359.25

验证: $(1+2x-x^2)e^x+(x^2-3)e^x+(2-2x)e^x=0$

$y_2''=2a \quad y_2'=2ax+b$

$\therefore\ (1+2x-x^2)(2a)+(x^2-3)(2ax+b)+2(1-x)(ax^2+bx+c)=0$

$\Rightarrow\ -bx^2+2(b-a-c)x+2a-3b+2c=0$

$\therefore\ b=0,\ a=-c.\ \therefore$ 取 $a=1,\ c=-1$

$y=C_1e^x+C_2(x^2-1)$

359.~~23~~24.

(2) $\frac{e^{2x}\cos5x}{e^{2x}\sin5x}=\cot5x\neq C$ 无关

(4) $\frac{\sin2x}{\sin x\cos x}=2$ ∴相关.

359.26

(2) $y''-\cancel{2}y'\frac{2}{x}+(1+\frac{2}{x^2})y=0,\quad y_1=x\cos x$

$\therefore\ y_2=x\cos x\int\frac{1}{x^2\cos^2x}e^{-\int-\frac{2}{x}dx}dx=x\sin x$

$y=\cancel{C_1x}\ x(C_1\cos x+C_2\sin x)$

(3) $y''-\frac{2x+1}{2x-1}y'+\frac{2}{2x-1}y=0,\quad y_1=e^x$

$\therefore\ y_2=e^x\int\frac{1}{e^{2x}}e^{\int\frac{2x+1}{2x-1}dx}dx=-1-2x$

$\therefore\ y=C_1e^x+\cancel{C_2(-1-2x)}\ C_2(2x+1)$

359.27

(1) $y_1=\ln x,\quad y''-\frac{1}{x(\ln x-1)}y'+\frac{1}{x^2(\ln x-1)}y=0$

$\therefore\ y_2=\ln x\int\frac{1}{\ln^2x}e^{\int\frac{1}{x(\ln x-1)}dx}dx$

$\int\frac{1}{\ln^2x}e^{\int\frac{1}{x(\ln x-1)}dx}dx=\int\frac{\ln x-1}{\ln^2x}dx=\frac{x}{\ln x}+\int\frac{1}{\ln^2x}dx-\int\frac{1}{\ln^2x}dx$
$=\frac{x}{\ln x}$

$\therefore\ y_2=x \quad \therefore\ y=C_1x+C_2\ln x$

(2) $y_1 = e^{x}$ ∴ $y'' - \frac{x+1}{x-1}y' + \frac{2}{x-1}y = 0$

$y_2 = e^x \int \frac{1}{e^{2x}} e^{\int \frac{x+1}{x-1}dx} dx = -(x^2+1)$

$y = C_1 e^x + C_2(x^2+1)$

359. 28.

$y'' = 2C_1 + 3C_2 + 2C_2 \ln x$, $y' = 2C_1 x + 2C_2 x\ln x + C_2 x + 1$

代入 $x^2y'' - 3xy' + 4y$

得 $\cancel{2C_1x^2} + \cancel{3C_2x^2} + \cancel{2C_2x^2\ln x} - \cancel{6C_1x^2} - \cancel{6C_2x^2\ln x} - \cancel{3C_2x^2} \overset{-3x}{+ 4x} + \cancel{4C_1x^2} \cancel{+4C_2x^2\ln x}$

$= \cancel{-4C_1x^2} \cancel{4x} \cancel{+x}\ x$

∴是.

359.29

(2) $y''-2y'-2y=0$ $\lambda^2-2\lambda-2=0 \Rightarrow \lambda=1\pm\sqrt{3}$

$\therefore y=C_1e^{(1+\sqrt{3})x}+C_2e^{(1-\sqrt{3})x}$

(4) $y''+6y'+9y=0 \Rightarrow (\lambda+3)^2=0 \Rightarrow \lambda=-3$

$y=C_1e^{-3x}+C_2xe^{-3x}$

(6) $y'''-y''-y'+y=0 \Rightarrow \lambda^3-\lambda^2-\lambda+1=0 \Rightarrow (\lambda+1)(\lambda-1)^2=0$

