3.34

$$h(n)=\begin{cases}3^n & n<0\\ (\frac{2}{5})^n & n\geq 0\end{cases} \qquad H(z)=\sum_{n=-\infty}^{-1}3^n z^{-n}+\sum_{n=0}^{+\infty}(\frac{2}{5})^n z^{-n}$$

$$=\frac{z}{3-z}+\frac{1}{1-\frac{2}{5}z^{-1}} \qquad \text{ROC: } \frac{2}{5}<|z|<3$$

$\forall\ y(n)=h(n)*x(n),\quad x(n)=u(n)$

$Y(z)=H(z)\cdot X(z),\quad X(z)=\frac{1}{1-z^{-1}}\quad \text{ROC: } |z|>1.$

$$\therefore\ Y(z)=-\frac{2}{3}\frac{1}{1-\frac{2}{5}z^{-1}}+\frac{5}{3}\frac{1}{1-z^{-1}}-\frac{3}{2}\frac{1}{1-3z^{-1}}+\frac{1}{2}\frac{1}{1-z^{-1}} \qquad \text{ROC: } 1<|z|<3$$

$$\therefore\ y(n)=-\frac{2}{3}(\frac{2}{5})^n u(n)+\frac{13}{6}u(n)+\frac{3}{2}3^n u(-n-1)$$

3.35

(b) $h(n)=(\frac{1}{2})^n u(n)\quad x(n)=(\frac{1}{3})^n u(n)+(\frac{1}{2})^{-n}u(-n-1)$

$y_{zs}(n)=h(n)*x(n)\cdot \Rightarrow Y_{zs}(z)=H(z)\cdot X(z).$

$$H(z)=\frac{1}{1-\frac{1}{2}z^{-1}}\quad \text{ROC: } |z|>\frac{1}{2},\qquad X(z)=\frac{1}{1-\frac{1}{3}z^{-1}}-\frac{1}{1-2z^{-1}}\quad \text{ROC: } \frac{1}{3}<|z|<2$$

$$\therefore\ Y_{zs}(z)=\frac{1}{1-\frac{1}{2}z^{-1}}\left(\frac{1}{1-\frac{1}{3}z^{-1}}-\frac{1}{1-2z^{-1}}\right)\quad \text{ROC: } \frac{1}{2}<|z|<2$$

$$=\frac{1}{1-\frac{1}{2}z^{-1}}\cdot\frac{10}{3}-\frac{2}{1-\frac{1}{3}z^{-1}}-\frac{4}{3}\frac{1}{1-2z^{-1}}$$

$$\Rightarrow y_{zs}(n)=\frac{10}{3}(\frac{1}{2})^n u(n)-2(\frac{1}{3})^n u(n)+\frac{4}{3}2^n u(-n-1).$$

(c) $Y(z)=-0.1z^{-1}Y(z)+0.2z^{-2}Y(z)+\bar{X}(z)+z^{-1}\bar{X}(z)$

$$\Rightarrow Y(z)=\frac{1+z^{-1}}{1+0.1z^{-1}+0.2z^{-2}}\bar{X}(z),\qquad \bar{X}(z)=\frac{1}{1-\frac{1}{3}z^{-1}}\quad \text{ROC: } |z|>\frac{1}{3}.$$

$$\Rightarrow Y(z)=\frac{-5(1+z^{-1})}{(z^{-1}-\frac{5}{2})(z^{-1}+2)}\cdot\frac{1}{1-\frac{1}{3}z^{-1}}=-\frac{1}{3}\cdot\frac{1}{1+\frac{1}{2}z^{-1}}+\frac{-8}{1-\frac{1}{5}z^{-1}}+\frac{28}{3}\frac{1}{1-\frac{2}{5}z^{-1}}$$

ROC: $|z|>\frac{1}{2}$

$$\Rightarrow y(n)=-\frac{1}{3}(-\frac{1}{2})^n u(n)-8(\frac{1}{5})^n u(n)+\frac{28}{3}(\frac{2}{5})^n u(n)$$

3.38

(a) $y(n)=\frac{3}{4}y(n-1)-\frac{1}{8}y(n-2)+x(n)$. ROC: $|z|>\frac{1}{2}$

$Y(z)=\frac{3}{4}z^{-1}Y(z)-\frac{1}{8}z^{-2}Y(z)+X(z)\Rightarrow H(z)=\frac{2}{1-\frac{1}{2}z^{-1}}-\frac{1}{1-\frac{1}{4}z^{-1}}$

稳定的. 包含单位圆.

$\delta(n)\Leftrightarrow 1$ $\therefore Y(z)=\frac{2}{1-\frac{1}{2}z^{-1}}-\frac{1}{1-\frac{1}{4}z^{-1}}\Rightarrow$ ROC同上

$\Rightarrow y(n)=2(\frac{1}{2})^n u(n)-(\frac{1}{4})^n u(n)$.

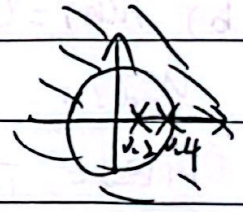
$u(n)\Leftrightarrow \frac{1}{1-z^{-1}}$ $\therefore Y(z)=\frac{8}{3}\frac{1}{1-z^{-1}}-2\frac{1}{1-\frac{1}{2}z^{-1}}+\frac{1}{3}\frac{1}{1-\frac{1}{4}z^{-1}}$. ROC: $|z|>1$

$\Rightarrow y(n)=\frac{8}{3}u(n)-2(\frac{1}{2})^n u(n)+\frac{1}{3}(\frac{1}{4})^n u(n)$.

