115.35

(2) $\frac{dy}{dx}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}\frac{dy}{dx}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial y}\frac{dy}{dx}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial x}$

$=\frac{\partial f}{\partial x}+\cos x\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial y}\frac{dy}{dx}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial x}$

$d\varphi(x^2,e^y,z)=\varphi_1 dx^2+\varphi_2 de^y+\varphi_3 dz=0$

$\Rightarrow 2x\varphi_1 dx+e^y\varphi_2 dy+\varphi_3 dz=0$ $\therefore \frac{\partial z}{\partial y}=-\frac{\varphi_2}{\varphi_3}e^y$ $\frac{\partial z}{\partial x}=-\frac{2\varphi_1}{\varphi_3}x$

$\therefore \frac{dy}{dx}=\frac{\partial f}{\partial x}+\cos x\cdot\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}\left(-\cos x\cdot\frac{\varphi_2}{\varphi_3}e^y-\frac{2\varphi_1}{\varphi_3}x\right)$.

(3) $e^{xy}-xy=2$ 两边d

$\therefore e^{xy}(xdy+ydx)-(xdy+ydx)=0\Rightarrow \frac{dy}{dx}=-\frac{y}{x}$.

$e^x=\cancel{\int_0^x\frac{\sin t}{t}dt-\int_0^z\frac{\sin t}{t}dt}$ $\therefore \cancel{e^x=\frac{\sin x}{x}-\frac{\sin z}{z}z'}$

$=\frac{\sin(x-z)}{x-z}\left(1-\frac{dz}{dx}\right)$. $\Rightarrow \cancel{z'=\left(\frac{\sin x}{x}-e^x\right)\frac{z}{\sin z}}$

$\frac{dy}{dx}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}\frac{dy}{dx}+\frac{\partial f}{\partial z}\frac{dz}{dx}=\frac{\partial f}{\partial x}-\frac{y}{x}\frac{\partial f}{\partial y}+\cancel{\frac{z}{\sin z}\left(\frac{\sin x}{x}-e^x\right)}\frac{\partial f}{\partial z}$

$\left(1-e^x\frac{x-z}{\sin(x-z)}\right)$.

115.36

(3) $\varphi\left(\frac{x}{z},\frac{y}{z}\right)=0$ 两边d

$\therefore \varphi_1 d\frac{x}{z}+\varphi_2 d\frac{y}{z}=0\Rightarrow dz=\frac{z}{x\varphi_1+y\varphi_2}(\varphi_1 dx+\varphi_2 dy)$.

$\therefore x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\frac{xz\varphi_1}{x\varphi_1+y\varphi_2}+\frac{zy\varphi_2}{x\varphi_1+y\varphi_2}=z$.

(4) 两边d

$adx+bdy+cdz=\varphi'\cdot(2xdx+2ydy+2zdz)$.

$\Rightarrow dz=\frac{1}{c-2\varphi' z}\left[(2x\varphi'-a)dx+(2y\varphi'-b)dy\right]$

$\therefore (cy-bz)\frac{\partial z}{\partial x}=(cy-bz)\frac{2x\varphi'-a}{c-2\varphi' z}$

$(az-cx)\frac{\partial z}{\partial y}=(az-cx)\frac{2y\varphi'-b}{c-2\varphi' z}$

$\therefore$ 原式 $=bx-ay$

115.37

(3). 两边d. ∴ $2xdx+2ydy+2zdz=4dz$

$\Rightarrow \quad dz=\frac{1}{2-z}(xdx+ydy)$

$\therefore \frac{\partial z}{\partial x}=\frac{x}{2-z} \quad \frac{\partial z}{\partial y}=\frac{y}{2-z} \quad \therefore \frac{\partial^2 z}{\partial y^2}=\frac{(2-z)+\frac{\partial z}{\partial y}y}{(2-z)^2}=\frac{(2-z)+\frac{y^2}{2-z}}{(2-z)^2}$

$=\frac{(2-z)^2+y^2}{(2-z)^3}.$

(4) $5z^4dz-z^4dx-4z^3dz\cdot x+z^3dy+3z^2ydz=0$

$\Rightarrow \quad dz=\cancel{\frac{1}{5z^4-4xz^3}}\frac{1}{5z^4-4xz^3+3z^2y}(z^4dx+z^3dy).$

