Unit 5 Homework

1 相关系数

t16

$$H_0: r_0 = 0; H_1: r_0 \neq 0$$

由于$z_r = \frac{1}{2}\ln\frac{1+r}{1-r} \sim N(\frac{1}{2}\ln(\frac{1+r}{1-r}), \frac{1}{n-3})$

$$\mathrm{P}(z \geq (z_r - \frac{1}{2}\ln\frac{1+r_0}{1-r_0})\sqrt{n-3}) \leq 0.025$$
$$\therefore n \geq (\frac{2z_{0.025}}{\ln(1+r) - \ln(1-r)})^2 + 3$$

In [93]:
```
import scipy.stats as stats
import numpy as np

print("z_0.025=", stats.norm.isf(0.025))
```

```
z_0.025= 1.9599639845400545
```

a. r=0.30, $n \geq 43.1$, n=44
b. r=0.25, $n \geq 61.9$, n=62
c. r=0.20, $n \geq 96.5$, n=97
当然这是与r=0进行NHST，无法说明总体的线性强度
实际上我也试过对r<=0.5进行检验，但是最后的结果确是r越大得到的n越大，这显然是不正确的，原因在于题目给出的r<0.5，会产生负号，这将改变不等号方向，导致错误的检验；而正确的那边是恒成立的，无论n的取值，这没有意义

如果说使用95%CI的下限与0比较，我认为和上面的方法结果是一致的（因为我一开始就是下限和0.5做比较），下面给出证明

$$z_L = z_r - z_{0.025} \cdot \frac{1}{\sqrt{n-3}}$$
$$\therefore r_L = \frac{e^{2z_L} - 1}{e^{2z_L} + 1} > 0$$
$$\Rightarrow z_L > 0 \Rightarrow n > (\frac{2z_{0.025}}{\ln(1+r) - \ln(1-r)})^2 + 3$$

t19

首先构造数组

In [94]:
```
rank = np.linspace(1, 11, 11)
grade = [1, 3, 1, 3, 3, 6, 9, 6, 6, 9, 11]

stats.spearmanr(rank, grade)
```

Out[94]:
```
SignificanceResult(statistic=0.907222105138509, pvalue=0.00011539867905256112)
```

样本的ρ=0.91，是非常大的，说明样本的rank和grade有很强的正相关性
而$p = 1e^{-4}$，说明总体的ρ显著不等于0，但无法用p值说明总体显著相关
还是使用95%CI来说明
$z_r = \frac{1}{2}\ln\frac{1+\rho}{1-\rho}$服从正态分布，z的CI为$z_\rho \pm z_{\alpha/2}\sqrt{\frac{1+\rho^2/2}{n-3}}$

$$\therefore \rho_L = \frac{e^{2z_L} - 1}{e^{2z_L} + 1}, \rho_U = \frac{e^{2z_U} - 1}{e^{2z_U} + 1}$$

只要$\rho_L \geq 0.5$，就能在$\alpha = 0.05$的水平上说明总体显著相关
对于本题，$\rho_L = 0.607 > 0.5$
故认为总体显著相关

2 简单线性回归模型

Julie Vu, Chapter 6/ 6.26

(a)R2 and correlation between lunch and helmet

根据相关系数不随量纲变化的性质和一阶线性回归模型的表示，有

$$r(x, y) = \pm r(\hat{y}, y) = \pm\sqrt{R^2} = -0.85$$

(b)slope and intercept

$$
\begin{gathered}
\text{sploe} \cdot = \frac{n\sum_i x_i y_i - \sum_i x_i \sum_i y_i}{n\sum_i x_i^2 - (\sum_i x_i)^2} = \frac{\text{COV(x,y)}}{\text{D}(x)} \\
\text{intercept} = \bar{y} - \text{sploe} \cdot \bar{x} \\
\because r_{x,y} = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y} = \frac{\sum_i (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_i (x_i - \overline{x})^2 \sum_i (y_i - \overline{y})^2}} \\
\therefore \text{sploe} = r \cdot \frac{SD(y)}{SD(x)} = -0.54, \text{intercept} = 55.4\%
\end{gathered}
$$

(c)what can intercept say?

