96.27

(1) $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^{\frac{x}{2}\cdot\frac{2x+6}{x}} = e^2$

(3) $\lim\limits_{x\to0}(1+x^2)^{\frac{1}{x^2}\cdot\frac{x^2}{1-\cos x}} = \cancel{1}\, e^{\lim\limits_{x\to0}\frac{x^2}{\frac{x^2}{2}}} = e^2$

(5) $\lim\limits_{x\to0}(1+\tan x)^{\cot x} = e$

(7) $\lim\limits_{x\to\infty}\left(\frac{2x-1}{2x+1}\right)^x = \lim\limits_{x\to\infty}\left(1-\frac{2}{2x+1}\right)^{-\frac{2x+1}{2}\left(-\frac{2x}{2x+1}\right)} = e^{-1}$

(9) $\lim\limits_{x\to0^+}\sqrt[x]{\cos\sqrt{x}} = \lim\limits_{x\to0^+}(\cos\sqrt{x})^{\frac{1}{x}} = \lim\limits_{x\to0^+}(1-\sin^2\sqrt{x})^{\frac{1}{2x}}$

$= \lim\limits_{x\to0^+}(1-\sin^2\sqrt{x})^{\frac{1}{-\sin^2\sqrt{x}}\left(-\frac{\sin^2\sqrt{x}}{2x}\right)} = e^{\lim\limits_{x\to0^+}-\frac{\sin^2\sqrt{x}}{2x}} = e^{-\frac{1}{2}}$

96.28

(2) $\lim\limits_{x\to0^+}\frac{x^{\frac{2}{3}}-x^{\frac{1}{2}}}{x^k} = \lim\limits_{x\to0^+}x^{\frac{2}{3}-k} - \lim\limits_{x\to0^+}x^{\frac{1}{2}-k} = C.$ $k=\frac{1}{2}$, $C=-1$

主: $-x^{\frac{1}{2}}$

(4) $\lim\limits_{x\to0}\frac{\sqrt{a+x^3}-\sqrt{a}}{x^k} = \lim\limits_{x\to0}\frac{x^3}{x^k(\sqrt{a+x^3}+\sqrt{a})} = \lim\limits_{x\to0}\frac{1}{x^{k-3}(2\sqrt{a})} = C$

$\therefore k=3,\ C=\frac{1}{2\sqrt{a}}$ $\therefore$ 主: $\frac{1}{2\sqrt{a}}x^3$

(6) $\lim\limits_{x\to0}\frac{\sqrt{x^2+x^{\frac{4}{3}}}}{x^k} = \lim\limits_{x\to0}\sqrt{\frac{x^2+x^{\frac{4}{3}}}{x^{2k}}} = \lim\limits_{x\to0}\sqrt{x^{2-2k}+x^{\frac{4}{3}-2k}} = C$

$\therefore k=\frac{2}{3},\ C=1$ 主: $x^{\frac{2}{3}}$

(8) $\lim\limits_{x\to0}\frac{\cos^x x-1}{x^k} = \lim\limits_{x\to0}\frac{x\ln\cos x}{x^k} = \lim\limits_{x\to0}\frac{\cos x-1}{x^{k-1}} = \lim\limits_{x\to0}-\frac{1}{2}\frac{1}{x^{k-3}} = C.$

$\therefore k=3,\ C=-\frac{1}{2}$ $\therefore$ 主 $-\frac{1}{2}x^3$

96.29

(1) $\lim\limits_{n\to\infty}\left(\frac{n+a}{n-a}\right)^n = \lim\limits_{n\to\infty}\left(1+\frac{2a}{n-a}\right)^{\frac{n-a}{2a}\cdot\frac{2an}{n-a}} = e^{\frac{2an}{n-a}} = e^{\frac{1}{2}}$

$\therefore \lim\limits_{n\to\infty}\frac{2an}{n-a} = \frac{1}{2} \Rightarrow a=\frac{1}{4}$

(2) $\lim\limits_{x\to\infty}\left(1+\frac{a}{x}\right)^x = \lim\limits_{x\to\infty}\left(1+\frac{a}{x}\right)^{\frac{x}{a}\cdot a} = e^a = 4 \Rightarrow a=\ln 4$

(5) $\lim\limits_{x\to0}\frac{\sqrt[4]{1+ax^2}-1}{\cos x-1} = \lim\limits_{x\to0}\frac{\frac{1}{4}ax^2}{-\frac{1}{2}x^2} = -\frac{1}{2}a = 1 \Rightarrow a=-2$

