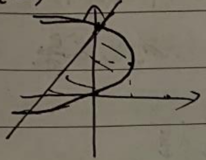
305.40

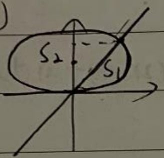
(2)



~~$S=\int$~~ $\begin{cases} x=2y-y^2 \\ x=y-2 \end{cases} \Rightarrow$ 交点 $(0,2)\ (-3,-1)$

$\therefore S=\int_{-1}^{2}(2y-y^2-y+2)dy=(\frac{1}{2}y^2-\frac{1}{3}y^3+2y)\Big|_{-1}^{2}=\frac{9}{2}$

(4)



$\begin{cases} \frac{x^2}{3}+(y-1)^2=1 \\ y=x \end{cases} \Rightarrow$ 交点 $(0,0),(\frac{3}{2},\frac{3}{2})$

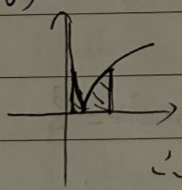
$S=\pi\cdot\sqrt{3}\cdot 1=\sqrt{3}\pi$, ~~$S_1=\int$~~

~~$\begin{cases} x=\sqrt{3}\cos\theta \\ y=\sin\theta+1 \end{cases}$ $\theta\in(\frac{\pi}{6},\frac{\pi}{2})$ $S_1=\left|\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(\sin\theta+1)(\sqrt{3}\sin\theta)d\theta\right|=\frac{9}{4}$~~

由 $\begin{cases} x=\sqrt{3(1-(y-1)^2)} \\ x=y. \end{cases}$ $\therefore S_1=\int_0^{\frac{3}{2}}y-\sqrt{3(1-(y-1)^2)}\,dy=\frac{\sqrt{3}}{3}\pi-\frac{3}{4}$

$S_2=\sqrt{3}\pi-S_1=\frac{2}{3}\sqrt{3}\pi+\frac{3}{4}$

(6)



$\begin{cases} y=-\ln x \\ x=\frac{1}{e} \end{cases} \Rightarrow y=1$. ~~$S_1=\int_0^1 e^{-y}dy=-e^{-y}\Big|_0^1=1-e^{-1}$~~

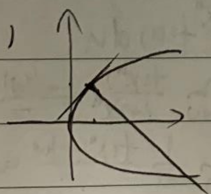
$\begin{cases} y=\ln x \\ x=e \end{cases} \Rightarrow y=1$ ~~$S_2=\int_0^1 e^{y}dy=e-1$~~

$\therefore$ ~~$S=S_1+S_2=e-e^{-1}$~~ $S_1=\left|\int_{\frac{1}{e}}^{1}\ln x\,dx\right|=\left|x(\ln x-1)\Big|_{\frac{1}{e}}^{1}\right|$

$S_2=\left|x(\ln x-1)\Big|_1^e\right|$ $\therefore S=2-\frac{2}{e}$

305.41

(1)

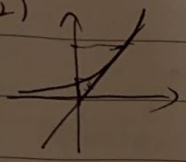


$\begin{cases} y=-x+\frac{3}{2}p \\ y^2=2px \end{cases} \Rightarrow$ 交点 $(\frac{p}{2},p)\ (\frac{9}{2}p,-3p)$.

$\therefore S=\int_{-3p}^{p}(\frac{3}{2}p-y-\frac{y^2}{2p})dy=\frac{16}{3}p^2$

305.41

(2)

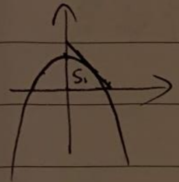


$$\begin{cases} y=ex \\ y=e^x \\ x=0 \end{cases}$$ 交点 $(0,1),(1,e),(0,0)$.

$\therefore$ $S_1=\int_1^e \ln y\, dy = (\ln y-1)y\Big|_1^e = 1$

$S_2=\int_0^e \frac{y}{e}\, dy = \frac{y^2}{2e}\Big|_0^e = \frac{e}{2}$ $\therefore S=\frac{e}{2}-1$.

