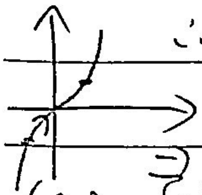
231.10

(1) $\vec{F}=(x^2y,-x)$. $\vec{e}_\tau=\frac{1}{\sqrt{1+9x^4}}(1,3x^2)$



∴ 原式 $=\int_C (x^2y-3x^3)\frac{1}{\sqrt{1+9x^4}}\,ds$.

(2) $\vec{F}=(P,Q,R)$. $\vec{e}_\tau=\frac{1}{\sqrt{1+4t^2+9t^4}}(1,2t,3t^2)$

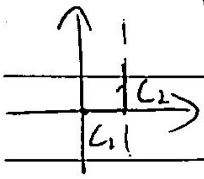
∴ 原式 $=\int_C (P+2tQ+3t^2R)\frac{1}{\sqrt{1+4t^2+9t^4}}\,ds$

231.11

(1) $y=x$ ∴ 原式 $=\int_0^1 x^2\,dx=\frac{1}{3}$

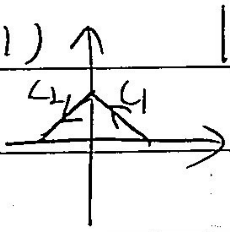
(2) $y=x^2$ ∴ 原式 $=\int_0^1 (x^2+x^4)\,dx=\frac{8}{15}$

(3) 原式 $=\int_{C_1} x^2\,dx+\int_{C_2} y\,dy=\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$



231.12

(1) $|x|+|y|=1$



∴ 原式 $=\int_C dx+dy=\int_{C_1} dx-dx+\int_{C_2} dx+dx$

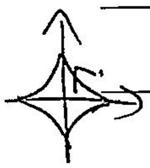
$=0+\int_0^{-1} 2\,dx=-2$

(3) $\int_0^1 (t^4-t^6)\,dt+2t^5\cdot 2t\,dt-t^2\cdot 3t^2\,dt=\frac{1}{35}$

(4) $\begin{cases} x=\cos\theta \\ y=\sin\theta \\ z=2+\sin\theta-\cos\theta \end{cases}$ $\theta: 2\pi\to 0$. ∴ 原式 $=-\int_0^{2\pi}(1-2\cos\theta-2\sin\theta)\,d\theta=-2\pi$.

233.19

(1) $A_D=\frac{1}{2}\int_{C^+} x\,dy-y\,dx=\frac{1}{2}\int_0^{2\pi} 3a^2\cos^4t\sin^2t+3a^2\cos^2t\sin^4t\,dt$

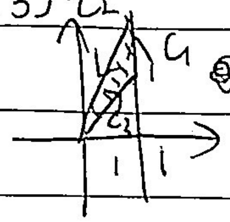


$=\frac{3}{8}\pi a^2$

233. 20

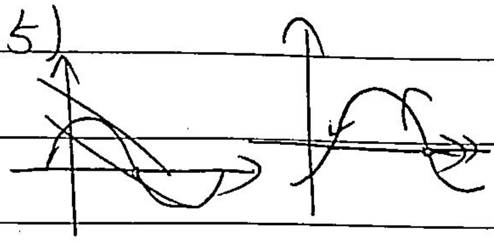
(1) 原式 $= \iint_D (2x\cos y + 1 - 2x\cos y + 4)\,d\sigma = 5\iint_D d\sigma = 15\pi$

(3)



原式 $= \iint_D -1+2\,d\sigma = \iint_D d\sigma = \times 1\times 1\times \frac{1}{2} = \frac{1}{2}$

(5)



令 $(1,0) \to (\pi+1,0)$ 为 C_2.

$\therefore$ 原式 $= \int_{C+C_2} - \int_{C_2}$

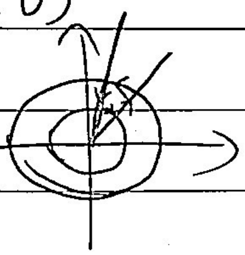
$\int_{C+C_2}(\cdots) = \iint_D \left(y^2 + y\frac{1}{\sqrt{x^2+y^2}} - \frac{y}{\sqrt{x^2+y^2}}\right)d\sigma = \iint_D y^2\,d\sigma$

$C_2: y=0,\ x\in[1,\pi+1]$. $= \int_1^{\pi+1} dx \int_0^{\sin(x-1)} y^2\,dy = \frac{4}{9}$

$\int_{C_2}\cdots = \int_{C_2} x\,dx + 0 = \int_1^{\pi+1} x\,dx = \frac{1}{2}[(\pi+1)^2 - 1]$

$\therefore$ 原式 $= \frac{4}{9} - \frac{1}{2}[\pi^2 + 2\pi] = \frac{4}{9} - \pi - \frac{\pi^2}{2}$

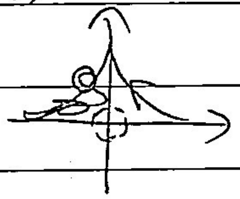
(6)



原式 $= \iint_D \frac{2}{x^2+y^2} - \frac{1}{x^2+y^2}\,d\sigma$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} d\theta \int_1^2 \frac{1}{r^2} r\,dr = \frac{\pi}{12}\ln 2$

(7). 挖去 $x^2+y^2=\varepsilon^2$ $\therefore$ 原式 $= \int_{D-\varepsilon} + \int_\varepsilon$.



$\int_{D-\varepsilon} = 0,\quad \int_\varepsilon = \iint_\varepsilon \frac{1}{\varepsilon^2}\cdot(-2)\,d\sigma = -\frac{\pi}{2}$

$\therefore$ 原式 $= -\frac{\pi}{2}$.