111.6

(1) $\lim\limits_{(x,y)\to(0,0)} \frac{2-\sqrt{x+y+4}}{x+y} = \lim\limits_{\cdots} \frac{1-\sqrt{\frac{1}{4}(x+y)+1}}{\frac{x+y}{2}} \overset{x+y=t}{=\!=} \lim\limits_{t\to0} \frac{-\frac{1}{8}t}{\frac{1}{2}t} = -\frac{1}{4}$

(2) $\lim\limits_{\substack{x\to0\\y\to0}} \frac{(2+x)\ln(1+xy)}{xy} \overset{xy=t}{=\!=} \lim\limits_{\substack{t\to0\\x\to0}} (x+2)\frac{\ln(1+t)}{t} = 2$

(4) $\lim\limits_{\substack{x\to0\\y\to0}} \sqrt{x^2+y^2}\sin\frac{1}{\sqrt{x^2+y^2}} \in [0,\sqrt{x^2+y^2}] = 0$

111.7

(1) $x=0$ 或 $y=0$ 或 $x=y=0$ 时 $f(x,y)=0$

$xy\neq0$ 时 $0\leq|f(x,y)| = |x\sin\frac{1}{y}+y\sin\frac{1}{x}| \leq |x|+|y| = 0$

$\therefore \lim\limits_{(x,y)\to(0,0)} f(x,y)=0$

(2) $x+y=0$ 时. $f(x,y)=0$.

$x+y\neq0$ 时. ~~$\lim\limits_{(x,y)\to(0,0)} |f(x,y)| = \lim\limits_{\substack{x\to0\\y\to0}} \left|\frac{xy}{x+y}\right| \leq \frac{xy+y^2}{x+y} = \frac{y(x+y)}{x+y} = y$~~

$\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x+y} \overset{x+y=t}{=\!=} \lim\limits_{\substack{t\to0\\y\to0}} \frac{(t-y)y}{t} = y-\frac{y^2}{t}$. 不存在.

111.8

(2) $\sin\pi x=0$ 或 $\sin\pi y=0$ 时. $x\in k$, $y\in k$

$\therefore$ 为 (k,y), (x,k) $k\in z$ (m,n) $m,n\in z$

(4) 在 $xyz=0$ 处

111.9

(2) $z(x,0)=\ln(x)$ $\therefore \frac{\partial z}{\partial x}\Big|_{(1,0)} = \frac{1}{x} = 1$

(3) $z(x,1) = (1+x)$ $\frac{\partial z}{\partial x}\Big|_{(1,1)} = 1$

$z(1,y) = (1+y)^y$ $\frac{\partial z}{\partial y} = (1+y)^y\left(\ln(1+y)+\frac{y}{1+y}\right)$

$\therefore \frac{\partial z}{\partial y}\Big|_{(1,1)} = 2\ln2+1$