305.42



$S_1=\int_0^1(-x^2+1)dx = (-\frac{1}{3}x^3+x)\Big|_0^1=\frac{2}{3}$

设 l: $y=-2x_0x+x_0^2+1$ $\therefore$ $(0, x_0^2+1)$, $(\frac{x_0^2+1}{2x_0}, 0)$

$\therefore S_2=\frac{1}{2}(x_0^2+1)(\frac{x_0^2+1}{2x_0})$ ~~≥ 1 $\therefore S\geq\frac{1}{3}$ $S_{min}=\frac{1}{3}$~~

305.43 $\therefore$ $x=\frac{\sqrt{3}}{3}$ 时 $\angle$ l: $y=-\frac{2\sqrt{3}}{3}x+\frac{4}{3}$

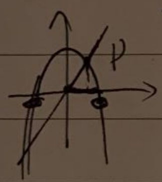
(2) $S=\int_0^{2\pi} a(2\sin t-\sin 2t)\cdot a(2\sin 2t-2\sin t)\,dt$

$=4a^2\left(\int_0^{2\pi}2\cos^4x\,dx-3\int_0^{2\pi}\cos^3x\,dx-\int_0^{2\pi}\cos^2x\,dx+3\int_0^{2\pi}\cos x\,dx-\int_0^{2\pi}1\,dx\right)$

$=4a^2\left(\int_0^{2\pi}(\sqrt{2}\,\frac{1+\cos 2x}{2})^2dx-\int_0^{2\pi}\frac{1+\cos 2x}{2}dx\right)-4a^2x\Big|_0^{2\pi}=6\pi a^2$.

305.44

$2x^2+y=2$.



(1) $y=2-2x^2$, $P(x_0, 2-2x_0^2)$

$\therefore$ l: $y=(\frac{2}{x_0}-2x_0)x$

$S=\frac{1}{2}x_0y_0+\int_{x_0}^1(2-2x^2)dx = \frac{4}{3}-x_0-\frac{1}{3}x_0^3$

(2) $x=\cos t$, $\therefore S=\frac{4}{3}-\cos t-\frac{1}{3}\cos t^3$, $S'=\sin t+\cos^2t\sin t$

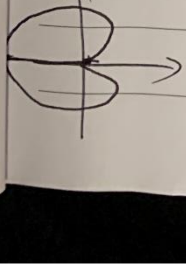
$S''=\cos t-2\cos t\sin^2t+\cos^3t=0 \Rightarrow \cos t=\sqrt{\frac{7}{12}}$

$\therefore$ ~~$P(\sqrt{\frac{7}{12}}, \frac{5}{6})$~~ $P(\frac{1}{\sqrt{3}}, \frac{4}{3})$

305.45.(1)

$S=2\int_0^{\pi}\frac{1}{2}r^2d\theta = \int_0^{\pi}4a^2(1-\cos\theta)^2d\theta = 4a^2\int_0^{\pi}(1-\cos\theta)^2d\theta$

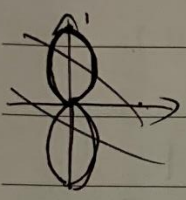
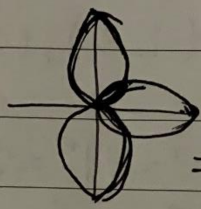
$=4a^2\int_0^{\pi}(1-2\cos\theta+\cos^2\theta)d\theta = 4a^2(\theta-2\sin\theta)\Big|_0^{\pi}+4a^2\int_0^{\pi}\cos^2\theta\, d\theta$



$=4a^2\pi+2a^2\pi=6a^2\pi$.

$8a^2\int_0^{\frac{\pi}{2}}\cos^2\theta\, d\theta$

306.46.(2)

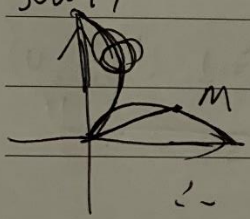
$\cancel{S = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{2} r_1^2(\theta) + \frac{1}{2} r_2^2(\theta)\, d\theta}$

$\cancel{= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (2\sin^2\theta + \cos 2\theta)\, d\theta = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (1 - \cos\theta + \cos\theta)\, d\theta}$

$\cancel{= \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}}$ $S_1 = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{2} \cos 2\theta\, d\theta.$

$S_1 = 2\int_0^{\frac{\pi}{6}} \sin^2\theta\, d\theta$ $\therefore S = \frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{2}$

306.47



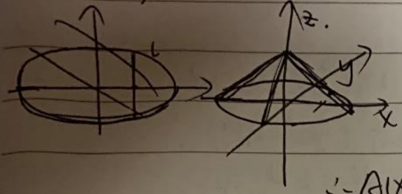
记 $M(r, \theta)$

$\therefore \int_0^{\theta_0} \frac{1}{2} r^2\, d\theta = \int_{\theta_0}^{\frac{\pi}{4}} \frac{1}{2} r^2\, d\theta$

$\therefore \sin 2\theta \Big|_0^{\theta_0} = \sin 2\theta \Big|_{\theta_0}^{\frac{\pi}{4}}$. $\therefore \theta_0 = \frac{\pi}{12}$.

