111.4

(4) $f(x,1)=x \quad f_x(x,1)=1$

111.10

(2) $\frac{\partial z}{\partial x}=-\left(\frac{1}{3}\right)^{-\frac{y}{x}} \ln 3 \cdot \frac{y}{x^2} \cancel{\left(\ln\frac{y}{x}\right)\cdot\frac{1}{x}}$ $\quad \cancel{\frac{\partial z}{\partial y}=\left(\frac{1}{3}\right)^{-\frac{y}{x}} \ln 3 \cdot \frac{1}{x} \left(\ln\frac{y}{x}\right)\cdot\frac{1}{x}}$ $\quad \frac{\partial z}{\partial y}=\left(\frac{1}{3}\right)^{-\frac{y}{x}}\frac{\ln 3}{x}.$

(4) $\frac{\partial z}{\partial x}=\frac{ye^{xy}(e^x+e^y)-e^xe^{xy}}{(e^x+e^y)^2}=\frac{(y-1)e^{xy+x}+ye^{xy+y}}{(e^x+e^y)^2}$

$\frac{\partial z}{\partial y}=\frac{(x-1)e^{xy+y}+xe^{xy+x}}{(e^x+e^y)^2}$

(5) $\frac{\partial z}{\partial x}=\frac{1}{\tan\frac{x}{y}}\frac{1}{\cos^2\frac{x}{y}}\frac{1}{y}=\frac{2}{y\sin\frac{2x}{y}}$ $\quad \frac{\partial z}{\partial y}=\frac{1}{\tan\frac{x}{y}}\frac{1}{\cos^2\frac{x}{y}}\cdot -\frac{x}{y^2}=-\frac{2x}{y^2\sin\frac{2x}{y}}$

(6) $\frac{\partial z}{\partial x}=\frac{-2y}{\sqrt{1-(3-2xy)^2}}-\cos\left(3-\frac{2x}{y}\right)\cdot\frac{2}{y}$

$\frac{\partial z}{\partial y}=\frac{-2x}{\sqrt{1-(3-2xy)^2}}+\cos\left(3-\frac{2x}{y}\right)\cdot\frac{2x}{y^2}$

(9) $\frac{\partial u}{\partial x}=\frac{y}{z}x^{\frac{y}{z}-1}$ $\quad \frac{\partial u}{\partial y}=\frac{1}{z}x^{\frac{y}{z}}\ln x;$ $\quad \frac{\partial u}{\partial z}=-\frac{y}{z^2}x^{\frac{y}{z}}\ln x$

(10) $\frac{\partial u}{\partial x}=(3x^2+y^2+z^2)e^{x^3+xy^2+xz^2}.$

$\frac{\partial u}{\partial y}=2yx\,e^{x^3+xy^2+xz^2}$ $\quad \frac{\partial u}{\partial z}=2zx\,e^{x^3+xy^2+xz^2}$

112.11

(2) $\frac{\partial z}{\partial y}=\frac{y}{\sqrt{1+x^2+y^2}}$ $\quad \therefore \frac{\partial z}{\partial y}\Big|_{(1,1)}=\frac{1}{\sqrt{3}}$

$\vec{r}=\left(0,1,\frac{1}{\sqrt{3}}\right)$ $\quad l: \frac{x-1}{0}=y-1=\cancel{\sqrt{3}(z-\sqrt{3})}=\frac{z-\sqrt{3}}{\frac{1}{\sqrt{3}}}$

$\pi: \; y+\frac{\cancel{\sqrt{3}}z}{\sqrt{3}}-4=0$

(5).

$$\begin{cases} z = x^2 + \frac{y^2}{6} \\ y = 2 \end{cases} \Rightarrow \frac{\partial z}{\partial x} = 2x \qquad \begin{cases} z = \frac{x^2+y^2}{3} \\ y = 2 \end{cases} \Rightarrow \frac{\partial z}{\partial x} = \frac{2}{3}x.$$

$$\begin{cases} z = x^2 + \frac{y^2}{6} \\ z = \frac{x^2}{3} + \frac{y^2}{3} \\ y = 2 \end{cases} \Rightarrow \begin{cases} x = \pm 1 \\ y = 2 \\ z = \frac{5}{3} \end{cases} \quad \therefore \quad \begin{aligned} l_1 &: x+1 = \frac{y-2}{0} = \frac{z-\frac{5}{3}}{2} \\ l_2 &: x+1 = \frac{y-2}{0} = \frac{z-\frac{5}{3}}{\frac{2}{3}} \end{aligned}$$

$$\Rightarrow \theta = \arccos \frac{7}{\sqrt{65}} \qquad \cos\theta = \frac{(1,0,2)\cdot(1,0,\frac{2}{3})}{|(1,0,2)|\cdot|(1,0,\frac{2}{3})|}$$

65=49+16.

