302.27

(2) $\int \frac{dx}{4+5\cos x} \overset{\tan\frac{x}{2}=t}{=} \int \frac{2dt}{4+5\frac{1-t^2}{1+t^2}} \frac{1}{1+t^2} = 2\int \frac{dt}{4+4t^2+5-5t^2} = 2\int \frac{dt}{9-t^2}$

$= \frac{1}{3}\int (\frac{1}{3-t} + \frac{1}{3+t}) dt = \frac{1}{3} \ln \left|\frac{3+t}{3-t}\right| + C$

$= \frac{1}{3} \ln \left|\frac{3+\tan\frac{x}{2}}{3-\tan\frac{x}{2}}\right| + C$

(4) $\int \frac{dx}{a^2\sin^2x + b^2\cos^2 x} = \int \frac{d(\tan x)}{a^2\tan^2 x + b^2} \overset{\tan x = u}{=\!=} \int \frac{du}{a^2u^2+b^2} = \frac{1}{b^2}\int \frac{du}{(\frac{a}{b}u)^2+1}$

$= \frac{1}{ab}\int \frac{d(\frac{a}{b}u)}{1+(\frac{a}{b}u)^2} = \frac{1}{ab} \arctan \frac{a}{b}u = \frac{1}{ab}\arctan(\frac{a}{b}\tan x) + C$

(6) $\int \frac{\sin x \cos x}{1+\sin^4 x} dx = \cancel{\frac{1}{2}\int \frac{\sin 2x}{\quad}} \int \frac{\sin x}{1+\sin^4 x} d(\sin x)$

$\overset{u=\sin x}{=\!=} \int \frac{u}{1+u^4} du = \frac{1}{2}\int \frac{du^2}{1+u^4} = \frac{1}{2}\arctan u^2$

$= \frac{1}{2}\arctan(\sin^2 x) + C$

(8) $\int \frac{\sin^2 x}{1+\sin^2 x} dx \cancel{= \int \frac{\sin^2 x}{2\sin^2 x + \cos^2 x} dx = \int \frac{\tan^2 x}{2\tan^2 x + 1}} dx$

$= \int (1 - \frac{1}{1+\sin^2 x}) dx = x - \int \frac{1}{1+\sin^2 x} dx$

$= x - \int \frac{1}{2\sin^2 x + \cos^2 x} dx = x - \int \frac{d\tan x}{2\tan^2 x + 1} = x - \frac{\sqrt{2}}{2}\int \frac{d(\sqrt{2}\tan x)}{(\sqrt{2}\tan x)^2+1}$

$= x - \frac{\sqrt{2}}{2}\arctan(\sqrt{2}\tan x) + C$

$\cancel{(10) \int \frac{\sin^2 x}{\cos^3 x} dx \overset{u=\cos x}{=\!=} \int \frac{1-u^2}{u^3} dx} \overset{\tan\frac{x}{2}=u}{=\!=} \cancel{\int \frac{(\frac{2t}{1+t^2})^2}{(\frac{1-t^2}{1+t^2})^3} \frac{2dt}{1+t^2}}$

$\cancel{= 8\int \frac{t^2}{(1-t^2)^3} dt}$

(10) $\int \frac{\sin^2 x}{\cos^4 x} d(\sin x) \overset{u=\sin x}{=\!=} \int \frac{u^2}{(1-u^2)^2} du$

$= \int [\frac{1}{2}(\frac{1}{u-1} - \frac{1}{u+1}) + \frac{1}{4}(\frac{1}{u-1} - \frac{1}{u+1})^2] du$

$= \frac{1}{2}\int (\frac{1}{u-1} - \frac{1}{u+1}) du + \frac{1}{4}\int (\frac{1}{(u-1)^2} - \frac{1}{u-1} + \frac{1}{u+1} + \frac{1}{(u+1)^2}) du$

