288.14

(1) $\left|\frac{\sin^n x}{n^2}\right| \le \frac{1}{n^2}$ ∴ $I = R$

(3) $|\ln x| < 1 \Rightarrow \frac{1}{e} < x < e$ ∴ $I = (\frac{1}{e}, e)$.

288.15

(1) $\frac{x^n}{n(n+1)} \le \frac{x^n}{n^2}$ ∴ $|x| \le 1 \Rightarrow I = [-1,1]$

(3) $\frac{(2x)^n}{n^2+1} \le \frac{(2x)^n}{n^2}$ ∴ $|x| \le \frac{1}{2} \Rightarrow I = [-\frac{1}{2}, \frac{1}{2}]$

(5) $\cancel{(x)^n}$ $p \le 0$时 $|x|^n < 1 \Rightarrow I = (-1,1)$ $p \in (0,1]$, $I = [-1,1)$

$\cancel{p \le 1}$ $p > 1$时, $|x|^n \le 1 \Rightarrow I = [-1,1]$

288.17

(1) $\left(\sum \frac{x^{2n+1}}{2n+1}\right)' = \sum x^{2n} = \sum (x^2)^n = \frac{(1-x^{2n})}{1-x^2}$

∴ $S(x) = \int \frac{(1-x^{2n})}{1-x^2} dx = \int \frac{1}{1-x^2} dx = \frac{1}{2} \ln \frac{1+x}{1-x}$ $x \in (-1,1)$

~~(3)~~ $\cancel{\sum_1^\infty n^2(-x)^n = \sum_1^\infty (n(n+1)-n)(-x)^n \quad \sum_1^\infty -(n(n-1)+n)(-x)^n}$

=

(2) $S''(x) = \sum_1^\infty x^{n-1} = \frac{1}{1-x}$ ∴ $S(x) = (1-x)\ln(1-x) + x + C_1 x + C_2$

$S(0) = 0 \Rightarrow C_2 = 0$. $S'(0) = 0 \Rightarrow C_1 = 0$ ∴ $S(x) = (1-x)\ln(1-x) + x$

$x = 1$时. $\sum \frac{1}{n} - \frac{1}{n+1}$ ∴ $S(x) = 1 - \frac{1}{n+1} = 1$ $x \in [-1,1)$.

(3) $\sum_1^\infty (-1)^{n-1}(n(n-1)+n)x^n = (-1)^{n-1}\left(x^2 \sum_1^\infty n(n-1)x^{n-2} + x \sum_1^\infty n x^{n-1}\right)$

$= (-1)^{n-1}\left(x^2(\sum x^n)'' + x(\sum x^n)'\right) = -x^2 \sum (-x)^{n''} - x \sum (-x)^{n'}$

$= \frac{x(1-x)}{(1+x)^3}$ $x \in (-1,1)$