$\therefore M(\sqrt{2\sqrt{3}}, \frac{\pi}{12})$

306.48.(1)



$A(x)=\frac{\sqrt{3}}{4}L^2$

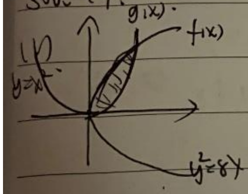
~~又~~ $\frac{L}{2}=b\sqrt{1-\frac{x^2}{a^2}}$

$\therefore A(x)=\frac{\sqrt{3}}{4}4b^2(1-\frac{x^2}{a^2})=\sqrt{3}b^2(1-\frac{x^2}{a^2})$

$\therefore V=\int_{-a}^{a}\sqrt{3}b^2(1-\frac{x^2}{a^2})dx=\sqrt{3}b^2\int_{-a}^{a}(1-\frac{x^2}{a^2})dx=\frac{4\sqrt{3}}{3}ab^2$

306.49.

(1)



$\begin{cases} y=x^2 \\ y^2=8x \end{cases}\Rightarrow$ 交点 $(0,0),(2,4)$

绕x: $V=\pi\int_0^2(f^2(x)-g^2(x))dx$

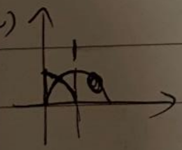
$=\pi\int_0^2(8x-x^4)dx=\pi(4x^2-\frac{1}{5}x^5)\Big|_0^2$

$=\cancel{\frac{40}{3}\pi}\ \frac{48}{5}\pi.$

绕y: $V=\pi\int_0^4((g^{-1}(y))^2-(f^{-1}(y))^2)dy=\pi\int_0^4(y-\frac{y^4}{64})dy$

$=\pi(\frac{1}{2}y^2-\frac{1}{64\times5}y^5)\Big|_0^4=\cancel{\frac{40}{5}\pi}\ \frac{24}{5}\pi$

(2)

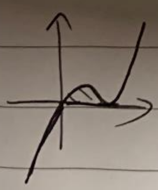


$V=2V'$, $V'=\pi\int_0^{\frac{\pi}{4}}(\cos^2x-\sin^2x)dx=\pi\int_0^{\frac{\pi}{4}}\cos2x\,dx$

$=\frac{\pi}{2}\sin2x\Big|_0^{\frac{\pi}{4}}=\frac{\pi}{2}$

$\therefore V=\pi$

50.(1)

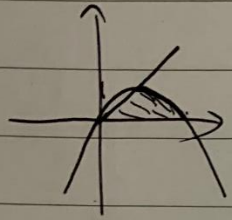


交点 $(0,0)$, $(1,0)$.

$\therefore V=2\pi\int_0^1x\cdot x(x-1)^2dx=2\pi(\frac{1}{5}x^5-\frac{2}{4}x^4+\frac{1}{3}x^3)\Big|_0^1$

$=\cancel{\frac{1}{30}}\ \frac{\pi}{15}$

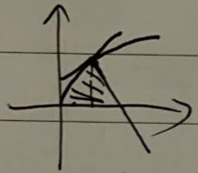
(2)



交点 (1, 1)

$$V=2\pi\int_0^1 x^2\,dx + 2\pi\int_1^2 x(2x-x^2)\,dx$$
$$=2\pi\ \tfrac{1}{3}x^3\Big|_0^1 + 2\pi\left(\tfrac{2}{3}x^3-\tfrac{1}{4}x^4\right)\Big|_1^2$$
$$=\tfrac{2}{3}\pi+\tfrac{11}{6}\pi=\tfrac{15}{6}\pi=\tfrac{5}{2}\pi$$

51.(2)



~~切~~ $y=\sqrt{8x}$ $y'=\sqrt{2}\sqrt{x}^{-1}$ $\therefore$ l: $y=x+2$

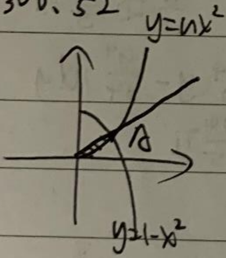
$\therefore$ l': $y=-x+6$ 交点 (2, 4)

$\therefore$ $\cancel{V=2\pi\int_0^2 x\sqrt{8x}\,dx+2\pi\int_2^6 x(-x+6)\,dx}$ 【概念错】.