(7) $\lim\limits_{x\to1}\frac{a(1-\sqrt[m]{x})}{1-x} = \lim\limits_{x\to1}\frac{\sqrt[m]{x}-1}{x-1}\cdot a \overset{u=\sqrt[m]{x}}{=\!=} \lim\limits_{u\to1}\frac{u-1}{u^m-1}\cdot a = \lim\limits_{u\to1}\frac{a}{u^{m-1}+u^{m-2}+\dots+1}$

$= \frac{a}{m} = 1 \Rightarrow a=m.$

96.30

(1) $\lim\limits_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{\sin 2x}=\lim\limits_{x\to 0}\frac{x+x^2}{2x(\sqrt{1+x+x^2}+1)}=\frac{1}{4}\lim\limits_{x\to 0}\frac{x+x^2}{x}=\frac{1}{4}\lim\limits_{x\to 0}(1+x)=\frac{1}{4}$

(3) $\lim\limits_{x\to 0}\frac{x^2\tan x}{\sqrt{1-x^2}-1}=\lim\limits_{x\to 0}\frac{x^2\tan x(\sqrt{1-x^2}+1)}{-x^2}=\lim\limits_{x\to 0}[-\tan x(\sqrt{1-x^2}+1)]=0$

(4) $\lim\limits_{x\to 0}\frac{\ln(\sin^2 x+e^{x})-x}{\ln(e^{2x}-x^2)-2x}=\lim\limits_{x\to 0}\frac{\ln(\frac{\sin^2 x}{e^x}+1)}{\ln(1-\frac{x^2}{e^{2x}})}=\lim\limits_{x\to 0}\frac{\frac{\sin^2 x}{e^x}}{-\frac{x^2}{e^{2x}}}=\lim\limits_{x\to 0}-\frac{x^2}{x^2}e^{x}.$

$=-1$

~~(6) $\lim\limits_{x\to 0}\frac{\sqrt[n]{1+\alpha x}-\sqrt[m]{1+\beta x}}{x}=\lim\limits_{x\to 0}\frac{\sqrt[n]{1+\alpha x}}{x}-\lim\limits_{x\to 0}\frac{\sqrt[m]{1+\beta x}}{x}=\lim\limits_{x\to 0}\frac{1+\frac{\alpha}{n}x}{x}-\lim\limits_{x\to 0}\frac{1+\frac{\beta}{m}x}{x}$~~

(6) $\lim\limits_{x\to 0}\frac{\sqrt[n]{1+\alpha x}-\sqrt[m]{1+\beta x}}{x}=\lim\limits_{x\to 0}\frac{(\sqrt[n]{1+\alpha x}-1)-(\sqrt[m]{1+\beta x}-1)}{x}=\lim\limits_{x\to 0}\frac{\frac{\alpha}{n}x}{x}-\lim\limits_{x\to 0}\frac{\frac{\beta}{m}x}{x}$

$=\frac{\alpha}{n}-\frac{\beta}{m}$

(7) $\lim\limits_{x\to 0}\frac{\ln(1+x)+\ln(1-x)}{1+\sin^2 x-\cos x}=\lim\limits_{x\to 0}\frac{\ln(1-x^2)}{1+\sin^2 x-\cos x}=\lim\limits_{x\to 0}\frac{-x^2}{1+\sin^2 x-\cos x}$

$=\lim\limits_{x\to 0}\frac{-x^2}{2-\cos^2 x-\cos x}=\lim\limits_{x\to 0}\frac{x^2}{(\cos x+2)(\cos x-1)}$

$=\frac{1}{3}\lim\limits_{x\to 0}\frac{x^2}{-\frac{x^2}{2}}=-\frac{2}{3}$

(8) $\lim\limits_{x\to 0}(3e^{\frac{x}{x-1}}-2)^{\frac{1}{x}}=\lim\limits_{x\to 0}[3(e^{\frac{x}{x-1}}-1)+1]^{\frac{1}{x}}$

$=\lim\limits_{x\to 0}[3(e^{\frac{x}{x-1}}-1)+1]^{\frac{1}{3e^{\frac{x}{x-1}}-3}\cdot\frac{3e^{\frac{x}{x-1}}-3}{x}}=e^{\lim\limits_{x\to 0}\frac{3(e^{\frac{x}{x-1}}-1)}{x}}$