结合一阶线性回归模型来看，截距是当自变量为0时的因变量值
在本题中指的是当 Rate of Receiving a Reduced-Fee Lunch=0时的Rate of Wearing a Helmet
当自变量没有做随机变量中心化时，截距往往是没有意义的，因为实际情况中自变量往往很少会等于0
当做了随机变量中心化，截距代表自变量为均值时的因变量值

(d)what can slope say?

斜率表示对于总体而言，当自变量增加时，因变量的平均增加值，即某样本自变量比另一个样本自变量大1单位，其因变量比另一个的平均大slope
在本题中指的是当不同社区之间，lunch观测值增加时，平均而言helmet减少量

(e)residual

$$
r = (y - \hat{y})^2 = (y - b_0 - b_1 x)^2 = 3.844 \times 10^{-3}
$$

残差表示预测模型与样本值的偏离程度，为了消除符号干扰所以平方
具体而言，残差越小表示模型和样本值越符合

3 多元线性回归模型

In [95]:
```
import pandas as pd
import numpy as np
import scipy.stats as stats
import seaborn as sns
import matplotlib.pyplot as plt
```

In [96]:
```
data = pd.read_csv("Datas/Student_Performance.csv")
data.head()
```

Out[96]:

	HoursStudied	PreviousScores	ExtracurricularActivities	SleepHours	SampleQuestionPapersPracticed	PerformanceIndex
0	7	99	Yes	9	1	91
1	4	82	No	4	2	65
2	8	51	Yes	7	2	45
3	5	52	Yes	5	2	36
4	7	75	No	8	5	66

In [97]:
```
data.shape
```

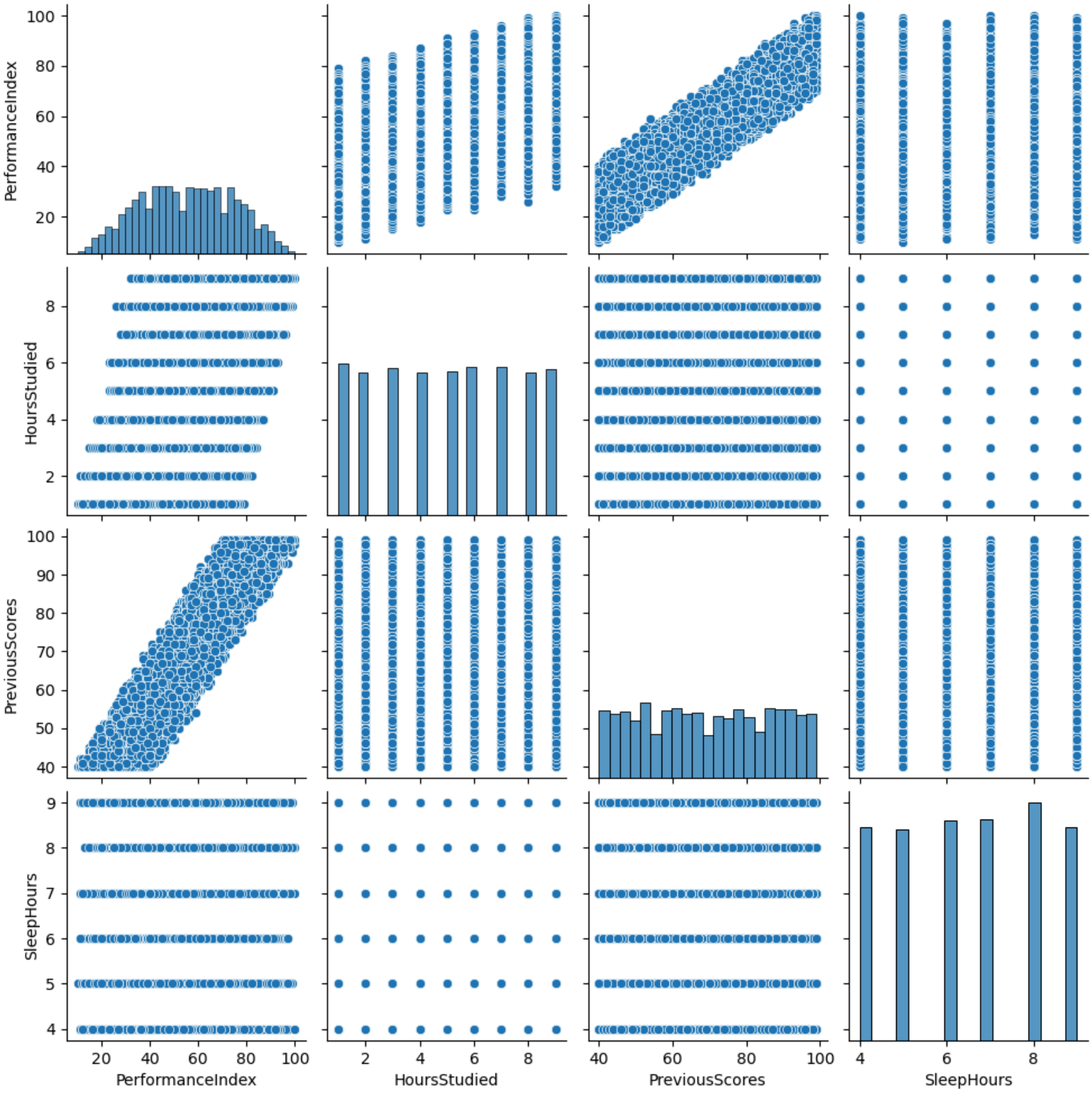
Out[97]: (10000, 6)

