P144.17

$X \sim U(0,1) \Rightarrow f_X(x) = \begin{cases} 1 & 0<x<1 \\ 0 & \text{o.w.} \end{cases}$, $\cancel{f(x,y) = \begin{cases} e^{-(y-5)} & 0<x<1 \end{cases}}$

$\therefore E(XY) = \cancel{\iint xy\, f(x,y)\, d\sigma =}$

$= E(X)\cdot E(Y) = \left(\int_0^1 x\,dx\right)\cdot\left(\int_5^{+\infty} y\, e^{-(y-5)}\,dy\right)$

$= 3.$

$D(XY) = E(X^2Y^2) - E^2(XY) = E(X^2)E(Y^2) - E^2(XY) = \frac{10}{3}$

$D(2X-Y) = 4D(X) - D(Y) = \frac{4}{3}$

P144.18

显然 X, Y 独立. $f_X(x) = \begin{cases} 2x & 0<x<1 \\ 0 & \text{o.w.} \end{cases}$ $f_Y(y) = \begin{cases} 3y^2 & 0<y<1 \\ 0 & \text{o.w.} \end{cases}$

$E(X) = \frac{2}{3}$, $E(X^2) = \frac{1}{2}$ $E(Y) = \frac{3}{4}$ $E(Y^2) = \frac{3}{5}$

$\therefore E(XY) = E(X)\cdot E(Y) = \frac{1}{2}$

$E(2X^2+3Y) = 2E(X^2) + 3E(Y) = \frac{13}{4}$

$D(X+Y) = D(X) + D(Y) = E(X^2) - E^2(X) + E(Y^2) - E^2(Y) = \cancel{0.0468}$ 0.09305 $= \frac{67}{720}.$

P144.19.

$X \sim N(1,2)$, $Y \sim N(2,1)$

$\therefore E(Z) = 2E(X) - E(Y) + 8 = 2\times1 + 2 + 8 = 12.$

$D(Z) = 4D(X) + D(Y) = 8 + 1 = 9$

P144.20

记 X 为距一端位置. $X \in [0,l]$, $X \sim U(0,l) \Rightarrow f(x) = \begin{cases} \frac{1}{l} & 0<x<l \\ 0 & \text{o.w.} \end{cases}$

Y 为另一点 $\therefore f(x,y) = \begin{cases} \frac{1}{l^2} & 0<x<l, 0<y<l \\ 0 & \text{o.w.} \end{cases}$ $Z = |X-Y|.$

$\therefore E(Z) = \iint |x-y| f(x,y)\, d\sigma = \int_0^l dx\left(\int_0^x (x-y)\frac{1}{l^2}dy + \int_x^l (y-x)\frac{1}{l^2}dy\right)$

$= \frac{1}{3}l.$

$E(Z^2) = \frac{1}{6}l^2$ $\therefore D(Z) = \frac{1}{18}l^2.$

P144. 22

(1) $E(X) = \int_a^b x f(x) dx \geq a \int_a^b f(x) dx = a$

$\leq b \int_a^b f(x) dx = b$ $\therefore$ $a \leq E(X) \leq b$

(2) $D(X) = E([X - E(X)]^2)$

$= \int_a^b (x - E(x))^2 f(x) dx = E(X^2) - E^2(X)$

$E(X^2) \in [a^2, b^2]$, , $D(X) \leq E[(X-C)^2] = E(X^2) - 2CE(X) + C^2$.

取 $C = \frac{a+b}{2}$ $\therefore$ $D(X) \leq \int_a^b (\frac{b-a}{2})^2 f(x) dx \leq \frac{(b-a)^2}{4}$.

∴得证

P144. 24

$D(XY) = E(X^2Y^2) - E^2(XY)$

$E(X^2Y^2) = E(X^2)E(Y^2)$, $E(XY) = E(X)E(Y)$.

$E(X) = 2$, $E(Y) = 0$, $E(X^2) = \frac{13}{3}$, $E(Y^2) = D(Y) + E(Y) = 1 + 0 = 1$.

