286.7

(1) $\int_1^{\infty}\frac{\ln x}{x}dx=\lim_{b\to\infty}\frac{1}{2}\ln^2x\Big|_1^b=\infty$ ∴发散.

(3) $\int_1^{\infty}\frac{\arctan n}{n^2+1}dn=\lim_{b\to\infty}\frac{1}{2}(\arctan n)^2\Big|_1^{\infty}=\frac{\pi^2}{8}-\frac{\pi^2}{32}=\frac{3}{32}\pi^2$ ∴收敛.

287.9

(1) $\frac{1+2^n}{1+3^n}<\frac{1+2^n}{3^n}<2(\frac{2}{3})^n$ ∴收敛.

(3) $\frac{[(n+1)!]^2}{(n+2)(n+1)^2}\cdot\frac{(n+1)n^2}{(n!)^2}=\frac{(n+1)n^2}{n+2}=\infty$ ∴发散.

(5) $\lim_{n\to\infty}\sqrt[n]{a_n}=\frac{n^{1+\frac{1}{n^2}}}{n+\frac{1}{n}}=\frac{n}{n+\frac{1}{n}}=1$

$\lim_{n\to\infty}a_n=(\frac{n}{n+\frac{1}{n}})^n=(\frac{n^2}{n^2+1})^n=(1-\frac{1}{n^2+1})^{-(n^2+1)\cdot\frac{n}{-n^2-1}}=e^{-\frac{n}{n^2+1}}=1$

∴ 为发散.

(7) $\int_1^{\infty}2^{-\sqrt{x}}dx\overset{\sqrt{x}=t}{=}2\int_1^{\infty}t2^{-t}dt=2(-\frac{t}{\ln2}2^{-t}+\int\frac{2^{-t}}{\ln2})$.

$=2(-\frac{t}{\ln2}2^{-t}-\frac{1}{\ln^22}2^{-t})\Big|_1^{\infty}=\frac{1}{\ln2}+\frac{1}{\ln^22}$

∴收敛.

287.10

(1) $\sum a_n$收 ∴$\lim_{n\to\infty}a_n=0$ ∴$\exists N\in\mathbb{Z}^+$, 当$n>N$时. $a_n^2\leq a_n$.

∴ $\sum a_n^2=\sum_1^N a_n^2+\sum_N^{\infty}a_n^2\leq\sum_1^N a_n^2+\sum_1^{\infty}a_n$ ∴收敛.

(3) $\lim_{n\to\infty}a_n=0$, $\lim_{n\to\infty}b_n=0$ ∴ $\exists N_1$, $n>N_1$时. $a_nb_n\leq a_n$

同上理, $\sum a_nb_n$收敛.

$(a_n+b_n)^2=a_n^2+2a_nb_n+b_n^2$, 由上, $\sum a_nb_n$, $\sum a_n^2$, $\sum b_n^2$收敛.

∴$\sum(a_n+b_n)^2$收敛.

(5) 设 $\lim_{n\to\infty}na_n=A$ ($A\in\mathbb{R}$).

$\sum n(a_n-a_{n-1})=-\sum a_{n-1}+na_n=A-S_{n-1}=T_n$.

∴ $A=T_n+S_{n-1}$ ($\lim n\to\infty$), 若 $\lim_{n\to\infty}S_{n-1}=\infty$ 则 $A=\infty$, 矛盾 ∴S_{n-1}收

∴ $\sum a_n=S$