3.1 数据可视化

In [98]:
```
sns.pairplot(data=data, y_vars=["PerformanceIndex", "HoursStudied", "PreviousScores", "SleepHours"], x_vars=["PerformanceIndex", "HoursStu
```

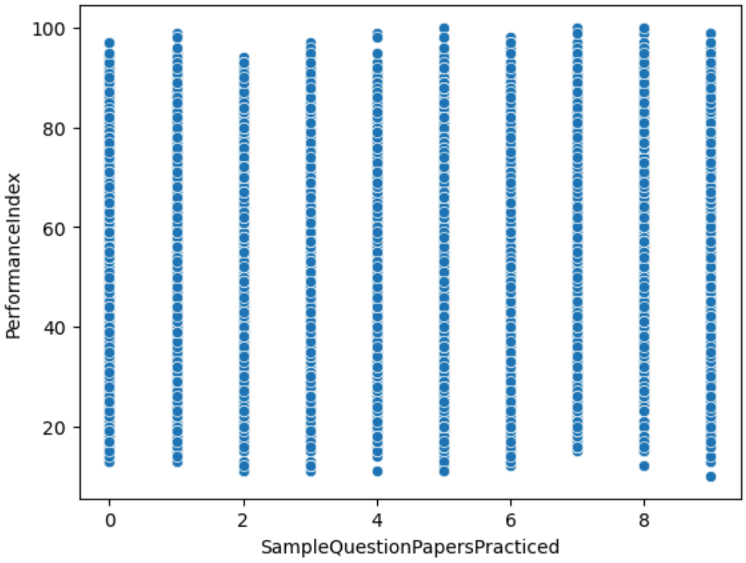
```
D:\mySoftwares\stem\Anaconda\Lib\site-packages\seaborn\axisgrid.py:118: UserWarning: The figure layout has changed to tight
  self._figure.tight_layout(*args, **kwargs)
```

