357.3

$y''+2y'+3y=e^{3x}$

当 $x=0$ 时. ∴ $y''=1$

$\lim_{x\to 0}\frac{\ln(1+x^2)}{f(x)} \underline{\underline{L'Hospital}} \lim_{x\to 0}\frac{\frac{2x}{1+x^2}}{f'(x)} \underline{\underline{L'Hospital}} \lim_{x\to 0}\frac{2}{f''(x)}=2$

357.4

(2) $\frac{x}{\cos y}dx+(x+1)dy=0$

$\cancel{\Rightarrow \frac{x}{x+1}dx=-\frac{1}{\cos y}dy \Rightarrow x-\ln(x+1)+C=-\ln|\frac{1}{\cos y}+\tan y|}$

$\cancel{\Rightarrow x-\ln(x+1)+C=-\ln|\frac{1+\sin y}{\cos y}| \Rightarrow \frac{1+\sin y}{\cos y}=-e^{x}} \frac{Ce^{x}}{x+1}$

$\Rightarrow \frac{x}{x+1}dx=-\cos y\,dy \Rightarrow x-\ln|x+1|+C=-\sin y$

(4) $y\frac{dy}{dx}+e^{y^2+3x}=0$

$\Rightarrow e^{-3x}\frac{dy}{dx}=-\frac{e^{y^2}}{y} \Rightarrow \frac{y}{e^{y^2}}dy=-e^{3x}dx$

$\Rightarrow -\frac{1}{2}e^{-y^2}=-\frac{1}{3}e^{3x} \Rightarrow \frac{1}{2}e^{-y^2}=\frac{1}{3}e^{3x}$

$\Rightarrow y=\sqrt{-(\ln\frac{2}{3}+3x)}$ $(y<0)$

357.5

(2) $(x+y)dx+(y-x)dy=0 \Rightarrow x+y+(y-x)\frac{dy}{dx}=0$

$\Rightarrow x+y+yy'-xy'=0 \Rightarrow 1+\frac{y}{x}+\frac{y}{x}\frac{dy}{dx}-\frac{dy}{dx}=0$

$\Rightarrow \frac{y}{x}=u$ ∴ $\frac{dy}{dx}=u+x\frac{du}{dx}$ ∴ $1+u+u(u+x\frac{du}{dx})-u-x\frac{du}{dx}=0$

$\Rightarrow \frac{(u-1)}{1+u^2}du=-\frac{dx}{x} \Rightarrow \frac{1}{2}\ln(1+u^2)-\arctan u=-\ln x+C$

$\Rightarrow \ln\sqrt{1+\frac{y^2}{x^2}}\cdot x=\arctan\frac{y}{x}$ ∴ $\sqrt{x^2+y^2}=e^{\arctan\frac{y}{x}}\cdot C$

(4) $\frac{dy}{dx}\sin\frac{y}{x}-\frac{y}{x}\sin\frac{y}{x}+1=0 \overset{u=\frac{y}{x}}{\Longrightarrow} (u+x\frac{du}{dx})\sin u-u\sin u+1=0$

$\Rightarrow \sin u\,du=-\frac{1}{x}dx \Rightarrow \cos u=\ln x+C$

$\Rightarrow \cos\frac{y}{x}=\ln x+C$

357.6

(1) $\frac{dy}{dx}=\left(\frac{2}{x+y}\right)^2 \xRightarrow{x+y=u} \frac{du}{dx}-1=\frac{4}{u^2} \Rightarrow \frac{du}{dx}=\frac{4+u^2}{u^2}$

$\Rightarrow \frac{u^2}{4+u^2}du=dx \Rightarrow u-2\arctan\frac{u}{2}=x+C$

$\Rightarrow \cancel{x}+y-2\arctan\frac{x+y}{2}=\cancel{x}+C \Rightarrow y=2\arctan\frac{x+y}{2}+C$

$\Rightarrow$ ~~tan(y-c)~~ $\tan\frac{y-C}{2}=\frac{x+y}{2}$

(2) $\frac{dy}{dx}=\frac{2x+4y+3}{x+2y+1} \xRightarrow{x+2y=u} \frac{1}{2}\frac{du-dx}{dx}=\frac{2u+3}{u+1} \Rightarrow \frac{du}{dx}-1=\frac{4u+6}{u+1}$

$\Rightarrow$ $\cancel{\frac{du}{dx}=\frac{\ }{u+1}}$ $\frac{du}{dx}=\frac{5u+7}{u+1} \Rightarrow \frac{u+1}{5u+7}du=dx$

$\Rightarrow \frac{u}{5}-\frac{2}{5}\ln(5u+7)=x+C \xRightarrow{u=x+2y} y-2x+C=\ln(5x+10y+7)$