$\therefore y=C_1e^{-x}+C_2e^{x}+C_3xe^{x}$

359.30

(2) $y''-4y'+3y=0 \Rightarrow \lambda^2-4\lambda+3=0 \Rightarrow \lambda=1,\ \lambda=3$

$\therefore y=C_1e^{x}+C_2e^{3x}$ $\begin{cases} y(1)=1 \Rightarrow \cancel{C_1+C_2=1}\ C_1e+C_2e^3=1 \\ y'(1)=2 \Rightarrow \cancel{C_1+3C_2=2}\ C_1e+3C_2e^3=2 \end{cases}$

$\therefore \cancel{y=\frac{1}{2}e^{x-1}+\frac{1}{2}e^{3x-3}}$ $y=-e^{x-1}+3e^{3x-3}$

(4) $y''+9y=0 \Rightarrow \lambda^2+9=0 \Rightarrow \lambda=\pm3i$

$\therefore y=C_1\sin3x+C_2\cos3x$ $\begin{cases} y(\frac{\pi}{3})=0 \Rightarrow -C_2=0 \\ y'(\frac{\pi}{3})=1 \Rightarrow -3C_1=1 \end{cases}$

$\therefore y=-\frac{1}{3}\sin3x$

360.31

(2) $6y''-y'-2y=xe^{x}$ $\alpha=1,\ P_n(x)=x$

$6\lambda^2-\lambda-2=0 \Rightarrow (3\lambda-2)(2\lambda+1)=0 \Rightarrow \lambda=\frac{2}{3},\ \lambda=-\frac{1}{2}$

$\therefore$ 齐次的解为 $e^{\frac{2}{3}x}, e^{-\frac{1}{2}x}$ ，$\therefore\alpha$不为解

$\therefore y=(Ax+B)e^{x}$ $y'=(Ax+A+B)e^{x},\ y''=(Ax+2A+B)e^{x}$

$\therefore$ ~~y~~ $A=\frac{1}{3},\ B=-\frac{11}{9}$ $\therefore y=(\frac{1}{3}x-\frac{11}{9})e^{x}+C_1e^{\frac{2}{3}x}+C_2e^{-\frac{1}{2}x}$

(4) $y''+y=x+\cos x$ $\lambda^2+1=0 \Rightarrow \lambda=\pm i$

$\therefore$ 齐次的解 $\sin x, \cos x$

$y''+y=x$, $\alpha=0$, $P_n(x)=x$ $\therefore k=0$

$\therefore y=Ax+B$ $\therefore y=x$

$y''+y=\cos x$ $P_n(x)=1$, $Q_n(x)=0$, $\alpha=0$, $\beta=1$ $\therefore \alpha\pm i\beta=\pm i$

$\therefore y=(A\cos x+B\sin x)x$ $\therefore k=1$

$\therefore y=\frac{1}{2}x\sin x$

$\therefore y=x+\frac{1}{2}x\sin x+C_1\sin x+C_2\cos x$

(6) $y''+2y'+2y=x+3+e^{-x}\sin x$ $\lambda^2+2\lambda+2=0 \Rightarrow \lambda=-1\pm i$

$\therefore$ 齐次的解 $e^{-x}\sin x$, $e^{-x}\cos x$

$y''+2y'+2y=x$ $\alpha=0$, $P_n(x)=x$

$\therefore y=Ax+B$ $\therefore y'=A$, $y''=0$ $\therefore 2A+2Ax+2B=x \Rightarrow \begin{cases} A=\frac{1}{2} \\ B=-\frac{1}{2} \end{cases}$

$\therefore y=\frac{1}{2}(x-1)$

$y''+2y'+2y=3$ $\alpha=0$, $P_n(x)=3$

$\therefore y=\frac{3}{2}$

$y''+2y'+2y=e^{-x}\sin x$ $\alpha=-1$, $\beta=1$ $\therefore \alpha\pm i\beta=-1\pm i$, $P_n(x)=1$, $Q_n($

$\therefore k=1$, $y=x(A\sin x+B\cos x)e^{-x}$

$\therefore y'=(A\sin x+B\cos x)e^{-x}+x(A\cos x-B\sin x)e^{-x}-x(A\sin x+B\cos x)e^{-x}$

$y''=\cdots$ $\therefore$ ~~$A=\frac{1}{4}$, $B=-\frac{1}{4}$~~ $A=0$, $B=-\frac{1}{2}$.