$= \frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| - \frac{1}{4}\ln\left|\frac{u-1}{u+1}\right| - \frac{1}{4}\frac{1}{u-1} - \frac{1}{4}\frac{1}{u+1} + C$

$= \frac{1}{4}(\ln\left|\frac{u-1}{u+1}\right| - \frac{1}{u-1} - \frac{1}{u+1}) \overset{u=\sin x}{=\!=} \frac{1}{4}(\ln\left|\frac{\sin x - 1}{\sin x + 1}\right| + \frac{2\sin x}{\cos^2 x}) + C$

302.28

(1) $\int \frac{x^{\frac{1}{2}}}{1+x^{\frac{3}{4}}} dx \xlongequal{t=x^{\frac{1}{4}}} 4\int \frac{t^3 t^2}{1+t^3} dt = 4\int \frac{t^5}{1+t^3} dt$

$= 4\int \frac{t^5+t^2-t^2}{1+t^3} dt = 4\int \left(t^2 - \frac{t^2}{1+t^3}\right) dt$

$= \frac{4}{3}t^3 - \frac{4}{3}\int \frac{dt^3}{1+t^3} = \frac{4}{3}t^3 - \frac{4}{3}\ln(1+t^3) = \frac{4}{3}x^{\frac{3}{4}} - \frac{4}{3}\ln(1+x^{\frac{3}{4}}) + C$

(3) $\int\sqrt{\frac{x+1}{x-1}} dx \xlongequal{\sqrt{\frac{x+1}{x-1}}=t} \int t\, d\left(\frac{t^2+1}{t^2-1}\right) = -4\int \frac{t^2}{(t^2-1)^2} dt$

$= \cancel{-4\int\left(\frac{1}{t-1} - \frac{1}{t+1}\right)} - 2\int\left(\frac{1}{t-1} - \frac{1}{t+1}\right)dt - \int\left(\frac{1}{(t-1)^2} + \frac{1}{(t+1)^2} - \frac{1}{t-1} + \frac{1}{t+1}\right)dt$

$= -\int\left(\frac{1}{t-1} - \frac{1}{t+1}\right)dt - \int \frac{1}{(t-1)^2} + \frac{1}{(t+1)^2} dt$

$= -\ln\left|\frac{t-1}{t+1}\right| + \frac{1}{t-1} + \frac{1}{t+1} = -\ln\left|\frac{\sqrt{\frac{x+1}{x-1}}-1}{\sqrt{\frac{x+1}{x-1}}+1}\right| + \frac{1}{\sqrt{\frac{x+1}{x-1}}-1} + \frac{1}{\sqrt{\frac{x+1}{x-1}}+1} + C$

(5) $\int \frac{x^2}{\sqrt{1+x-x^2}} dx = \int \frac{x^2-x-1+x+1}{\sqrt{1+x-x^2}} dx = -\int\sqrt{1+x-x^2} + \int \frac{x-\frac{1}{2}}{\sqrt{1+x-x^2}}dx + \frac{3}{2}\int\frac{1}{\sqrt{1+x-x^2}}dx$

$= -\sqrt{1+x-x^2} + \frac{3}{2}\arcsin\frac{2}{\sqrt{5}}\left(x-\frac{1}{2}\right) + \frac{1}{8}\arcsin\frac{2}{\sqrt{5}}\left(x-\frac{1}{2}\right) - \frac{2x-1}{4}\sqrt{1+x-x^2}$

$= -\frac{2x+3}{4}\sqrt{1+x-x^2} + \frac{7}{8}\arcsin\frac{2}{\sqrt{5}}\left(x-\frac{1}{2}\right) + C$

(7) $\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}} dx = \int \frac{(\sqrt{x+1}-\sqrt{x-1})^2}{2} = \frac{1}{2}\int(x+1+x-1-2\sqrt{x^2-1})dx$

