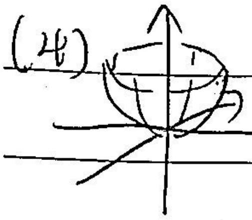
231. 6

(2) S: $z=\sqrt{R^2-x^2-y^2}$, $D_{xy}:\{x^2+y^2\le R^2\}$. $z_x=\frac{-x}{\sqrt{R^2-x^2-y^2}}$ $z_y=\frac{-y}{\sqrt{R^2-x^2-y^2}}$

∴ $\iint_\Sigma y\,dS = \iint_{D_{xy}} y\sqrt{\frac{R^2}{R^2-x^2-y^2}}\,d\sigma = \int_0^{2\pi}d\theta\int_0^R \frac{r\sin\theta\cdot R}{\sqrt{R^2-r^2}}\, r\,dr$

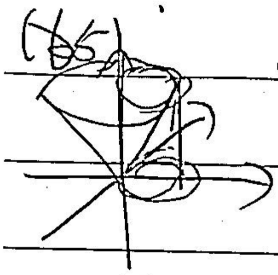
$=0$

(4) 

原式 $=4\iint_{\Sigma_1} xyz\,dS = 4\iint_{D_{xy}} xy(x^2+y^2)\sqrt{1+4x^2+4y^2}\,d\sigma$

$D_{xy}=\{x^2+y^2\le1\}$. $=4\int_0^{\frac{\pi}{2}}d\theta\int_0^1 r^4\sin\theta\cos\theta\sqrt{1+4r^2}\cdot r\,dr$

$=\frac{125\sqrt5-1}{420}$.

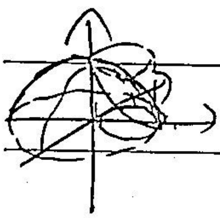
(5) 

S: $z=\sqrt{x^2+y^2}$, $D_{xy}:\{x^2+y^2\le 2ax\}$.

∵ 关于xoz对称 ∴ 原式 $=\iint_\Sigma zx\,dS$

$=\sqrt2\iint_{D_{xy}}\sqrt{x^2+y^2}\,x\,d\sigma = \frac{64}{15}\sqrt2\, a^4$

231.7

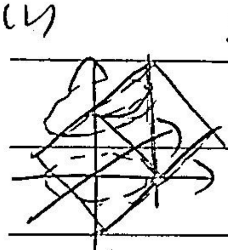


S: $z=\sqrt{R^2-x^2-y^2}$, $D_{xy}:\{x^2+y^2\le Rx\}$.

∴ $A_0=\iint_{D_{xy}}\sqrt{z_x^2+z_y^2+1}\,d\sigma=\int_{-\frac\pi2}^{\frac\pi2}d\theta\int_0^{R\cos\theta}\frac{Rr}{\sqrt{R^2-r^2}}dr$

$=(\pi-2)R^2$

232.16

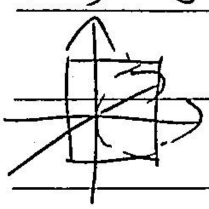
(1) 

S: $z=a-x$, $D_{xy}:\{x^2+y^2\le a^2\}$. $z_x=-1$, $z_y=0$

∴ $\cancel{\text{原式}=\iint_\Sigma(P,Q,R)(1,0,1)dS}$ $\vec n^0_+=\frac{\sqrt2}{2}(1,0,1)$.

$=\cancel{\iint_\Sigma(P+R)dS}$ 原式 $=\frac{\sqrt2}{2}\iint_\Sigma(P+R)dS$.

(2) S: $y=x^2+2z^2$, $D_{xz}:\{x^2+2z^2\le2\}$. $y_x=2x$, $y_z=4z$.



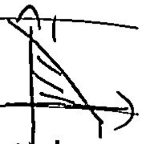
∴ $\cancel{\text{原式}=\iint_\Sigma(P,Q,R)(2x,-1,4z)dS}$ $\vec n^0_-=(2x,-1,4z)$

$=\cancel{\iint_\Sigma(2xP-Q+4zR)dS}$

原式 $=\iint_\Sigma \frac{1}{\sqrt{1+4x^2+16z^2}}(2xP-Q+4zR)\,dS$

232.17

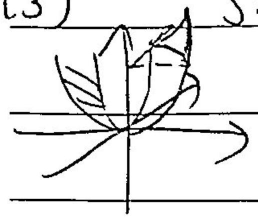
(1) $\vec{F}=(0,0,z^2)$. $z=1-x-y$ $\quad z_x=-1$, $z_y=-1$, D_{xy}:



∴ $\iint_{\Sigma} z^2\,dxdy = \iint_{D_{xy}}(0,0,z^2)(1,1,1)\,d\sigma = \int_0^1 du\int_0^u (1-u)^2\,dv$

$=\frac{1}{12}$

(3) S: $z=x^2+y^2$, ~~D_{xy} {$x^2+y^2\le1$}~~ D_{xy}: $[0,1]\times[0,1]$

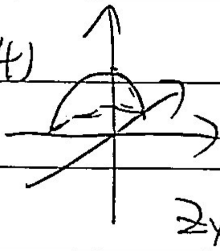


$z_x=2x$, $z_y=2y$

原式 $=\iint_{D_{xy}}(e^y, e^0\cdot y, x^2y)(-2x,-2y,1)\,d\sigma$

$=\frac{11}{6}-\frac{5}{3}e$.

(4)



S: $z=\sqrt{1-x^2-y^2}$. D_{xy}: $\{x^2+y^2\le 1,\ x\le 0,\ y\ge 0\}$.

$z_x=\frac{-x}{\sqrt{1-x^2-y^2}}$ $\quad z_y=\frac{-y}{\sqrt{1-x^2-y^2}}$.

∴原式 $=\iint_{D_{xy}}(x^2,y^2,z^2)\left(\frac{x}{\sqrt{1-x^2-y^2}},\frac{y}{\sqrt{1-x^2-y^2}},1\right)d\sigma$

$=\int_{\frac{\pi}{2}}^{\pi} d\theta\int_0^1 \frac{x^3+y^3}{\sqrt{1-x^2-y^2}}+1-x^2-y^2\,dr^2$ 又∵ x与$-y$对称

$=\int_{\frac{\pi}{2}}^{\pi} d\theta\int_0^1 (1-r^2)\,r\,dr=\frac{\pi}{8}$