P160. 2

~~解~~ $E(X)=7300$, $D(X)=700^2$.

$P(5200\le X\le 9400)=P(|X-7300|\le 2100)\ge 1-\frac{D(X)}{(2100)^2}=\cancel{0.000155873}=\cancel{0.16\%}$ ~~99.84%~~

$\ge \cancel{99.84\%}\ \frac{8}{9}$

P160. 5

即证 $\lim\limits_{n\to\infty}P\left(\left|\frac{1}{n}\sum\limits_{i=1}^{n}X_i-\frac{1}{n}\sum\limits_{i=1}^{n}E(X_i)\right|\ge\varepsilon\right)=0$, $\forall\varepsilon>0$

~~$\Sigma E(X_i)$~~ $E(X_i)=0$, $D(X_i)=E(X_i^2)=\cancel{\ln n}\cdot\ln i$

$\Sigma E(X_i)=0$, $\Sigma D(X_i)=\ln(\prod\limits_{i=1}^{n}i)<n\ln n$.

~~$\frac{1}{n}\Sigma$~~ $D(\frac{1}{n}\Sigma X_i)=\frac{1}{n^2}D(\Sigma X_i)<\frac{\ln n}{n}$

$0\le\lim\limits_{n\to\infty}P\left(\left|\frac{1}{n}\Sigma X_i-\frac{1}{n}\Sigma E(X_i)\right|\ge\varepsilon\right)\le\frac{D(\frac{1}{n}\Sigma X_i)}{\varepsilon^2}<\frac{\ln n}{n\varepsilon^2}=0$

∴ ———

P160. 6

即 $\lim\limits_{n\to\infty}P\left(\left|\frac{b-a}{n}\sum\limits_{i=1}^{n}f(X_i)-\int_a^b f(x)dx\right|>\varepsilon\right)=0$. $\forall\varepsilon>0$.

$E\left(\frac{b-a}{n}\sum\limits_{i=1}^{n}f(X_i)\right)=\frac{b-a}{n}E\left(\sum\limits_{i=1}^{n}f(X_i)\right)=\frac{b-a}{n}\Sigma E(f(X_i))$

$Ef(X_i)=\int_a^b f(x)\cdot\frac{1}{b-a}dx$ ∴ ~~原式=~~ $\int_a^b$

∴ $E\left(\frac{b-a}{n}\sum\limits_{i=1}^{n}f(X_i)\right)=\frac{b-a}{n}\cdot n\int_a^b f(x)\frac{1}{b-a}dx=\int_a^b f(x)dx$. ＜vvvvvd. 常数 为常数.

∴ $P\left(\left|\frac{b-a}{n}\Sigma f(X_i)-\int_a^b f(x)dx\right|>\varepsilon\right)\le\frac{D(\frac{b-a}{n}\Sigma f(X_i))}{\varepsilon^2}$.

$D\left(\frac{b-a}{n}\Sigma f(X_i)\right)=\frac{(b-a)^2}{n^2}\Sigma D(f(X_i))=\frac{(b-a)^2}{n}\left(\int_a^b f^2(x)\frac{1}{b-a}dx-\left(\int_a^b f(x)\frac{1}{b-a}dx\right)^2\right)$

∵ $f(x)\in C(a,b)\Rightarrow f(x)$ 在 (a,b) 上有界 ∴ $\int_a^b f(x)dx$ 有界

∴ $\lim\limits_{n\to\infty}D\left(\frac{b-a}{n}\Sigma f(X_i)\right)=\frac{(b-a)^2}{n}\cdot A=0$, $A<+\infty$

∴ 得证。

P161.7

~~X_i 服从大数定律~~

$\mu = \frac{1}{n}E(\sum X_i)$. 即证 $\lim_{n\to\infty} P\left(\left|\frac{1}{n}\sum_{i=1}^{n}X_i - \frac{1}{n}E(\sum X_i)\right| < \varepsilon\right) = 1$. $\forall \varepsilon > 0$

由 $P\left(\left|\frac{1}{n}\sum X_i - \frac{1}{n}\sum E(X_i)\right| < \varepsilon\right) > 1 - \frac{D(\frac{1}{n}\sum X_i)}{\varepsilon^2}$

$D(\frac{1}{n}\sum X_i) = \frac{1}{n^2}D(\sum X_i) = \frac{1}{n^2}\left(\sum_{i=1}^{n} D(X_i) + 2\,cov(X_i, X_j)\right)$.