$=e^{\lim\limits_{x\to 0}3\frac{x}{x(x-1)}}=e^{-3}$

(9) $\lim\limits_{x\to 0}\left(\frac{2+e^{\frac{1}{x}}}{1+e^{\frac{2}{x}}}+\frac{\sin x}{|x|}\right)$

$\lim\limits_{x\to 0^+}$原式$=\lim\limits_{x\to 0^+}\frac{2+e^{\frac{1}{x}}}{1+e^{\frac{2}{x}}}+\lim\limits_{x\to 0^+}\frac{\sin x}{x}=1+\lim\limits_{x\to 0^+}\frac{2+2e^{\frac{2}{x}}+e^{\frac{1}{x}}-2e^{\frac{2}{x}}}{1+e^{\frac{2}{x}}}$

$=1+\lim\limits_{x\to 0^+}(2+\frac{e^{\frac{1}{x}}-2e^{\frac{2}{x}}}{1+e^{\frac{2}{x}}})\overset{u=e^{\frac{1}{x}}}{=\!=}\lim\limits_{u\to +\infty}\frac{u-2u^2}{1+u^2}+3=1$

$\lim\limits_{x\to 0^-}$原式$=\lim\limits_{x\to 0^-}\frac{2+e^{\frac{1}{x}}}{1+e^{\frac{2}{x}}}+\lim\limits_{x\to 0^-}-\frac{\sin x}{x}\overset{u=e^{\frac{1}{x}}}{=\!=}\lim\limits_{u\to 0^+}\frac{2+u}{1+u^2}-1$

$=\lim\limits_{u\to 0^+}2-1=1$

$\therefore\lim\limits_{x\to 0}$原式$=1$

97.30

(10) $\lim\limits_{x\to 0}\left(\frac{3^x+2^x}{2}\right)^{\frac{1}{x}} = \lim\limits_{x\to 0}\left(\frac{3^x+2^x-2}{2}+1\right)^{\frac{2}{3^x+2^x-2}\cdot\frac{3^x+2^x-2}{2}\cdot\frac{1}{x}}$

$= e^{\lim\limits_{x\to 0}\frac{3^x-1+2^x-1}{2x}} = e^{\lim\limits_{x\to 0}\frac{3^x-1}{2x}+\lim\limits_{x\to 0}\frac{2^x-1}{2x}} = e^{\lim\limits_{x\to 0}\frac{x\ln 3}{2x}+\lim\limits_{x\to 0}\frac{x\ln 2}{2x}}$

$= e^{\frac{\ln 3+\ln 2}{2}} = \sqrt{6}$

97.31

(2) $\lim\limits_{x\to\infty}\left(\frac{x^2+1}{x+1}-ax-b\right) = \lim\limits_{x\to\infty}\frac{x^2+1-ax^2-ax}{x+1}-b=0$

$\therefore a=1,\ b=-1$ $\quad \frac{(1-a)x^2+1-ax}{x+1}$ 若 $(1-a)\neq 0$，则 $\lim\limits_{x\to\infty}$原式 $=\infty$.

~~(3)~~

(3) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+ax}-\sqrt{bx^2-1}\right) = \lim\limits_{x\to+\infty}\frac{(1-b)x^2+ax+1}{\sqrt{x^2+ax}+\sqrt{bx^2-1}}$

又$\because$ $\lim\limits_{x\to+\infty}$原式存在 $\therefore$ $b=1$ $\therefore \lim\limits_{x\to+\infty}\frac{ax+1}{\sqrt{x^2+ax}+\sqrt{x^2-1}} = \lim\limits_{x\to\infty}\frac{a+\frac{1}{x}}{\sqrt{1+\frac{a}{x}}+\sqrt{1-\frac{1}{x}}}$

$= \frac{a}{2} = 1$

$\therefore a=2$

97.32

$\lim\limits_{x\to 0}\frac{\ln\left(1+\frac{f(x)}{\sin 2x}\right)}{3^x-1}$ $\therefore\ 2\lim\limits_{x\to 0}\frac{f(x)}{2x} = \lim\limits_{x\to 0}\frac{f(x)}{\sin 2x} = 0$

