P80.4

令 $z=\cancel{x+iy}$ $re^{i\theta}$ ∴ $\frac{\bar{z}}{|z|}=\frac{re^{-i\theta}}{r}=e^{-i\theta}$

(1) $r=2$, $\theta: -\pi\to\pi$ ∴ $\int_{-\pi}^{\pi}e^{-i\theta}2ie^{i\theta}d\theta=4\pi i$

(2) $r=4$, $\theta: -\pi\to\pi$ ∴ $\int_{-\pi}^{\pi}e^{-i\theta}\cdot 4ie^{i\theta}d\theta=8\pi i$

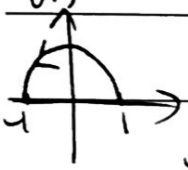
P80.5

$\left|\int_C\frac{dz}{z-i}\right|\le\int_C\left|\frac{1}{z-i}\right||dz|$

∴ $\left|\frac{1}{z-i}\right|\le 1$ ∴ $\int_C\left|\frac{1}{z-i}\right||dz|\le\int_C|dz|=\sqrt{2}\le 2$

P81.6

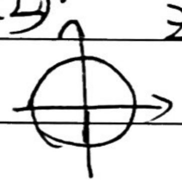
(1) $z=e^{i\theta}$, $\theta: 0\to\pi$ ∴



$\cancel{\int_0^{\pi}3e^{2i\theta}+8e^{i\theta}}$ $f(z)$解析. $\Phi(z)=z^3+4z^2+z+C$

∴ 原式 $=\Phi(-1)-\Phi(1)=-4$

(2). $z=Re^{i\theta}$ ∴ $\int_{-\pi}^{\pi}R\,d\theta-\oint_C 2e^z\cos z\,dz$



$\theta: -\pi\to\pi$

$=2\pi R-0=2\pi R$.

P81.7

(1) $\oint_C\frac{1}{z+2}dz=\int_{-\pi}^{\pi}\frac{1}{(\cos\theta+2)^2+\sin^2\theta}(\cos\theta+2-i\sin\theta)(-\sin\theta+i\cos\theta)d\theta$

$=\int_{-\pi}^{\pi}i\frac{1+2\cos\theta}{5+4\cos\theta}-\frac{2\sin\theta}{5+4\cos\theta}d\theta$.

$\int\frac{1}{z+2}dz=\ln(z+2)$ 解析

∴ $\oint_C\frac{dz}{z+2}=0$

(2) ∴ $\int_0^{\pi}\frac{1}{z+2}dz=i\int_0^{\pi}\frac{1+2\cos\theta}{5+4\cos\theta}d\theta-\int_0^{\pi}\frac{2\sin\theta}{5+4\cos\theta}d\theta=-\ln 3$ ∴ $\int_0^{\pi}\frac{1+2\cos\theta}{5+4\cos\theta}d\theta=0$

P81.8

(1) $\oint_{|z|=1} \frac{e^z}{z} dz = \oint_{|z|=5} \frac{e^z}{z} dz = 2\pi i e^0 = 2\pi i$

(2) $\oint_0^{\pi} e^{\cos\theta}(\cos(\sin\theta) + i\sin(\sin\theta)) = e^{\cos\theta} \cdot e^{i\sin\theta} = e^z$, $z = e^{i\theta}$

$\therefore I = \int_0^{2\pi} e^z d\theta = \frac{1}{i}\oint_C \frac{e^z}{z} dz = \frac{2\pi i}{i} = 2\pi$

$\int_0^\pi e^{\cos\theta}(\cos(\sin\theta))d\theta = \frac{1}{2}\int_0^{2\pi} e^{\cos\theta}(\cos(\sin\theta))d\theta = \frac{1}{2}\mathrm{Re}(I) = \pi$