$= \frac{1}{n^2}\left(\sum_{i=1}^{n} D(X_i) + 2\sum_{i=1}^{n-1} cov(X_i, X_{i+1}).\right)$

故 $\cancel{\frac{1}{n^2(n\sigma^2}}$ $0 \le \cdots \le \frac{1}{n^2}(n\sigma^2 + 2(n-1)\sigma^2) = \frac{\sigma^2}{n^2}(3n-2)$.

$\therefore P(|\cdots| < \varepsilon) > 1 - \frac{D()}{\varepsilon^2} > 1 - \frac{3n-2}{n^2}\cdot\frac{\sigma^2}{\varepsilon^2} \xrightarrow{n\to+\infty} 1$.

$\therefore \cdots$

P161.8

(1) 每户用电 X_i. $X_i \sim U(0, 20)$

$E(X_i) = 10$, $D(X_i) = \frac{100}{3}$, $E(X) = 75000$, $D(X) = 250000$.

$P(\sum X_i > 76000) = 1 - \Phi\left(\frac{76000-75000}{\sqrt{250000}}\right) = 0.0228$.

(2). $P(\sum X_i \le Y) \ge 99.9\%$

$P(\sum X_i \le Y) = \Phi\left(\frac{Y-75000}{500}\right) \ge 99.9\% \Rightarrow Y = 76540$ kw·h

P161.10.

$P\left(\left|\frac{\varepsilon_n}{n} - p\right| < \varepsilon\right)$, $p = \frac{1}{n}E(\sum \xi_i)$, $\varepsilon_n = \sum \xi_i$. $D(\xi_i) = p(1-p)$

$D(\varepsilon_n) = np(1-p)$.

$= P\left(\left|\frac{1}{n}\sum\xi_i - \frac{1}{n}E(\sum\xi_i)\right| < \varepsilon\right) = P\left(p-\varepsilon < \frac{\varepsilon_n}{n} < p+\varepsilon\right)$

记 $\frac{1}{n}\sum\xi_i = Y_n$. $\frac{1}{n}E(\sum\xi_i) = E(Y_n)$, $Y_n \sim B(n,p)$. $D(\frac{1}{n}\sum\xi_i) = \frac{p(1-p)}{n} = D(Y_n)$.

$P(|Y_n - E(Y_n)| < \varepsilon) = P\left(\frac{|Y_n - E(Y_n)|}{\sqrt{D(Y_n)}} < \frac{\sqrt{n}\cdot\varepsilon}{\sqrt{p(1-p)}}\right)$

$= 2\Phi\left(\varepsilon\sqrt{\frac{n}{p(1-p)}}\right) - 1$.

P161. 11

(1).
$$f(x)=\begin{cases} kx & 0\le x<30 \\ k(60-x) & 30\le x\le 60 \\ 0 & \text{o.w.} \end{cases}$$

$$F(+\infty)=\int_{-\infty}^{+\infty} f(x)dx=1 \Rightarrow k=\frac{1}{900}$$

(2). $E(X_i)=\int_{-\infty}^{+\infty} x f(x) dx=30$, $D(X_i)=150.$

$E(X)=6000$ $D(X)=30000.=(\sqrt{3}\times 100)^2.$

$P(|X-6000|<200)\ge 1-\frac{30000}{40000}=1-75\%.=\cancel{0.75}\ 0.25.$

(3). $P(5800<X<6200)=\Phi(\frac{200}{100\sqrt{3}})-\Phi(\frac{-200}{100\sqrt{3}})$

$=2\Phi(\frac{2}{\sqrt{3}})-1=74.98\%.=0.7498$

补 1.

