289.20

(2) $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n$, ~~$x\in(-1,1]$~~ ~~$x\in(0,2]$~~ $x\in(-1,1]$

当 $x=0.2$ 时. 收敛.

$\left|\sum_{n=i}^{\infty} \frac{(-1)^{n+1}}{n}(x)^n\right| \le \cancel{\frac{(x-1)^i}{i}}\ \frac{0.2^i}{i} \le 10^{-4} \Rightarrow i = 5$

$\therefore \ln(1.2) \approx 0.2 - \frac{0.2^2}{2} + \frac{0.2^3}{3} - \frac{0.2^4}{4} = 0.1826\overline{7} - 0.0004$

$= 0.18227 = 0.1823$

289.21

(1) $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \quad x\in R.$

$\therefore \int_0^{0.8} x^{10} \sin x\,dx = \int_0^{0.8} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+11} dx$

$= \sum_{n=0}^{\infty} \int_0^{0.8} \frac{(-1)^n}{(2n+1)!} x^{2n+11} dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+12}}{(2n+12)(2n+1)!}\Big|_0^{0.8}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n (0.8)^{2n+12}}{(2n+12)(2n+1)!}$

$\left|\frac{(0.8)^{2i+12}}{(2i+12)(2i+1)!}\right| \le 10^{-3} \Rightarrow i = 0 \quad \therefore 原式 \approx \frac{1}{12}(0.8)^{12}$

(3). $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n!} x^{2n} = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$

$\therefore 原式 = \int \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{(2n)!} dx = \sum_{n=1}^{\infty} \int \frac{(-1)^{n+1} x^{2n-1}}{(2n)!} dx$

$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n}}{(2n)! \cdot 2n} + C$