$\therefore \lim\limits_{x\to 0}\frac{\ln\left(1+\frac{f(x)}{\sin 2x}\right)}{3^x-1} = \lim\limits_{x\to 0}\frac{f(x)}{(3^x-1)\sin 2x} = \lim\limits_{x\to 0}\frac{f(x)}{2x\cdot x\ln 3} = \lim\limits_{x\to 0}\frac{f(x)}{2x^2\ln 3} = 5.$

$\therefore \lim\limits_{x\to 0}\frac{f(x)}{x^2} = 10\ln 3$

P97.33

(1) $y=\frac{x^2-1}{(x-1)(x-2)}=\frac{(x-1)(x+1)}{(x-1)(x-2)}$

$\because \lim\limits_{x\to 1} y=-2$ $\therefore$ $x=1$ 为可去间断点， $x=2$ 为无穷间断点

$y=\frac{x+1}{x-2}\ (x\neq 2)$ 修正

(3) $y=\frac{\sqrt[3]{1+4x}-1}{2\sin x}$ $x=\pi k$, $k\in N$.

$\because$ 当 ~~x=0~~ $x\to 0$ 时， $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+4x}-1}{2\sin x}=\lim\limits_{x\to 0}\frac{\frac{4}{3}x}{2x}=\frac{2}{3}$

$\therefore$ $x=0$ 是可去间断点， $x=\pi k$ $k\in Z$ 时 无穷间断点.

修正. $y=\begin{cases}\frac{\sqrt[3]{1+4x}-1}{2\sin x} & x\neq 0\\ \frac{2}{3} & x=0\end{cases}$

(5) $\lim\limits_{x\to 1^+}\frac{1}{1+e^{\frac{1}{1-x}}}=1$ $\lim\limits_{x\to 1^-}\frac{1}{1+e^{\frac{1}{1-x}}}=0$

$\therefore$ $x=1$ 是跳跃间断点

(6) $\frac{1}{|x|+1}\leqslant 1$ $\cancel{\lim\limits_{x\to 0}\frac{1}{|x|+1}}$ $x=0$ 时. $[\frac{1}{|x|+1}]=1$

$\lim\limits_{x\to 0^-}[\frac{1}{|x|+1}]=0$ $\lim\limits_{x\to 0^+}[\frac{1}{|x|+1}]=0$

$\therefore$ $x=0$ 时可断间断点 $y=\begin{cases}[\frac{1}{|x|+1}] & x\neq 0\\ 0 & x=0\end{cases}$

P97.34

(2) $x=1$ 时 无穷间断点 $D(f)=(-\infty,1)\ \cancel{(1,2]}\cup[2,+\infty)$

$\therefore$ $f(x)\in C(-\infty,1)$ $f(x)\in C[2,+\infty)$

(3) $D(f)=(0,1]$ $\therefore$ ~~f(x)~~ 无间断点 $\therefore$ $f(x)\in(0,1]$

97.36

$\forall \varepsilon>0$，$\exists\delta$，$|x-0|<\delta$时．有$|\varphi(x)-\varphi(0)|<\varepsilon$

又$\because$ $\varphi(0)=0$ $\therefore$ $|\varphi(x)|<\varepsilon$ $\therefore$ $|f(x)|\le|\varphi(x)|<\varepsilon$.

$\therefore$ $|f(0)|\le|\varphi(0)|=0$ $\therefore$ $f(0)=0$

$\therefore$ $|f(x)-f(0)|<\varepsilon$ 对 $\forall$ $|x-0|<\delta$ 成立．

$\therefore$ $f(x)$在$x=0$处连续．$\delta=\delta_1$．

97.37

(1) $\lim\limits_{x\to0^-}\frac{\sin ax}{x}=\lim\limits_{x\to0^-}\frac{ax}{x}=a=1$

$\lim\limits_{x\to0^+}\frac{b(\sqrt{1+x}-1)}{x}=b\lim\limits_{x\to0^+}\frac{\frac{1}{2}x}{x}=\frac{b}{2}=1\Rightarrow b=2$

(2) $\cancel{f(x)=\lim\limits_{n\to\infty}\frac{x^{2n-1}+ax^2+bx}{x^{2n}+1}=x+\lim\limits_{n\to\infty}\frac{}{x^{2n}+1}}$

