

P142. 1.

X 1 2 3 4

P

$$P(X=1) = \cancel{\frac{1}{3}} \cancel{\frac{1}{4}} \left(\binom{1}{3} \times \frac{1}{4} \times \left(\frac{3}{4}\right)^2 + \binom{2}{3} \times \left(\frac{1}{4}\right)^2 \times \frac{1}{4} + \binom{3}{3} \times \left(\frac{1}{4}\right)^3 \right) = \frac{57}{64}$$

$$\text{同理 } P(X=2) = \frac{19}{64}, \quad P(X=3) = \frac{7}{64}, \quad P(X=4) = \frac{1}{64}$$

$$\therefore E(X) = \frac{25}{16}$$

P142. 3.

X 局. X = 3, 4, 5.

$$P(X=3) = 2 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(X=4) = 2 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \left(\binom{2}{3} \times \frac{1}{2} \right) = \frac{3}{8}$$

$$P(X=5) = 2 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \left(\binom{2}{4} \times \frac{1}{2} \right) = \frac{3}{8}$$

$$\therefore E(X) = \frac{33}{8}$$

P142.5.

第 X 层. 停了 1, 不停 0

X_i	0	1
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P	$(\frac{n-2}{n-1})^m$	$1 - (\frac{n-2}{n-1})^m$
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$$E(X_i) = 1 - (\frac{n-2}{n-1})^m \quad \therefore E(X) = \sum_{i=2}^n E(X_i) = (n-1) \left(1 - (\frac{n-2}{n-1})^m \right).$$

P142.6

设缴纳 X , 收益 Y . n 个人, 2个 \bar{z} , $n \geq 2$.

$$\therefore Y = n \cdot X - m \cdot z \quad \therefore E(Y) = ~~nE(X)~~ nX - mE(z).$$

$$E(z) = np = np \quad \therefore E(Y) = n(X - mp) \geq 5\% \cdot nmp.$$

$$\geq) X \geq ~~\frac{21}{20}m~~ m(p + 0.05).$$

P143.11

$$E(X) = \int_{-\infty}^{+\infty} X f(x) dx = \int_0^1 x^2 dx + \int_1^2 x(2-x) dx = 1$$

$$E(2X+1) = 2E(X) + 1 = 3$$

$$E(e^{-X}) = \int_{-\infty}^{+\infty} e^{-X} f(x) dx = \int_0^1 e^{-X} x dx + \int_1^2 e^{-X} (2-x) dx \\ = 1 + 0^2 - 2e^{-1}$$

P143.13

$$X \sim N(\mu, 1)$$

$$E(T) = \int_{-\infty}^{+\infty} T(x) \cdot f(x) dx = \int_{-\infty}^{10} -f(x) dx + \int_{10}^{12} 20 f(x) dx + \int_{12}^{+\infty} -5 f(x) dx \\ = -\Phi(10-\mu) + 20[\Phi(12-\mu) - \Phi(10-\mu)] - 5(1 - \Phi(12-\mu)) \\ = 25\Phi(12-\mu) - 21\Phi(10-\mu) - 5.$$

$$\text{令 } F(\mu) = E(T) \quad \therefore F'(\mu) = 25 \frac{1}{\sqrt{2\pi}} e^{-\frac{(12-\mu)^2}{2}} - 21 \frac{1}{\sqrt{2\pi}} e^{-\frac{(10-\mu)^2}{2}}$$

$$\therefore F'(\mu) = 0 \text{ 时 } \mu = 11 - \frac{1}{2}(\ln \frac{21}{25}) \quad \therefore \text{此时 } E(T)_{\max}.$$

$$\approx 10.91 \text{ mm.}$$