$\therefore y=\frac{1}{2}(x-1)+\frac{3}{2}+$ ~~$\frac{1}{4}x(\sin x-\cos x)e^{-x}$~~ ~~$\frac{1}{2}x\sin x e^{-x}$~~ $-\frac{1}{2}x\cos x e^{-x}+C_1e^{-x}\sin x+C_2e^{-x}\cos x$

360. 32

(2) $y''+y+\sin 2x=0 \Rightarrow y''+y=-\sin 2x$ $\quad \lambda^2+1=0 \Rightarrow \lambda=\pm i$

$\alpha=0,\ \beta=2$ $\quad \therefore \alpha\pm i\beta=\pm 2i$ $\quad \therefore$ 不是，$k=0$ $\quad P_n(x)=1,\ Q_n(x)=0$

$\therefore\ y=A\sin 2x+B\cos 2x$ $\quad \therefore\ y''=\cancel{A\sin}\ -4A\sin 2x-4B\cos 2x$

$\therefore\ A=\frac{1}{3},\ B=0$ $\quad \therefore\ y=\frac{1}{3}\sin 2x$

齐次的解：$y=\sin x$，$y=\cos x$

$\therefore\ y=\frac{1}{3}\sin 2x+C_1\sin x+C_2\cos x$

$\begin{cases} y(\pi)=1 \\ y'(\pi)=1 \end{cases} \Rightarrow \begin{cases} C_2=-1 \\ \frac{2}{3}-C_1=1 \end{cases}$ $\quad \therefore\ y=\frac{1}{3}\sin 2x-\frac{1}{3}\sin x-\cos x$

(4) $4y''+16y'+15y=4e^{-\frac{3}{2}x}$ $\quad 4r^2+16r+15=(2r+3)(2r+5)=0$

$\therefore\ r_1=-\frac{3}{2},\ r_2=-\frac{5}{2},$ $\quad \alpha=-\frac{3}{2}$ $\quad \therefore k=1$ $\quad P_n(x)=4$

$\therefore\ y=Axe^{-\frac{3}{2}x}$ $\quad \therefore\ y'=-\frac{3}{2}Axe^{-\frac{3}{2}x}+Ae^{-\frac{3}{2}x}$

$y''=\frac{9}{4}Axe^{-\frac{3}{2}x}-\frac{3}{2}Ae^{-\frac{3}{2}x}-\frac{3}{2}Ae^{-\frac{3}{2}x}$

$\therefore\ A=1$ $\quad \therefore\ y=xe^{-\frac{3}{2}x}+C_1e^{-\frac{3}{2}x}+C_2e^{-\frac{5}{2}x}$

$\begin{cases} y(0)=3 \\ y'(0)=-\frac{11}{2} \end{cases} \Rightarrow \begin{cases} C_1+C_2=3 \\ 3C_1+5C_2=13 \end{cases} \Rightarrow \begin{cases} C_1=1 \\ C_2=2 \end{cases}$

$\therefore\ y=xe^{-\frac{3}{2}x}+e^{-\frac{3}{2}x}+2e^{-\frac{5}{2}x}$

360.33

$a\lambda^2+b\lambda+c=0 \Rightarrow \lambda=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

① 两个~~单~~实根. $\lambda_1<0,\ \lambda_2<0 \therefore f(x)=C_1e^{\lambda_1 x}+C_2e^{\lambda_2 x},\ \lim\limits_{x\to+\infty}f(x)=0$

② 重根. $\lambda=-\frac{b}{2a}<0 \therefore f(x)=C_1e^{\lambda x}+C_2xe^{\lambda x} \therefore \lim\limits_{x\to+\infty}f(x)=0$

③ 一对复根 记 $\lambda=\alpha\pm\beta i \therefore f(x)=C_1e^{\alpha x}\sin\beta x+C_2e^{\alpha x}\cos\beta x \therefore \lim\limits_{x\to+\infty}f(x)=0$

360.35.

