

P156. 6

① $(A \in B \wedge B \subseteq C) \rightarrow A \in C$, 真

(1) $A \in B$ 前提

(5) $A \in C$ (1)(4) 分离.

(2) $B \subseteq C$ 前提.

~~(4)~~

(3) $(\forall y)(y \in B \rightarrow y \in C)$ (2) 全称特化.

(4) $A \in B \rightarrow A \in C$ (3) \forall

② $(A \in B \wedge B \subseteq C) \rightarrow A \subseteq C$ 假.

令 $A = \{1, 2\}$, $B = \{1, 3, \{1, 2\}\}$, $C = B$. $\therefore A \notin C$

③ $(A \subseteq B \wedge B \in C) \rightarrow A \in C$ 假.

令 $A = \{1, 2\}$, $B = \{1, 2, 3\}$, $C = \{\{1, 2, 3\}, 1\}$. $\therefore A \notin C$

④ $(A \in B \wedge B \notin C) \rightarrow A \notin C$ 假.

令 $A = \{1, 2\}$, $B = \{\{1, 2\}, 3\}$, $C = \{\{1, 2\}\} \cup A \in C$.

P156. 8

$B = P(P(P(\emptyset)))$, $P(\emptyset) = \{\emptyset\}$, $P(P(\emptyset)) = \{\emptyset, \{\emptyset\}\}$.

$B = \{\emptyset, \{\{\emptyset\}\}, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$.

(1) $\emptyset \in B$, $\emptyset \subseteq B$ (2) $\{\emptyset\} \in B$, $\{\emptyset\} \subseteq B$

(3) $\{\{\emptyset\}\} \in B$, $\{\{\emptyset\}\} \subseteq B$

P157, 15.

$$\text{PPP}(\emptyset) = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}.$$

$$\text{PP}(\emptyset) = \{\emptyset, \{\emptyset\}\}, \quad \text{P}(\emptyset) = \{\emptyset\}, \quad \emptyset = \emptyset$$

$$(1) \text{PPPP}(\emptyset) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\} = \text{PPP}(\emptyset)$$

$$(2) \text{PP}(\emptyset) = \{\emptyset\}.$$

P157, 16 $A = \{\{\emptyset\}, \{\{\emptyset\}\}\}.$

$$(1) \text{P}(A) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\{\emptyset\}, \{\{\emptyset\}\}\}\}.$$

$$\text{UP}(A) = \{\{\emptyset\}, \{\{\emptyset\}\}\}$$

$$(2) \text{UA} = \{\emptyset, \{\emptyset\}\}.$$

$$\text{P(UA)} = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}.$$

P158, 21

$$A \neq \emptyset \xrightarrow{\wedge \exists} (A \subseteq \text{P(UA)}) \quad * x \in A.$$

$$(1) A \neq \emptyset \text{ 前提.}$$

$$A \subseteq \text{P(UA)} \Leftrightarrow (\forall y)(y \in A \rightarrow y \in \text{P(UA)})$$

$$(2) (\exists y)(y \in A) \quad (1) \text{ 重复结论.}$$

$$\Leftrightarrow (\forall y)(y \in A \rightarrow y \subseteq \text{UA})$$

$$(3) x \in A \quad (1) \exists -$$

$$\Leftrightarrow (\forall y)(y \in A \rightarrow (\exists x)x(y \in A \wedge x \in y).)$$

$$\Leftrightarrow (\forall y)(y \in A \rightarrow (\exists x)x \in y)$$

$$(\exists y)(\exists x)(y \in A \wedge x \in y) \rightarrow (A \subseteq \text{P(UA)}).$$

$$A \subseteq \text{P(UA)} \Leftrightarrow (\forall y)(y \in A \rightarrow y \in \text{P(UA)}) \Leftrightarrow (\forall y)(y \in A \rightarrow y \subseteq \text{UA})$$

$$\Leftrightarrow (\forall y)(\forall x)(y \in A \rightarrow (x \in y \rightarrow x \in \text{UA})) \Leftrightarrow \text{True 在前提下.}$$

$$\text{UA} \Leftrightarrow \{x \mid (\exists y)(y \in A \wedge x \in y)\}. \quad \neq * y_0 \in A \wedge x_0 \in y_0 \therefore x_0 \in \text{UA}.$$

$$\therefore (\forall x)(x \in \text{UA} \rightarrow x \in \text{UA}) \therefore (\forall y)(y \in A \rightarrow (x \in y \rightarrow x \in \text{UA})).$$

P158. 28.

$$\cancel{x/2} \quad x \% 2 = 0, \quad x \% 3 = 0, \quad x \% 5 = 0, \quad 2, 3, 5 \text{ 互质}$$

$$\therefore x \% 30 = 0 \Rightarrow |A| = 8.$$