$\cancel{=\tfrac{64}{5}\pi+\tfrac{160}{3}\pi=}$ $V=\pi\int_0^2 8x\,dx+\pi\int_2^6(-x+6)^2\,dx$

$=\tfrac{112}{3}\pi$

306、52



$y=ax^2=1-x^2 \Rightarrow x=\sqrt{\tfrac{1}{a+1}}$ $\left(\sqrt{\tfrac{1}{a+1}},\ \tfrac{a}{a+1}\right)$

$\cancel{V=\pi\int_0^{\sqrt{\frac{1}{a+1}}}}$ l: $y=\tfrac{a}{\sqrt{a+1}}x$

$\therefore$ $V=\pi\int_0^{\sqrt{\frac{1}{a+1}}}\left[\left(\tfrac{a}{\sqrt{a+1}}x\right)^2-(ax^2)^2\right]dx$

$=\tfrac{2\pi}{15}a^2\ \tfrac{1}{(a+1)^2\sqrt{a+1}}$

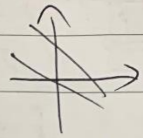
令 $g(a)=\tfrac{2\pi}{15}\ \tfrac{a^2}{(a+1)^2\sqrt{a+1}}$ $\underline{\underline{\sqrt{a+1}=t}}$ $g(t)=\tfrac{2\pi}{15}\ \tfrac{(t^2-1)^2}{t^5}$

$\therefore$ $g'(t)=0$ 时, $t=\sqrt{5}$ $\therefore$ $a=4$ 时 $g(a)_{max}=\tfrac{32\sqrt{5}}{1875}\pi$

306.55

(1) $S=\int_1^e \sqrt{1+y'^2(x)}\,dx = \frac{1}{2}\int_1^e (x+\frac{1}{x})\,dx = \frac{1}{2}(\frac{1}{2}x^2+\ln x)\Big|_1^e = \frac{1}{4}(e^2+1)$

(3) $S(x)=\int_{-\sqrt{3}}^{x}\sqrt{3-t^2}\,dt.$



$\therefore\ S'(x)=\sqrt{3-x^2}$ $\therefore\ \cancel{S=\int_{-\sqrt{3}}^{\sqrt{3}}\sqrt{3-x^2}\,dx}$

$S=\int_{-\sqrt{3}}^{\sqrt{3}}\sqrt{1+S'^2(x)}\,dx = \int_{-\sqrt{3}}^{\sqrt{3}}\sqrt{4-x^2}\,dx$

$= \cancel{2\int}\ 2\int_0^{\sqrt{3}}\sqrt{4-x^2}\,dx \overset{x=2\sin t}{=\!=} 2\int_0^{\frac{\pi}{3}} 2\cos^2 t\, 2\,dt = 4\int_0^{\frac{\pi}{3}}(1+\cos 2t)\,dt$

$= 4\left(t+\frac{1}{2}\sin 2t\right)\Big|_0^{\frac{\pi}{3}} = \frac{4}{3}\pi+\sqrt{3}$

(5) $S=\int_0^{2\pi}\sqrt{e^{4\theta}+4e^{4\theta}}\,d\theta = \frac{\sqrt{5}}{2}\int_0^{2\pi} e^{2\theta}\,d2\theta = \frac{\sqrt{5}}{4}e^{2\theta}\Big|_0^{2\pi} = \frac{\sqrt{5}}{4}(e^{4\pi}-1)$

307.56

$S_0=\int_0^{2\pi}\sqrt{a^2(1-\cos t)^2+a^2\sin^2 t}\,dt = \sqrt{2}a\int_0^{2\pi}\sqrt{1-\cos t}\,dt$

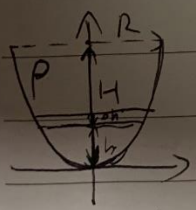
$= \sqrt{2}a\int_0^{2\pi}\sqrt{2}\sin\frac{t}{2}\,dt = 4a\int_0^{2\pi}\sin\frac{t}{2}\,d\frac{t}{2} = 4a\int_0^{\pi}\sin x\,dx = 8a.$

