34.21

$m = k l^2$

$8 = k\times 16 \Rightarrow k=\frac{1}{2}$ $\therefore$ $m=\frac{1}{2}l^2$, $(l>0)$

34.22

(2) $4x^2-4x+y^2+2y=(2x-1)^2+(y+1)^2-2$

$\therefore \begin{cases} x = \frac{1}{2}(\sqrt{2}\cos\theta+1) \\ y=\sqrt{2}\sin\theta-1 \end{cases}$ $\theta\in[0,2\pi]$

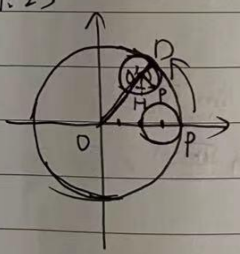
(4) $\frac{(x+1)^2}{4}=\cosh^2\theta$, $\frac{(y-2)^2}{9}=\sinh^2\theta$

$\therefore \begin{cases} x=\pm 2\cosh\theta-1 \\ y= 3\sinh\theta+2 \end{cases}$ $\theta\in R$.

$\sinh\theta$ 有正有负

(6) $x=\frac{1-e^y-y^3}{2}$ $\angle$ $\begin{cases} x=\frac{1}{2}-\frac{1}{2}e^t-\frac{1}{2}t^3 \\ y=t \end{cases}$ $t\in R$.

34.23



记切点为D. 令$\angle DO'P=\theta$.

$\therefore \overset{\frown}{DP'}=\frac{a}{4}\theta=\overset{\frown}{DP}=a\cdot\theta' \Rightarrow \theta'=\frac{\theta}{4}=\angle DOP$

$\therefore O'(\frac{3}{4}a\cos\frac{\theta}{4}, \frac{3}{4}a\sin\frac{\theta}{4})$

过O'作$O'H\perp x$轴交$\odot O'$于H.

$\therefore \angle OO'H=\frac{\pi}{2}-\frac{\theta}{4}$ $\therefore \angle HO'P+\angle PO'D=\frac{1}{2}\pi+\frac{\theta}{4}$

$\therefore \angle HO'P=\frac{\pi}{2}-\frac{3}{4}\theta$

$\therefore P'(\frac{3}{4}a\cos\frac{\theta}{4}+\frac{a}{4}\sin(\frac{\pi}{2}-\frac{3}{4}\theta), \frac{3}{4}a\sin\frac{\theta}{4}-\frac{a}{4}\cos(\frac{\pi}{2}-\frac{3}{4}\theta))$

$\therefore P'(\frac{3}{4}a\cos\frac{\theta}{4}+\frac{a}{4}\cos\frac{3}{4}\theta, \frac{3}{4}a\sin\frac{\theta}{4}-\frac{a}{4}\sin\frac{3}{4}\theta)$

$\therefore \begin{cases} x=a\cos^3\frac{\theta}{4} \\ y=a\sin^3\frac{\theta}{4} \end{cases}$

34.24.

(2) $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \Rightarrow r^2\cos^2\theta - 4r\cos\theta + r^2\sin^2\theta = 0$

$\Rightarrow r^2 = 4r\cos\theta \Rightarrow r = 4\cos\theta$. $\theta \in [0, \frac{\pi}{2}] \cup [\frac{3}{2}\pi, 2\pi]$

(4) $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \Rightarrow r^3 = r^2\cos 2\theta \Rightarrow r = \cos 2\theta$

$\theta \in [0, \frac{\pi}{4}] \cup [\frac{3}{4}\pi, \frac{5}{4}\pi] \cup [\frac{7}{4}\pi, 2\pi]$

9 2. ~~(1)~~ 2

(2) $\left|\frac{3n-2}{2n+1} - \frac{3}{2}\right| = \frac{7}{4n+2} < \frac{7}{4n} < \frac{2}{n} < \varepsilon \Rightarrow n > [\frac{2}{\varepsilon}] + 1$