$= \int(x-\sqrt{x^2-1})dx = \frac{1}{2}x^2 - \int\sqrt{x^2-1}dx \xlongequal{x=\frac{1}{\cos t}} \frac{1}{2}x^2 - \int \tan^2 t \sec t\, dt$

记 $I = \int\sqrt{x^2-1}\,dx \quad \therefore I = x\sqrt{x^2-1} - \int \frac{x^2}{\sqrt{x^2-1}}dx = x\sqrt{x^2-1} - I - \int\frac{1}{\sqrt{x^2-1}}dx$

$\therefore I = \frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}\ln|x+\sqrt{x^2-1}| \quad \therefore$ 原式 $= \frac{1}{2}x^2 - \frac{1}{2}x\sqrt{x^2-1} + \frac{1}{2}\ln|x+\sqrt{x^2-1}| + C$

303.24

(2) $\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}} = \arcsin x\Big|_0^{\frac{1}{2}} = \arcsin\frac{1}{2} = \frac{\pi}{6}$

(4) $\int_0^{\frac{\pi}{4}} \tan^2 x\, dx = \int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{1-\sin^2 x} dx \cancel{= \int_0^{\frac{\pi}{4}} \frac{\frac{1}{2}-\frac{1}{2}\cos 2x}{\frac{1}{2}+\frac{1}{2}\cos 2x} dx =}$

$\xlongequal{\tan x=t} \cancel{\int_0 \left(\frac{2t}{1-t^2}\right)^2 \cdot \frac{2}{1+t^2} dt} \int_0^1 \frac{t^2}{1+t^2} dt = \int_0^1 \frac{t^2+1-1}{t^2+1} dt = \int_0^1 \left(1-\frac{1}{t^2+1}\right) dt$

$= t\Big|_0^1 - \arctan t\Big|_0^1 = 1 - \frac{\pi}{4}$

(6) $\int_0^3 \sqrt{(x-2)^2}\, dx = \int_0^2 (2-x)dx + \int_2^3 (x-2)dx$

$= \left(2x - \frac{1}{2}x^2\right)\Big|_0^2 + \left(\frac{1}{2}x^2 - 2x\right)\Big|_2^3 = \frac{5}{2}$

303.31.(10) $\int_0^{\pi} x\sin^6 x\cos^4 x\, dx$, $f(x)=x$, $g(x)=\sin^6 x\cos^4 x$. $f(x)+f(\pi-x)=\pi$, $g(x)=g(\pi-x)$. 一直降幂14角.

$\therefore$ 原式 $= \frac{\pi}{2}\cdot\int_0^{\pi}\sin^6 x\cos^4 x\, dx = \pi\int_0^{\frac{\pi}{2}} \cos^6 x\sin^4 x\, dx$

$= \left(\frac{1}{2}\right)^5 \pi \int_0^{\frac{\pi}{2}} \sin^4 2x(1+\cos 2x)dx = \frac{\pi}{2^5}\int_0^{\frac{\pi}{2}}\sin^4 2x\, dx = \frac{3}{2^9}\pi$

303.29

(8) $\int_0^3 x^2[x]\,dx = \int_1^2 x^2\,dx + \int_2^3 2x^2\,dx = \frac{1}{3}x^3\Big|_1^2 + \frac{2}{3}x^3\Big|_2^3$

$= 15.$

303.30

(2) $\int_0^1 (e^x-1)^4 e^x\,dx \overset{e^x=t}{=\!=} \int_1^e (t-1)^4\,d(t-1) = \frac{1}{5}(t-1)^5\Big|_1^e = \frac{1}{5}(e-1)^5$

(4) $2\int_0^{\frac{\pi}{2}} \cos^6 x\sin x\,dx = -2\int_0^{\frac{\pi}{2}} \cos^6 x\,d\cos x \overset{\cos x=t}{=\!=} -2\int_1^0 t^6\,dt = \frac{2}{7}t^7\Big|_0^1 = \frac{2}{7}$