$\cancel{\lim=\frac{1}{x}+\lim\limits_{n\to\infty}\frac{}{x^{2n}+1}}$

(2) $\lim\limits_{x\to1^-}\frac{x^{2n-1}+ax^2+bx}{x^{2n}+1}=\lim\limits_{x\to1^-}\lim\limits_{n\to\infty}\frac{a+b}{1}=a+b.$

$\lim\limits_{x\to1^+}\frac{x^{2n-1}+ax^2+bx}{x^{2n}+1}=\frac{1}{x}+\lim\limits_{n\to\infty}\frac{ax^2+bx-\frac{1}{x}}{x^{2n}+1}=1$

$\lim\limits_{x\to1}f(x)=\frac{1+a+b}{2}=1=a+b\Rightarrow a+b=1$

$\lim\limits_{x\to-1^-}f(x)=-1$，$\lim\limits_{x\to-1^+}f(x)=a-b$ $\therefore$ $a-b=-1$ $\Rightarrow\begin{cases}a=0\\b=1\end{cases}$

98.38

$f(0)=1+b$，$\nexists\lim\limits_{x\to0^+}x^a\sin\frac{1}{x}$ ① $a>0$时．$\lim\limits_{x\to0^+}x^a\sin\frac{1}{x}=0$

~~① $b=-1$时~~ ② $a\le0$时 $x\to0^+$时振荡

$\therefore$ $b=-1$，$a>0$时，连续．

$b=-1$，$a\le0$时，左连续

$b\ne-1$，$a>0$时，左连续．跳跃间断点 $\lim\limits_{x\to0^+}f(x)\ne\lim\limits_{x\to0^-}f(x)$，但$\exists$

$b\ne-1$，$a\le0$时，左连续．

98.39

(1) $f(1)f(2)<0$

(2) $f(x)=x-a\sin x-b$.

$\because f(0)=-b<0$，当 $x\to+\infty$ 时. $f(x)\to+\infty$

$f(a+b)=a+b-a\sin(a+b)-b=a-a\sin(a+b)=a(1-\sin(a+b))\geq 0$

$\therefore f(0)f(a+b)\leq 0 \therefore x_0\in(0,a+b]$

98.40

$F(x)=f(x)-x$

$\because F(x)\in C[0,1]$ $F(0)=f(0)>0$, $F(1)=f(1)-1<0$

$\therefore F(0)F(1)<0 \Rightarrow \exists\,\xi\in(0,1)$，使 $F(\xi)=0$ 即 $f(\xi)=\xi$.

98.42

$f(x)\in C[0,+\infty)$，$\lim\limits_{x\to+\infty}f(x)=A<0$. $\forall\varepsilon>0$，$\exists X$，当 $x>X$ 时.

$|f(x)-A|<\varepsilon$ $\therefore A-\varepsilon<f(x)<A+\varepsilon$ 取 $\varepsilon=\frac{|A|}{2}$

$\therefore \frac{3}{2}A<f(x)<\frac{A}{2}$ $\therefore x_1>X$ 时，$f(x_1)<\frac{A}{2}<0$

$\therefore f(0)f(x_1)<0 \Rightarrow \exists\,\xi\in[0,x_1)$，使 $f(\xi)=0$

98.46

$f(x)\in C(R)$，$\lim\limits_{x\to+\infty}f(x)=+\infty$，$\lim\limits_{x\to-\infty}f(x)=+\infty$

$\therefore \forall M$，$\exists X_1>0$，当 $|x|>X_1$ 时. $f(x)>M$

$\because f(x)\in C(R)$ $\therefore f(x)\in C[-X_1,X_1]$ $\therefore \exists\,\delta\in[-X_1,X_1]$

使 $f(\delta)=\min f(x)$.

98.43

令 $a<x_1\le x_2\le\cdots\le x_n<b$ ∴ ~~f(x)在~~ $f(x)\in C[x_1,x_n]$

∴ 记 $max=f(x_i)$, $min=f(x_j)$ 令 $x_i<x_i$

∴ $f(x_j)\le$ $\frac{1}{n}(f(x_1)+\cdots+f(x_n))\le f(x_i)$

∴ 介值定理，一定 $\exists\,\xi\in[x_i,x_j]$ ，使 $f(\xi)=\frac{1}{n}(f(x_1)+\cdots+f(x_n))$

98.44

~~$mf(c)+nf(d)=(m+n)f(\xi)$ 即 $m\{f(c)-f(\xi)\}=n\{f(c)-f(d)\}$~~

令 $F(x)=$ ~~$F(x)=f(x)-f(\xi)$ ∴ 即证 $\exists\,\xi$，使 $F(c)=-\frac{n}{m}$~~ $F(d)$

~~① $f(d)=f(c)$ ∴ $\xi=c$ 或 $\xi=d$~~

~~② $f(d)\ne f(c)$ 令 $f(c)<f(d)$~~

∴ ~~$\exists\,\xi$，使 $\exists\,\xi\in(c,d)$ ，使 $f(c)<f(\xi)<f(d)$~~

∴ ~~$F(c)<0$, $F(d$~~ $)>0$

98.44.