$\therefore$ 对 $\forall \varepsilon > 0$, $\exists N = [\frac{2}{\varepsilon}] + 1 \in N_+$, 当 $n > N$ 时. $\left|\frac{3n-2}{2n+1} - \frac{3}{2}\right| < \varepsilon$

$\therefore \lim\limits_{n\to\infty} \frac{3n-2}{2n+1} = \frac{3}{2}$

(4) $\left|\sqrt[3]{x_n} - \sqrt[3]{a}\right| = \left|\frac{x_n - a}{x_n^{\frac{2}{3}} + x_n^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}}\right|$

当 $\forall \varepsilon > 0$, $\exists N_1 \in N_+$, $|x_n - a| < \varepsilon$ 对 $\forall n > N_1$ 成立.

① $a = 0$ $\therefore$ $n > N_1$ 时. $x_n < \varepsilon$, $\therefore x^{\frac{1}{3}} < \varepsilon^{\frac{1}{3}}$ $\therefore \lim\limits_{n\to+\infty} x_n^{\frac{1}{3}} = 0 = \sqrt[3]{a}$

② $a \neq 0$

$\lim\limits_{n\to\infty} \left|\frac{a - x_n}{x_n^{\frac{2}{3}} + x_n^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}}\right| = 0$ $\therefore \lim\limits_{n\to\infty}(\sqrt[3]{x_n} - \sqrt[3]{a}) = 0$ $\therefore \lim\limits_{n\to+\infty}\sqrt[3]{x_n} = \sqrt[3]{a}$.

(4) $\sin\sqrt{n+1} - \sin\sqrt{n} = 2\cos\frac{\sqrt{n+1}+\sqrt{n}}{2}\sin\frac{\sqrt{n+1}-\sqrt{n}}{2} < 2\sin\frac{\sqrt{n+1}-\sqrt{n}}{2} < \sqrt{n+1} - \sqrt{n} < \frac{1}{\sqrt{n}}$

$\therefore$ ~~lim~~ $\left|\frac{1}{\sqrt{n}}\right| < \varepsilon \Rightarrow n > [\frac{1}{\varepsilon^2}] + 1$

$\therefore \forall \varepsilon > 0$, $\exists N = [\frac{1}{\varepsilon^2}] + 1$, 当 $n > N$ 时, $|\sin\sqrt{n+1} - \sin\sqrt{n}| < \varepsilon$ 成立

$\therefore \lim\limits_{n\to+\infty}(\sin\sqrt{n+1} - \sin\sqrt{n}) = 0$

9.2.3

$\forall \varepsilon>0, \exists N_1 \in N_+, n>N_1$ 时 $\left|\frac{a_{n+1}}{a_n}\right|<\varepsilon$

要，则 $\frac{a_{n+1}}{a_n}<1$，当n充分大时. 又∵ $n>N_1$ 时. $\frac{a_{n+1}}{a_n}<\varepsilon$ 取 $\varepsilon \in (0,1)$

∴得证.

9.2.4

$\frac{a_n}{a_{n-1}} \leq q$ ∴ $\frac{a_n}{a_1} \leq q^{n-1}$ ∴ $a_n \leq q^{n-1} a_1$

当 $|q^{n-1} a_1|<\varepsilon$ 时 ∴ $n > [\log_{|q|} \frac{\varepsilon}{a_1}] + 2$

∴ $\forall \varepsilon>0, \exists N=[\log_{|q|} \frac{\varepsilon}{a_1}]+2 \in N_+$，当 $n>N$ 时. $|q^{n-1} a_1|<\varepsilon$

∴ $\lim\limits_{n \to +\infty} a_n = 0$

9.2.5

$\forall \varepsilon>0, \exists N_1 \in N_+$，当 $n>N_1$ 时. $|a_n - a|<\varepsilon$

又∵ $\varepsilon > |a_n - a| > ||a_n| - |a||$ ∴ $\lim\limits_{n \to \infty} |a_n| = |a|$

$a_n = (-1)^n$，$\{|a_n|\}$ 收敛于 $A=1$，$\{a_n\}$ 不收敛.