$\therefore\ S_1=\frac{1}{4}S_0=2a.$

$S_1 = 4a\int_0^{t}\sin\frac{x}{2}\,d\frac{x}{2} = 4a\int_0^{\frac{t}{2}}\sin x\,dx = (1-\cos\frac{t}{2})4a = 2a$

$\therefore\ t=\frac{2}{3}\pi$ $\therefore\ \left(a(\frac{2}{3}\pi-\frac{\sqrt{3}}{2}),\ \frac{3}{2}a\right)$

307.59



令 $y=\frac{H}{R^2}x^2$. ∴ 距底高h的液体. $x_h=R\sqrt{\frac{h}{H}}$

$\therefore\ S=\pi x_h^2$ $\Delta V=\Delta h\cdot S=\pi\frac{R^2}{H}h\Delta h.$

所做功为 $\Delta W=\Delta V\cdot\rho g\cdot(H-h)$

$\therefore\ \Delta W=\rho g\pi\frac{R^2}{H}h(H-h)\Delta h.$ $\therefore\ W=\int_0^H\rho g\pi\frac{R^2}{H}(Hh-h^2)\,dh$

$=\frac{1}{6}\rho g\pi R^2H^2$

$kg/m^3\times m/s^2\times m^2\times m$

$W = kg\, m/s^2\cdot m$

不用乘 cosα吗

307.61

$F = P_\perp \cdot S \quad S = ab$

对位于 y 深的一小块.

$P = \rho g y \qquad P_\perp = P\cos\alpha$

$\therefore F(y) = \rho g a y \cos\alpha \frac{\partial y}{\sin\alpha} = \rho g a y \partial y \frac{\cos\alpha}{\sin\alpha}$

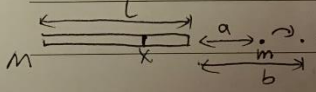
$\therefore F = \int_{h-b\sin\alpha}^{h} \rho g a b \cos\alpha\, y\, dy = \rho g a b\cos\alpha \left(\frac{1}{2}y^2\right)\Big|_{h-b\sin\alpha}^{h}$

$= \rho g a b\cos\alpha\,(\cancel{hb\sin\alpha - \frac{1}{2}b^2\sin^2\alpha})$ $\therefore F = \sum_{y=h-b\sin\alpha}^{h} F(y) = \int_{h-b\sin\alpha}^{h} \frac{F(y)}{\partial y}\, dy$

$= \frac{1}{2}\rho g a b\,(2h + b\sin\alpha)\cos\alpha$

不用 cosα

307.62



距点 X 处一段对点的引力为

$F = \frac{G m \frac{\partial x}{L} M}{x^2} = \frac{GmM}{L}\frac{dx}{x^2}$

$\therefore F = \frac{dE}{dx} \Rightarrow dE = F\,dx \quad \therefore E = \int F\,dx = \int \frac{GMm}{L}\frac{d^2x}{x^2} = -\frac{GMm}{L}\frac{dx}{x}$

$E(x) = -\frac{GMm}{L}\frac{\partial x}{x} \qquad E(x+b-a) = -\frac{GMm}{L}\frac{\partial x}{x+b-a}$

$\sum_{x=a}^{a+l} E(x) + W = \sum_{x=a}^{a+l} E(x+b-a) \qquad \sum_{x=a}^{a+l} E(x) = \int \frac{E(x)}{\partial x} = \int_a^{a+l} -\frac{GMm}{L}\frac{1}{x}dx$

$\therefore W = \frac{GMm}{L}\ln\frac{b(a+l)}{a(b+l)} \qquad = -\frac{GMm}{L}\ln\frac{a+l}{a}$

~~E(x+b-a)~~ 307.64.

~~(5) $\int_1^{+\infty}\frac{\arctan x}{x^2}dx \overset{x=\tan t}{=} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{t}{\tan^2 t\cos^2 t}dt = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{t}{\sin^2 t}dt$~~

~~$= 2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{t}{1-\cos 2t}dt$ 令 $f(x)=x,\ g(x)=\frac{1}{1-\cos 2x}$~~

(5) $\int_1^{+\infty}\frac{\arctan x}{x^2}dx = \lim_{b\to+\infty}\int_1^b \frac{\arctan x}{x^2}dx = \lim_{b\to+\infty}\left(-\frac{1}{x}\arctan x\Big|_1^b + \int_1^b \frac{1}{x(1+x^2)}dx\right)$