即证 $\exists\,\xi$，使 $f(\xi)=\frac{mf(c)+nf(d)}{m+n}$.

① $f(c)=f(d)$ 时 $f(\xi)=f(c)=f(d)$ ∴ $\xi=c\,|\,d$

② $f(c)\ne f(d)$ 令 $f(c)<f(d)$

∴ $f(c)<\frac{mf(c)+nf(d)}{m+n}<f(d)$ 又 ∵ $f(x)\in C[c,d]$

∴ $\exists\,\xi\in[c,d]$ ，使 $f(\xi)=\frac{mf(c)+nf(d)}{m+n}$ ∴得证

142.3

(1) $\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)-f(0)}{\Delta x}=f'(0)=\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)}{\Delta x}=\lim\limits_{x\to 0}\frac{f(x)}{x}$

(4) $\lim\limits_{h\to 0}\frac{f(x_0+3h)-f(x_0-h)}{h}\left(f(x_0+3h)+f(x_0-h)\right)=\cancel{4}\ 8f(x_0)f'(x_0)$

(5) $\lim\limits_{x\to x_0}\frac{xf(x_0)-x_0f(x_0)+x_0f(x_0)-x_0f(x)}{x-x_0}=\lim\limits_{x\to 0}\frac{f(x_0)-f(x)}{x-x_0}x_0+f(x_0)$

$=-f'(x_0)\cdot x_0+f(x_0)$

143.4.

(2) $\lim\limits_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{e^{a(x+\Delta x)}-e^{ax}}{\Delta x}=e^{ax}\lim\limits_{\Delta x\to 0}\frac{e^{a\Delta x}-1}{\Delta x}=e^{ax}\lim\limits_{\Delta x\to 0}\frac{a\Delta x}{\Delta x}$

$=ae^{ax}$

(4) $\lim\limits_{\Delta x\to 0}\frac{f(x+\Delta x)-f(\Delta x)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{(x+\Delta x)\sin(x+\Delta x)-x\sin x}{\Delta x}$

$=\frac{(x+\Delta x)(\sin x\cos\Delta x+\sin\Delta x\cos x)-x\sin x}{\Delta x}$

$=\lim\limits_{\Delta x\to 0}\sin x\cos\Delta x+\sin\Delta x\cos x+\lim\limits_{\Delta x\to 0}\frac{x(\sin x\cos\Delta x+\sin\Delta x\cos x-\sin x)}{\Delta x}$

$=\sin x+x\lim\limits_{\Delta x\to 0}\frac{\sin x(\cos\Delta x-1)}{\Delta x}+x\lim\limits_{\Delta x\to 0}\frac{\sin\Delta x\cos x}{\Delta x}$

$=\sin x+x\lim\limits_{\Delta x\to 0}\sin x\frac{-\frac{\Delta x^2}{2}}{\Delta x}+x\lim\limits_{\Delta x\to 0}\cos x$

$=\sin x+x\cos x.$

143.5

(1) $\lim\limits_{\Delta x\to 0}\frac{f^2(x+\Delta x)-f^2(x)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}(f(x+\Delta x)+f(x))=2f(x)f'(x)$

$\therefore 2f(0)f'(0)=A\Rightarrow f'(0)=\frac{A}{2f(0)}=\lim\limits_{\Delta x\to 0}\frac{A}{2f(0+\Delta x)}$ $\therefore$ $f(0)\neq 0$时，$f'(0)$存在.