$\therefore$ $D(XY) = \frac{13}{3}$

P144. 26

易知 X, Y 独立. ~~$f_X(x) = \{ e$~~ $f_X(x) = \begin{cases} e^{-x} & x>0 \\ 0 & x\leq 0 \end{cases}$, $f_Y(y) = \begin{cases} e^{-y} & y>0 \\ 0 & y\leq 0 \end{cases}$.

$X \sim E(1)$, $Y \sim E(1)$

$\therefore$ $E(X) = E(Y) = 1$, $D(X) = D(Y) = 1$

$cov(X, Y) = E(XY) - E(X)E(Y) = 0$. $\rho_{XY} = 0$

$C = \begin{pmatrix} D(X) & 0 \\ 0 & D(Y) \end{pmatrix} = \Lambda(1, 1)$

P145.27

$\rho_{XY}=\frac{Cov(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}}$, $Cov(X,Y)=E(XY)-E(X)E(Y)$.

$E(X)=\sum x_i p_i=0$, $E(Y)=0$, $E(XY)=\sum\sum x_i y_j p_{ij}=0$

$D(X)=E(X^2)-E(X)^2=\frac{3}{4}=D(Y)$.

$\therefore Cov(X,Y)=0$, $\rho_{XY}=0 \Rightarrow$ X, Y 不相关.

$P(X=0,Y=0)=0$. $P(X=0)=\frac{2}{8}$, $P(Y=0)=\frac{2}{8}$

$\therefore P(X=0,Y=0)\neq P(X=0)P(Y=0)$ $\therefore$ 不独立.

P145.28

$E(X)=P(A)$, $E(Y)=P(B)$

X, Y 不相关 $\Rightarrow Cov(X,Y)=E(XY)-E(X)E(Y)=0$

$\therefore E(XY)=E(X)E(Y)$

XY	0	1
P	$1-P(A)P(B)$	$P(A)P(B)$

$P(X=1,Y=1)=P(A)\cdot P(B)=P(X=1)P(Y=1)$.

$X\backslash Y$	1	0	
1	$P(A)P(B)$	$P(A)-P(A)P(B)$	$P(A)$
0	$P(B)-P(A)P(B)$	$P(\bar{B})P(\bar{A})$	$P(\bar{A})$
	$P(B)$	$P(\bar{B})$.	

$\therefore$ 如右所示. X, Y 独立

$\forall\ P(X=i,Y=j)=P(X=i)P(Y=j)$.

P145.29

$E(Z)=\frac{1}{3}E(X)+\frac{1}{2}E(Y)$, $E(X)=1$, $E(Y)=0$. $\Rightarrow E(Z)=\frac{1}{3}$

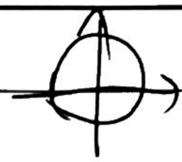
$D(Z)=\frac{1}{9}D(X)+\frac{1}{4}D(Y)+\frac{1}{6}\times 2Cov(X,Y)$. $D(X)=9$, $D(Y)=16$.

$Cov(X,Y)=\rho_{XY}\cdot\sqrt{D(X)}\sqrt{D(Y)}=-\frac{1}{2}\times 3\times 4=-6$. $\therefore D(Z)=3$

$\rho_{XZ}=\frac{Cov(X,Z)}{\sqrt{D(X)}\sqrt{D(Z)}}$, $Cov(X,Z)=\frac{1}{2}Cov(X,Y)+Cov(X,X)\cdot\frac{1}{3}=\frac{1}{3}D(X)-3=0$.

$\therefore \rho_{XZ}=0$

P145. 30.



$f(x,y)=\begin{cases}\frac{1}{\pi} & x^2+y^2\le 1\\ 0 & \text{o.w.}\end{cases}$ 易知不独立.

$P(X\le 0, Y\le \frac{1}{2}) = 0 \ne P(X\le 0)\cdot P(Y\ge \frac{1}{2})$.

下证不相关. $\rho_{XY}=\frac{cov(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}}$, $cov(X,Y)=E(XY)-E(X)E(Y)$

$E(XY)=\iint xy\frac{1}{\pi}d\sigma=0$, $\underset{=}{\overset{0}{E(X)}}\underset{=}{\overset{0}{E(Y)}}=0$

∴ $\rho_{XY}=0$ 不相关

P145. 31.