(6) $\int_0^1 \frac{dx}{1+e^x} = \int_0^1 \frac{de^x}{e^x(1+e^x)} \overset{e^x=t}{=\!=} \int_1^e \frac{dt}{t(t+1)} = \int_1^e (\frac{1}{t}-\frac{1}{t+1})\,dt = \ln\frac{t}{t+1}\Big|_1^e = \ln\frac{2e}{1+e}$

(8) $\int_0^1 \frac{dx}{\sqrt{1+e^{2x}}} = \frac{1}{2}\int_0^1 \frac{de^{2x}}{e^{2x}\sqrt{1+e^{2x}}} \overset{e^{2x}=t}{=\!=} \frac{1}{2}\int_1^{e^2} \frac{dt}{t\sqrt{1+t}} \overset{\sqrt{1+t}=u}{=\!=} \frac{1}{2}\int_{\sqrt{2}}^{\sqrt{1+e^2}} \frac{2u\,du}{(u^2-1)u} = \frac{1}{2}\int_1^{\sqrt{1+e^2}} \frac{1}{u-1}-\frac{1}{u+1}$

$= \frac{1}{2}\ln\left|\frac{u-1}{u+1}\right|\Big|_{\sqrt{2}}^{\sqrt{1+e^2}} = \ln\frac{\sqrt{1+e^2}-1}{e(\sqrt{2}-1)}$

303.31

(2) $\int_1^4 \frac{dx}{x(1+\sqrt{x})} \overset{\sqrt{x}=t}{=\!=} \int_1^2 \frac{2t\,dt}{t^2(1+t)} = 2\int_1^2 \frac{dt}{t(1+t)} = 2\int_1^2 (\frac{1}{t}-\frac{1}{1+t})\,dt$

$= 2\left(\ln\left|\frac{t}{1+t}\right|\right)\Big|_1^2 = \cancel{\ln\frac{8}{9}}\ 2\ln\frac{4}{3}$

(4) $\int_0^1 \frac{x^2\,dx}{\sqrt{2x-x^2}} = \int_0^1 \frac{x^2\,dx}{\sqrt{1-(x-1)^2}} \overset{t=x-1}{=\!=} \int_{-1}^0 \frac{(t+1)^2}{\sqrt{1-t^2}}\,dt$

$= \int_0^1 \frac{x^2-2x+2x-2+2}{\sqrt{2x-x^2}}\,dx = \int_0^1 \left(-\sqrt{2x-x^2} + \frac{2x-2}{\sqrt{2x-x^2}} + \frac{2}{\sqrt{2x-x^2}}\right)dx$

$= -2\sqrt{2x-x^2}\Big|_0^1 + 2\arcsin(x-1)\Big|_0^1 + \int_0^1 \sqrt{2x-x^2}\,dx \overset{x-1=\sin u}{=\!=} -\int_{-\frac{\pi}{2}}^0 \frac{1}{2}u+\frac{1}{2}\cos\sin u$

$= -2+\frac{3}{4}\pi$

(6) $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}} \overset{x=a\cos t}{=\!=} \int_{\frac{\pi}{2}}^0 \frac{a\,d\cos t}{a\cos t+a\sin t} = \int_{\frac{\pi}{2}}^0 \frac{d\cos t}{\cos t+\sin t} = \int_0^{\frac{\pi}{2}} \frac{\sin t}{\cos t+\sin t}\,dt$

$I_1 = \int \frac{\sin t}{\sin t+\cos t}\,dt,\quad I_2 = \int \frac{\cos t}{\sin t+\cos t}\,dt \Rightarrow I_1 = \frac{1}{2}(t-\ln(\cos t+\sin t))$

$\therefore$ 原式 $= \frac{\pi}{4},\quad f(x) = \frac{\sin x}{\cos x+\sin x},\ f(\frac{\pi}{2}-x) + f(x) = 1 \quad \therefore \int_0^{\frac{\pi}{2}} f(x)\,dx = \frac{\pi}{2}\times\frac{1}{2}$

