P208. 2.

$$f(x;\theta)=\begin{cases}\frac{2\theta^2}{(\theta^2-1)x^3} & x\in(1,\theta)\\ 0 & \text{o.w}\end{cases}$$

$$E(X)=\int_1^{\theta}\frac{2\theta^2}{\theta^2-1}\cdot\frac{1}{x^2}dx=\frac{2\theta}{\theta+1}=\overline{X}$$

$\therefore\ \hat{\theta}=\frac{\overline{X}}{2-\overline{X}},\quad \hat{\theta}=\frac{\bar{x}}{2-\bar{x}}.$

P209. 3

$E(X)=\lambda \qquad \therefore\ \hat{\lambda}=\overline{X}=1.$ 矩估计值.

$E(X^2)=\cancel{\ }\lambda+\lambda^2$

$P(X=x)=e^{-\lambda}\frac{\lambda^x}{x!} \qquad L(\lambda)=\prod_{x_i=0}^{4}e^{-\lambda}\frac{\lambda^{x_i}}{x_i!}$

$\therefore\ \ln L(\lambda)=-5\lambda+\sum x_i\ln\lambda+C. \qquad \therefore\ \frac{d\ln L(\lambda)}{d\lambda}=-5+\frac{\sum x_i}{\lambda}=0$

$\therefore\ \cancel{\lambda}\ \hat{\lambda}_{mle}=\overline{X}=1.$

P209. 4.

① $L(\theta)=\prod_{i=1}^{8}P(X=x_i)=(1-2\theta)^4\cdot[2\theta(1-\theta)]^2\cdot\theta^2\cdot\theta^2$

$\therefore\ \ln L(\theta)=4\ln(1-2\theta)+2[\ln 2\theta+\ln(1-\theta)]+4\ln\theta.$

$\frac{d\ln L(\theta)}{d\theta}=\frac{-8}{1-2\theta}+\frac{6}{\theta}-\frac{2}{1-\theta}=0 \Rightarrow \theta=\frac{7\pm\sqrt{13}}{12} \quad \because\ p_i\geq 0$

$\therefore\ \theta=\frac{7-\sqrt{13}}{12}\Rightarrow\hat{\theta}_{mle}=\frac{7-\sqrt{13}}{12}.$

② 矩估计.

$E(X)=2\theta-2\theta^2+2\theta^2+3-6\theta=3-4\theta=\frac{3\times4}{8}+\frac{1\times2}{8}+\frac{2}{8}=2.$

$\therefore\ \hat{\theta}=\frac{1}{4}$

P249, 6

(2) 矩估计. $E(X)=\int_0^{+\infty}\frac{x}{\theta^2}e^{-\frac{x}{\theta}}\cdot x\,dx=2\theta := \overline{X} \Rightarrow \hat{\theta}=\frac{1}{2}\overline{X}$

最大似然估计. $L(\theta)=\prod_{i=1}^{n}$ 写错符号}.

~~写~~ $L(\theta)=\prod_{i=1}^{n}f(x_i;\theta)=\begin{cases}\prod_{i=1}^{n}\frac{x_i}{\theta^2}e^{-\frac{x_i}{\theta}} & x_i>0\\ 0 & o.w.\end{cases}$

$\therefore \ln L(\theta)=\sum_{i=1}^{n}x_i-2n\ln\theta-\frac{1}{\theta}\sum_{i=1}^{n}x_i$, $\frac{d\ln L(\theta)}{d\theta}=0 \Rightarrow \hat{\theta}_{MLE}=\frac{1}{2}\overline{X}$

(1) 矩估计, $E(X)=kp=\overline{X} \Rightarrow \hat{p}=\frac{1}{k}\overline{X}$

最大似然估计. $L(p)=\prod_{i=1}^{n}P(X=x_i)=\prod_{i=1}^{n}C_k^{x_i}p^{x_i}(1-p)^{k-x_i}$

$\ln L(p)=\sum_{i=1}^{n}x_i\ln p+\sum_{i=1}^{n}(k-x_i)\cdot\ln(1-p)+C.$

$\frac{d\ln L(p)}{dp}=\frac{1}{p}\sum_{i=1}^{n}x_i-\frac{1}{1-p}(nk-\sum_{i=1}^{n}x_i)=0 \Rightarrow \hat{p}_{MLE}=\frac{1}{k}\overline{X}$

(3) 矩估计. $E(X)=\int_0^1\sqrt{\theta}x^{\sqrt{\theta}}\,dx=\frac{\sqrt{\theta}}{\sqrt{\theta}+1}=\overline{X} \Rightarrow \hat{\theta}=\left(\frac{\overline{X}}{1-\overline{X}}\right)^2$

最大似然估计 $L(\theta)=\prod_{i=1}^{n}f(x;\theta)=\begin{cases}\sqrt{\theta}^{\,n}\prod_{i=1}^{n}x_i^{\sqrt{\theta}-1} & 0<x<1\\ 0 & o.w.\end{cases}$

$\therefore \ln L(\theta)=\frac{n}{2}\ln\theta+(\sqrt{\theta}-1)\sum_{i=1}^{n}\ln x_i$. $\frac{d\ln L(\theta)}{d\theta}=\frac{n}{2\theta}+\frac{\sum_{i=1}^{n}\ln x_i}{2\sqrt{\theta}}=0$

$\therefore \hat{\theta}_{MLE}=\frac{n^2}{(\sum_{i=1}^{n}\ln x_i)^2}$

(5) 矩估计 $E(X)=\int_\mu^{+\infty}\frac{1}{\theta}e^{-\frac{x-\mu}{\theta}}\cdot x\,dx=\theta+\mu := \overline{X}$

$E(X^2)=\int_\mu^{+\infty}\frac{1}{\theta}e^{\frac{\mu}{\theta}}\cdot e^{-\frac{x}{\theta}}x^2\,dx := \mu^2+2\mu\theta+2\theta^2=\frac{1}{n}\sum_{i=1}^{n}x_i^2$

$\therefore \begin{cases}\hat{\theta}=\left[\frac{1}{n}\sum_{i=1}^{n}x_i^2-\left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)^2\right]^{\frac{1}{2}}=\left[\frac{1}{n}\sum_{i=1}^{n}x_i^2-\overline{X}^2\right]^{\frac{1}{2}}\\ \hat{\mu}=\overline{X}-\left[\frac{1}{n}\sum_{i=1}^{n}x_i^2-\overline{X}^2\right]^{\frac{1}{2}}\end{cases}$

最大似然估计 $L(\theta, \mu) = \prod_{i=1}^{n} f(x_i; \theta, \mu) = \cancel{\frac{1}{\theta^n} \prod_{i=1}^{n} e^{-\frac{x_i-\mu}{\theta}}} \frac{1}{\theta^n} \prod_{i=1}^{n} e^{-\frac{x_i-\mu}{\theta}}$

$\therefore \ln L(\theta, \mu) = -n\ln\theta - \sum_{i=1}^{n} \frac{x_i-\mu}{\theta} = -n\ln\theta - \sum_{i=1}^{n} \frac{x_i}{\theta} + \frac{n\mu}{\theta}$

$\therefore \frac{\partial \ln L(\theta, \mu)}{\partial \mu} = 0 \Rightarrow$ 无解. $\frac{\partial \ln L(\theta, \mu)}{\partial \theta} = 0 \Rightarrow \theta + \mu = \frac{1}{n} \sum_{i=1}^{n} x_i = \overline{X}$

$\therefore \hat{\mu}_{MLE} = X_{(1)}, \quad \hat{\theta}_{MLE} = \overline{X} - X_{(1)}$