(3) $\frac{dy}{dx}=\frac{1}{(4x+y+1)(4x+y)} \xRightarrow{u=4x+y} \frac{du}{dx}-4=\frac{1}{(u+1)u}$

$\Rightarrow \frac{(u+1)u}{4u(u+1)+1}du=dx \Rightarrow \frac{u}{4}+\frac{1}{8}\frac{1}{2u+1}=x+C$

$\Rightarrow 2u+\frac{1}{2u+1}=8x+C \xRightarrow{u=4x+y} \cancel{8x}+2y+\frac{1}{8x+2y+1}=\cancel{8x}+C$

$\Rightarrow 2y+\frac{1}{8x+2y+1}=C$

(4) $\frac{dy}{dx}=\frac{y-x+1}{y+x+5}$

$\begin{cases} y-x+1=0 \\ y+x+5=0 \end{cases} \Rightarrow \begin{cases} x=-2 \\ y=-3 \end{cases}$ 令 $\begin{cases} X=x+2 \\ Y=y+3 \end{cases}$

$\therefore \frac{dY}{dX}=\frac{Y-X}{Y+X}=\frac{\frac{Y}{X}-1}{\frac{Y}{X}+1} \xRightarrow{u=\frac{Y}{X}} u+X\frac{du}{dX}=\frac{u-1}{u+1}$

$\Rightarrow \frac{u}{1+u^2}du+\frac{1}{1+u^2}du=-\frac{dX}{X} \Rightarrow \frac{1}{2}\ln(1+u^2)+\arctan u=-\ln|X|+C$

$\therefore \ln\sqrt{(x+2)^2+(y+3)^2}=-\arctan\frac{y+3}{x+2}+C$

357. 7.

(2) $\frac{dy}{dx}=\frac{xy+3x}{x^2+1} \Rightarrow \frac{dy}{y+3}=\frac{x}{x^2+1}dx \Rightarrow \ln|y+3|+C=\frac{1}{2}\ln(1+x^2)$

$\because y|_{x=2}=2 \therefore C=-\frac{1}{2}\ln 5.$

$\therefore y=\sqrt{5+5x^2}-3$

(4) $\frac{dy}{dx}=\frac{x}{y}+\frac{y}{x} \xRightarrow{\frac{y}{x}=u} u+x\frac{du}{dx}=\frac{1}{u}+u.$

$\Rightarrow x\frac{du}{dx}=\frac{1}{u} \Rightarrow u\,du=\frac{1}{x}dx$

$\Rightarrow \frac{1}{2}u^2=\ln x+C \Rightarrow \frac{y^2}{x^2}=2\ln x+C$

$\Rightarrow y^2=2x^2\ln x+Cx^2 \quad \because y(1)=2 \therefore y=\sqrt{2x^2\ln x+4x^2}$

357.8

(2) $y'+4y=x$

$\therefore\ y=e^{-\int 4dx}(\int xe^{\int 4dx}+C)=e^{-4x}(\frac{1}{4}(x-\frac{1}{4})e^{4x}+C)=\frac{1}{4}(x-\frac{1}{4})+Ce^{-4x}$

(4) $\frac{dy}{dx}+y\cos x=e^{-\sin x}$

$y=e^{-\int\cos xdx}(\int e^{-\sin x}e^{\int\cos xdx}dx+C)$

$=e^{-\sin x}(x+C)$

(6) $dx+(x+y^2)dy=0 \Rightarrow \frac{dx}{dy}+x=-y^2$

$\therefore\ x=e^{-\int 1dy}(\int -y^2\cdot e^{\int 1dy}dy+C)$

$=(2y-y^2-2)+Ce^{-y}$

358.9

(3) $t\frac{dx}{dt}=-x+\sin t \Rightarrow \frac{dx}{dt}+\frac{x}{t}=\frac{\sin t}{t}$

$\therefore\ x=e^{-\int\frac{1}{t}dx}(\int\frac{\sin t}{t}e^{\int\frac{1}{t}dt}dt+C)$

$=\frac{C-\cos t}{t}$ $\quad x(\pi)=1\quad \therefore C=\pi-1$

$\therefore\ x=\frac{\pi-1-\cos t}{t}$

(4) $y'+y\cot x=5e^{\cos x}$

$\therefore\ y=e^{-\int\cot xdx}(\int 5e^{\cos x}e^{\int\cot xdx}dx+C)=\frac{C-5e^{\cos x}}{\sin x}$

$\therefore C=1\quad\therefore\ y=\frac{1-5e^{\cos x}}{\sin x}$

358.10

$f(x)=\int_0^{2x}f(\frac{t}{2})dt+\ln 2 \xlongequal{\frac{t}{2}=a} 2\int_0^x f(a)da+\ln 2 \quad f(0)=\ln 2$

$\therefore\ f'(x)=2f(x)\quad\therefore\ \frac{df}{dx}-2f=0\Rightarrow f=e^{\int 2dx}=e^{2x}\cdot C$

$\therefore C=\ln 2\quad\therefore\ f(x)=\ln 2\cdot e^{2x}$

358.12

$\int_0^1 f(ux)\,du \xlongequal{ux=t} \frac{1}{x}\int_0^x f(t)\,dt = \frac{1}{2}f(x)+1$.