P81.9

(1) $\oint_C \frac{2z^2-z+1}{z-1}dz = 2\pi i(2z^2-z+1)|_{z=1} = 4\pi i$

(2) $\oint_C \frac{2z^2-z+1}{(z-1)^2}dz = \frac{2\pi i}{1!}(2z^2-z+1)'|_{z=1} = 6\pi i$

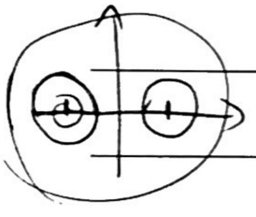
(7) 无奇点. $I=0$

(8) $\oint_C \frac{z^{2n}}{(z+1)^n}dz = \frac{2\pi i}{(n-1)!}(z^{2n})^{(n-1)}|_{z=-1} = \frac{2\pi i}{(n-1)!}\frac{2n!}{(n+1)!}(-1)^{n+1}$

p

P81.10

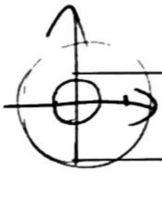
(1) $\oint_{C'} \frac{\sin\frac{\pi}{4}z}{(z+1)(z-1)}dz = 2\pi i \frac{\sin\frac{\pi}{4}z}{z-1}|_{z=-1} = \frac{\sqrt{2}}{2}\pi i$



(2) $\oint_{C''} \frac{\sin\frac{\pi}{4}z}{(z+1)(z-1)}dz = 2\pi i \frac{\sin\frac{\pi}{4}z}{z+1}|_{z=1} = \frac{\sqrt{2}}{2}\pi i$

(3). $I = (1)+(2) = \sqrt{2}\pi i$

P81.11



(1) $\oint_{C'}\frac{\sin^2 z}{z^2(z-1)}dz = \frac{2\pi i}{1!}\left(\frac{\sin^2 z}{z-1}\right)'|_{z=0} = 0$

(2) $\oint_{C'}+\oint_{C''} = 0 + \frac{\sin^2 z}{z^2}2\pi i|_{z=1} = 2\pi i\sin^2 1$

P82. 12 $\oint_C \frac{e^z}{(z+1)^2(z-3)}dz$.

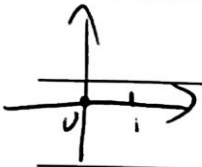
(5) $\oint_{C_1^+}-\oint_{C_2^+} = 2\pi i\cdot\frac{e^z}{(z+1)^2}\Big|_{z=3} + \frac{2\pi i}{1!}\left(\frac{e^z}{z-3}\right)'\Big|_{z=-i} - \frac{2\pi i}{1!}\left(\frac{e^z}{z-3}\right)'\Big|_{z=-i}$

$= \frac{\pi i}{8}e^3$

(7) $\oint_{|z|=\frac{1}{2}}\cdots=0$ ∴ 原式 $=\oint_{|z|=5}\cdots dz$

$= \frac{\pi i}{8}e^3+\frac{2\pi i}{1!}\left(\frac{e^z}{z-3}\right)'\Big|_{z=-1} = \frac{\pi i}{8}(e^3-5e^{-1})$

P82. 13



① (0,0), (1,0) ∉ C ∴ I=0

② (0,0), (1,0) ∈ C

∴ $\cancel{I} = \frac{1}{2\pi i}\oint_C\frac{e^z}{z(1-z)^3}dz = -\frac{1}{2\pi i}\oint\frac{e^z}{(z-1)^3 z}dz$

∴ $I = -\frac{1}{2\pi i}\cdot 2\pi i\frac{e^z}{(z-1)^3}\Big|_{z=0} - \frac{1}{2\pi i}\cdot\frac{2\pi i}{2!}\left(\frac{e^z}{z}\right)''\Big|_{z=1} = 1-\frac{1}{2}e$.

③ (0,0) ∈ C, (1,0) ∉ C

∴ I = 1

④ (1,0) ∈ C, (0,0) ∉ C ∴ $I = -\frac{1}{2}e$

P82. 4.