$= \frac{\pi}{4}.$

(8) $\int_{-1}^1 \cos x\ln\frac{2+x}{2-x}\,dx,\quad f(x) = \cos x\ln\frac{2+x}{2-x}$ 为奇 $\therefore$ 原式 $= 0$

(9) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin^2 x}{1+e^x}\,dx \quad f(x) = \sin^2 x,\ g(x) = \frac{1}{1+e^x}$

$\therefore \cancel{f(x)+f(x-\frac{\pi}{2})f(\frac{\pi}{2}-x) = 1 \cdot g(x)+g(\frac{\pi}{2}-x) =}$

$f(x) = f(-x),\quad g(x)+g(-x) = 1 \quad \therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} f(x)\cdot g(x)\,dx = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\,dx$

$= \frac{\pi}{4}$

304. 32

(1) $\int_0^{\frac{1}{2}} \arcsin x\,dx = x\arcsin x\Big|_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}}\,dx = x\arcsin x\Big|_0^{\frac{1}{2}} + \sqrt{1-x^2}\Big|_0^{\frac{1}{2}} = \frac{\pi}{12}+\frac{\sqrt{3}}{2}-1$

(3) $\int_1^e (x\ln x)^2 dx = \frac{1}{3}x^3\ln^2 x\Big|_1^e - \frac{2}{3}\int x^2\ln x = \frac{1}{3}e^3 - \frac{2}{3}\int_1^e x^2\ln x\,dx$

$= \frac{1}{3}e^3 - \frac{2}{3}\left(\frac{1}{3}x^3\ln x\Big|_1^e - \frac{1}{3}\int_1^e x^2 dx\right) = \frac{5}{27}e^3 - \frac{2}{27}$

(5) $\int_0^1 e^{\sqrt{x}}dx \overset{u=\sqrt{x}}{=\!=} \int_0^1 e^u\,du^2 = 2\int_0^1 e^u u\,du = 2(u-1)e^u\Big|_0^1 = 2$

(7) $\int_0^{\sqrt{\ln 2}} x^3 e^{-x^2}dx \overset{x=\sqrt{u}}{=\!=} \int_0^{\ln 2} u^{\frac{3}{2}} e^{-u}\,d\sqrt{u} = \frac{1}{2}\int_0^{\ln 2} ue^{-u}du$

$= -\frac{1}{2}(u+1)e^{-u}\Big|_0^{\ln 2} = \frac{1}{4} - \frac{1}{4}\ln 2$

$\cancel{(4)\ \int_0^4 x^2\sqrt{4x-x^2}\,dx = \int_0^4 x^2\sqrt{4-(x-2)^2}\,dx \overset{x-2=t}{=\!=} \int_{-2}^{2} (t+2)^2\sqrt{4-t^2}\,dt}$

$\cancel{= \frac{1}{3}x^3\sqrt{4x-x^2}\Big|_0^4 - \frac{1}{3}\int_0^4 \frac{x^3(2-x)}{\sqrt{4x-x^2}}\,dx}$

(9) $\int_0^4 x^2\sqrt{4x-x^2}\,dx \overset{x=2+2\sin t}{=\!=} 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2+2\sin t)^2|\cos t|\,d2\sin t$

$= 4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2+2\sin t)^2\cos^2 t\,dt = 4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2+2\sin t)^2(1-\sin^2 t)\,dt$

$\cancel{\overset{\sin t=u}{=\!=} 2\int_{-1}^{1} (2+2u)^2(1-u^2)\,d}$

$= 8\times 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1+2\sin t-2\sin^3 t-\sin^4 t)\,dt = 2\times 8(t-2\cos t)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - 16\times 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 t\,dt - 8\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 t\,dt$

