P55. 13

(10) $w = \mathrm{Arcsin}\, i \Rightarrow i = \sin w = \frac{e^{iw} - e^{-iw}}{2i}$

$\therefore e^{iw} = -1 \pm \sqrt{2} \Rightarrow iw = \mathrm{Ln}(-1 \pm \sqrt{2})$.

$iw = \mathrm{Ln}(-1+\sqrt{2}) \Rightarrow iw = \ln(\sqrt{2}-1) + i2\pi k \Rightarrow w = -i\ln(\sqrt{2}-1) + 2\pi k$, $k \in Z$

$iw = \mathrm{Ln}(-1-\sqrt{2}) \Rightarrow w = -i\ln(1+\sqrt{2}) + 2\pi k + \pi$, $k \in Z$.

P55.15

(1) $e^{2z} = 1 - \sqrt{3}i \Rightarrow 2z = \mathrm{Ln}(1-\sqrt{3}i) = \ln 2 + i(2\pi k - \frac{\pi}{3})$

$\therefore z = \frac{1}{2}\ln 2 + i(\pi k - \frac{\pi}{6})$ $k \in Z$.

(3) $\cos z = 2 = \frac{e^{iz} + e^{-iz}}{2} \Rightarrow e^{iz} = 2 \pm \sqrt{3}$.

$\therefore iz = \mathrm{Ln}(2 \pm \sqrt{3}) = \ln(2\pm\sqrt{3}) + i2\pi k \Rightarrow z = -i\ln(2\pm\sqrt{3}) + 2\pi k$, $k \in Z$.

P56. 5.

$\nabla^2 u = 0 \Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $u \in C^2(D(u))$

$f(z) = u_x - i u_y$. $\because u_{xx} = -u_{yy}$ $\therefore \frac{\partial u_x}{\partial x} = \frac{\partial u_y}{\partial y}$ 且 $u \in C^2(D(u))$

$\frac{\partial u_x}{\partial y}$, $-\frac{\partial u_y}{\partial x}$ 显然相等. $\therefore u_x, -u_y$ 满足 (C-R) 式

$\therefore f(z)$ 调和.

P56. 7

$\because \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$, $u, v \in C^1(D)$

(1) $\overline{if(z)} = v - iu$ $\therefore \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = \frac{\partial(-u)}{\partial y}$, $\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = -\frac{\partial(-u)}{\partial x}$

$\therefore$ 解析.

(2) $\because g(z) = v + i(-u)$ 解析 $\therefore -u$ 是 $v$ 的 ---

P56. 9.

(1) $\sin\sqrt{z} = \frac{e^{i\sqrt{z}} - e^{-i\sqrt{z}}}{2i}$ , ~~$\sqrt{z}$ 有两个~~

令 $z = re^{i\theta}$ ∴ $\sqrt{z} = \sqrt{r}\,e^{\frac{i\theta}{2}+\pi k i}$ , ~~多值函数~~

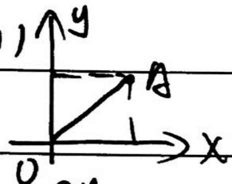
$= \pm\sqrt{r}\,e^{\frac{i\theta}{2}}$ ∴ $\sin\sqrt{z} = \pm\sin(\sqrt{r}\,e^{\frac{i\theta}{2}})$ ∴多值

(2) 同上, $\cos\sqrt{z} = \cos(\pm\sqrt{r}\,e^{\frac{i\theta}{2}}) = \cos(\sqrt{r}\,e^{\frac{i\theta}{2}})$ ∴单值

(3). $\frac{\sin\sqrt{z}}{\sqrt{z}} = \frac{\sin(\sqrt{r}\,e^{\frac{i\theta}{2}})}{\sqrt{r}\,e^{\frac{i\theta}{2}}}$ ∴单值.

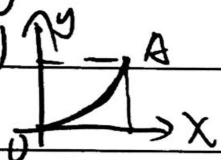
(4). $\mathrm{Ln}\sin z = \ln|\sin z| + i(z+2\pi k)$ ∴多值.

P80. 2

(1)


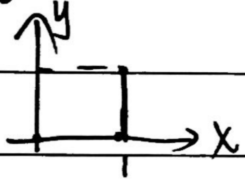
$\mathrm{Re}\,z = x$ ∴ $\int_c \mathrm{Re}\,z\,dz = \int_0^1 x\,dz$

$= \int_0^1 t(1+i)\,dt = \frac{1}{2}(1+i)$.

(2)


$z = x + ix^2$ ∴ $dz = (1+2xi)dx$

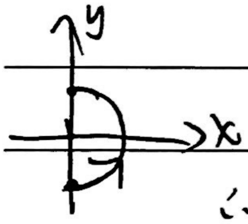
∴ 原式 $= \int_0^1 x(1+2xi)dx = \frac{1}{2} + \frac{2}{3}i$

(3)


$(0,0)\to(1,0)$. $z = x$, $\int_0^1 x\,dz = \int_0^1 x\,dx = \frac{1}{2}$

$(1,0)\to(1,1)$, $z = iy+1$, $\int_0^1 x\,dz = i$ ∴ 原式 $= \frac{1}{2} + i$

P80. (2)



$x = \cos\theta$, $y = \sin\theta$. $\theta: -\frac{\pi}{2} \to \frac{\pi}{2}$

∴ $f(z) = \cos^2\theta + i\sin^2\theta$ , $z'(\theta) = -\sin\theta + i\cos\theta$.

∴ $\int_c f(z)dz = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos^2\theta + i\sin^2\theta)(-\sin\theta + i\cos\theta)\,d\theta$

$= 2\int_0^{\frac{\pi}{2}} i\cos^3\theta - \sin^2\theta\cos\theta\,d\theta$

$= 2(\frac{2}{3}i - \frac{1}{3})$.

∴ 原式 $= \frac{2}{3}\sqrt{5} < \pi$