285.1

(1) $a_n=\frac{n}{2^n}$

285.2

(2). $S_n=1-\frac{1}{2^n}$, $S_{n+1}-S_n=a_{n+1}=\frac{1}{2^n}-\frac{1}{2^{n+1}}=\frac{1}{2^{n+1}}\ (n\geq1)$.

$\therefore a_n=\frac{1}{2^n}$ $\therefore \sum_1^\infty \frac{1}{2^n}=1$

286.3

(2) $\sum \ln n-\ln(n+1)=1-\ln(n+1)$ ∴ 发散

(4) $\frac{2^n}{n}>1$ ∴ 发散.

(6) $\left(\frac{n}{1+n}\right)^n=\left(1+\frac{1}{n}\right)^{-n}=\left(1+\frac{1}{n}\right)^{n\cdot(-1)}=\frac{1}{e}$

$\therefore \sum a_n=\sum_1^N a_n+\sum_N^\infty \frac{1}{e}$ 发散. $\therefore \sum a_n=\frac{1}{4}$

(7) $a_n=\frac{2}{n+2}-\frac{\frac{1}{2}}{n+3}-\frac{\frac{1}{2}}{n+1}$ ~~$\frac{2}{n+2}$ ∴ 发散~~ $=\left(\frac{\frac{1}{2}}{n+2}-\frac{\frac{1}{2}}{n+1}\right)-\left(\frac{\frac{1}{2}}{n+3}-\frac{\frac{1}{2}}{n+2}\right)$

(8) $a_n=(\sqrt{n+2}-\sqrt{n+1})-(\sqrt{n+1}-\sqrt{n})$ $\therefore \sum a_n=(\sqrt{n+2}-\sqrt{n+1})-(\sqrt{2}-1)$

$\sqrt{n+2}-\sqrt{n+1}=\frac{\sqrt{n+2}-\sqrt{n+1}}{1}=\frac{1}{\sqrt{n+2}\sqrt{n+1}}\to0$ $\therefore \sum a_n=1-\sqrt{2}$.

286.4

~~证: $\sum(a_n+b_n)=\sum a_n+\sum b_n$ ∴ 发散.~~

$\sum_{n+1}^\infty b_n$ 与 $\sum_1^\infty b_n$ 共敛散 ∴ 只证 $\sum_1^\infty(a_n+b_n)$ ∴ $\sum_1^\infty a_n+\sum_1^\infty b_n$ 发散.

不一定. 若 $a_n\equiv1$, $b_n\equiv-1$, 就不发散. $=S+\sum_1^\infty b_n$

286.5

(1) $\ln(n+1)<n$ $\therefore \frac{1}{\ln(n+1)}>\frac{1}{n}$ ∴ 发散.

(2) $\exists N>0$, $n>N$时 $|\sin\frac{\pi}{2^n}|<\varepsilon$. $\therefore \sin\frac{\pi}{2^n}<\frac{\pi}{2^n}\ (n>N)$

$\therefore 0<\sum\sin\frac{\pi}{2^n}<\sum_1^N\sin\frac{\pi}{2^n}+\frac{\frac{\pi}{2}}{1-\frac{1}{2}}$ 收敛.

(4) $\arctan n<n$, 且$\exists N$, $n>N$时 $\frac{\arctan n}{n}>\frac{1}{n}$

$\therefore \sum\frac{\arctan n}{n}>\sum_1^N\frac{\arctan n}{n}+\sum_N^\infty\frac{1}{n}$ ∴ 发散.

(5) $\exists N>0$, 当 $n>N$ 时. ~~lim~~ $\sqrt[n]{n}=1$ ∴ $n>N$ 时 $\sqrt[n]{n}>\frac{1}{2}$

∴ $n>N$ 时 $\frac{1}{n\sqrt[n]{n}}>\frac{1}{2n}$ ∴ $\sum a_n > \sum_{1}^{N} a_n + \sum_{N}^{\infty}\frac{1}{2n}$ ∴发散.

(8) ~~$\exists N$,~~ $n>3$ 时. $\sqrt[n]{n}-1 > e^{\frac{1}{n}}-1 > \frac{1}{n}$ ∴发散.

286.6

(1) $\exists N$, 当 ~~ㄓ~~ $n>N$ 时. $\frac{n^3}{3^n}<k\frac{1}{3^{n-2}}$ ∴ $\sum a_n$ 收敛.

(3) $\frac{n^{n+1}}{(n+1)!}=\frac{n^{n+1}}{n!}\cdot\frac{n}{n+1}$ $\exists N$, $n>N$ 时. ~~ㄓ~~ $\frac{n^{n+1}}{n!}\frac{n}{n+1}=\frac{n^n}{n!}$

$\exists N_2$, $n>N_2$ 时. $\frac{n^n}{n!}=\frac{n^N}{1\cdots N}\cdot\frac{n^{n-N}}{N\cdots n}>n$ ∴发散.

(5) $\frac{a^n}{\ln(n+1)}$ ~~ㄓ~~ $>\frac{a^n}{n}$ ~~$\exists N>0$.~~

1) $0<a\leq 1$ ∴ ~~且~~ $a^n\to 0$ ∴~~收敛~~, $\ln(1+n)>\frac{n}{n+1}$ ∴ $\frac{a^n}{\ln(n+1)}<a^n\to 0$

~~且~~ ∴收敛.

2) $a\geq 1$ 时. $\frac{a^n}{\ln(n+1)}>\frac{1}{n}$ ∴发散.

(7) $\lim\limits_{n\to\infty}\sqrt[n]{3}-1=0$ ∴ $\sum(\sqrt[n]{3}-1)^n$ 收敛.

(9) ~~ㄓ~~ $\lim\limits_{n\to\infty}(2n\arcsin\frac{1}{n})^{\frac{1}{2}}=\sqrt{2}>1$ ∴发散.