P118.12 ($k\in Z$).

(2). ~~$f(z)=\frac{}{z+}$~~ $z^2+i=0 \Rightarrow z=e^{i(\pi k-\frac{\pi}{4})}$

$\therefore z=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$ 或, $z=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$ 为二阶极点.

(3). $\sin z=0 \Rightarrow z=\pi k,\ k\in Z$ 为一阶极点.

(4). $\lim\limits_{z\to 1}\frac{\tan(z-1)}{z-1}=1$ $\therefore z=1$ 为可去奇点. $z=1+(k+\frac{1}{2})\pi$ 为一阶极点

(5). $\sin\frac{1}{z}$ ~~$z=0 \Rightarrow z=0,\ z=\frac{1}{\pi k}$~~ $\therefore$ ~~…~~

~~$z=0$ …~~ ~~$z=\frac{1}{\pi k}$ 为一阶极点.~~ $\lim\limits_{z\to 0}e^{-z}\sin\frac{1}{z}$ = 不定. $z=0$ 为本性.

(7). $\sin z=\cos z$, $z=\frac{\pi}{4}+\pi k$. 为一阶极点.

(9). $\lim\limits_{z\to 0}\frac{z-\sin z}{z^2(e^{\pi z}-1)}=\lim\limits_{z\to 0}\frac{z-\sin z}{z^2\pi z}$ ~~$\frac{\sin z}{z^2\pi z}$~~ $=\lim\limits_{z\to 0}\frac{z-z+\frac{1}{3!}z^3+O(z^3)}{z^3\pi}=\frac{1}{3!\pi}$

$\therefore z=0$ 为可去奇点, $z=2ki$ 为一阶.

P118.14.

$\lim\limits_{z\to 0}f(z)=\lim\limits_{z\to 0}\frac{z^2-2+2\cos z}{z^2(1-\cos z)}=\lim\limits_{z\to 0}\frac{z^2-2+2(1-\frac{z^2}{2}+O(z^2))}{z^2(1-1+\frac{z^2}{2}+O(z^2))}=\lim\limits_{z\to 0}\frac{O(z^2)}{\frac{1}{2}z^4}$

$=\lim\limits_{z\to 0}\frac{\frac{1}{12}z^4}{\frac{1}{2}z^4}=\frac{1}{6}$ $\therefore z=0$ 可去.

$1-\cos z=0 \Rightarrow z=2\pi k,\ k\in Z$.

$\lim\limits_{z\to 2\pi k}\frac{1}{1-\cos z}=$ ~~$\lim\limits_{z\to 2\pi k}\frac{1}{1-(1-\frac{z^2}{2}+O(z^2))}=$~~

$=\lim\limits_{z\to 2\pi k}\frac{1}{1-\cos(z-2\pi k)}=\lim\limits_{z\to 2\pi k}\left(1-(1-\frac{1}{2}(z-2\pi k)^2)\right)^{-1}$

$\lim\limits_{z\to 2\pi k}1-\cos z=\frac{1}{2}(z-2\pi k)^2-\frac{1}{24}(z-2\pi k)^4-\dots=(z-2\pi k)^2g(z)$.

$\therefore$ $z=2\pi k$ 为二阶极点.

P119. 15

$f(z)=(z-a)^{-m}\cdot h(z)$.

$g(z)=(z-a)^{-n}T(z)$. , $h(z)$, $T(z)$在 $U(z_0)$解析且 $h(z_0)\neq$, $T(z_0)\neq 0$

(1) $f(z)g(z)=(z-a)^{-m-n}h(z)T(z)$ ∴ a为$(m+n)$阶极点.

(2) $f(z)/g(z)=(z-a)^{n-m}\dfrac{h(z)}{T(z)}$. ∴ a为$(m-n)$阶极点.$(m-n>0)$或非奇点$(0,w)$

(3) $f(z)+g(z)=(z-a)^{-R}((z-a)^{-m+R}h(z)+(z-a)^{-n+R}T(z))$, $R=\max\{m,n\}$.

后面因子在a处$\neq 0$ ∴ a为 $\max\{m,n\}$阶极点.

