298.1

(2) $\int_0^1 e^x dx = \lim\limits_{\lambda\to 0}\sum\limits_{i=1}^{n} e^{\xi_i}\Delta x_i$ ~~令 $\xi_i = x_i$，$x_n = 1$，$\Delta x = \frac{1}{n}$~~

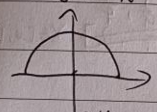
∴ $\cancel{\lim\limits_{\lambda\to 0}\sum\limits_{i=1}^{n} e^{\xi_i}\Delta x_i = \lim\limits_{\lambda\to 0}\frac{1}{n}(e^{\frac{1}{n}}+e^{\frac{2}{n}}+\cdots+e^{1}) = \lim\limits_{n\to\infty}\frac{e^{\frac{1}{n}}+e^{\frac{2}{n}}+\cdots+e^{1}}{n}}$

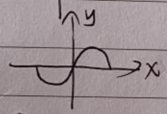
令 $\xi_i = x_i$，$x_i = \ln(1+\frac{i}{n}(e-1))$. ∴ $\int_0^1 e^x dx = \sum\limits_{i=1}^{n}\frac{n+(e-1)i}{n}\cdot\frac{e-1}{n+(e-1)i} = e-1$.

∵ $\Delta x_i = \ln(1+\frac{e-1}{n+(e-1)i}) \overset{\lim\limits_{n\to\infty}}{=} \frac{e-1}{n+(e-1)i}$，$e^{x_i} = \frac{n+(e-1)i}{n}$

298.2

(3) 令 $y=\sqrt{a^2-x^2}$ ∴ $y^2+x^2=a^2$ $(y\geq 0)$

∴  ∴ $\int_0^a y\,dx = \frac{1}{4}\pi a^2$.

(4)  $\int_{-\pi}^{\pi} y\,dx = 0$

298.3

(1) $\int_0^{\pi}\sin x\,dx = \lim\limits_{\lambda\to 0}\sum\limits_{i=1}^{n}\sin\xi_i\cdot\Delta x_i$ 取 $\xi_i = \frac{i}{n}$，$\Delta x_i = \frac{1}{n}$

∴ $\lim\limits_{\lambda\to 0}\frac{1}{n}\sum\limits_{i=1}^{n}\sin\frac{i}{n} = 2$

又 ∵ $\lim\limits_{n\to\infty}\frac{1}{n}(\sin\frac{\pi}{n}+\cdots+\sin\frac{n-1}{n}\pi) = \frac{1}{\pi}\lim\limits_{n\to\infty}\frac{\pi}{n}(\sum\limits_{i=1}^{n-1}\sin\frac{i}{n}\pi) = \frac{2}{\pi}$.

(3). $\int_1^2 \ln x\,dx = \lim\limits_{\lambda\to 0}\sum\limits_{i=1}^{n}\ln\xi_i\cdot\Delta x_i$ 取 $\xi_i = x_i$ $x_1 = 1$，$x_n = 2$. 取 $\Delta x = \frac{1}{n}$.

$x_i = \frac{i}{n}+1$.

$\lim\limits_{n\to\infty}\frac{\sqrt[n]{(n+1)\cdots(2n)}}{n} = e^{\lim\limits_{n\to\infty}\frac{1}{n}(\ln(n+1)+\cdots+\ln(2n))-\ln n}$.

∴ $\lim\limits_{n\to\infty}\frac{1}{n}(\ln(\frac{1}{n}+1)+\cdots+\ln(2))] = 2\ln 2-1$ ∴ 原式 $= e^{2\ln 2-1} = \frac{4}{e}$

298.5

$f(x)\in C[a,b]$.

(1) ∵ $f(x)\not\equiv 0$ ∴ ∃ x_0，δ，在 $U(x_0,\delta)$ 中，$f(x)>0$ ∴ $\int_a^b f(x)dx \geq \int_{x_0-\delta}^{x_0+\delta} f(x)dx > 0$

(2) 令 $F(x)=f(x)-g(x)$ ∴ 同(1)理，有 $\int_a^b F(x)dx > 0$

∴ $\int_a^b f(x)dx > \int_a^b g(x)dx$.

298.6

(2) $e^x \geq 1+x$ 且 $x=0$ 时取等.

$\therefore \int_0^1 e^x dx > \int_0^1 (1+x) dx$

(3). $f(x)=\ln x$，$x\in(1,e)$ 时 $f(x)<1$.

$\therefore f(x) > f^2(x)$ 又 $\because f(x)$ 在 $R\uparrow$ $\therefore f(x^2) > f(x)$ 在 $[1,+\infty)$ 成立.

$\therefore f(x^2) > f(x) > f^2(x)$ $\therefore$ - - -

298.7.

(1) 令 $f(x)= -x^2+x+2 = -(x-\frac{1}{2})^2+\frac{9}{4}$ $\therefore f(x)\in[\overset{2,\ \frac{9}{4}}{\cancel{\frac{9}{4},2}}]$ $x\in[0,1]$ 时.

$\therefore \frac{1}{\sqrt{2+x-x^2}} \in [\frac{2}{3}, \frac{\sqrt{2}}{2}]$.

$\therefore \frac{2}{3} = \frac{2}{3}(1-0) \leq \int_0^1 \frac{dx}{\sqrt{2+x-x^2}} \leq \frac{\sqrt{2}}{2}(1-0) = \frac{\sqrt{2}}{2}$.

(2) 令 $f(x)= \frac{\sin x}{x}$ $x\in[\frac{\pi}{4},\frac{\pi}{2}]$ $\therefore f'(x)= \frac{x\cos x-\sin x}{x^2}$

令 $g(x) = x\cos x-\sin x$ $\therefore g'(x) = \cancel{\cos x} - x\sin x < 0$，$g(0)=0$

$\therefore g(x)\leq 0$ 在 $[\frac{\pi}{4},\frac{\pi}{2}]$ 成立. $\therefore f(x)$ 在 $[\frac{\pi}{4},\frac{\pi}{2}]\downarrow$

$f(\frac{\pi}{4}) = \frac{2\sqrt{2}}{\pi}$，$f(\frac{\pi}{2}) = \frac{2}{\pi}$.

$\therefore \frac{1}{2} = \frac{2}{\pi}\cdot\frac{\pi}{4} < \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x}{x} dx \leq \frac{2\sqrt{2}}{\pi}\cdot\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.