$= 2\times 8\pi - 0 - 2\times 16\int_0^{\frac{\pi}{2}} \sin^4 t\,dt = 2\times 8\pi - 2\times 16\,\frac{3\times 1}{4\times 2}\cdot\frac{\pi}{2} = \cancel{5\pi}\times 2\ \ 10\pi$

304. 33

(2) $f(x) = 2\ln x - x^2\int_1^e \frac{f(x)}{x}dx \Rightarrow \frac{f(x)}{x} = 2\frac{\ln x}{x} - x\int_1^e \frac{f(x)}{x}dx$

令 $\int_1^e \frac{f(x)}{x}dx = a$ ∴ $\frac{f(x)}{x} = 2\frac{\ln x}{x} - ax$ 两边积分

∴ $a = 2\int_1^e \frac{\ln x}{x}dx - a\int_1^e x\,dx$

$\int \frac{\ln x}{x}dx = \ln^2 x - \int \frac{\ln x}{x}dx \Rightarrow \int \frac{\ln x}{x}dx = \frac{\ln^2 x}{2}$

∴ $a = \frac{2}{e^2+1}$

∴ $f(x) = 2\ln x - \frac{2x^2}{e^2+1}$

33.(3)

$f(x)=x^2-x\int_0^2 f(x)dx+2\int_0^1 f(x)dx$ 令 $\int_0^2 f(x)dx=a$, $\int_0^1 f(x)dx=b$

$\therefore f(x)=x^2-ax+2b$.

$\therefore \begin{cases} \int_0^2 f(x)dx=\frac{1}{3}x^3\Big|_0^2-\frac{a}{2}x^2\Big|_0^2+2bx\Big|_0^2=a \\ \int_0^1 f(x)dx=\frac{1}{3}x^3\Big|_0^1-\frac{a}{2}x^2\Big|_0^1+2bx\Big|_0^1=b \end{cases}$

$\therefore a=\frac{4}{3},\ b=\frac{1}{3}$ $\therefore f(x)=x^2-\frac{4}{3}x+\frac{2}{3}$

304.34

(2) $\int_1^4 f(x-2)dx \xlongequal{x-2=t} \int_{-1}^{2} f(t)dt=\int_{-1}^{0}f(t)dt+\int_0^2 f(t)dt$

$=\int_{-1}^{0}\frac{1}{1+e^x}dx+\int_0^2 xe^{-x^2}dx$

$\cancel{=\int_{-1}^{0}}$ $\int\frac{1}{1+e^x}dx \xlongequal{e^x=t} \int\frac{t}{t(1+t)}d(\ln t)=\int\frac{1}{(1+t)t}dt=\int(\frac{1}{t}-\frac{1}{1+t})dt=\ln\left|\frac{t}{1+t}\right|$

$\int xe^{-x^2}dx \xlongequal{x^2=t} \int\sqrt{t}e^{-t}d\sqrt{t}=\frac{1}{2}\int e^{-t}dt=-\frac{1}{2}e^{-t}=-\frac{1}{2}e^{-x^2}$ $=\ln\left|\frac{e^x}{1+e^x}\right|$

$\therefore$ 原式 $=\ln\frac{1+e}{2}+\frac{1}{2}-\frac{1}{2}e^{-4}$

~~304.35~~

$\cancel{(1)\int_0^{\frac{\pi}{2}}\frac{f(x)}{\sqrt{x}}dx,\ f(x)=\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{x}}\frac{dt}{1+\tan t^2}}$ $\cancel{\therefore f'(x)=\frac{1}{2(1+\tan\sqrt{x})\sqrt{x}}}$

$\cancel{\int\frac{1}{1+\tan t^2}dt \xlongequal{\tan t^2=u} \int\frac{1}{(1+u^2)}d\sqrt{u}}$ $\xlongequal{t^2=u}$ $\cancel{\int\frac{1}{1+\tan u^2}d\sqrt{u}}$).

$\cancel{\int\frac{1}{1+\tan t^2}dt=\int\frac{\cos t^2}{\sin t^2+\cos t^2}}dt$

304.35.