(4). $\dfrac{f(z)}{g(z)}+\dfrac{g(z)}{f(z)}=(z-a)^{n-m}\dfrac{h(z)}{T(z)}+(z-a)^{m-n}\dfrac{T(z)}{h(z)}$.

$m-n>0$时. $(m-n)$阶极点. ; $n-m>0$时. $(n-m)$阶极点.

$m=n$时. 非极点.

P163. 1

(1) ~~$\dfrac{1}{1+z^4}=\sum_{n=0}^{+\infty}(-z^4)^n$~~ $z^4+1=0 \Rightarrow z=e^{2i(\frac{\pi}{2}k+\frac{\pi}{8})}$

∴ $z_1=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$, $z_2=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$, $z_3=-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$, $z_4=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$

均为一阶极点

~~∴ $\operatorname{Res}[f(z),z_1]=\lim_{z\to z_1}(z-z_1)f(z)=-\dfrac{2+i}{4\sqrt{2}}$~~

$\operatorname{Res}[f(z),z_1]=-\dfrac{2+i}{4\sqrt{2}}$ $\quad\operatorname{Res}[f(z),z_3]=\dfrac{1+i}{4\sqrt{2}}$

$\operatorname{Res}[f(z),z_2]=\dfrac{1-i}{4\sqrt{2}}$ $\quad\operatorname{Res}[f(z),z_4]=\dfrac{i-1}{4\sqrt{2}}$

(2). $z_0=2\pi k i$为一阶极点.

$\operatorname{Res}[f(z),2\pi k i]=\lim_{z\to 0} z\cdot\dfrac{1}{1-e^z}=-1$

(4) $\lim_{z\to 0}\dfrac{1-e^{2z}}{z^4}=-\dfrac{1}{z^3}$ ∴ $z=0$为三阶极点.

∴ $\operatorname{Res}[f(z),0]=\dfrac{1}{2!}\lim_{z\to 0}\left((z)^3\dfrac{1-e^{2z}}{z^4}\right)^{(2)}=$ 不算了.

$1-e^{2z}=-\sum_{n=1}^{+\infty}\dfrac{2^n}{n!}z^n$. ∴ $\dfrac{1-e^{2z}}{z^4}=-\sum_{n=1}^{+\infty}\dfrac{2^n}{n!}z^{n-4}$ ∴ $C_{-1}=-\dfrac{8}{6}=-\dfrac{4}{3}$

(12) z=0为本性奇点. $\sin\frac{1}{z}=\sum_{n=0}^{+\infty}\frac{1}{(2n+1)!}z^{-2n-1}$ ∴ $\mathrm{Res}[f(z),0]=1$.

~~(12) z=和~~ $\cancel{\frac{1}{\pi k}}$ ~~k∈Z 为孤立奇点，均为一阶极点~~

∴ $\cancel{\sin\frac{1}{z}=\sum_{n=0}^{+\infty}\frac{1}{(2n+1)!}}$

$\cancel{\mathrm{Res}[f(z),\frac{1}{\pi k}]=\lim_{z\to\frac{1}{\pi k}}(z-\frac{1}{\pi k})\sin\frac{1}{z}=\lim_{z\to\frac{1}{\pi k}}(z-\frac{1}{\pi k})\sin(\frac{1}{z}-\pi k)(-1)^k}$

$\cancel{=(-2)\cdot(-1)^k\cdot 2(-1)^{k+1}}$.

P143. 2

(2) $1-2\sin^2 z=0\Rightarrow\sin z=\pm\frac{\sqrt{2}}{2}\Rightarrow z=\frac{\pi}{4}+\frac{k\pi}{2},\ k\in Z$.

在 $|z|=1$ 内. $z=\frac{\pi}{4}$, $-\frac{\pi}{4}$, 均为一阶极点.

∴ 原式 $=\oint_{C_1}\frac{z}{1-2\sin^2 z}dz+\oint_{C_2}\frac{z}{1-2\sin^2 z}dz$

$=2\pi i(\mathrm{Res}[f(z),\frac{\pi}{4}]+\mathrm{Res}[f(z),-\frac{\pi}{4}])$.