Out[98]: <seaborn.axisgrid.PairGrid at 0x1b6c479f910>



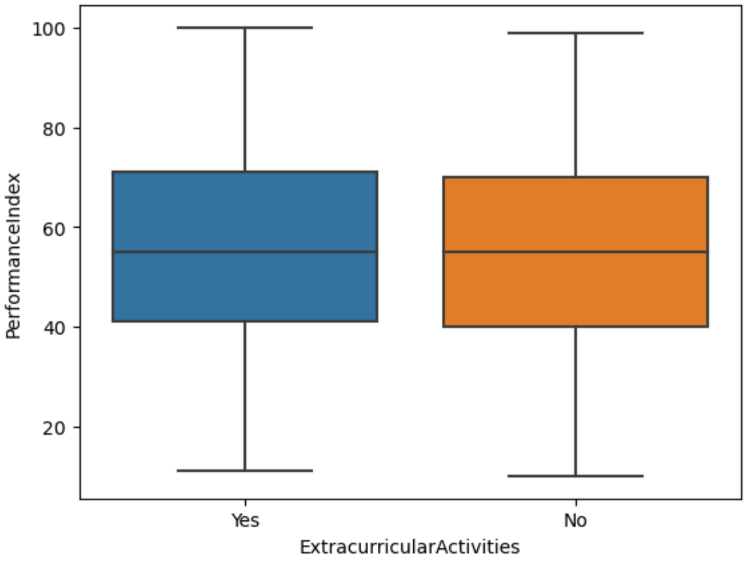
In [99]:
```
sns.scatterplot(data=data, y="PerformanceIndex", x="SampleQuestionPapersPracticed")
```

Out[99]: <Axes: xlabel='SampleQuestionPapersPracticed', ylabel='PerformanceIndex'>



In [100]:
```
sns.boxplot(data=data, y="PerformanceIndex", x="ExtracurricularActivities")
```

Out[100]: <Axes: xlabel='ExtracurricularActivities', ylabel='PerformanceIndex'>



3.2 用线性模型尽可能地预测PerformanceIndex

In [101]:
```
import statsmodels.formula.api as smf
model = smf.ols("PerformanceIndex~PreviousScores+SleepHours+HoursStudied", data=data)
results = model.fit()
results.summary()
```

Out[101]:

OLS Regression Results

Dep. Variable:	PerformanceIndex	**R-squared:**	0.988
Model:	OLS	**Adj. R-squared:**	0.988
Method:	Least Squares	**F-statistic:**	2.665e+05
Date:	Thu, 30 May 2024	**Prob (F-statistic):**	0.00
Time:	16:11:05	**Log-Likelihood:**	-21774.
No. Observations:	10000	**AIC:**	4.356e+04
Df Residuals:	9996	**BIC:**	4.359e+04
Df Model:	3		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-32.9146	0.127	-258.414	0.000	-33.164	-32.665
PreviousScores	1.0188	0.001	827.342	0.000	1.016	1.021
SleepHours	0.4776	0.013	37.928	0.000	0.453	0.502
HoursStudied	2.8572	0.008	346.406	0.000	2.841	2.873

Omnibus:	0.836	**Durbin-Watson:**	2.002
Prob(Omnibus):	0.658	**Jarque-Bera (JB):**	0.803
Skew:	-0.002	**Prob(JB):**	0.669
Kurtosis:	3.044	**Cond. No.**	431.

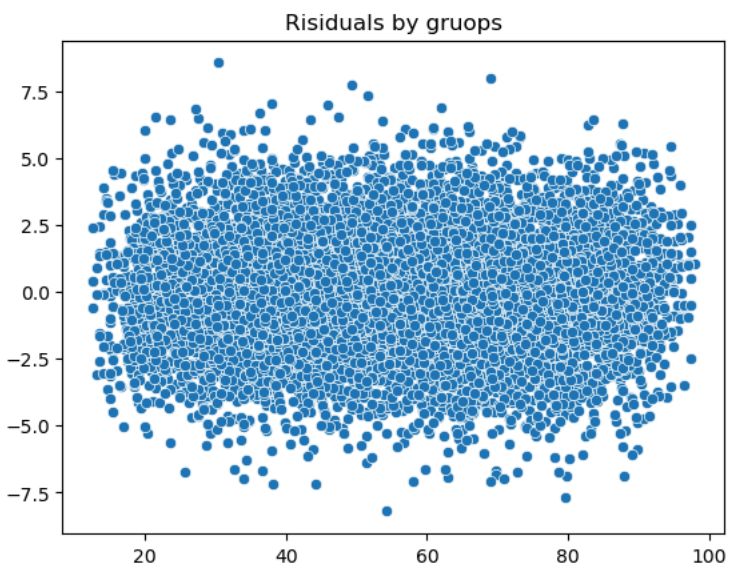
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