$\int_0^{\frac{\pi}{2}}\frac{f(x)}{\sqrt{x}}dx=2(\sqrt{x}f(x)\Big|_0^{\frac{\pi}{2}}-\int\sqrt{x}f'(x)dx)$

$=\cancel{-2}\cancel{f}\int\sqrt{x}\frac{1}{1+\tan x}\frac{1}{2}\sqrt{x}^{-1}dx=\cancel{2}-\int_0^{\frac{\pi}{2}}\frac{1}{1+\tan x}dx$

$=-\int_0^{\frac{\pi}{2}}\frac{\cos x}{\sin x+\cos x}dx$ 令 $g(x)=\frac{\cos x}{\sin x+\cos x}$ $\therefore g(\frac{\pi}{2}-x)=\frac{\sin x}{\cos x+\sin x}$

$\cancel{\frac{\cos(\frac{\pi}{2}-x)}{\sin(\frac{\pi}{2}-x)+\cos(\frac{\pi}{2}-x)}=}$ $\frac{\cos x}{\sin x+\cos x}=\frac{\cos x}{\cos x+\cos(\frac{\pi}{2}-x)}$ $\therefore$ 原式 $=-\frac{\pi}{4}$.

35.(2)

$$\cancel{\int_0^{\pi} f(x)dx = xf(x)\Big|_0^{\pi} - \int_0^{\pi} xf'(x)dx}$$

$$\cancel{= \pi f(\pi) - \int_0^{\pi} x\frac{\sin x}{\pi - x}dx = \pi f(\pi) = \pi f(\pi) - \frac{\pi}{2}\int_0^{\pi}\frac{\sin x}{\pi - x}dx}$$

$$\cancel{f(\pi) = \int_0^{\pi}\frac{\sin t}{\pi - t}dt \overset{u=\frac{\pi}{2}-t}{=} \int_{\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos t}{u+\frac{\pi}{2}}}$$

$$\cancel{f(x) = \int_0^{x}\frac{\sin t}{\pi - t}dt \overset{u=\pi-t}{=} -\int_{\pi}^{\pi-x}\frac{\sin u}{u}du = \int_{\pi-x}^{\pi}\frac{\sin u}{u}du}$$

$$\int_0^{\pi} f(x)dx = (x-\pi)f(x)\Big|_0^{\pi} - \int_0^{\pi}(x-\pi)f'(x)dx = -\int_0^{\pi}(x-\pi)\frac{\sin x}{\pi - x}dx$$

$$= \int_0^{\pi}\sin x dx = 2$$

36.

(1) $\int_0^1 x^2 f''(2x)dx = \cancel{\frac{1}{3}x^3 f''(2x)\Big|_0^1 - \int_0^1 \frac{1}{3}x}$

$$\int_0^2 f(x)dx \overset{x=2t}{=} 2\int_0^1 f(2t)dt = 2tf(2t)\Big|_0^1 - 4\int_0^1 tf'(2t)dt$$

$$4 = 2 - 4\int_0^1 tf'(2t)dt = 2 - 4\left(\frac{1}{2}t^2 f'(2t)\Big|_0^1 - \int_0^1 t^2 f''(2t)dt\right)$$

$\therefore \int_0^1 x^2 f''(2x)dx = \frac{1}{2}$

(2) $\int_0^{\pi}(f(x)+f''(x))\sin x dx = -\int_0^{\pi}(f(x)+f''(x))d\cos x$

$$\cancel{\int_0^{\pi} f'(x)dx = f(\pi) - f(0) = -1}$$

$$\cancel{\int_0^{\pi} f'(x)dx = xf'(x)\Big|_0^{\pi} - \int_0^{\pi} xf''(x)dx}$$

$$\int_0^{\pi}(f'(x)\cos x - \sin x f(x))dx = f(\pi)\cos\pi - f(0)\cos 0 = -3$$

$$\int_0^{\pi} f'(x)\cos x dx - \int_0^{\pi} f(x)\sin x dx = -3.$$

$$\sin x f'(x)\Big|_0^{\pi} - \int_0^{\pi}\sin x f''(x)dx - \int_0^{\pi} f(x)\sin x dx = -3$$

$\therefore$ 原式 $= \sin x f'(x)\Big|_0^{\pi} + 3 = 3$

37.

