1. $h(n) \in R$, FIR, 线性相位. 证明: $H(N-k) = H^*(k)$.

$\because H(k) = \sum_{n=0}^{N-1} h(n) e^{-jn\frac{2\pi}{N}k}$, $H(k) = |H(k)| e^{j\varphi(k)}$ $\varphi(k) = C\cdot k$. C是const.

$H^*(k) = \sum_{n=0}^{N-1} h(n) e^{jn\frac{2\pi}{N}k}$, $H(N-k) = \sum_{n=0}^{N-1} h(n) e^{-jn\frac{2\pi}{N}(N-k)}$

$= \sum_{n=0}^{N-1} h(n) e^{jn\frac{2\pi}{N}k}$

2. 低通, $N=9$. 已知 $|H_d(k)|$

$H(k) = |H(k)| e^{j\varphi(k)}$ $\cancel{\varphi(k)=}$ $N = 2M+1 \Rightarrow M=4$

$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{4} |H(k)| \cos\frac{2\pi k(n-4)}{N}\right]$ $n = 0,1,2,3,4$

$\cancel{h(n)=}$ $\varphi(k) = -(\frac{8}{2})\frac{2\pi}{N}k = -\frac{8\pi}{9}k$, 低通

$H(0) = 1$, $H(1) = e^{-j\frac{8\pi}{9}}$, $H(2) = e^{-j\frac{16}{9}\pi}$, $H(3) = 0.4e^{-j\frac{24}{9}\pi}$ $H(4) = 0$

$|H(0)| = 1$ $|H(1)| = 1$ $|H(2)| = 1$ $|H(3)| = 0.4$ $|H(4)| = 0$

$h(n) = \frac{1}{9}\left[1 + 2\left[\cos\frac{2\pi(n-4)}{9} + \cos\frac{4\pi(n-4)}{9} + 0.4\cos\frac{6\pi(n-4)}{9}\right]\right]$

$n = 0, 1, \cdots, 8$

3. FIR 线性高通 $f_c = 2000\,Hz$. $f_s = 8000\,Hz$, $N=5$. 求 $h(n)$

$\omega = \frac{2\pi}{N} = \frac{2\pi}{5}$. $\cancel{H_d(0)=0}$. $\because \omega_c = 2\pi \cdot f_c/f_s = \frac{\pi}{2}$ $\therefore H_d(\omega) = \begin{cases} 1 & \frac{\pi}{2} \le \omega \le \frac{3}{2}\pi \\ 0 & \text{o.w.} \end{cases}$

$\therefore |H(0)| = 0$. $|H(1)| = 0$, $|H(2)| = 1$ $|H(3)| = 1$. $|H(4)| = 0$

由于是高通. 故 $h(n) = h(4-n)$ $h(n) = \frac{1}{5}\{0 + 2\sum_{k=1}^{2} H_k \cos[\frac{2\pi}{N}k(n-\frac{N-1}{2})]$

$\therefore h(n) = \frac{2}{5}\cos[\frac{4\pi}{5}(n-2)]$ $n = 0 \sim 4$.

4. FIR线性带通. $\cancel{\omega_L}$ $f_L = 1500\,Hz$. $f_H = 3000\,Hz$. $f_s = 8000\,Hz$. $N=5$.

$\omega_L = 2\pi \cdot f_L / f_s = \frac{3}{8}\pi$. $\omega_H = \frac{6}{8}\pi$ $\therefore$ $H_d(\omega) = \begin{cases} 1 & \frac{3\pi}{8} \le \omega \le \frac{6}{8}\pi,\ \frac{10}{8}\pi \le \omega \le \frac{13}{8}\pi \\ 0 & o.w. \end{cases}$

$h(n) = h(4-n)$ 是全能. 可以用

$\cancel{H_k}$ $H_0 = 0$. $H_1 = 1$. $H_2 = 0$. $H_3 = 0$. $H_4 = 1$

$h(n) = \frac{1}{5}\left\{2\sum_{k=1}^{2} H_k \cos\left[\frac{2\pi}{N}k(n-M)\right]\right\} = \frac{2}{5}\cos\left[\frac{2\pi}{5}(n-2)\right]$ $n = 0\sim4$.

也能用 $h(n) = -h(4-n)$. 可以带通.

$h(n) = \frac{2}{N}\sum_{k=1}^{M} H_k \sin\left[\frac{2\pi}{N}k(M-n)\right] = \frac{2}{5}\sin\left[\frac{2\pi}{5}(2-n)\right]$ $n = 0\sim4$.