$f(x)=0$ 且 $A=0$时 $f'(0)$存在；$f(x)=0$ 且 $A\neq 0$时，$f'(0)$不存在

(2) $\lim\limits_{\Delta x\to 0}\frac{(x_0+\Delta x)f(x_0+\Delta x)-x_0f(x_0)}{\Delta x}=x_0\lim\limits_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}+\lim\limits_{\Delta x\to 0}\frac{\Delta x}{\Delta x}f(x_0+\Delta x)$

又∵ $\lim\limits_{\Delta x\to 0}f(x_0+\Delta x)\ \exists$ ∴ 前者 $=x_0f'(x_0)\ \exists$ ∴ $f'(x_0)$存在.

143.6

$\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)-f(0)}{\Delta x}=f'(0)=2=\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)-1}{\Delta x}$

$\therefore$ $x\to 0$时 $f(x)=2x+1$ $\therefore \lim\limits_{x\to 0}[f(x)]^{\frac{1}{x}}=(2x+1)^{\frac{2}{2x}}=e^2$

143.7

$\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)-f(0)}{\Delta x}=f'(0)$ $\lim\limits_{\Delta x\to 0^-}\frac{f(\Delta x)-f(0)}{\Delta x}=-\lim\limits_{\Delta x\to 0^+}\frac{f(\Delta x)-f(0)}{\Delta x}=f'(0)=-f'(0)$

$\therefore f'(0)=0$

143.9

(1) $\forall x,y\in R$, $\therefore$ 令 $y=0$ $\therefore f(x)=f(x)\cdot f(0)\Rightarrow f(0)=1$ 或 $f(x)\equiv 0$

若 $f(x)\equiv 0$ $\therefore f'(x)=1$ 若 $f(0)=1$，则

$\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)-f(0)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)-1}{\Delta x}=1\Rightarrow \lim\limits_{x\to 0}f(x)=x+1.$

2 $\because f(0)=f(x)\cdot f(-x)\Rightarrow f(x)=\frac{1}{f(-x)}$

$\therefore \lim\limits_{y\to 0}f(x+y)=f(x)\cdot f(y)=f(x)\cdot(y+1)$ $\therefore \lim\limits_{y\to 0}\frac{f(x+y)-f(x)}{y}=\frac{yf(x)}{y}=f(x).$

$\therefore f'(x)=f(x)$

(2) $f(x)=f(x)+f(0)\Rightarrow f(0)=0$ $\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)}{\Delta x}\ \exists$ $\therefore \lim\limits_{x\to 0}f(x)=0$

$\lim\limits_{y\to 0}\frac{f(x+y)-f(x)}{y}=\lim\limits_{y\to 0}\frac{f(y)+2xy}{y}=2x+f'(0).$ $\therefore f'(x)=2x+f'(0)$

(3) $f(1)=af(0)$ $f'(0)=\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)-f(0)}{\Delta x}=b\Rightarrow \lim\limits_{x\to 0}f(x)=bx+f(0)$

$f'(1)=\lim\limits_{\Delta x\to 0}\frac{f(1+\Delta x)-f(1)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{af(\Delta x)-f(1)}{\Delta x}=a\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)-f(0)}{\Delta x}=af'(0)=ab.$

143.10

(b) $y=x^2\Rightarrow y'=2x$ $\therefore 2x_0=\tan(\arctan 3+\frac{\pi}{4})\Rightarrow x_0=-1$

$2x_0=\tan(\arctan 3-\frac{\pi}{4})\Rightarrow x_0=\frac{1}{4}$ $\therefore P(-1,1)$ 或 $P(\frac{1}{4},\frac{1}{16})$

143.11

$f'(x)=\lim\limits_{\Delta x\to 0}\frac{\frac{1}{x+\Delta x}-\frac{1}{x}}{\Delta x}=-\frac{1}{x^2}$ $\therefore l: y=-x+2$. $l_\perp: y=x$

143.13

$y = \frac{a^2}{x}$ ∴ $y' = a^2 \lim\limits_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x} = -a^2 \lim\limits_{\Delta x \to 0} \frac{\Delta x}{x(x+\Delta x)\Delta x} = -\frac{a^2}{x^2}$

∴ $l: y = -\frac{a^2}{x_0^2} x + \frac{2a^2}{x_0}$ ∴ $\left(0, \frac{2a^2}{x_0}\right), \left(2x_0, 0\right)$

∴ $S = \left|\frac{1}{2} \frac{2a^2}{x_0} 2x_0\right| = 2a^2$.

143.14.

(1) $\forall \varepsilon > 0, \exists \delta$, 当 $|x-a| < \delta$ 时. $|F(x) - F(a)| < \varepsilon$ 成立.

