23.6 1

(1) $K_1=\{a+b\sqrt{3} \mid a,b\in Z\}$ ∵ $\{0,1\}\subseteq K_1$

∴ $a+b\sqrt{3} \pm c+d\sqrt{3} = (a\pm c)+(b\pm d)\sqrt{3} \in K_1$

$(a+b\sqrt{3})(c+d\sqrt{3}) = (ac+3bd)+\sqrt{3}(bc+ad) \in K_1$

$\frac{a+b\sqrt{3}}{c+d\sqrt{3}} = \frac{(ac-3bd)+(bc-ad)\sqrt{3}}{c^2-3d^2} = \frac{ac-3bd}{c^2-3d^2} + \frac{bc-ad}{c^2-3d^2}\sqrt{3}$ 不一定 $\in K_1$

∴ K_1 不是

(2) 同理，$(a+b\sqrt[3]{3})\pm(c+d\sqrt[3]{3}) \in K_2$，$\{0,1\}\subseteq K_2$

$(a+b\sqrt[3]{3})(c+d\sqrt[3]{3}) = ac+bd\,3^{\frac{2}{3}}+(bc+ad)\sqrt[3]{3}$ 不一定 $\in K_2$

∴ K_2 不是

(3) K_3 同 K_1 理，$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{c^2+d^2} = \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i$ 不一定 $\in K_3$

∴ K_3 不是.

(4) 易知 $K_4=C$，C 是一个数域 ∴ K_4 是.

23.2

(1) $A_m = \begin{pmatrix} a_1 & a_1 & a_1 & \cdots & a_1 \\ a_2 & a_2 & \cdots & \cdots & a_2 \\ a_3 & \vdots & \vdots & \cdots & \vdots \\ \vdots & & & & \\ a_m & a_m & & & a_m \end{pmatrix}$

(4) $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

(2) $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$

(3) $A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$

23.3 $\begin{cases} a+2b=4 \\ 2a-b=-2 \end{cases}$ $\begin{cases} 2c+d=4 \\ c-2d=-3 \end{cases}$

$\Rightarrow (a,b,c,d)=(0,2,1,2)$

24.9 证明:

易知 $\{0,1\}\subseteq Q(\sqrt{3})$

且 $a+b\sqrt{3}\pm c+d\sqrt{3} = (a\pm c)+(b\pm d)\sqrt{3} \in Q(\sqrt{3})$

$(a+b\sqrt{3})(c+d\sqrt{3}) = (ac+3bd)+(ad+bc)\sqrt{3} \in Q(\sqrt{3})$

$\frac{a+b\sqrt{3}}{c+d\sqrt{3}} = \frac{ac-3bd+(bc-ad)\sqrt{3}}{c^2-3d^2} = \frac{ac-3bd}{c^2-3d^2} + \frac{bc-ad}{c^2-3d^2}\sqrt{3} \in Q(\sqrt{3})$

(Q 有封闭性)

24.11 证明.

设 P 是一个数域 ∴ $\{0,1\}\subseteq P$

又∵ P 有封闭性 ∴ $0-1=-1\in P$，∴ $Z\subseteq P$ 令 $m,n\in Z$

∴ $\frac{m}{n}\in P$ 又∵ $Q=\{x \mid x=\frac{m}{n}, m\in Z, n\in N^*$ 且 m,n 互质$\}$

∴ $Q\subseteq\{x \mid x=\frac{m}{n}, m,n\in Z\ (n\neq 0)\}$.

∴ $Q\subseteq P$