93.13

(2) $\sum_{i=1}^{n}\frac{1}{\sqrt{n^2+i}} \le \frac{n}{\sqrt{n^2+1}}=\frac{1}{\sqrt{1+\frac{1}{n^2}}}$, $\sum_{i=1}^{n}\frac{1}{\sqrt{n^2+i}} \ge \frac{n}{\sqrt{n^2+n}}=\frac{1}{\sqrt{1+\frac{1}{n}}}$

$\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{1}{n^2}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{1}{n}}}=1$ ∴ $\lim_{n\to\infty}$原式$=1$

(3) 记 $x_n=\frac{(2n-1)!!}{2n!!}$, $0<x_n<1$ $x_n^2=\frac{1\times1}{2\times2}\times\frac{3\times3}{4\times4}\cdots\frac{(2n-1)(2n-1)}{2n\cdot2n}$

∴ $0<x_n^2<x_n$ 记 $y_n=\frac{2}{3}\cdot\frac{4}{5}\cdots\frac{2n-2}{2n-1}>x_n$

∴ $x_n\cdot y_n=\frac{(2n-1)!!}{2n!!}\cdot\frac{(2n-2)!!}{(2n-1)!!}=\frac{1}{2n}$

∴ $0<x_n^2<x_n\cdot y_n=\frac{1}{2n}$ $\lim_{n\to\infty}0=\lim_{n\to\infty}\frac{1}{2n}=0$

∴ $\lim_{n\to\infty}x_n^2=0$ ∴ $\lim_{n\to\infty}x_n=0$

(4) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i}{n^2+n+i}$

$\frac{i}{n^2+2n}\le\frac{i}{n^2+n+i}\le\frac{i}{n^2+n}$ ∴ $\frac{\frac{1}{2}n(n+1)}{n^2+2n}\le\cancel{\lim}\sum_{i=1}^{n}\frac{i}{n^2+n+i}\le\frac{\frac{1}{2}n(n+1)}{n^2+n}$

$\lim_{n\to\infty}$左边$=\frac{1}{2}$, $\lim_{n\to\infty}$右边$=\frac{1}{2}$ ∴ $\lim_{n\to\infty}$原式$=\frac{1}{2}$

94.16.

(1) $a_{n+1}-a_n=\frac{1}{3^{n+1}+1}>0$ ∴ $a_{n+1}>a_n$ ∴ $\{a_n\}$单增.

又∵ $0<a_n<\frac{1}{3}+\frac{1}{3^2}+\cdots+\frac{1}{3^n}<\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{1}{2}$

∴ $a_n\in(0,\frac{1}{2})$ ∴ $\lim_{n\to\infty}a_n$ 存在.

(3) $a_{n+1}-a_n=\frac{1}{n^2}>0$ ∴ $a_{n+1}>a_n$ ∴ $\{a_n\}$单增

$\frac{1}{n^2}<\frac{1}{(n+\frac{1}{2})(n-\frac{1}{2})}=\frac{1}{n-\frac{1}{2}}-\frac{1}{n+\frac{1}{2}}$ ∴ $a_n<2-\frac{1}{n+\frac{1}{2}}<2$.

∴ $0<a_n<2$ ∴ $\lim_{n\to\infty}a_n$存在

9.4.17

(3) $\cancel{\text{又}}$ $a_{n+1}^2 = 6 + a_n$ ∴ $a_{n+1}^2 - 9 = a_n - 3$ ∴ $\frac{a_{n+1}-3}{a_n-3} = \frac{1}{a_{n+1}+3} > 0$

∴ $|a_{n+1}-3| = \frac{1}{a_{n+1}+3}|a_n-3| < \frac{1}{3}|a_n-3|$

∴ $|a_{n+1}-3| < (\frac{1}{3})^n |a_1-3|$ 记 $N = [\log_{\frac{1}{3}} \frac{\varepsilon}{a_1-3}] + 1$ $\varepsilon \in (0,1)$

当 $n > N$ 时. ∴ $|a_{n+1}-3| < \varepsilon$ ∴ $\lim\limits_{n\to\infty} a_n = 3$

(4) $a_1 = 1$, $a_2 = \frac{3}{2} > 0$ ⟹ 归纳 $a_n > 0$

$a_{n+1} = \frac{1+2a_n}{1+a_n} = 2 - \frac{1}{1+a_n} < 2$ ∴ $1 < a_n < 2$

$\cancel{a_{n+1} - a_n = \frac{1+2a_n}{1+a_n} - a_n = \frac{1+a_n-a_n^2}{1+a_n}}$

$a_{n+1} - \frac{1+\sqrt{5}}{2} = \frac{1}{1+a_n} \frac{3-\sqrt{5}}{2}(a_n - \frac{\sqrt{5}+1}{2}) < \frac{3-\sqrt{5}}{2}(a_n - \frac{\sqrt{5}+1}{2})$

又 $\frac{3-\sqrt{5}}{2} < 1$ ∴ $|a_{n+1} - \frac{1+\sqrt{5}}{2}| < (\frac{3-\sqrt{5}}{2})^n (a_1 - \frac{\sqrt{5}+1}{2})$

∴ $\forall \varepsilon > 0$, $\exists N = [\log_{\frac{3-\sqrt{5}}{2}} \frac{\varepsilon}{a_1 - \frac{\sqrt{5}+1}{2}}] + 1$, 当 $n > N$ 时. $|a_{n+1} - \frac{1+\sqrt{5}}{2}| < \varepsilon$.