$= -\frac{\arctan b}{b} + \frac{\pi}{4} + \lim_{b\to+\infty}\left(\ln x - \frac{1}{2}\ln(1+x^2)\right)\Big|_1^b = \frac{\pi}{4} + \frac{1}{2}\ln 2$

(6) $\int_2^{+\infty}\frac{dx}{x\sqrt{x-1}} = \lim_{b\to+\infty}\int_2^b \frac{dx}{x\sqrt{x-1}} \overset{\sqrt{x-1}=t}{=} \lim_{b\to+\infty}\int_1^b \frac{2t\,dt}{(t^2+1)t} = 2\arctan t\Big|_1^b$

$= \frac{\pi}{2}$

(7) $\int_1^{+\infty}\frac{dx}{x\sqrt{2x^2-2x+1}} \overset{x-\frac{1}{2}=t}{=} \int_{\frac{1}{2}}^{b}\frac{\sqrt{2}\,dt}{(t+\frac{1}{2})\sqrt{\ldots}}$ $\overset{t=\tan u}{=} \sqrt{2}\int \frac{du}{(\tan u+1)\cos u} = \sqrt{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{du}{1+\tan u}$

~~$= \sqrt{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos u}{\sin u+\cos u}du = \frac{\sqrt{2}}{8}\pi - \frac{\sqrt{2}}{4}\ln 2$~~ $= \sqrt{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{du}{\sin u+\cos u}$

~~$\int\frac{\cos u}{\sin u+\cos u}du = \frac{1}{2}x + \frac{1}{2}\ln|\sin u+\cos u|$~~ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{du}{\sin(u+\frac{\pi}{4})}$

$= \ln(\sqrt{2}+1)$

(8) $\int_0^{+\infty}\ldots$

(9) $\int_{-\infty}^{+\infty}\ldots$

$\int_{-\infty}^{+\infty}\frac{dx}{(1+x^2)\ldots}$

$\int_0^{+\infty}\frac{dx}{(1+x^2)\ldots}$

(10) $\int_0^{+\infty}\ldots$

307.65

(2) $\lim_{x\to-1}\ldots$

$\int_{-1}^{0}\frac{x\,dx}{\sqrt{1+x}}$

(4) $\int_0^2 \frac{d\ldots}{(x-\ldots}$

$\lim_{x\to 1}\ldots$

$\therefore$ 不存在

(6) $\int_0^1 \ln\ldots$

$= \lim_{x\to}\ldots$

$\lim_{x\to 1}\frac{f(x)}{g(x)}\ldots$

$\int_0^1 \ln(\ldots$

(8) $\int_{\frac{1}{2}}^{\frac{3}{2}}\ldots$

$\lim_{x\to 1}\int\ldots$

$\therefore \int\ldots$

$\int\frac{dx}{\sqrt{3x\ldots}}$

一直算√2−1

$\lim_{x\to+\infty} \frac{x^{p+1}e^{-x}}{(1+e^{x})^2} = \frac{x^{p+1}}{e^{x}+2+e^{-x}} = \frac{x^{p+1}}{e^{x}} = 0$

(8) $\int_0^{+\infty} \frac{xe^{-x}}{(1+e^{x})^2}dx$ · $\cancel{\frac{xe^{-x}}{(1+e^{x})^2} \le \frac{x}{1} = x}$ ∴收敛.

$\int_0^{+\infty} \frac{xe^{-x}}{(1+e^{-x})^2}dx = x\frac{1}{(1+e^{-x})}\Big|_0^b - \int \frac{1}{1+e^{-x}}dx$

$= b - \ln(1+e^{x})\Big|_0^b$

$= b - \ln(1+e^{b}) + \ln 2$

$= \ln 2$.

(9) $\int_{-\infty}^{+\infty} \frac{dx}{(1+x^2)^2}$

$\int_{-\infty}^{+\infty} \frac{dx}{(1+x^2)^2} = \int_0^{+\infty} \frac{dx}{(1+x^2)^2} + \int_{-\infty}^{0} \frac{dx}{(1+x^2)^2}$

$\int_0^{+\infty} \frac{dx}{(1+x^2)^2} \xlongequal{x=\tan t} \int_0^{\frac{\pi}{2}} \cos^2 t\,dt = \frac{\pi}{4}$ ∴ $\int$ 原 $= \frac{\pi}{2}$.