∴ $\int_0^x f(t)\,dt = \frac{x}{2}f(x)+x$

∴ $f(x)=\frac{1}{2}f(x)+\frac{1}{2}xf'(x)+1$.

∴ $f(x)=xf'(x)+2 \Rightarrow f = x\frac{df}{dx}+2$

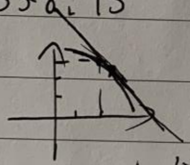
∴ $\frac{dx}{x}=\frac{df}{f-2} \Rightarrow \ln|x|+C=\ln|f-2|$

∴ $f-2=C\cdot x$ ∴ $f(x)=Cx+2$

又∵ $\lim_{x\to0}\frac{\int_0^x f(t)\,dt}{x} \xlongequal{L'Hospital} \frac{f(0)}{1} = \frac{1}{2}f(0)+1 \Rightarrow f(0)=2$

∴ $f(x)=Cx+2$

358.15



设 $y=f(x)$.

∴ l: $y-f(x_0)=f'(x_0)(x-x_0)$

∴ 与坐标轴交点 $(0,\ f(x_0)-x_0f'(x_0))$, $(x_0-\frac{f(x_0)}{f'(x_0)},\ 0)$

∴ $(f(x_0)-x_0f'(x_0))=2f(x_0)$, $(x_0-\frac{f(x_0)}{f'(x_0)})=2x$

∴ $\cancel{\frac{1}{2}f(x)=xf'(x)}$, $\cancel{\frac{1}{2}xf'(x)=f(x)}$.

$\cancel{f(x)=2xf'(x)}$ $f(x)+xf'(x)=0$.

∴ $xf(x)=C \Rightarrow f(x)=\frac{C}{x}$ 又∵ 过 $(2,3)$ ∴ $f(x)=\frac{6}{x}$

358.20

(2) $y''=\frac{1}{1+x^2}$ 设 $y'=p(x)$ ∴ $y''=\frac{dp}{dx}$

∴ $\frac{dp}{dx}=\frac{1}{1+x^2} \Rightarrow dp=\frac{dx}{1+x^2} \Rightarrow p+C=\arctan x$

∴ $\frac{dy}{dx}=\arctan x+C \Rightarrow dy=(\arctan x+C)\,dx$.

∴ $y=x\arctan x-\frac{1}{2}\ln(1+x^2)+C_1x+C_2$

(3) $4xy'' - y'' - 4y' = 0$ 令 $y' = p(x)$ ∴ $y'' = \frac{dp}{dx}$

∴ $4x\frac{dp}{dx} - \frac{dp}{dx} - 4p = 0 \Rightarrow \frac{dp}{4p} = \frac{dx}{4x-1}$

$\Rightarrow \frac{1}{4}\ln|p| + C = \frac{1}{4}\ln|4x-1| \Rightarrow 4x-1 = C\cdot p$

∴ $4x-1 = C\frac{dy}{dx} \Rightarrow (4x-1)dx = C_1 dy \Rightarrow 2x^2 - x + C_2 = C_1 y$

∴ $y = \frac{2x^2-x}{C_1} + C_2$ ✓

(4) $yy'' - (y')^2 = 0$ 令 $y' = p(y)$ ∴ $y'' = p\frac{dp}{dy}$

∴ $yp\frac{dp}{dy} - p^2 = 0 \Rightarrow y\frac{dp}{dy} - p = 0 \Rightarrow p = cy$

∴ $\frac{dy}{dx} = cy \Rightarrow \frac{dy}{y} = c\,dx \Rightarrow \ln|y| = C_1 x + C_2$

∴ $y = C_2 e^{C_1 x}$ ~~讨 $y' = 0$~~

(5) $y'' = (y')^3 + y'$ 令 $y' = p(y)$ ∴ $y'' = p\frac{dp}{dy}$

∴ $p\frac{dp}{dy} = p^3 + p \Rightarrow \frac{dp}{dy} = p^2 + 1$

$\Rightarrow \frac{dp}{p^2+1} = dy \Rightarrow \arctan p = y + C$

∴ $p = \tan(y+C) \Rightarrow \frac{dy}{dx} = \tan(y+C) \Rightarrow \frac{dy}{\tan(y+C)} = dx$

$\Rightarrow \ln|\sin(y+C_1)| = x + C_2 \Rightarrow \sin(y+C_1) = C_2 e^x$ ✓