

P158. 28.

~~X/2~~ $X \% 2 = 0, X \% 3 = 0, X \% 5 = 0, 2, 3, 5$ 互质

$\therefore X \% 30 = 0 \Rightarrow |A| = 8.$

P188. 4

$M(R)$ 不同 $|R|$ 不同, $|M(R)| = |A|^2, m_{ij} = 1/0 \therefore 2^{|A|^2} = 2^{n^2} = 2^9$

P189. 8

$R = \{ \langle 0, 1 \rangle, \langle 0, 2 \rangle, \langle 0, 3 \rangle, \langle 1, 2 \rangle, \langle 1, 3 \rangle, \langle 2, 3 \rangle \}.$

$R \circ R = \{ \langle 0, 2 \rangle, \langle 0, 3 \rangle, \langle 1, 3 \rangle \}.$

$R \upharpoonright \{1\} = \{ \langle 1, 2 \rangle, \langle 1, 3 \rangle \}, R^{-1} \upharpoonright \{1\} = \{ \langle 1, 0 \rangle \}.$

$R \upharpoonright \{1\} = \{ 2, 3 \}, R^{-1} \upharpoonright \{1\} = \{ 0 \}$

P189. 10

$M(R \circ (S \cup T)) = M(R) \cdot M(S \cup T) = M(R) \cdot (M(S) + M(T))$
 $M[(R \circ S) \cup (R \circ T)] = M(R \circ S) + M(R \circ T) = M(R) (M(S) + M(T))$

P190. 18

(1) ~~$M(R_1)$~~ $m_{ii}(R_1) = m_{ii}(R_2) = 1 \quad \forall i \therefore m_{ii}(R_1 \circ R_2) = 1 \quad \therefore \checkmark$

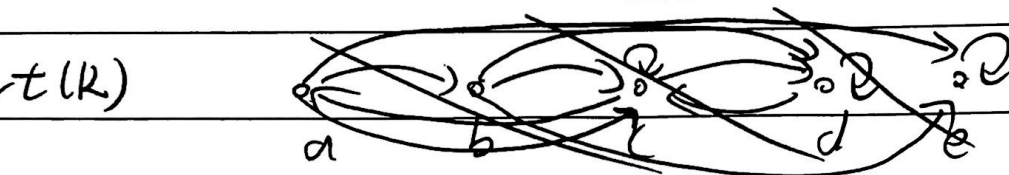
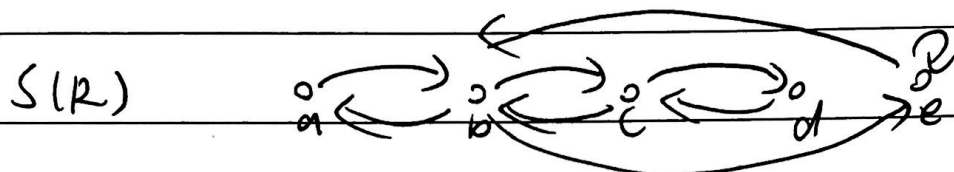
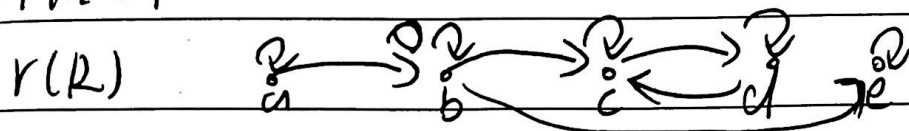
(2) $\times, M(R_1) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, M(R_2) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(3) ~~$M(R_1) = M^T(R_1)$~~ \times ~~$M(R_1) = (x R_1 y, y R_1 z \Rightarrow x R_1 \circ R_1 z, z \in \text{dom}(R_1)$~~

(4) $\times, R_1 \neq \langle, R_2 \neq \rangle, 1 \langle 3 \rangle 2, 2 \langle 4 \rangle 1, 1 \langle ? \rangle 1 \rightarrow$