(10) $\int_0^{+\infty} \frac{dx}{\sqrt{x}(4+x)} \xlongequal{\sqrt{x}=t} \int_0^{+\infty} \frac{2t\,dt}{t(4+t^2)} = \frac{1}{2}\int_0^{+\infty} \frac{dt}{1+\frac{t^2}{4}} = \int_0^{+\infty} \frac{d\frac{t}{2}}{1+\frac{t^2}{4}} = \arctan\frac{t}{2}\Big|_0^{+\infty}$

$= \frac{\pi}{2}$

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(2) $\lim_{x\to -1^+} (1+x)^p \frac{x}{\sqrt{1+x}} = (1+x)^{p-\frac{1}{2}} x$ ∴ $p=\frac{1}{2}$ ∴收敛.

$\int_{-1}^{0} \frac{x\,dx}{\sqrt{1+x}} = \lim_{\varepsilon\to 0^+} \int_{-1+\varepsilon}^{0} \frac{x\,dx}{\sqrt{1+x}} = \lim_{\varepsilon\to 0^+} \left(2\sqrt{1+x}\,x\Big|_{-1+\varepsilon}^{0} - 2\int_{-1+\varepsilon}^{0}\sqrt{1+x}\,dx\right)$.

$= -2\times\frac{2}{3}(1+x)^{\frac{3}{2}}\Big|_{-1+\varepsilon}^{0} = -\frac{4}{3}$

(4) $\int_0^2 \frac{dx}{(x-1)(x-3)} = \int_0^1 \frac{dx}{(x-1)(x-3)} + \int_1^2 \frac{dx}{(x-1)(x-3)}$

$\lim_{x\to 1^-} (x-1)^{p-1}(x-3) = 1 \Rightarrow p=1$ ∴发散.

∴不存在.

(6) $\int_0^1 \ln(1-x)\,dx$ $\cancel{= \lim_{\varepsilon\to 0^+}\int_0^{1-\varepsilon}\ln(1-x)dx = \lim_{\varepsilon\to 0^+}\left(x\ln(1-x)\Big|_0^{1-\varepsilon} + \int\frac{x}{1-x}dx\right)}$

$= \lim_{x\to 1} (1-x)^p \ln(1-x) = \frac{\ln(1-x)}{(1-x)^{-p}} = \frac{\frac{-1}{1-x}}{p(1-x)^{-p-1}} = -\frac{1}{p}\frac{1}{(1-x)^{-p}}$ ∴ $p=0$.

$\lim_{x\to 1} \frac{f(x)}{g(x)} = \infty$ ∴ 我不好说

$\int_0^1 \ln(1-x)dx = \lim_{\varepsilon\to 0^+}\left(x\ln(1-x)\Big|_0^{1-\varepsilon} - x\Big|_0^{1-\varepsilon} - \ln(1-x)\Big|_0^{1-\varepsilon}\right) = -1$

(8) $\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{dx}{\sqrt{|x^2-x|}} = \int_{\frac{1}{2}}^{1} \frac{dx}{\sqrt{x-x^2}} + \int_1^{\frac{3}{2}} \frac{dx}{\sqrt{x^2-x}}$

$\lim_{x\to 1^+} (1-x)^p \frac{1}{\sqrt{x(x-1)}} = \frac{(1-x)^p}{\sqrt{x-1}}\frac{1}{\sqrt{x}}$ ∴ $p=\frac{1}{2}$ ∴收敛.

$\int \frac{dx}{\sqrt{x-x^2}} = \int \frac{dx}{\sqrt{\frac{1}{4}-(x-\frac{1}{2})^2}} = 2\int \frac{dx}{\sqrt{1-(2x-1)^2}} \xlongequal{2x-1=t} \int \frac{dt}{\sqrt{1-t^2}} = \arcsin t$.

∴ $\int_{\frac{1}{2}}^{1} \frac{dx}{\sqrt{x-x^2}} = \arcsin t\Big|_0^1 = \frac{\pi}{2}$

$\int \frac{dx}{\sqrt{x^2-x}} = \int \frac{dt}{\sqrt{t^2-1}} = \ln|t+\sqrt{t^2-1}|$ ∴ $\int_1^{\frac{3}{2}} \frac{dx}{\sqrt{x^2-x}} = \ln|t+\sqrt{t^2-1}|\Big|_1^2 = \ln(2+\sqrt{3})$

∴ 原 $= \frac{\pi}{2} + \ln(2+\sqrt{3})$