theta}]$.

$\therefore$ 原式 $= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\theta \int_0^{a\sqrt{\cos 2\theta}} r^3\, dr + \int_{\frac{3}{4}\pi}^{\frac{5}{4}\pi} d\theta \int_0^{a\sqrt{\cos 2\theta}} r^3\, dr$

$= \frac{a^4}{16}\pi \cancel{\theta} \times 2$ 关于y对称轴. 关于x倍.

$= \frac{\pi}{8} a^4$.

(5)



$(\frac{1}{2}, \frac{\sqrt{3}}{2})$.

$\theta \in [0, \cancel{\arctan} \frac{\pi}{3}]$, $r \in [1, 2\cos\theta]$.

$\therefore$ 原式 $= \int_0^{\frac{\pi}{3}} d\theta \int_1^{2\cos\theta} r^3 \cos\theta \sin\theta\, dr$

$= -\int_0^{\frac{\pi}{3}} (4\cos^4\theta - \frac{1}{4}) \cos\theta\, d\cos\theta$

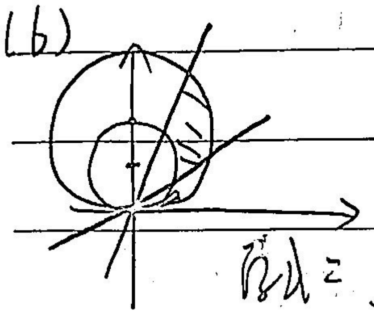
$\overset{\cos\theta = u}{=} \cancel{-} \int_{\frac{1}{2}}^{1} (4u^4 - \frac{1}{4}) u\, du$

$= \cancel{2} (\frac{2}{3}u^6 - \frac{1}{8}u^2) \Big|_{\frac{1}{2}}^{1}$

$= \frac{9}{16}$

171.14

(6)



$\theta \in [\frac{\pi}{6}, \frac{\pi}{3}]$, $r \in [2\sin\theta, 4\sin\theta]$

$$原式 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} d\theta \int_{2\sin\theta}^{4\sin\theta} r^3 dr$$

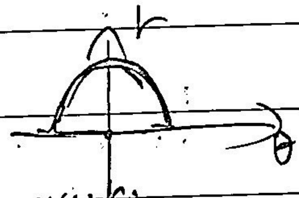
$$= 60 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin^4\theta \, d\theta$$

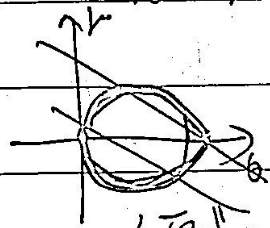
$\frac{15}{4}\pi + \frac{15}{8}\sqrt{3}$?

$$= 15 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (1 - 2\cos 2\theta + \frac{1}{2} + \frac{1}{2}\cos 4\theta) d\theta = \frac{15}{4}\pi - \frac{15}{8}\sqrt{3}$$

171.15

(1) $D_{r\theta} = \{(r,\theta) \mid -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}, 0 \le r \le 2a\cos\theta\}$:

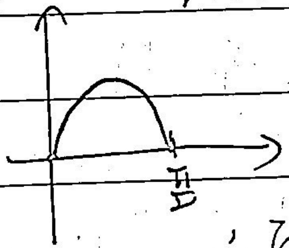




$\therefore D_{\theta r} = \{\cancel{(\theta, r)}(r,\theta) \mid 0 \le r \le 2a, -\arccos\frac{r}{2a} \le \theta \le \arccos\frac{r}{2a}\}$.

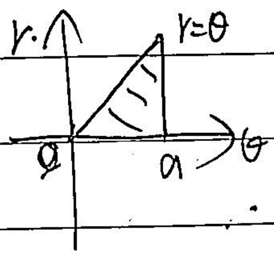
$$\therefore 原式 = \int_0^{2a} dr \int_{-\arccos\frac{r}{2a}}^{\arccos\frac{r}{2a}} f(r,\theta) d\theta.$$

(2) $D_{r\theta} = \{(r,\theta) \mid 0 \le \theta \le \frac{\pi}{2}, 0 \le r \le a\sqrt{\sin 2\theta}\}$.