∴ $\lim\limits_{n\to\infty} a_{n+1} = \frac{1+\sqrt{5}}{2}$

94.18

(1)

$\left|\frac{x^2-6x+5}{x-5}-4\right| = |x-5| < \varepsilon$

∴ $\forall \varepsilon>0$, $\exists\, \delta=\varepsilon$, 当 $|x-5|<\delta$ 时, $|f(x)-4|<\varepsilon$.

∴ $\lim\limits_{x\to 5} f(x)=4$

~~(3) $|\cos x-1| \le$~~ $|x-0|<\frac{\pi}{2}$

~~∴ $\cos x > 0$ 又∵ $\cos x \le 1+|x|$~~

~~∴ cos~~

(3) $1-|x| \le \cos x \le 1+|x| \Rightarrow |\cos x-1| \le |x| < \varepsilon$.

不等易证

∴ $|x|<\varepsilon$

∴ $\forall \varepsilon>0$, $\exists\, \delta=\varepsilon$, 当 $|x|<\delta$ 时, $|\cos x-1|<\varepsilon$.

∴ $\lim\limits_{x\to 0}\cos x=1$

(5).

$\left|\sin x-\frac{\sqrt2}{2}\right| = \left|\sin x-\sin\frac{\pi}{4}\right| = \left|2\cos\frac{x+\frac{\pi}{4}}{2}\sin\frac{x-\frac{\pi}{4}}{2}\right| \le \left|x-\frac{\pi}{4}\right| < \varepsilon$.

∴ $\forall \varepsilon>0$, $\exists\, \delta=\varepsilon$, 当 $\left|x-\frac{\pi}{4}\right|<\delta$ 时, $\left|\sin x-\frac{\sqrt2}{2}\right|<\varepsilon$.

∴ $\lim\limits_{x\to \frac{\pi}{4}}\sin x = \frac{\sqrt2}{2}$

(6) $\left|\frac{1}{2^x}-1\right| = \left|\frac{1-2^x}{2^x}\right| = \frac{|2^x-1|}{2^x}$ $|x-0|<1$ 时.

$\left|\frac{1}{2^x}-1\right| < 2|x|$ (不等式易证)

∴ $2|x|<\varepsilon \Rightarrow |x|<\frac{\varepsilon}{2}$ ∴ 取 $\delta=\min\{\frac{\varepsilon}{2}, 1\}$.

$\forall \varepsilon>0$, $|x-0|<\delta$, $\left|\frac{1}{2^x}-1\right|<\delta$ ∴ $\lim\limits_{x\to 0}\left(\frac{1}{2^x}-1\right)=0$

94.20.

(1) $\lim\limits_{x\to 1}\frac{\sqrt{x-}}{x}$

(2) $\lim\limits_{x\to 2^-} x+$

(4) 若 $f(x)=$

$\lim\limits_{x\to 0^+}$

94.5.21

(1) $\left|\frac{3x^2-1}{x+3}\right.$

∴ $\forall \varepsilon>0$,

∴ $\lim\limits_{x\to\infty} f$

(3) $a^x<$

∴ $\forall \varepsilon>0$

当 $x<$

94.22

$\forall \varepsilon>0$,

要, 则 |

$|\sqrt{f(x)}-$

对 $\forall \varepsilon$

94.20.

(1) $\lim\limits_{x\to 1}\frac{\sqrt{(x-1)^2}}{x-1}=\frac{|x-1|}{x-1}$ 当 $x\to 1^+$ 时. $\lim\limits_{x\to 1^+}f(x)=1$.

右

左 $x\to 1^-$，$\lim\limits_{x\to 1^-}f(x)=-1$ ∴无

(2) $\lim\limits_{x\to 2^-}x+2=4$，$\lim\limits_{x\to 2^+}\frac{1}{x-2}=+\infty$ ∴无

(4) ~~令~~ $f(x)=\cancel{\frac{1-2^x}{2^x+1}=-1+\frac{2}{2^x+1}}$ $f(x)=\frac{2^{\frac{1}{x}}-1}{2^{\frac{1}{x}}+1}$

$\lim\limits_{x\to 0^+}f(x)=1$，$\lim\limits_{x\to 0^-}f(x)=-1$ ∴ $\cancel{\lim\limits_{x\to 0}f(x)=0}$ 无.