11.2.12

(2) $\frac{\partial z}{\partial x} = \frac{1}{1+(\frac{x+y}{1-xy})^2} \frac{1+y^2}{(1-xy)^2} = \frac{1+y^2}{(1-xy)^2+(x+y)^2} = \frac{1+y^2}{(x^2+1)(y^2+1)} = \frac{1}{1+x^2}$

$\frac{\partial z}{\partial y} = \frac{1}{1+(\frac{x+y}{1-xy})^2} \frac{1+x^2}{(1-xy)^2} = \frac{1+x^2}{(1-xy)^2+(x+y)^2} = \frac{1}{1+y^2}$

$\frac{\partial^2 z}{\partial x^2} = -\frac{2x}{(1+x^2)^2}$, $\frac{\partial^2 z}{\partial y^2} = -\frac{2y}{(1+y^2)^2}$, $\frac{\partial^2 z}{\partial x \partial y} = 0$

(3) $\frac{\partial z}{\partial x} = y x^{y-1}$, $\frac{\partial z}{\partial y} = \cancel{x^y \ln y}\ x^y \ln x$.

$\frac{\partial^2 z}{\partial x^2} = y(y-1)x^{y-2}$ $\frac{\partial^2 z}{\partial y^2} = \cancel{x^y \ln^2 y + \frac{1}{y} x^y}\ x^y \ln^2 x$.

$\frac{\partial^2 z}{\partial x \partial y} = \cancel{x^{y-1} + y x^{y-1} \ln y}\ x^{y-1} + y x^{y-1} \ln x$.

11.2.13

~~(2) $\frac{\partial z}{\partial x} = \frac{2x}{\sqrt{x^2+y^2}}$, $\frac{\partial z}{\partial y} = \frac{2y}{\sqrt{x^2+y^2}}$~~

~~$\frac{\partial^2 z}{\partial x^2} = \frac{2\sqrt{x^2+y^2} - (\sqrt{x^2+y^2})' \cdot 2x}{x^2+y^2} =$~~

(2) $\frac{\partial z}{\partial x} = \frac{x}{x^2+y^2}$, $\frac{\partial z}{\partial y} = \frac{y}{x^2+y^2}$.

$\frac{\partial^2 z}{\partial x^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$, $\frac{\partial^2 z}{\partial y^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$

$\therefore \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

112.13

(3) $\frac{\partial u}{\partial x} = 3x^2 - 3yz$, $\frac{\partial u}{\partial y} = 3y^2 - 3xz$, $\frac{\partial u}{\partial z} = 3z^2 - 3xy$

$\frac{\partial^2 u}{\partial x^2} = 6x$, $\frac{\partial^2 u}{\partial y^2} = 6y$ $\frac{\partial^2 u}{\partial z^2} = 6z$

$\therefore (\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2 + (\frac{\partial u}{\partial z})^2 = 9[(x^2-yz)^2 + (y^2-xz)^2 + (z^2-xy)^2]$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 6(x+y+z)$

112.14

(2) $\frac{\partial u}{\partial x} = \frac{1}{1+\frac{y^2}{x^2}} \cdot -\frac{y}{x^2} + \cdots = -\frac{y}{x^2+y^2} - \frac{z}{x^2+z^2}$

$\frac{\partial u}{\partial y} = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x} = \frac{x}{(x^2+y^2)}$ $\frac{\partial u}{\partial z} = \frac{x}{x^2+z^2}$

$\frac{\partial^2 u}{\partial x^2} = \frac{2xy}{(x^2+y^2)^2} + \frac{2xz}{(x^2+z^2)^2}$ $\frac{\partial^2 u}{\partial y^2} = -\frac{2yx}{(x^2+y^2)^2}$ $\frac{\partial^2 u}{\partial z^2} = -\frac{2zx}{(x^2+z^2)^2}$