$=2\pi i(\lim_{z\to\frac{\pi}{4}}(z-\frac{\pi}{4})f(z)+\lim_{z\to-\frac{\pi}{4}}(z+\frac{\pi}{4})f(z))$.

$=-\frac{\pi^2}{2}i$

(3) ~~$e^{\frac{1}{z}}$的奇点 z=0, $\lim_{z\to0^+}e^{\frac{1}{z}}=+\infty$, $\lim_{z\to0^-}e^{\frac{1}{z}}=0$ ∴~~

~~z=0为本性奇点~~

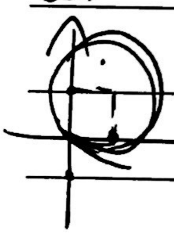
$\lim_{z\to0}ze^{\frac{1}{z}}=\cancel{\lim_{z\to0}z\sum_{n=0}^{+\infty}}\lim_{z\to0}\frac{e^{\frac{1}{z}}}{\frac{1}{z}}\overset{t=\frac{1}{z}}{=\!=}\lim_{t\to\infty}\frac{e^t}{t}=+\infty/0$

∴ z=0为本性奇点.

$e^{\frac{1}{z}}=\sum_{n=0}^{+\infty}\frac{1}{n!}z^{-n}$ ∴ $\oint_C ze^{\frac{1}{z}}dz=\sum_{n=0}^{+\infty}\oint\frac{1}{n!}z^{-n+1}dz$

$=2\pi i\cdot\frac{1}{2}=\pi i$.

(6). $f(z)=\frac{1}{(z+i)(z-i)(z-1)^2}$. z=1为二阶极点. z=i为一阶极点



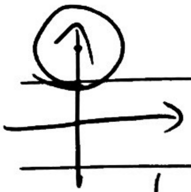
z=-i不在C内.

∴ $\oint\cdots=2\pi i[\mathrm{Res}[f(z),1]+\mathrm{Res}[f(z),i]]$

$=\cancel{\frac{\pi}{2}i}-\frac{\pi}{2}i$

P143.2

(7) $\tanh z = \frac{e^z-e^{-z}}{e^z+e^{-z}}$. ~~lim~~ $e^z+e^{-z}=0 \Rightarrow z=i(\frac{\pi}{2}+k\pi)$



∴ $z=\frac{\pi}{2}i$, $\tanh z = \frac{e^{2z}-1}{e^{2z}+1} = \frac{e^{2z}-1}{(e^z+i)(e^z-i)}$

一阶极点.

∴ $\oint_C \tanh z\,dz = 2\pi i\,\mathrm{Res}[f(z),\frac{\pi}{2}i] = 2\pi i$.

P144.3

(1) $z=0$ 为本性奇点, $z=-1$ 为一阶极点.

原式 $= 2\pi i\{\mathrm{Res}[f(z),0]+\mathrm{Res}[f(z),-1]\}$

$\mathrm{Res}[f(z),0]=C_{-1}$, $f(z)=\frac{z^3}{1+z}e^{\frac{1}{z}} = z^3\left(\sum_{n=0}^{+\infty}(-1)^n z^n\right)\cdot\left(\sum_{n=0}^{+\infty}\frac{1}{n!}\frac{1}{z^n}\right)$.

∴ $C_{-1}=\sum_{n=0}^{+\infty}\frac{(-1)^n}{(n+4)!} = \sum_{n=4}^{+\infty}\frac{(-1)^n}{n!}$

$\mathrm{Res}[f(z),-1] = \lim_{z\to-1}(z+1)\frac{z^3}{1+z}e^{\frac{1}{z}} = -e^{-1}$.

∴ 原式 $= 2\pi i\left(\left(\sum_{n=4}^{+\infty}\frac{(-1)^n}{n!}\right)-e^{-1}\right)$

又 $e^x=\sum_{n=0}^{+\infty}\frac{1}{n!}x^n$, $e^{-x}=\sum_{n=0}^{+\infty}\frac{(-1)^n}{n!}x^n$.

∴ $\sum_{n=4}^{+\infty}\frac{(-1)^n}{n!} = e^{-1}-1+1-\frac{1}{2}+\frac{1}{6} = e^{-1}-\frac{1}{3}$

∴ 原式 $= -\frac{2\pi}{3}i$.