$z(0,0)=1$

$\frac{\partial z}{\partial x}=\frac{z^4}{5z^4-4xz^3+3z^2y} \quad \frac{\partial^2 z}{\partial x^2}=\frac{\frac{\partial z^4}{\partial x}(\cdots)-\frac{\partial(\cdots)}{\partial x}\cdot z^4}{(\cdots)^2}$

$\frac{\partial z}{\partial x}=\frac{1}{5} \quad dz^4=4z^3dz \quad \therefore \frac{\partial z^4}{\partial x}=4z^3\frac{\partial z}{\partial x}=4\frac{\partial z}{\partial x}=\frac{4}{5}.$

$\frac{\partial z}{\partial y}=-\frac{1}{5}. \quad d(\cdots)=20z^3dz-4z^3dx-12z^2xdz+6zydz+3z^2dy$

$=(20z^3-12z^2x+6zy)dz-4z^3dx+3z^2dy$

$\therefore \frac{\partial(\cdots)}{\partial x}=\frac{20\partial z}{\partial x}-4=0$

$\therefore \frac{\partial^2 z}{\partial x^2}=\frac{4}{5} \quad \frac{\partial^2 z}{\partial x\partial y}=-\frac{3}{25}$

116.39

(2) $\begin{cases} \cancel{xdu+udx-ydv-vdy=0} \\ \cancel{udy+ydu+xdv+vdx=0} \end{cases}$

$\Rightarrow \begin{cases} \cancel{du=} \\ \cancel{dv=} \end{cases} \quad \begin{cases} xu-yv=0 \\ yu+xv=1 \end{cases} \Rightarrow \begin{cases} \frac{y^2}{x}v+xv=1 \\ yu+\frac{x^2}{y}u=1 \end{cases}$

$\therefore \frac{\partial u}{\partial x}=-\frac{y}{y^2+x^2}\frac{2ux}{y}=-\frac{2ux}{y^2+x^2}.$

$\frac{\partial u}{\partial y}=\frac{u(x^2-y^2)}{y(y^2+x^2)}.$

$\frac{\partial v}{\partial x}=v\frac{y^2-x^2}{x(y^2+x^2)}$

$\frac{\partial v}{\partial y}=-v\frac{2y}{y^2+x^2}.$

116.39

(4)

$\begin{cases} x = e^u \cos v \\ y = e^u \sin v \end{cases} \Rightarrow \ln(x^2+y^2) = u,\quad v = \arctan\frac{y}{x}$

$\therefore\ z = \ln(x^2+y^2)\arctan\frac{y}{x}\quad \therefore \frac{\partial z}{\partial x} = \left(\frac{2x}{x^2+y^2}\arctan\frac{y}{x} - \frac{y}{x^2+y^2}\ln(x^2+y^2)\right)\times\frac{1}{2}$

$\frac{\partial z}{\partial y} = \left(\frac{2y}{x^2+y^2}\arctan\frac{y}{x} + \ln(x^2+y^2)\frac{x}{x^2+y^2}\right)\times\frac{1}{2}$?

?

116.40

(2) $u^2 = x^2+y^2+z^2$ $\therefore$ $udu = xdx+ydy+zdz$

$\Rightarrow du = \frac{x}{u}dx+\frac{y}{u}dy+\frac{z}{u}dz$ 取 $dx=\frac{1}{3}t$ $dy=\frac{2}{3}t$, $dz=\frac{2}{3}t$.

$\therefore \frac{\partial u}{\partial l}\Big|_{(1,0,1)} = \frac{\sqrt{2}}{2}$.

(4). $\vec{l} = \overrightarrow{M_0M} = (3,4,12)$

$du = xdy+ydx+ydz+zdy+xdz+zdx = (y+z)dx + (x+z)dy + (x+y)dz$.

$\therefore \frac{\partial u}{\partial l}\Big|_{M_0} = \frac{1}{13}(4\times3+5\times4+12\times3) = \frac{68}{13}$

116.41

(3) $u^2 = z^4(x^2+2y^2)$ $\therefore$ $2udu = 4z^3(x^2+2y^2)dz + z^4(2xdx+4ydy)$

$\Rightarrow du = \frac{xz^4}{u}dx + \frac{2yz^4}{u}dy + \frac{2z^3(x^2+2y^2)}{u}dz$.

$\therefore \nabla u = (\frac{\sqrt{2}}{2}, 1, 2\sqrt{2})$