$\lim\limits_{\Delta x \to 0^+} \frac{f(a+\Delta x) - f(a)}{\Delta x} = \lim\limits_{\Delta x \to 0^+} \frac{|\Delta x| F(a+\Delta x)}{\Delta x} = \lim\limits_{\Delta x \to 0^+} F(a+\Delta x) = F(a) = 0$

$\lim\limits_{\Delta x \to 0^-} \frac{f(a+\Delta x) - f(a)}{\Delta x} = -F(a) \neq 0 \neq \lim\limits_{\Delta x \to 0^+} \frac{f(a+\Delta x) - f(a)}{\Delta x}$ ∴ $f(x)$ 在 a 处不可导

(2) $\lim\limits_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x} = \lim\limits_{\Delta x \to 0^+} F(a)$ ∴ $f'(a)$ 存在

143.15

(1) $\lim\limits_{\Delta x \to 0^+} \frac{\sin(x_0+\Delta x) - \sin x_0}{\Delta x} = \cos x_0 = 1$ 右

$\lim\limits_{\Delta x \to 0^-} \frac{(x_0+\Delta x)^2 - x_0^2}{\Delta x} = \lim\limits_{\Delta x \to 0^-} \frac{2x_0\Delta x + \Delta x^2}{\Delta x} = 2x_0 = 0 \neq \lim\limits_{\Delta x \to 0^+}$ — 左.

∴ 不可导

(3) $\lim\limits_{\Delta x \to 0^+} \frac{\frac{x+\Delta x}{1+e^{\frac{1}{x+\Delta x}}} - \frac{x}{1+e^{\frac{1}{x}}}}{\Delta x} = 0$ $\lim\limits_{\Delta x \to 0^-} \cdots = 1$

$\lim\limits_{\Delta x \to 0^+} f(x) \neq \lim\limits_{\Delta x \to 0^-} f(x)$ ∴ $f(x_0)$ 不存在

144.16

$\forall \varepsilon > 0$, $|\sin x - \sin 0| = \left|2\sin\frac{x}{2}\cos\frac{x}{2}\right| \leq \left|2\sin\frac{x}{2}\right|$

$x > 0$ 时. ~~$\lim\limits_{x \to 0^+} |2\sin\frac{x}{2}|$~~ $\left|2\sin\frac{x}{2}\right| = 2\sin\frac{x}{2} < x$

$x < 0$ 时. $\left|2\sin\frac{x}{2}\right| = -2\sin\frac{x}{2} > -x$ ∴ $\left|2\sin\frac{x}{2}\right| < |x|$

∴ $\left|2\sin\frac{x}{2}\right| \leq |x| \leq \delta$ ∴ $\delta = \varepsilon$, $|x| < \delta$ 时. $|f(x) - f(0)| < \varepsilon$ 成立

∴ $f(x)$ 在 0 处连续.

$\lim\limits_{\Delta x \to 0^+} \frac{|\sin \Delta x|}{\Delta x} = 1$ $\lim\limits_{\Delta x \to 0^-} \frac{|\sin \Delta x|}{\Delta x} = -1$ ∴ $f'(0)$ 不存在

144.18

充分性: 又∵ $f(0)=0$.

又∵ $\lim\limits_{\Delta x\to 0}\frac{F(\Delta x)-F(0)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{(1+|\sin\Delta x|)f(\Delta x)-f(0)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{(1+|\sin\Delta x|)f(\Delta x)}{\Delta x}$

又∵ $\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)}{\Delta x}$ 存在 ∴ $\lim\limits_{\Delta x\to 0}f(\Delta x)=0$

∴ $F'(0)=\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)}{\Delta x}\cdot\lim\limits_{\Delta x\to 0}(1+|\sin\Delta x|)=f'(0)$

必要性. ∵ $F'(0)$存在.

又∵ $F'(0)=\lim\limits_{\Delta x\to 0}\left(\frac{f(\Delta x)}{\Delta x}(1+|\sin\Delta x|)-\frac{f(0)}{\Delta x}\right)$.

又∵ $\lim\limits_{\Delta x\to 0}\frac{f(\Delta x)}{\Delta x}(1+|\sin\Delta x|)=f'(0)$ 存在. ∴ $\lim\limits_{\Delta x\to 0}\frac{f(0)}{\Delta x}$ 存在.

∴ $f(0)=0$

∴ 得证