(4) \times ~~\times~~ $\{3\} \{3\}.$

Prq. 24

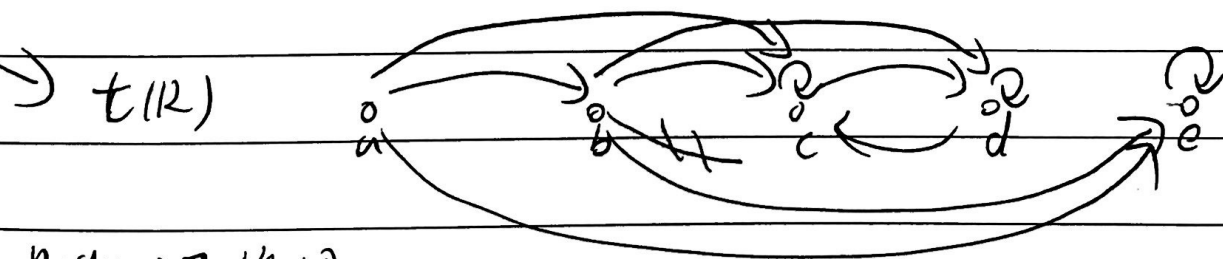


Prq. 27

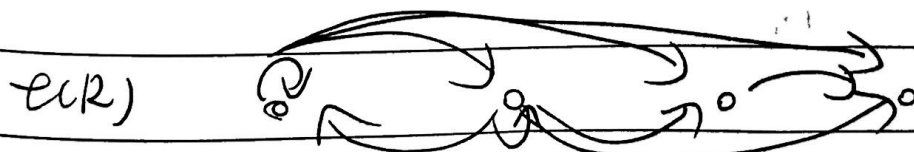
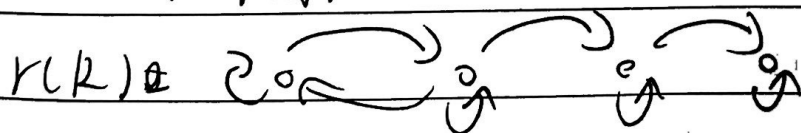
(1) $M(R) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\therefore r(R) = M(R) + I_{4 \times 4} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$s(R) = M(R) + M^T(R) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ ~~$M(R)^2$~~

$t(R) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$



Prq. 27 作图.



(2).

$$M(R) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow T(R) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

P190. 28.

最多: $I, X=X$

最少: 空集 $R = \{ \langle x, y \rangle \mid x \in A \wedge y \notin A \}$.

P190. 29

~~$aRb \wedge bRa \Rightarrow aRc$~~ $aTa \Leftrightarrow aRa \wedge aRa$ True.

$aTb \Leftrightarrow aRb \wedge bRa, bTa \Leftrightarrow bRa \Leftrightarrow aRb$. Δ 有交换律.

$\therefore aTb \Leftrightarrow bTa$.

$aTb \Leftrightarrow aRb \wedge bRa, bTc \Leftrightarrow bRc \wedge cRb$.

$\therefore aTb \wedge bTc \Leftrightarrow aRb \wedge bRa \wedge bRc \wedge cRb \Leftrightarrow aRc \wedge cRa$

$\therefore T$ 是等价关系. $\Leftrightarrow aTc$.

P191. 33.

划分的个数.

① 1个划分 1个

② 2个划分: $(1+3) \vee (2+2), C_4^3 + C_4^2 = 4+6=10$ 个

③ 3个划分: $1+1+2$ ~~$C_4^1 + C_3^1$~~ $C_4^2 = 6$ 个

④ 4个划分: 1个

\therefore 共 $1+10+6+1=18$ 个

P191.34

证明: $aSb \Leftrightarrow aRc \wedge cRb \quad (\exists c)$.

R 等价 $\therefore aRa, aRb \Rightarrow bRa, aRb \wedge bRc \rightarrow aRc$

$aSa \Leftrightarrow aRc \wedge cRa \Leftrightarrow aRa \therefore aSa \rightarrow aRa \text{ True.}$

$aSb \Leftrightarrow aRc \wedge cRb, bSa \Leftrightarrow bRc \wedge cRa \quad \wedge$

$aSb \wedge bSc \Leftrightarrow aRd \wedge dRb \wedge bRe \wedge eRc \therefore aSb \Leftrightarrow bSa.$

$\Rightarrow aRe \wedge eRc \Leftrightarrow aSc \therefore S$ 等价.

P191.38

$R = \{ \langle x_1, x_2 \rangle, \langle x_2, x_1 \rangle, \langle x_2, x_3 \rangle, \langle x_3, x_2 \rangle \}$

$B_1 = \{x_1, x_2, x_3\}, B_2 = \{x_1, x_3, x_6\}, B_3 = \{x_3, x_6, x_5\}$

$B_4 = \{x_3, x_5, x_4\}$. B_1, B_2, B_3, B_4 是一个完全覆盖.

P192.44

$\langle a_1, b_1 \rangle R \langle a_2, b_2 \rangle \Leftrightarrow a_1 R a_2 \wedge b_1 R b_2 \text{ True.}$

$\langle a_1, b_1 \rangle R \langle a_2, b_2 \rangle \wedge \langle a_2, b_2 \rangle R \langle a_3, b_3 \rangle$

$\Leftrightarrow a_1 R a_2 \wedge b_1 R b_2 \wedge a_2 R a_3 \wedge b_2 R b_3 \Rightarrow a_1 R a_3 \wedge b_1 R b_3 \Rightarrow \langle a_1, b_1 \rangle R \langle a_3, b_3 \rangle$

$\langle a_1, b_1 \rangle R \langle a_2, b_2 \rangle \wedge \langle a_2, b_2 \rangle R \langle a_1, b_1 \rangle$

$\Leftrightarrow a_1 R a_2 \wedge a_2 R a_1 \dots \text{False.}$

$\therefore R$ 是偏序关系.