$\therefore D = \{(r,\theta) \mid 0 \le r \le a, \frac{1}{2}\arcsin\frac{r^2}{a^2} \le \theta \le \frac{\pi}{2} - \frac{1}{2}\arcsin\frac{r^2}{a^2}\}$.

$$\therefore 原式 = \int_0^a dr \int_{\frac{1}{2}\arcsin\frac{r^2}{a^2}}^{\frac{\pi}{2} - \frac{1}{2}\arcsin\frac{r^2}{a^2}} f(r,\theta) d\theta$$



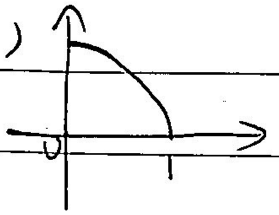
(3) $D_{r\theta} = \{(r,\theta) \mid 0 \le \theta \le a, 0 \le r \le \theta\}$.

$\therefore D_{r\theta} = \{(r,\theta) \mid 0 \le r \le a, r \le \theta \le a\}$.

$$\therefore 原式 = \int_0^a dr \int_r^a f(r,\theta) d\theta.$$

172.16

(1)

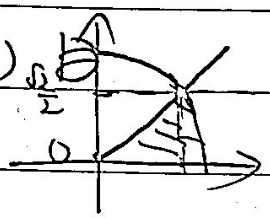


∴ $D_{r\theta}=\{(r,\theta)\mid 0\le\theta\le\frac{\pi}{2},\ 0\le r\le 1\}$.

∴ 原式 $=\int_0^{\frac{\pi}{2}}d\theta\int_0^1 e^{r^2} r\,dr$

$=\frac{\pi}{4}(e-1)$.

(2)

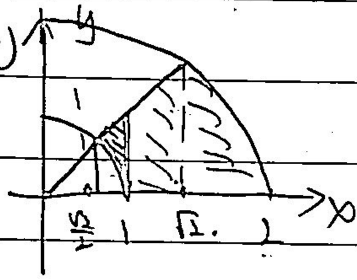


∴ $D_{r\theta}=\{(r,\theta)\mid 0\le\theta\le\frac{\pi}{4},\ 0\le r\le 1\}$.

∴ 原式 $=\int_0^{\frac{\pi}{4}}d\theta\int_0^1 \arctan(\tan\theta)\, r\,dr$

$=\frac{1}{4}\theta^2\Big|_0^{\frac{\pi}{4}}=\frac{\pi^2}{64}$

(3)



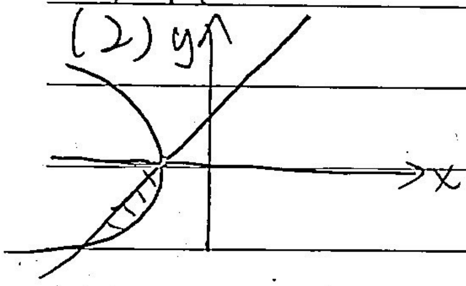
$D_{r\theta}=\{(r,\theta)\mid 0\le\theta\le\frac{\pi}{4},\ 1\le r\le 2\}$.

∴ 原式 $=\int_0^{\frac{\pi}{4}}d\theta\int_1^2 r^3\sin\theta\cos\theta\,dr$

$=\frac{15}{16}\int_0^{\frac{\pi}{2}}\sin\theta\,d\theta=\frac{15}{16}$.

172.19

(2)

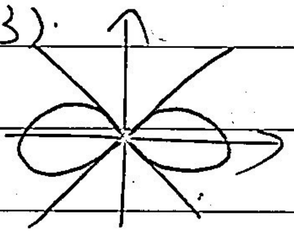


交点: $(-1,0),(-2,-1)$

∴ $A_D=\int_{-1}^{0}-(y^2-1-y+1)\,dy$

$=-\frac{1}{3}y^3+\frac{1}{2}y^2\Big|_{-1}^{0}=\left|-\frac{1}{6}\right|=\frac{1}{6}$

(3)

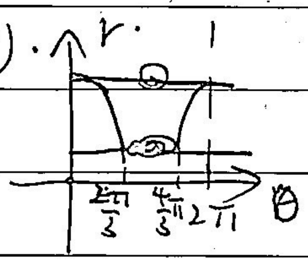


$D_{r\theta}=\{(r,\theta)\mid -\frac{\pi}{4}\le\theta\le\frac{\pi}{4},\ 0\le r\le 2\sqrt{\cos 2\theta}\}$.