$\therefore \Delta u = 0$

(4) $\ln r = \frac{1}{2}\ln(x^2+y^2+z^2)$

$\frac{\partial(\ln r)}{\partial x} = \frac{x}{x^2+y^2+z^2}$ $\frac{\partial(\ln r)}{\partial y} = \frac{y}{x^2+y^2+z^2}$ $\frac{\partial(\ln r)}{\partial z} = \frac{z}{x^2+y^2+z^2}$

$\frac{\partial^2(\ln r)}{\partial x^2} = \frac{y^2+z^2-x^2}{(x^2+y^2+z^2)^2}$ $\frac{\partial^2(\ln r)}{\partial y^2} = \frac{x^2+z^2-y^2}{(x^2+y^2+z^2)^2}$, $\frac{\partial^2(\ln r)}{\partial z^2} = \frac{x^2+y^2-z^2}{(x^2+y^2+z^2)^2}$

$\therefore$ 原式 $= \frac{1}{x^2+y^2+z^2} = \frac{1}{r^2}$

112.15

$\frac{\partial z}{\partial x} = -\frac{y}{x^2}$, $\frac{\partial z}{\partial y} = \frac{1}{x}$ $\therefore \exists\, dz$.

$\Delta z = z(2.1, 1.2) - z(2, 1) = \frac{1}{14}$ ✓

$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = \frac{3}{40}$ ✓

0.0714
$14 \,)\, 100$
98
20
14
60

11.2.13

(3) $\frac{\partial u}{\partial x} = 3x^2 - 3yz$, $\frac{\partial u}{\partial y} = 3y^2 - 3xz$, $\frac{\partial u}{\partial z} = 3z^2 - 3xy$

$\frac{\partial^2 u}{\partial x^2} = 6x$, $\frac{\partial^2 u}{\partial y^2} = 6y$, $\frac{\partial^2 u}{\partial z^2} = 6z$

$\therefore \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 9[(x^2-yz)^2 + (y^2-xz)^2 + (z^2-xy)^2]$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 6(x+y+z)$

11.2.14

(2) $\frac{\partial u}{\partial x} = \frac{1}{1+\frac{y^2}{x^2}} \cdot -\frac{y}{x^2} + \cdots = -\frac{y}{x^2+y^2} - \frac{z}{x^2+z^2}$

$\frac{\partial u}{\partial y} = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x} = \frac{x}{(x^2+y^2)}$ $\quad \frac{\partial u}{\partial z} = \frac{x}{x^2+z^2}$

$\frac{\partial^2 u}{\partial x^2} = \frac{2xy}{(x^2+y^2)^2} + \frac{2xz}{(x^2+z^2)^2}$ $\quad \frac{\partial^2 u}{\partial y^2} = -\frac{2yx}{(x^2+y^2)^2}$ $\quad \frac{\partial^2 u}{\partial z^2} = -\frac{2zx}{(x^2+z^2)^2}$

$\therefore \Delta u = 0$

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$\frac{\partial(\ln r)}{\partial x} = \frac{x}{x^2+y^2+z^2}$ $\quad \frac{\partial(\ln r)}{\partial y} = \frac{y}{x^2+y^2+z^2}$ $\quad \frac{\partial(\ln r)}{\partial z} = \frac{z}{x^2+y^2+z^2}$

$\frac{\partial^2(\ln r)}{\partial x^2} = \frac{y^2+z^2-x^2}{(x^2+y^2+z^2)^2}$ $\quad \frac{\partial^2(\ln r)}{\partial y^2} = \frac{x^2+z^2-y^2}{(x^2+y^2+z^2)^2}$, $\quad \frac{\partial^2(\ln r)}{\partial z^2} = \frac{x^2+y^2-z^2}{(x^2+y^2+z^2)^2}$

$\therefore$ 原式 $= \frac{1}{x^2+y^2+z^2} = \frac{1}{r^2}$

11.2.15

$\frac{\partial z}{\partial x} = -\frac{y}{x^2}$, $\frac{\partial z}{\partial y} = \frac{1}{x}$ $\therefore \exists\, dz$.

$\Delta z = z(2.1, 1.2) - z(2, 1) = \frac{1}{14}$ ✓

$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = \frac{3}{40}$ ✓

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