(1) $\int_0^x t f(x-t)\,dt \xlongequal{x-t=u} -\int_x^0 (x-u)f(u)\,du = \int_0^x (x-u)f(u)\,du$

$= x\int_0^x f(u)\,du - \int_0^x u f(u)\,du = 1-\cos x$ ，两边求导.

$\therefore \int_0^x f(u)\,du + f(x)\cdot x - xf(x) = \sin x$ ，再导.

$\therefore \quad f(x) = \cos x \quad \therefore \int_0^{\frac{\pi}{2}} \cos x\,dx = \sin x\Big|_0^{\frac{\pi}{2}} = 1$

(2) $\int_0^x t f(2x-t)\,dt = \frac{\arctan x^2}{2}$

$\int_0^x t f(2x-t)\,dt \xlongequal{u=2x-t} \int_{2x}^x (2x-u)f(u)\,d(-u) = \int_x^{2x}(2x-u)f(u)\,du$

$= \int_0^{2x}(2x-u)f(u)\,du - \int_0^x (2x-u)f(u)\,du$

$= 2x\int_0^{2x} f(u)\,du - \int_0^{2x} u f(u)\,du - 2x\int_0^x f(u)\,du + \int_0^x u f(u)\,du$

$= \frac{\arctan x^2}{2}$ 两边导.

$\therefore 2\int_0^{2x} f(u)\,du + \cancel{4x f(2x)} - \cancel{4x f(2x)} - 2\int_0^x f(u)\,du - 2xf(x) + xf(x)$

$= 2\int_0^{2x} f(u)\,du - 2\int_0^x f(u)\,du - xf(x) = \frac{x}{1+x^4}$ ~~再导~~.

$\cancel{4f(2x) - 2f(x) - f(x) - xf'(x) = \frac{1-3x^4}{(1+x^4)^2}}$

$2\int_x^{2x} f(u)\,du = x\left(f(x) + \frac{1}{1+x^4}\right)$

$\therefore 2\int_1^2 f(u)\,du = 1\left(1+\frac{1}{2}\right) = \frac{3}{2} \quad \therefore \int_1^2 f(x)\,dx = \frac{3}{4}$

38.

$\lim_{x\to 0} \frac{\int_0^x t^{n-1} f(x^n-t^n)\,dt}{x^{2n}} \qquad \cancel{\int_0^x t^{n-1} f(x^n-t^n)\,dt \xlongequal{u=x^n-t^n} \int_{x^n}^0}$

$\cancel{\int_0^x t^{n-1} f(x^n-t^n)\,dt = \frac{1}{n}t^n f(x^n-t^n) + \frac{1}{n}\int_0^x t^{2n-1} f'(x^n-t^n)}$

$= \frac{1}{n}\int_0^x f(x^n-t^n)\,dt^n \xlongequal{x^n-t^n=u} \frac{1}{n}\int_{x^n}^0 f(u)\,d(-u) = \frac{1}{n}\int_0^{x^n} f(u)\,du$

$\therefore \lim_{x\to 0}\frac{\int_0^{x^n} f(u)\,du}{n x^{2n}} \xlongequal{\text{L'Hospital}} \lim_{x\to 0}\frac{n f(x^n) x^{n-1}}{2n^2 x^{2n-1}} = \lim_{x\to 0}\frac{f(x^n)}{2n x^n}$

$= \lim_{x\to 0}\frac{1}{2n}\,\frac{f(x^n)-f(0)}{x^n} = \frac{f'(0)}{2n}$