$A_D=2A_{D_{r\theta}}=2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}d\theta\int_0^{2\sqrt{\cos 2\theta}} r\,dr$

$=4$

(5)



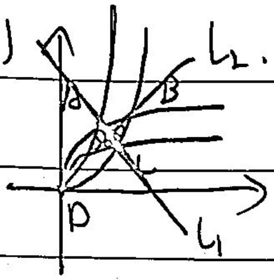
$A_D=\cancel{\int_0^{2\pi}d\theta\int_{\frac{1}{2}}^{1+\cos\theta}dr}$

$\int_0^{\frac{2}{3}\pi}d\theta\int_{\frac{1}{2}}^{1+\cos\theta}r\,dr+\int_{\frac{4}{3}\pi}^{2\pi}d\theta\int_{\frac{1}{2}}^{1+\cos\theta}r\,dr$

$=\left(\frac{7\sqrt{3}}{16}+\frac{5}{12}\pi\right)\times 2=\frac{7}{8}\sqrt{3}+\frac{5}{6}\pi$

173.19 $\quad L_2: y=x, \quad L_1: y=-x+2^{-\frac{3}{4}}+2^{-\frac{1}{4}}.$

(8)



交点：$A(2^{-\frac{3}{4}}, 2^{-\frac{1}{4}}) \quad C(2^{-\frac{1}{4}}, 2^{-\frac{3}{4}}).$

$B(1, 1) \quad D(\frac{1}{2}, \frac{1}{2}).$

$\therefore A_D = 4A_{D_1}.$ $\quad x \in [2^{-\frac{3}{4}}, 2^{-\frac{7}{4}}+2^{-\frac{5}{4}}]$

$D_1 = \{(x,y) \mid 2^{-\frac{3}{4}} \le x \le 2^{-\frac{1}{4}}, (\frac{x}{4})^{\frac{1}{3}} \le y \le x$

$(\frac{x}{4})^{\frac{1}{3}} \le y \le -x+2^{-\frac{3}{4}}+2^{-\frac{1}{4}}\}.$

$$\therefore A_{D_1} = 4\int_{2^{-\frac{3}{4}}}^{2^{-\frac{7}{4}}+2^{-\frac{5}{4}}} dx \int_{(\frac{x}{4})^{\frac{1}{3}}}^{x} dy$$

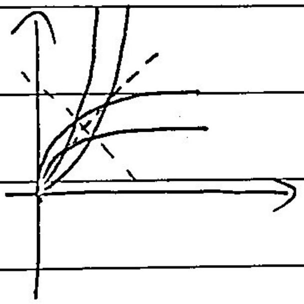
$$= 4\left(\frac{1}{2}x^2 - \frac{3}{4}(\frac{1}{4})^{\frac{1}{3}}x^{\frac{4}{3}}\right)\Big|_{2^{-\frac{3}{4}}}^{2^{-\frac{7}{4}}+2^{-\frac{5}{4}}}$$

$$+ 4\left(-\frac{1}{2}x^2+(2^{-\frac{3}{4}}+2^{-\frac{1}{4}})x - \frac{3}{4}(\frac{1}{4})^{\frac{1}{3}}x^{\frac{4}{3}}\right)\Big|_{2^{-\frac{7}{4}}+2^{-\frac{5}{4}}}^{2^{-\frac{1}{4}}}$$

$$= 1+2^{-\frac{1}{2}} - 3\cdot 2^{-\frac{3}{2}} + 3\cdot 2^{-\frac{5}{3}} - 3\cdot 2^{-1}$$

173.19.

(8)



令 $u = \frac{y}{x^3}, \quad v = \frac{x}{y^3}$

$\therefore D_{uv} = \{(u,v) \mid 1 \le u \le 4, 1 \le v \le 4\}.$

$\therefore A_D = \cancel{A_{D_{uv}}} \int_1^4 du \int_1^4 |J| \, dv$

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{8}u^{-\frac{1}{2}}v^{-\frac{1}{2}}$$

$\therefore A_D = \frac{1}{8}$