117.54

(2) $2x^2+2y^2+z^2+8xz-z+8=0$ 求极值

$\therefore\ 4x\,dx+4y\,dy+2z\,dz+8x\,dz+8z\,dx-dz=0$

$\Rightarrow dz=\dfrac{1}{2z+8x-1}\left((-4x-8z)dx-4y\,dy\right)$

$\therefore\ \dfrac{\partial z}{\partial x}=-\dfrac{4x+8z}{2z+8x-1}=0,\quad \dfrac{\partial z}{\partial y}=-\dfrac{4y}{2z+8x-1}=0$

$\Rightarrow \begin{cases}x=-2\\ y=0\\ z=1\end{cases}$ 或 $\begin{cases}x=\frac{16}{7}\\ y=0\\ z=-\frac{8}{7}\end{cases}$ $\quad \dfrac{\partial^2 z}{\partial x^2}=-\dfrac{(4+8\frac{\partial z}{\partial x})(2z+8x-1)-(2\frac{\partial z}{\partial x}+8)(4x+8z)}{(2z+8x-1)^2}$

$\dfrac{\partial^2 z}{\partial x\partial y}=-\dfrac{8\frac{\partial z}{\partial y}(2z+8x-1)-(4x+8z)(2\frac{\partial z}{\partial y})}{(2z+8x-1)^2}\quad \dfrac{\partial^2 z}{\partial y^2}=-\dfrac{4(2z+8x-1)-4y(2\frac{\partial z}{\partial y})}{(2z+8x-1)^2}$

对于 $\begin{cases}x=-2\\ y=0\\ z=1\end{cases}$ $\quad H=\begin{vmatrix}-\frac{4}{15} & 0\\ 0 & -\frac{4}{15}\end{vmatrix}$ $\quad\therefore$ $z_{极大}=z(-2,0)=1$

对于 $\begin{cases}x=\frac{16}{7}\\ y=0\\ z=-\frac{8}{7}\end{cases}$ $\quad H=\begin{vmatrix}-\frac{4}{2z+8x-1} & 0\\ 0 & -\frac{4}{2z+8x-1}\end{vmatrix}$ $\quad\therefore\ z_{极大}=z(\frac{16}{7},0)=-\frac{8}{7}$

118.55.

(3) $L(x,y,z,\lambda) = x-2y+2z+\lambda(x^2+y^2+z^2-1)$

$\frac{\partial L}{\partial x} = 1+2x\lambda \quad \frac{\partial L}{\partial y} = -2+2y\lambda \quad \frac{\partial L}{\partial z} = 2+2z\lambda \quad \frac{\partial L}{\partial \lambda} = x^2+y^2+z^2-1$

$\Rightarrow \begin{cases} x = \mp\frac{1}{3} \\ y = \pm\frac{2}{3} \\ z = \mp\frac{2}{3} \end{cases}$

$\therefore f(x,y,z)$ 极大 $= \frac{1}{3}+\frac{4}{3}+\frac{4}{3} = 3 \quad f(x,y,z)$ 极小 $= -\frac{1}{3}-\frac{4}{3}-\frac{4}{3} = -3$

118.56

$d = \sqrt{x^2+y^2+z^2} \Rightarrow d^2 = x^2+y^2+z^2$

$\Rightarrow L(x,y,z,\lambda) = x^2+y^2+z^2+\lambda_1(x+y+z-1)+\lambda_2(x^2+y^2-z)$

~~$\frac{\partial L}{\partial x} = 2x+\lambda \quad \frac{\partial L}{\partial y} = 2y+\lambda \quad \frac{\partial L}{\partial z} = 2z+\lambda \quad \frac{\partial L}{\partial \lambda} = x+y+z-1$~~

$\Rightarrow \frac{\partial L}{\partial x} = 2x+\lambda_1+2x\lambda_2 \quad \frac{\partial L}{\partial y} = 2y+\lambda_1+2y\lambda_2 \quad \frac{\partial L}{\partial z} = 2z+\lambda_1-\lambda_2.$

$\frac{\partial L}{\partial \lambda_1} = x+y+z-1 \quad \frac{\partial L}{\partial \lambda_2} = x^2+y^2-z$

$\Rightarrow \begin{cases} x = \frac{-1\pm\sqrt{3}}{2} \\ y = \frac{-1\pm\sqrt{3}}{2} \\ z = 2\mp\sqrt{3} \end{cases} \quad d_{max} = \sqrt{9+5\sqrt{3}} \quad d_{min} = \sqrt{9-5\sqrt{3}}$

118.57

(1)

$f(x,y,z) = (x-1)^2+(y-1)^2+(z-1)^2+(x-2)^2+(y-3)^2+(z-4)^2$

$= 2x^2-6x+2y^2-8y+2z^2-10z+32.$

$L(x,y,z,\lambda) = 2x^2-6x+2y^2-8y+2z^2-10z+32+\lambda(3x-2z)$

$\frac{\partial L}{\partial x} = 4x-6+3\lambda \quad \frac{\partial L}{\partial y} = 4y-8 \quad \frac{\partial L}{\partial z} = 4z-10-2\lambda \quad \frac{\partial L}{\partial \lambda} = 3x-2z.$

$\Rightarrow \begin{cases} x = \frac{21}{13} \\ y = 2 \\ z = \frac{63}{26} \end{cases} \quad f(x,y,z)_{min} =$

118.57

(3)

$L(x, y, z, \lambda) = xyz^3 + \lambda(x^2+y^2+z^2-5R^2)$.

$\frac{\partial L}{\partial x} = yz^3 - 2x\lambda$ $\frac{\partial L}{\partial y} = xz^3 - 2y\lambda$ $\frac{\partial L}{\partial z} = 3xyz^2 - 2z\lambda$ $\frac{\partial L}{\partial \lambda} = x^2+y^2+z^2-5R^2$.

$\Rightarrow \begin{cases} x = R \\ y = R \\ z = \sqrt{3}R \end{cases}$. $u_{max} = 3\sqrt{3}R^5$

118.58

(1) $y' = 2x = 1 \Rightarrow x = \frac{1}{2}$ 点 $(\frac{1}{2}, \frac{1}{4})$ 到 l:

$d = \frac{|\frac{1}{2}-\frac{1}{4}-2|}{\sqrt{1+1}} = \frac{7}{4\sqrt{2}}$

118.60

$Q = 2x_1^{\alpha}x_2^{\beta}$, $P = P_1x_1 + P_2x_2$

~~$L(\alpha, \beta, \lambda) = 2x_1^{\alpha}x_2^{\beta}$~~

~~$L(\alpha, \beta, \lambda_1, \lambda_2) = P$~~

$L(x_1, x_2, \lambda) = P_1x_1 + P_2x_2 + \lambda(2x_1^{\alpha}x_2^{1-\alpha} - 12)$

$\frac{\partial L}{\partial x_1} = P_1 + 2x_2^{1-\alpha}\alpha\lambda x_1^{\alpha-1}$ $\frac{\partial L}{\partial x_2} = P_2 + 2\lambda x_1^{\alpha}(1-\alpha)x_2^{-\alpha}$

$\frac{\partial L}{\partial \lambda} = 2x_1^{\alpha}x_2^{1-\alpha} - 12$

$\Rightarrow \begin{cases} x_1 = (\frac{P_2}{P_1}\frac{\alpha}{\beta})^{\beta} \cdot 6 \\ x_2 = 6(\frac{P_2}{P_1}\frac{\alpha}{\beta})^{\alpha} \end{cases}$