经过遍历的尝试，PerformanceIndex~PreviousScores+SleepHours+HoursStudied是最佳的模型

1. R^2_{adj} 和 R^2 此时最大，F-value=1.757E5,非常大，对应的p-value=.00
2. 此时的AIC、BIC和Cond.No.也最小，表明模型兼顾简洁性和精准性，对于预测和拟合（解释）都有效
3. 残差分析见下

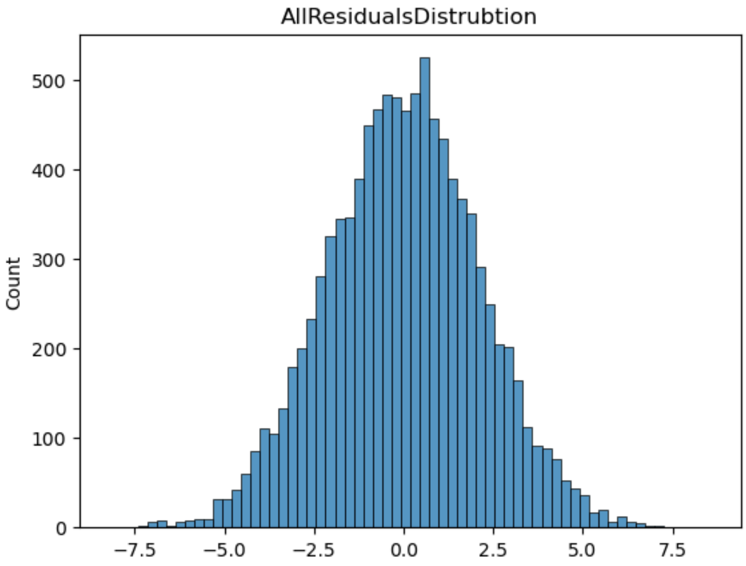
In [102]:
```
#残差可视化
residuals = results.resid
fitted_value = results.fittedvalues
sns.scatterplot(x=fitted_value, y=residuals)
plt.title("Risiduals by gruops")
```

Out[102]: Text(0.5, 1.0, 'Risiduals by gruops')



In [103]:
```
sns.histplot(x=residuals)
plt.title("AllResidualsDistrubtion")
```

Out[103]: Text(0.5, 1.0, 'AllResidualsDistrubtion')



做NHST，p很大，不能认为显著不正态，偏度≈0，峰度≈3，JB检验也说明了正态性较好
再观察两幅图
综合以上，可以认为残差的分布非常正态

观察Risiduals by gruops可以发现，不同x对应的残差分布的方差基本相同
且Durbin-Watson≈2,说明残差不存在自相关性，即方差是基本不变的
故认为残差**方差齐性好**

Cond.No.却非常大，表明了自变量之间的线性关系很强
这说明模型的**解释性并不好，但是预测性仍是足够好的**
但是在pairplot上并没有发现其他三个变量有很强的线性关系

报告如下:
PreviousScores(b=1.0,t(9998)=827.34,p=.00)、SleepHours(b=0.48,t(9998)=39.93,p=.00)、HoursStudied(b=2.9,t(9998)=346.4,p=.00)显著地预测了PerformanceIndex，纵截距(-32.92,t(9998)=-258.4,p=.00)（可以发现，当n很大时，p会非常显著，在统计上）， 这些变量也一定程度上解释了成绩(R2=0.988,F(3,9996)=2.7E05,p=.00)