P236.1

(2) $\mathcal{L}[f(t)] = \int_0^{\pi} \sin t\, e^{-pt} dt = \frac{1}{2i}\left(\frac{e^{\pi(i-p)}-1}{i-p} + \frac{e^{-\pi(i+p)}-1}{i+p}\right)$

P236.2.

$= \frac{1}{p^2+1}(1+e^{-\pi p})$.

(4) $\mathcal{L}[f(t)] = \int_0^{+\infty} \sin(t-2)\, e^{-pt} dt = \frac{1}{1+p^2}\cdot(\cos 2 - p\sin 2)$

(5) $\mathcal{L}[f(t)] = \int_2^{+\infty} \sin t\, e^{-pt} dt = \frac{1}{1+p^2}(\cos 2 + p\sin 2)e^{-2p}$.

(6) $\mathcal{L}[f(t)] = \int_2^{+\infty} e^{2t} e^{-pt} dt = \frac{1}{p-2} e^{4-2p}$

P236.3

(4). $\mathcal{L}^{-1}[F(p)] = 2\mathcal{L}^{-1}[\frac{e^{-p}}{p}] - \mathcal{L}^{-1}[\frac{e^{-2p}}{p}]$

$\mathcal{L}^{-1}[\frac{1}{p}] = 1$ ∴ 原式 $= 2u(t-1) - u(t-2)$

P237.4

(1) $\mathcal{L}[(t-1)^2 e^t] = F_1(p-1)$. $F_1(p) = \mathcal{L}[(t-1)^2] = \mathcal{L}[t^2-2t+1]$

$\mathcal{L}[t^2] = F_2^{(2)}(p)$, $\mathcal{L}[t] = -F_2'(p)$, $\mathcal{L}[1] = F_2(p) = \frac{1}{p}$.

∴ 原式 $= \frac{1}{p-1} - \frac{2}{(p-1)^2} + \frac{2}{(p-1)^3}$

(3). $\mathcal{L}[te^{-at}\sin\beta t] = F_1(p+a)$., $\mathcal{L}[t\sin\beta t] = -F_2'(p)$.

$\mathcal{L}[\sin\beta t] = F_2(p) = \frac{1}{2i}\mathcal{L}[e^{i\beta t} - e^{-i\beta t}] = \frac{1}{2i}\left(F_3(p-i\beta) - F_3(p+i\beta)\right)$

$F_3(p) = \mathcal{L}[1] = \frac{1}{p}$ ∴ 原式 $= \frac{1}{2i}\left[\frac{1}{(p+a-i\beta)^2} - \frac{1}{(p+a+i\beta)^2}\right]$

P237.5

$= \frac{2\beta(p+a)}{((p+a)^2+\beta^2)^2}$

(3). $\mathcal{L}^{-1}[\frac{1}{(p+2)^2}] = \mathcal{L}^{-1}[-(\frac{1}{p+2})'] = -\mathcal{L}^{-1}[(\frac{1}{p+2})'] = -t\mathcal{L}^{-1}[\frac{1}{p+2}]$.

$= t\cdot e^{-2t}$.

(6). $\frac{p+3}{(p+1)(p-3)} = \frac{A}{p+1} + \frac{B}{p-3} = -\frac{1}{2}\frac{1}{p+1} + \frac{3}{2}\frac{1}{p-3}$ ∴ 原式 $= -\frac{1}{2}e^{-t} + \frac{3}{2}e^{3t}$.

P237.6

(1) $\mathcal{L}^{-1}\left[\frac{1}{p(p-a)}\right]=\mathcal{L}^{-1}\left[\frac{1}{a}\left(\frac{1}{p-a}-\frac{1}{p}\right)\right]=\frac{1}{a}e^{at}-\frac{1}{a}$

(3) $=\text{Res}\left[\frac{e^{pt}}{p^3(p-a)},0\right]+\text{Res}\left[\frac{e^{pt}}{p^3(p-a)},a\right]$

$=\frac{1}{a^3}\left(e^{at}-\frac{1}{2}t^2a^2-at-1\right).$

(6) $=\text{Res}[\cdots,-a]$, a, ai, $-ai$

$=\frac{1}{4a^3}\left(e^{at}-e^{-at}+ie^{ait}-ie^{-iat}\right).$

$=\frac{1}{4a^3}\left(e^{at}-e^{-at}-2\sin at\right).$

P237.7

(1) $\frac{4}{p(2p+3)}=\frac{A}{p}+\frac{B}{2p+3}=\frac{4}{3}\frac{1}{p}-\frac{8}{3}\frac{1}{2p+3}=\frac{4}{3}\frac{1}{p}-\frac{8}{6}\frac{1}{p+\frac{3}{2}}$

$\therefore \mathcal{L}[\cdots]=\frac{4}{3}-\frac{8}{6}e^{-\frac{3}{2}t}=\frac{4}{3}\left(1-e^{-\frac{3}{2}t}\right).$

(4). ~~原式 $=\mathcal{L}[\cdots]=$ Res~~

~~原式~~ $=\mathcal{L}[\cdots]=\text{Res}[\cdots,0]+\text{Res}[\cdots,1]$

$=t^2+5t+8+(3t-8)e^t.$