(1) $\lim_{n\to\infty} P(|\frac{1}{n}\sum_{i=1}^{n} X_i-\frac{1}{n}\sum_{i=1}^{n} E(X_i)|<\varepsilon)=0$ $\forall \varepsilon>0$ 成立.

$E(X_i)=\sum_{k=1}^{+\infty} x_k P(x_k)=\sum_{k=1}^{+\infty}\frac{2^k}{k^2}\cdot\frac{1}{2^k}=\sum_{k=1}^{+\infty}\frac{1}{k^2}$ 存在.

$D(X_i)=E(X_i^2)-E(X_i)^2=\sum_{k=1}^{+\infty}\frac{2^k}{k^4}-\frac{1}{k^4}$ 不存在.

X_n i.i.d. 且 $E(X_i)$ 存在. $\therefore \{X_n\}$ 符合辛钦大数定律.

补2.

记 $X_i=\begin{cases}1 & A出现\\ 0 & 不出现\end{cases}$ ~~$P(X_i)=P_i$~~ $P(X_i=1)=P_i$, $P(X_i=0)=1-P_i$

$\therefore E(X_i)=P_i$, $D(X_i)=P_i(1-P_i)$. 由 $D(S_n)=\Sigma D(X_i)=\Sigma P_i(1-P_i)$.

$\frac{1}{n}S_n=\frac{1}{n}\sum_{i=1}^{n}X_i$, $\frac{1}{n}\sum_{i=1}^{n}P_i=\frac{1}{n}\Sigma E(X_i)=\frac{1}{n}E(S_n)=E(\frac{S_n}{n})$.

~~$\therefore$ 即证 $\lim_{n\to\infty}P(|\frac{1}{n}\sum_{i=1}^{n}X_i-\frac{1}{n}\sum_{i=1}^{n}E(X_i)|<\varepsilon)=1$.~~

$D(\frac{S_n}{n})=\frac{1}{n^2}\sum_{i=1}^{n}P_i(1-P_i)\le\frac{n}{4n^2}=\frac{1}{4n}$

$1\ge P(|\frac{1}{n}S_n-E(\frac{1}{n}S_n)|<\varepsilon)\ge 1-\frac{D(\frac{1}{n}S_n)}{\varepsilon^2}\ge 1-\frac{1}{4n\varepsilon^2}$

$\therefore \lim_{n\to\infty}P(|\frac{1}{n}S_n-E(\frac{1}{n}S_n)|<\varepsilon)=1$.

补3.

记一根 X_i $\therefore P(X_i\ge 3)=0.8$, $P(X_i<3)=0.2$

记 $Y_i=\begin{cases}1 & X_i<3\\ 0 & X_i\ge 3\end{cases}$ $\therefore E(Y_i)=0.2$, $D(Y_i)=0.16$.

$\therefore E(\Sigma Y_i)=20$, $D(\Sigma Y_i)=16$.

$P(\Sigma Y_i\ge 30)=1-P(\Sigma Y_i\le 30)=1-\Phi(\frac{30-20}{4})=0.0062$.

补4.

一间用 X_i. $\therefore$ ~~$P(X_i$~~ $X_i=\begin{cases}2 & p=0.8\\ 0 & p=0.2\end{cases}$ $\therefore E(X_i)=1.6$, $D(X_i)=0.64$.

$E(X)=800$, $D(X_i)=320$.

$P(\Sigma X_i\le Y)\ge 99\%$.

~~$P(0\le\Sigma X_i\le Y)=\Phi(\frac{Y-1.6}{0.8})-\Phi(\frac{-1.6}{0.8})\ge 0.99$~~

~~$\Rightarrow Y=$~~

$P(\Sigma X_i\le Y)\ge 99\%\Rightarrow \Phi(\frac{Y-800}{\sqrt{320}})-\cancel{\Phi(\frac{-800}{\sqrt{320}})}\ge 99\%$

$\Rightarrow Y\ge 841.68$ KW