$D(X)=1$, $D(Y)=5$ $cov(X,Y)=2$

$\rho_{UV}=\frac{cov(U,V)}{\sqrt{D(U)}\sqrt{D(V)}}$, $cov(U,V)=cov(X-2Y, 2X-Y)$

$=cov(X,2X-Y)-2cov(Y,2X-Y)$

$=2cov(X,X)-cov(X,Y)-4cov(Y,X)+2cov(Y,Y)=2$.

$D(U)=D(X-2Y)=D(X)+D(2Y)-2cov(X,2Y)=13$.

$D(V)=D(2X-Y)=4D(X)+D(Y)-2cov(2X,Y)=1$.

∴ $\rho_{UV}=\frac{2}{13}\sqrt{13}$

补 1.

X_i 为一个 X_i 独立. $E(X_i)=\frac{1+\cdots+6}{6}=3.5$. $D(X_i)=E(X_i^2)-E(X_i)^2=\frac{35}{12}$.

$E(X)=\sum E(X_i)=3.5n$. $D(X)=\sum D(X_i)=\frac{35}{12}n$.

补2.

记第一个X，第二个Y. ~~$f(x,y)=\{$~~ X, Y独立.

$Z=|X-Y|$.

$$E(Z)=\sum\sum|X-Y|\cdot \overset{p_{ij}}{\cancel{f(x,y)}}=\sum_{x=0}^{n}\left(\sum_{y=0}^{x}(x-y)\frac{1}{(n+1)^2}+\sum_{j=x+1}^{n}(y-x)\frac{1}{(n+1)^2}\right).$$

$$=\sum_{x=0}^{n}\frac{1}{2(n+1)^2}\left(x^2+x+x^2-x-2nx+n^2+n\right)$$

$$=\frac{n^2+2n}{3(n+1)}$$

补3.

$U\sim U(-2,2)$. $f(u)=\begin{cases}\frac{1}{4} & -2<u<2\\ 0 & \text{o.w.}\end{cases}$

$X=Y$. $\therefore D(X+Y)=4D(X)$

$$E(X)=\int_{-\infty}^{+\infty}g(u)f(u)\,du=\int_{-2}^{-1}-\frac{1}{4}\,du+\int_{-1}^{2}\frac{1}{4}\,du=\frac{1}{2}$$

$$E(X^2)=\int_{-2}^{2}\frac{1}{4}\,du=1$$

$\therefore D(X)=1-\frac{1}{4}=\frac{3}{4}$ $\therefore D(X+Y)=3$

补4.

$X+Y=n$. $\therefore \rho_{XY}=-1$

$D(X)=D(n-Y)=D(Y)$ $\quad Cov(X,Y)=\rho_{XY}\cdot D(X)=-D(X)$.

记每次X_i

X_i	0	1
p	$\frac{1}{2}$	$\frac{1}{2}$

$\therefore X=\sum X_i$ $\quad E(X_i)=\frac{1}{2}$

$E(X)=\frac{n}{2}$

$E(X_i^2)=\frac{1}{2}$, $D(X_i)=\frac{1}{4}$ $\quad D(X)=\frac{n}{4}$

$\therefore Cov(X,Y)=-\frac{n}{4}$

补.5.

$X \sim P(\lambda)$, $Y \sim P(\lambda)$, X, Y 独立.

$\therefore E(X)=\lambda$, $D(X)=\lambda$, $D(Y)=\lambda$, ~~D~~ $E(Y)=\lambda$.

$\rho_{UV} = \dfrac{cov(U,V)}{\sqrt{D(U)}\sqrt{D(V)}}$

$cov(U,V) = cov(2X+Y, 2X-Y) = cov(2X, 2X-Y) + cov(Y, 2X-Y)$

$= 4cov(X,X) - \cancel{2cov(X,Y)} + \cancel{2cov(X,Y)} - cov(Y,Y)$.

$= 4D(X) - D(Y) = 3\lambda$.

$D(U) = D(2X+Y) = 4D(X) + D(Y) = 5\lambda$.

$D(V) = D(2X-Y) = D(U) = 5\lambda$.

$\therefore \rho_{U,V} = \frac{3}{5}$