(1) $\oint_C\frac{e^{\zeta^2}}{\zeta^2(\zeta-z)}d\zeta$

① $|z|>3$ ∴ $f(z) = \frac{1}{2\pi i}\cdot 2\pi i\cdot\frac{e^{z^2}}{z^2} = \frac{e^{z^2}}{z^2}$.

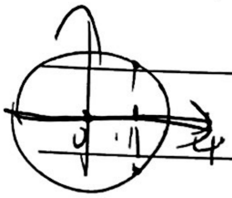
② $0<|z|\leq 3$ ∴ $f(z) = \frac{e^{z^2}}{z^2}+\frac{1}{2\pi i}\cdot\frac{2\pi i}{1}\frac{-1}{z^2} = \frac{e^{z^2}-1}{z^2}$.

③ $z=0$ $f(z) = \frac{1}{2\pi i}\cdot\frac{2\pi i}{2!}(e^{z^2})''\Big|_{z=0} = 1$

(2) $f'(i) = 2i(1-2e^{-1})$

P82.5

$$I = \oint_{|z|=2} \frac{e^z}{z^2(z-1)(\bar{z}-1)}dz. \qquad z\cdot\bar{z}=4 \Rightarrow \bar{z}=\frac{4}{z}$$



$$\cancel{= 2\pi i \left(\frac{e^z}{(z-1)(\bar{z}-1)}\right)\bigg|_{z=0}}$$

$$= \oint_{|z|=2} \frac{-e^z}{z(z-1)(z-4)}dz$$

$$= 2\pi i \frac{-e^z}{(z-1)(z-4)}\bigg|_{z=0} + 2\pi i \frac{-e^z}{z(z-4)}\bigg|_{z=1}$$

$$= 2\pi i\left(\frac{e}{3}-\frac{1}{4}\right)$$

P83.8

(1) $$\oint_C f(z)dz = \oint_C \frac{a_1}{z-a} + \sum_{k=2}^{n}\frac{a_k}{(z-a)^k} + \varphi(z)\,dz$$

$$= 2\pi i a_1 + 0 + 0 \quad \angle 证$$

(2) $$\oint_C \frac{f(z)}{z-b}dz = \oint_C \frac{a_1}{(z-b)(z-a)} + \sum_{k=2}^{n}\frac{a_k}{(z-a)^k(z-b)} + \frac{\varphi(z)}{z-b}\,dz.$$

② $$= \oint_C \sum_{k=1}^{n}\frac{a_k}{(z-b)(z-a)^k}dz$$

$$= \oint_C \frac{a_1}{(z-b)(z-a)} + \sum_{k=2}^{n}\frac{a_k}{(z-b)(z-a)^k}dz$$

$$= 2\pi i \frac{a_1}{z-b}\bigg|_{z=a} + \sum_{k=2}^{n}\frac{2\pi i}{(k-1)!}\left(\frac{a_k}{z-b}\right)^{(k-1)}$$

$$= 2\pi i \frac{a_1}{a-b} + \sum_{k=2}^{n}(-1)^{k-1}2\pi i\frac{a_k}{(a-b)^k}$$

$$= 2\pi i \frac{a_1}{a-b} + \sum_{k=2}^{n}(-1)^{k-1}\cdot(-1)^k 2\pi i\frac{a_k}{(b-a)^k}$$

$$= -\sum_{k=1}^{n}\frac{a_k}{(b-a)^k}.$$

P83、9　　　　　　$\exists z_0$,

(1) $|f(z)-1|<1$，设 $f(z_0)=0$ ∴ $|f(z_0)-1|=1$ 矛盾 ∴ 不存在z_0.

(2). 只需证 $\frac{f'(z)}{f(z)}$ 解析.

$f(z)\neq 0$ ∴ $\frac{1}{f(z)}$ 解析，$f'(z)$ 解析

∴ $\frac{f'(z)}{f(z)}$ 解析 ∴ 得证.