$y=\int_0^x t\,y(x-t)dt+\sin x \overset{x-t=u}{=} \int_x^0 (x-u)y(u)d(-u)+\sin x$

$y=x\int_0^x y(u)du-\int_0^x u\,y(u)du+\sin x$

$\therefore y'=\int_0^x y(u)du+xy-xy+\cos x=\int_0^x y(u)du+\cos x$

$\therefore y''=y-\sin x \Rightarrow y''-y=-\sin x$

$\lambda^2-1=0 \Rightarrow \lambda=\pm1 \therefore$ 齐次的解 e^x, e^{-x}

$\alpha=0,\ \beta=1 \therefore \alpha\pm i\beta=\pm i$ 不是解 $\therefore k=0 \therefore y=A\sin x+B\cos x$

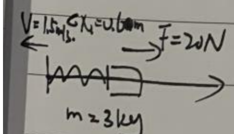
$y''=-A\sin x-B\cos x \therefore -2A\sin x-2B\cos x=-\sin x \Rightarrow A=\frac{1}{2}, B=0$

$\therefore y=\frac{1}{2}\sin x+C_1e^x+C_2e^{-x}$ 又 $\because y(0)=0,\ y'(0)=1$

$\therefore y=\frac{1}{2}\sin x+\frac{1}{4}e^x-\frac{1}{4}e^{-x}$

360.36

$X(t),\ X'(0)=-1.5\,m/s\quad X''(0)=0$



$F=mX''(t),\ F=-kX(t),\ F_0=\Delta x\cdot k \Rightarrow k=\frac{100}{3}$

$\therefore -kX(t)=mX''(t) \Rightarrow X''(t)+\frac{100}{9}X(t)=0$

~~$\lambda^2-\frac{100}{9}=0 \Rightarrow \lambda=\pm\frac{10}{3} \therefore X(t)=C_1e^{-\frac{10}{3}t}+C_2e^{\frac{10}{3}t}$~~

~~$X(0)=0,\ X'(0)=-1.5\,m/s \Rightarrow X(t)=\frac{9}{40}e^{-\frac{10}{3}t}-\frac{9}{40}e^{\frac{10}{3}t}$.~~

$\lambda^2+\frac{100}{9}=0 \Rightarrow \lambda=\pm\frac{10}{3}i \therefore X(t)=C_1\sin\frac{10}{3}t+C_2\cos\frac{10}{3}t$.

$X(0)=0,\ X'(0)=-1.5 \Rightarrow X(t)=\pm\frac{9}{20}\sin\frac{10}{3}t$.

360、39

(1) $(D(D-1)-3D+3)y=0 \Rightarrow \lambda^2-4\lambda+3=0 \Rightarrow \lambda_1=1, \lambda_2=3$

$\therefore y = C_1e^t + C_2e^{3t} = C_1X + C_2X^3$

(3) $(9D(D-1)+3D+1)y=0 \Rightarrow 9\lambda^2-6\lambda+1=0 \Rightarrow \lambda=\frac{1}{3}$

$\therefore y = C_1e^{\frac{1}{3}t} + C_2te^{\frac{1}{3}t} = C_1X^{\frac{1}{3}} + C_2X^{\frac{1}{3}}\ln|x|$

(5) $(D(D-1)+D+4)y = 2x\ln x \Rightarrow \lambda^2+4=0 \Rightarrow \lambda=\pm 2i$

$\therefore$ ~~齐次的解~~ ~~$\sin 2x, \cos 2x$~~ $\sin 2t, \cos 2t$

$2x\ln x = 2te^t \Rightarrow \alpha=1, P_n=t \quad \therefore y = (At+B)e^t$

$\therefore y''+4y = 2te^t \quad y' = (At+B+A)e^t, \; y'' = (At+2A+B)e^t$

$\therefore y = \cancel{\frac{2}{5}t+B}\;(\frac{2}{5}t-\frac{4}{25})e^t + C_1\sin 2t + C_2\cos 2t.$

$\therefore y = (\frac{2}{5}\ln|x| - \frac{4}{25})X + C_1\sin 2\ln|x| + C_2\cos 2\ln|x|$