~~21~~.95.21

(1) $\left|\frac{3x^2-1}{x^2+3}-3\right|=\frac{10}{x^2+3}\leq\frac{10}{x^2}<\varepsilon \Rightarrow |x|>\sqrt{\frac{10}{\varepsilon}}$

∴ $\forall\varepsilon>0$，$\exists X=\sqrt{\frac{10}{\varepsilon}}$ 当 $|x|>X$ 时 $|f(x)-3|<\varepsilon$.

∴ $\lim\limits_{x\to\infty}f(x)=3$

(3) $a^x<\varepsilon \Rightarrow x<\log_a\varepsilon$ $\varepsilon\in(0,1)$

∴ $\forall\varepsilon>0$，$X_1=\log_a\varepsilon<0$ ~~当 $x<\log_a X$ 即 $|x|>\log_a\frac{1}{\varepsilon}$ 时~~

当 $x<X_1$ 时. $a^x<\varepsilon$ ∴ $\lim\limits_{x\to-\infty}a^x=0$

94.22

$\forall\varepsilon>0$，~~$f(x)-A$~~ $\exists X_1\in R^+$，当 $|x|>X_1$ 时，$|f(x)-A|<\sqrt{A}\varepsilon$.

要，则 $|\sqrt{f(x)}-\sqrt{A}|<\varepsilon$

$|\sqrt{f(x)}-\sqrt{A}|=\frac{|f(x)-A|}{\sqrt{f(x)}+\sqrt{A}}\leq\frac{|f(x)-A|}{\sqrt{A}}<\varepsilon$. ∴ 当 $|x|>X_1$ 时.

对 $\forall\varepsilon>0$，$|\sqrt{f(x)}-\sqrt{A}|<\varepsilon$ ∴ $\lim\limits_{x\to\infty}\sqrt{f(x)}=\sqrt{A}$.

95.25

(1) $$\lim_{x\to 4}\frac{x^2-6x+8}{x^2-5x+4}=\lim_{x\to 4}\frac{(x-4)(x-2)}{(x-1)(x-4)}=\lim_{x\to 4}\frac{x-2}{x-1}=\frac{2}{3}$$

(3) $$\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=\lim_{h\to 0}(3hx+3x^2+h^2)=3x^2$$

(5) $$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}\overset{u=\sqrt{1+x}}{=\!=}\lim_{u\to 1}\frac{u-1}{u^2-1}=\lim_{u\to}\frac{1}{u+1}=\frac{1}{2}$$

(7) $$\lim_{x\to 4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}=\lim_{x\to 4}\frac{2x-8}{x-4}\,\frac{\sqrt{x-2}+\sqrt{2}}{\sqrt{2x+1}+3}=2\lim_{x\to 4}\frac{\sqrt{x-2}+\sqrt{2}}{\sqrt{2x+1}+3}=\frac{2\sqrt{2}}{3}$$

(9) $$\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)=\lim_{x\to 1}\frac{x-1}{x^2-1}=\lim_{x\to 1}\frac{1}{x+1}=\frac{1}{2}$$

(11) $$\lim_{x\to 1}\frac{x-1+x^2-1+\cdots+x^n-1}{x-1}=\lim_{x\to 1}(1+x+1+x^2+x+1+\cdots+x^{n-1}+\cdots+1)$$
$$=\frac{n}{2}(n+1)$$

95.26

(2) $$\lim_{x\to 0}\frac{\arcsin x}{x}\overset{u=\sin x}{=\!=}\lim_{u\to 0}\frac{u}{\sin u}=\lim_{u\to 0}\frac{1}{\frac{\sin u}{u}}=1$$

(4) $$\lim_{x\to\pi}\frac{\sin x}{\pi-x}=\lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi-x}\overset{u=\pi-x}{=\!=}\lim_{u\to 0}\frac{\sin u}{u}=1$$

(6) $$\lim_{x\to 0}\frac{\cos x-\sqrt[3]{\cos x}}{\sin^2 x}\overset{u=\cos x}{=\!=}\lim_{u\to 1}\frac{u-\sqrt[3]{u}}{1-u^2}\overset{a=\sqrt[3]{u}}{=\!=}\lim_{a\to 1}\frac{a^3-a}{1-a^6}$$
$$=\lim_{a\to 1}-\frac{a^2+a}{a^5+a^4+a^3+a^2+a+1}=-\frac{1}{3}$$

(8) $$\lim_{x\to\frac{\pi}{6}}\frac{\sin(x-\frac{\pi}{6})}{\frac{\sqrt{3}}{2}-\cos x}=\lim_{x\to\frac{\pi}{6}}\frac{2\sin\frac{x-\frac{\pi}{6}}{2}\cos\frac{x-\frac{\pi}{6}}{2}}{-2\sin\frac{\frac{\pi}{6}+x}{2}\sin\frac{\frac{\pi}{6}-x}{2}}=\lim_{x\to\frac{\pi}{6}}\frac{\cos\frac{x-\frac{\pi}{6}}{2}}{\sin\frac{\frac{\pi}{6}+x}{2}}=2$$