(b) $y(n)=0.6y(n-1)-0.08y(n-2)+x(n)$

$\Rightarrow Y(z)=0.6z^{-1}Y(z)-0.08z^{-2}Y(z)+X(z)$

$\Rightarrow H(z)=\frac{1}{(1-0.2z^{-1})(1-0.4z^{-1})}$ ROC: $|z|>0.4$



稳定.

$\delta(n)\Leftrightarrow 1$, ROC: All

$Y(z)=-\frac{1}{1-0.2z^{-1}}+\frac{2}{1-0.4z^{-1}}$ ROC同上. $y(n)=2(0.4)^n u(n)-(0.2)^n u(n)$

$u(n)\Leftrightarrow \frac{1}{1-z^{-1}}$, ROC: $|z|>1$. $Y(z)=\frac{1}{4}\frac{1}{1-0.2z^{-1}}-\frac{4}{3}\frac{1}{1-0.4z^{-1}}+\frac{25}{12}\frac{1}{1-z^{-1}}$

$\Rightarrow y(n)=\frac{1}{4}(0.2)^n u(n)-\frac{4}{3}(0.4)^n u(n)+\frac{25}{12}u(n)$ ROC: $|z|>1$

例1.

$y(n)=\frac{3}{4}y(n-1)-\frac{1}{8}y(n-2)+x(n)$，$x(n)=\delta(n)$，$y(-1)=-1$，$y(-2)=1$

两边单边Z变换：$Y^+(z)=\frac{3}{4}z^{-1}[Y^+(z)+zy(-1)]-\frac{1}{8}z^{-2}[Y^+(z)+zy(-1)+z^2y(-2)]+X^+(z)$.

$\Rightarrow \frac{Y^+(z)}{X^+(z)}$ $\Rightarrow Y^+(z)=\frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}X^+(z)+\frac{\frac{3}{4}y(-1)-\frac{1}{8}z^{-1}y(-1)-\frac{1}{8}y(-2)}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$

$X(n)=\delta(n)$.

代入 $y(-1)=-1$，$y(-2)=1$，得到

$Y^+(z)=\frac{1}{8}\frac{1+z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}=\frac{3}{4}\frac{1}{1-\frac{1}{2}z^{-1}}-\frac{5}{8}\frac{1}{1-\frac{1}{4}z^{-1}}$ ROC: $|z|>\frac{1}{2}$

$y(n)=\frac{3}{4}(\frac{1}{2})^nu(n)-\frac{5}{8}(\frac{1}{4})^nu(n)$.

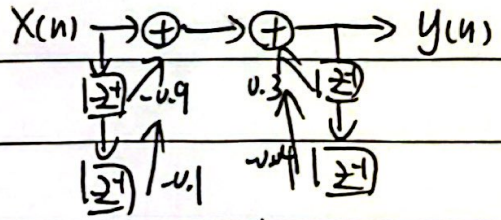
代入 $y(-1)=y(-2)=0$，得到

$Y^+(z)=2\frac{1}{1-\frac{1}{2}z^{-1}}-\frac{1}{1-\frac{1}{4}z^{-1}}$ ROC: $|z|>\frac{1}{2}$.

$\therefore y(n)=2(\frac{1}{2})^nu(n)-(\frac{1}{4})^nu(n)$

例2: $H(z)=\frac{1-0.9z^{-1}-0.1z^{-2}}{1+0.3z^{-1}-0.04z^{-2}}$

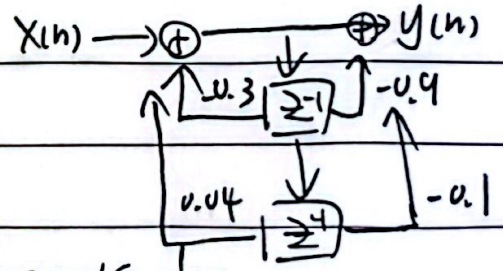
I型.

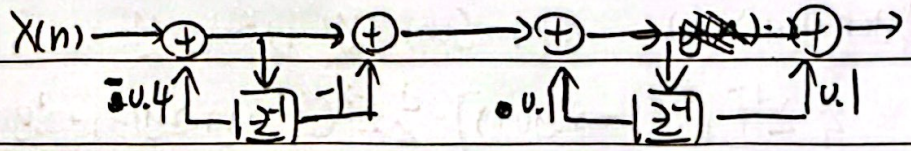


$H(z)=\frac{1-0.9z^{-1}-0.1z^{-2}}{(1+0.4z^{-1})(1-0.1z^{-1})}=2.8\frac{1}{1+0.4z^{-1}}-1.8\frac{1}{1-0.1z^{-1}}$

$=\frac{(1-z^{-1})(1+0.1z^{-1})}{(1+0.4z^{-1})(1-0.1z^{-1})}=\frac{1-z^{-1}}{1+0.4z^{-1}}\cdot\frac{1+0.1z^{-1}}{1-0.1z^{-1}}$

II型.




X(n)
-0.4
z⁻¹
-1
+
0.1
z⁻¹
0.1
y(n).