P108. 10

~~P(X=0)~~

X\Y	1	2	3	
0	0.1	0.2	0.1	0.4
1	0.3	0.1	0.2	0.6
	0.4	0.3	0.3	

$\therefore P(X=0|Y=1) = \frac{0.1}{0.4} = 0.25$

$P(X=0|Y\neq 1) = \frac{1}{2}$

$P(X=1|Y=1) = \frac{0.3}{0.4} = 0.75$

$P(X=1|Y\neq 1) = \frac{1}{2}$

P108. 12

(1) $1 - P(X=0, Y=0) = 0.9$

(2) $P(Y=0|X=0) = \frac{0.10}{0.16} = \frac{5}{8}$

$P(Y=1|X=0) = \frac{0.04}{0.16} = \frac{2}{8}$

$P(Y=2|X=0) = \frac{0.02}{0.16} = \frac{1}{8}$

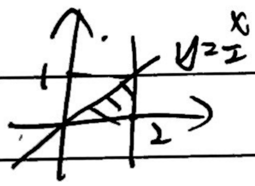
(3) $P(X=0|Y=2) = \cancel{\frac{0.06}{0.24}}\ \frac{0.02}{0.38} = \frac{1}{19}$

$P(X=1|Y=2) = \frac{0.06}{0.38} = \frac{3}{19}$

$P(X=2|Y=2) = \frac{0.30}{0.38} = \frac{15}{19}$

P108. 13

$f(x,y) = \begin{cases} \frac{3}{4}x & (x,y)\in G \\ 0 & \text{others} \end{cases}$



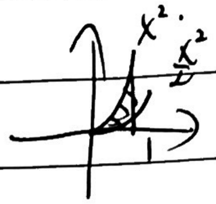
(1) $f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)}$, $f_X(x) = \int_{-\infty}^{+\infty} f(x,y)dy = \begin{cases} \frac{3}{8}x^2 & 0\le x<2 \\ 0 & \text{others.} \end{cases}$

$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)dx = \begin{cases} \frac{3}{2}(1-y^2) & 0\le y<1 \\ 0 & \text{others.} \end{cases}$

$\therefore f_{Y|X}(y|x) = \begin{cases} \frac{2}{x} & 0<y<\frac{x}{2} \\ 0 & \end{cases}$ $\quad f_{X|Y}(x|y) = \begin{cases} \frac{x}{2(1-y^2)} & 2y<x<2 \\ 0 & \text{others} \end{cases}$

(2).

$$f(x,y)=\begin{cases}6 & (x,y)\text{ in }G\\ 0 & \text{others}\end{cases}$$



$$f_X(x)=\int_{-\infty}^{+\infty} f(x,y)\,dy=\begin{cases}3x^2 & 0\le x<1\\ 0 & \text{others.}\end{cases}$$

$$f_Y(y)=\int_{-\infty}^{+\infty} f(x,y)\,dx=\begin{cases}6\sqrt{2y}-6\sqrt{y} & 0<y\le\frac{1}{2}\\ 6(1-\sqrt{y}) & \frac{1}{2}<y\le 1\\ 0 & \text{others.}\end{cases}$$

$$\therefore f_{Y|X}(y|x)=\begin{cases}\frac{2}{x^2} & \cancel{(x,y)\text{ in }G}\ \frac{x^2}{2}<y<x^2.\\ 0 & \text{others}\end{cases}$$

$$f_{X|Y}(x|y)=\begin{cases}\frac{1}{\sqrt{2y}-\sqrt{y}} & 0<y\le\frac{1}{2},\ \sqrt{y}<x<\sqrt{2y}\\ \frac{1}{1-\sqrt{y}} & \frac{1}{2}<y\le 1,\ \sqrt{y}<x\le 1\\ 0 & \text{others.}\end{cases}$$

P108. 14

(1) $f_X(x)=\int_{-\infty}^{+\infty} f(x,y)\,dy$

$$=\begin{cases}\frac{x^2}{2}e^{-x} & x\ge 0\\ 0 & x<0\end{cases}$$

(2) $$f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}=\begin{cases}xe^{-xy} & x>0,\ y>0\\ 0 & \text{others}\end{cases}$$

$$P(Y\le y\mid X=\tfrac{1}{2})=\int_{-\infty}^{y} f_{Y|X}(y|\tfrac{1}{2})\,dy=\begin{cases}1-e^{-\frac{1}{2}y} & y>0\\ 0 & y\le 0\end{cases}$$

(3) $P(Y\ge 1\mid X=0.5)=1-P(Y<1\mid X=0.5)=1-(1-e^{-\frac{1}{2}})=e^{-\frac{1}{2}}$

P109. 15

(1) $f(x,y) = f_X(x) \cdot f_{Y|X}(y|x) = \begin{cases} \frac{1}{3}(1+xy) & 0<x<2,\ 0<y<1 \\ 0 & \text{others.} \end{cases}$

(2) $f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \begin{cases} \frac{2}{3}(1+y) & 0<y<1 \\ 0 & \text{others} \end{cases}$

(3) $f_{X|Y}(x,y) = \frac{f(x,y)}{f_Y(y)} = \begin{cases} \frac{1+xy}{2(1+y)} & 0<x<2,\ 0<y<1 \\ 0 & \text{others} \end{cases}$

P109. 17

(1) ~~$P(X=y_j \mid X=0)$~~ $P(X=0 \mid Y=1) = \frac{P(X=0,\ Y=1)}{P(Y=1)} = 1$

X \ Y	0	1	
-1	$\frac{1}{4}$	0	$\frac{1}{4}$
0	0	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{4}$	0	$\frac{1}{4}$
	$1-p$	p	

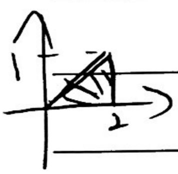
$\therefore p = \frac{1}{2}$

$\therefore P(Y=1 \mid X=0) = 1$

$P(Y=0 \mid X=0) = 0$.

(2) 不独立. $P(Y=0 \mid X=0) = 0 \neq P(Y=0)$

P110. 20



(1) $f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \begin{cases} \frac{3}{8}x^2 & 0<x<2 \\ 0 & \text{others} \end{cases}$

$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \begin{cases} \frac{3}{2}(1-y^2) & 0<y<1 \\ 0 & \text{others.} \end{cases}$

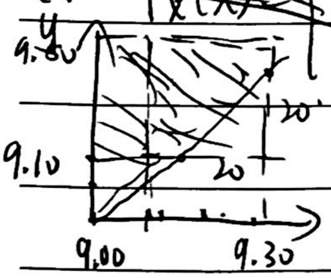
(2) $f(x,y) \neq f_X(x) \cdot f_Y(y)$

$\therefore$ 不独立.

P110、22.

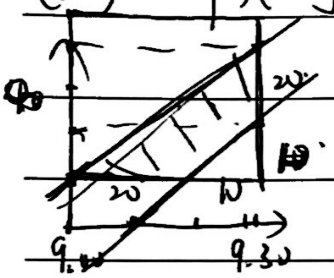
(1) 设甲 X 到，乙 Y 到.

∴ ~~$f_X(x)=$~~ $\left\{\right.$ $x \le y \Rightarrow$



∴ $p = 1 - \frac{S_\text{不}}{S_\text{总}} = 1 - \frac{20\times20\times\frac{1}{2}}{30\times40} = \frac{5}{6}$.

(2) $|x-y| \le 10 \Rightarrow -10 \le x-y \le 10 \Rightarrow x-10 \le y \le x+10$.



∴ $p = \frac{S_\text{阴}}{S} = \frac{1}{3}$