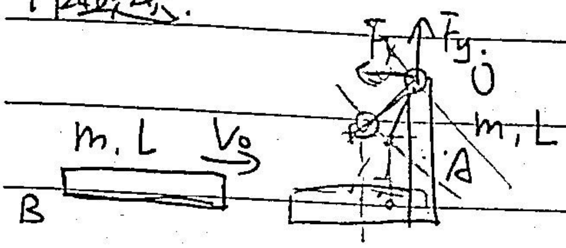
作业题.



碰撞前相对 O 的 $\vec{L}$

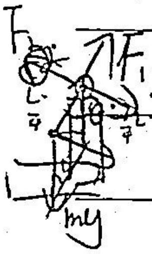
$$\vec{L} = \vec{L_A} + \vec{L_B},\quad \vec{L_A} = 0$$

$$\vec{L_B} = \vec{L_{BC}} + \vec{L'_{BC}},\quad \vec{L'_{BC}} = 0$$

$$\vec{L_{BC}} = \vec{r_{BC}} \times m\vec{V_0} = mV_0L$$

撞后 AB 整体的 $J = J_A + J_B = \frac{1}{3}mL^2 + \frac{1}{12}mL^2 + \frac{5}{4}mL^2 = \cancel{\frac{17}{12}mL^2}\ \frac{5}{3}mL^2$

$J\omega_0 = |\vec{L}| \Rightarrow \omega_0 = \cancel{\frac{12}{17}\frac{V_0}{L}}\ \frac{3}{5}\frac{V_0}{L}$ $\qquad 2mg\frac{L}{4} = J\beta \Rightarrow \beta = \frac{3}{10}\frac{g}{L}$



由质心运动定理 $\qquad \rho = \sqrt{x_c^2 + y_c^2} = \frac{\sqrt{10}}{4}L$

$\tan\theta = 3,\quad F_1 - 2mg\cos\theta = 2ma_{c法} = 2m\omega_0^2\rho$

$F_2 + 2mg\sin\theta = 2a_{c切}m = 2m\cdot\beta\cdot\rho$

$\Rightarrow F = \sqrt{F_1^2 + F_2^2} = \sqrt{\frac{81}{250}\left(\frac{mV_0^2}{L}\right)^2 + \frac{29}{8}m^2g^2 + \frac{54}{25}\frac{m^2gV_0^2}{L}}$

177. 4-13

(1) $J = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$

$\frac{1}{2}J\omega_0^2 = mgR \Rightarrow \omega_0 = \sqrt{\frac{4g}{3R}}$ $\quad \therefore V_C = \omega_0 R = \frac{2}{\sqrt{3}}\sqrt{gR}$.

(2) 同上理. $F_x = 0$. $\qquad V_A = 2V_C = \frac{4}{\sqrt{3}}\sqrt{gR}$

$F_y - mg = m\omega^2R \Rightarrow F_y = \frac{7}{3}mg$ $\quad \therefore F = F_y = \frac{7}{3}mg$ 方向 $-\frac{\vec{g}}{|\vec{g}|}$

177. 4-15.

$\frac{1}{2}mV_{块}^2 = mg\mu\cdot s \Rightarrow V_{块} = \sqrt{2g\mu s}$

$$\begin{cases} \frac{1}{2}J\omega_1^2 = \frac{1}{2}mgL \\ J\omega_2 + mV_{块}\cdot L = J\omega_1 \\ J = \frac{1}{3}mL^2 \end{cases} \Rightarrow \omega_2 = \frac{3\sqrt{\frac{1}{3}gL} - 3\sqrt{2g\mu s}}{L}$$

$\frac{1}{2}J\omega_2^2 = mg\frac{1}{2}(L - L\cos\theta)$

$h = L(1 - \frac{1}{2}\cos\theta) = 3\mu s + L - \sqrt{6\mu sL}$

177. 4-16

(1) $J_{总} = J_{环}+J_m$, $J_m = m(R\sin\varphi)^2$

∴ $J_\varphi = \frac{1}{2}mR^2 + mR^2\sin^2\varphi$

$J_{\varphi_1}\cdot\omega_0 = J_{\varphi_2}\cdot\omega_\varphi \Rightarrow \omega_\varphi = \frac{\omega_0}{1+2\sin^2\varphi}$

∴ $\omega_{\varphi_B} = \frac{1}{3}\omega_0$, $\omega_{\varphi_C} = \omega_0$

(2) $\frac{1}{2}J_{\varphi_B}\omega_{\varphi_B}^2 - \frac{1}{2}J_{\varphi_0}\omega_0^2 + \frac{1}{2}mv_B^2 = mgR$

$\Rightarrow v_B = \sqrt{2gR+\frac{1}{3}R^2\omega_0^2}$ $v_{B地} = \sqrt{v_B^2+\omega_{\varphi_B}^2R^2} = \sqrt{2gR+\frac{4}{9}\omega_0^2R^2}$

$\frac{1}{2}J_{\varphi_C}\omega_{\varphi_C}^2 - \frac{1}{2}J_0\omega_0^2 + \frac{1}{2}mv_C^2 = 2mgR$

$\Rightarrow v_C = 2\sqrt{gR}$ $v_{C地} = \sqrt{v_C^2+0} = 2\sqrt{gR}$

178. 4-17

(1) $\frac{1}{2}mv_0^2 = mgL \Rightarrow v_0 = \sqrt{2gL}$

相对O点,角动量守恒: 能量守恒:
$$\begin{cases} mv_0\cdot L = J\omega \\ \frac{1}{2}mv_0^2 = \frac{1}{2}J\omega^2 \\ J = \frac{1}{3}ML^2 \end{cases} \Rightarrow \begin{cases} M = \frac{3mv_0}{\sqrt{2gL}} = 3m. \\ \omega = \sqrt{\frac{2g}{L}} \end{cases}$$

(2) $\frac{1}{2}J\omega^2 = \frac{1}{2}Mg(1-\cos\theta)L \Rightarrow \theta = \arccos\frac{1}{3}$

178. 4-18

(1) $J = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2$ 关于O角动量守恒.

$\Rightarrow mv_0\cdot L = \frac{1}{2}mv_0L + J\omega_0 \Rightarrow \omega_0 = \frac{3}{8}\frac{mv_0}{ML}$

$\frac{1}{2}J\omega_0^2 \geq Mg2L + MgL \Rightarrow v_0 \geq \frac{4M}{m}\sqrt{2gL}$

(2)· $M = J\beta \Rightarrow M = mg\frac{L}{2} + mgL = \frac{4}{3}mL^2\beta \Rightarrow \beta = \frac{9}{8}\frac{g}{L}$

$\begin{cases} F_x = 2m a_{C切} \\ -F_y + 2mg = 2m a_{C法} \end{cases}$ $|r_C| = \frac{\frac{1}{2}mL+mL}{2m} = \frac{3}{4}L$ ∴ $\begin{cases} F_x = \frac{3}{2}M\omega_1 L \\ F_y = \cancel{\frac{59}{16}mg}\ \frac{5}{16}Mg \end{cases}$