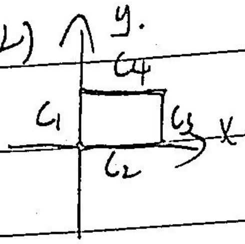
230.

(2)

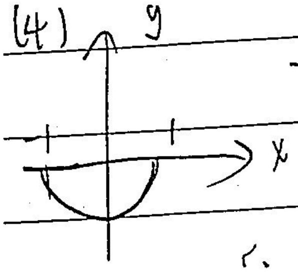


原式 $= \int_{C_1} xy\,ds + \int_{C_2} xy\,ds + \int_{C_3} xy\,ds + \int_{C_4} xy\,ds$.

C_1: $\cancel{y}\ x=0$ C_2: $y=0$, C_3: $x=4$, $0\le y\le 2$

C_4: $0\le x\le 4$, $y=2$ ∴ 原式 $=24$.

(4)

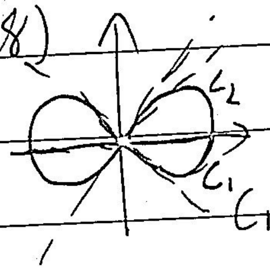


~~C: $\cos^2\theta$~~ $\begin{cases} x=\cos\theta \\ y=\sin\theta \end{cases}$ $\theta\in[-\pi,0]$.

∴ 原式 $= \int_C ds = \pi$

(5) $x^2+y^2=a^2$ ∴ 原式 $= \int_C a^{2n} ds = a^{2n}\int_C ds = 2\pi a^{2n+1}$

(8)



原式 $= 2\int_{C_2} y\,ds = 2\int_{C_1} y\,ds$

C_1: $\begin{cases} x = a\sqrt{\cos 2\theta}\cos\theta \\ y = a\sqrt{\cos 2\theta}\sin\theta \end{cases}$ $\theta\in[-\frac{\pi}{4},0]$.

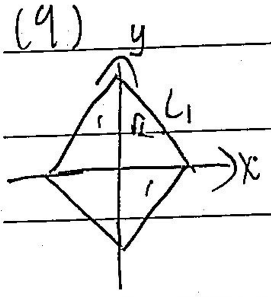
C_2: $\begin{cases} x = a\sqrt{\cos 2\theta}\cos\theta \\ y = a\sqrt{\cos 2\theta}\sin\theta \end{cases}$ $\theta\in[0,\frac{\pi}{4}]$.

∴ 原式 $= 2\int_0^{\frac{\pi}{4}} a\sqrt{\cos 2\theta}\sin\theta \; a\sqrt{\cos 2\theta}\,(1+\tan^2 2\theta)^{\frac{1}{2}}\,d\theta - 2\int_{-\frac{\pi}{4}}^{0} a\sqrt{\cos 2\theta}\sin\theta \; a\sqrt{\cos 2\theta}\,(1+\tan^2 2\theta)^{\frac{1}{2}}\,d\theta$

$\quad = 4(1-\frac{\sqrt{2}}{2})a^2$

$\parallel$ $a^2\sin\theta$.

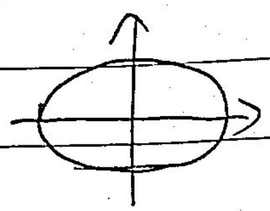
(9)



原式 $= 4\int_{C_1}(x+y)\,ds$ C_1: $\begin{cases} x=x \\ y=1-x \end{cases}$ $x\in[0,1]$.

∴ 原式 $= 4\cancel{\int_0^1 d}\ 4\int_{C_1} ds = 4\sqrt{2}$

(10)

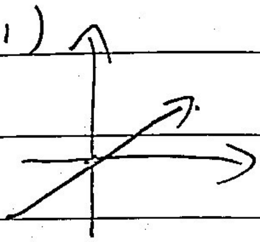


原式 $= \int_c 3x^2+4y^2\,ds = 12\oint_c ds = 12a.$

$3x^2+4y^2=12$

230.2

(1)



$l: \frac{x}{2}=\frac{y}{1}=\frac{z}{2}=t \Rightarrow \begin{cases} x=2t\\ y=t\\ z=2t\end{cases}$ $\cancel{t\in[0,1]}$

∴ 原式 $=25\int_0^1 t^2\sqrt{4+1+4}\,dt=25.$

(2).

原式 $= \int_0^2 \frac{1}{2}e^{-2t}\sqrt{3}e^{t}\,dt = \frac{\sqrt{3}}{2}(1-e^{-2})$

230.3

$L(x,y,\lambda)= x^3y+\lambda(3x+4y-12)$

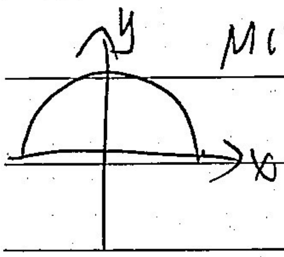
$\begin{cases}\frac{\partial L}{\partial x}=0\\ \frac{\partial L}{\partial y}=0\\ \frac{\partial L}{\partial \lambda}=0\end{cases} \Rightarrow \begin{cases}x=0\\ y=3\end{cases}$ 或 $\begin{cases}x=3\\ y=\frac{3}{4}\end{cases}$ ∴ $f(3,\frac{3}{4})=\frac{81}{4}$ 为最大值.

证明. $\sqrt{x^3y}\in[0,\frac{9}{2}]$ ∴ $e^{-\sqrt{x^3y}}\in[e^{-\frac{9}{2}},1]$.

∴ 原式 $\in[e^{-\frac{9}{2}}\int_c ds, \int_c ds]$ $\int_c ds=5.$

∴ 得证.

230.4



$\mu(x,y)=y$ $M=\int_c \mu(x,y)\,ds=\int_0^{\pi} a^2\sin t\,dt$

$=2a^2.$

$C: \begin{cases}x=a\cos t\\ y=a\sin t\end{cases}$

$t\in[0,\pi].$