9.2.6

$\forall \varepsilon>0, \exists N_1 \in N_+, n>N_1$ 时. $|a_n - a|<\varepsilon \Rightarrow a-\varepsilon < a_n < \varepsilon + a \quad \varepsilon \in (0, |a|)$

$\left|\frac{a_{n+1}}{a_n} - 1\right| = \left|\frac{a_{n+1}-a_n}{a_n}\right| < \frac{a+\varepsilon-a+\varepsilon}{a-\varepsilon} = \left|\frac{2\varepsilon}{a-\varepsilon}\right|$ 取 $\varepsilon^* = |a|\varepsilon' > 0$

∴ 原式 $= \frac{2\varepsilon'}{1 \pm \varepsilon'}$ ① $\frac{2\varepsilon'}{1+\varepsilon'} < 2\varepsilon'$ ∴得证. ② $\frac{2\varepsilon'}{1-\varepsilon'} = \frac{2}{1-\varepsilon'} - 2$

对 $f(\varepsilon') = \frac{2}{1-\varepsilon'} - 2$，$f(\varepsilon')$ 在 $(0, 1)$ ↗，$D(f) = R(f)$ ∴ ε' 与 $f(\varepsilon')$ 双射 ∴ 得证.

当 $a=0$，$\lim\limits_{n \to \infty} \frac{a_{n+1}}{a_n}$ 存在，但不一定为1 或不存在.

$\left|\frac{a_{n+1}}{a_n} - 1\right| = \left|\frac{a_{n+1}-a_n}{a_n}\right| \leq \frac{|a_{n+1}-a| + |a_n - a|}{|a_n|}$ 取 $|a_{n+1} - a| < \frac{|a|}{3}\varepsilon$，$|a_n| > \frac{\varepsilon}{2}$

∴ 原式 $\leq \frac{\frac{7}{?}\varepsilon}{\frac{\varepsilon}{2}} = \frac{7}{6}\varepsilon$ ∴ 得.

92.7

$\forall \varepsilon>0$，$\exists N_1\in N_+$，当 $n>N_1$ 时，$\left|\frac{a_n}{b_n}-a\right|<\varepsilon$ 成立.

$\left|\frac{a_n}{b_n}-a\right|=\left|\frac{a_n-ab_n}{b_n}\right|$

$\forall \varepsilon>0$，$\exists N_2\in N_+$，当 $n>N_2$ 时，$|a_n|<\varepsilon$ 　若 $b_n\neq 0$.

$\therefore$ 当 $n>\max\{N_1,N_2\}$ 时. $\left|\frac{a_n-ab_n}{b_n}\right|>\left|\left|\frac{a_n}{b_n}\right|-\left|\frac{ab_n}{b_n}\right|\right|=\left|\frac{ab_n}{b_n}\right|=|a|>0$

此时 对 $\forall \varepsilon>0$，$|a|<\varepsilon$ 不成立. 即 $\lim\limits_{n\to\infty}\left|\frac{a_n}{b_n}-a\right|=0$ 不成立

$\therefore$ 矛盾 $\therefore$ ~~$b_n=0$~~ $n>\max\{N_1,N_2\}$ 时 $\therefore \lim\limits_{n\to\infty} b_n=0$

92.8.(3)

$\frac{n+(-1)^n}{n^2-1}<\frac{n+1}{n^2-1}=\frac{1}{n-1}<\varepsilon \Rightarrow n>[\frac{1}{\varepsilon}]+2$

$\therefore \forall \varepsilon>0$，$\exists N=[\frac{1}{\varepsilon}]+2\in N_+$，当 $n>N$ 时，$|a_n-0|<\varepsilon$.

$\therefore \lim\limits_{n\to\infty} a_n=0$ 　$\therefore \{a_n\}$ 为无穷小.

93.9.(2)

$\forall G>0$，$\frac{n^2+1}{3n-1}>\frac{n^2+1}{3n}>\frac{n^2}{3n}>n$ $\therefore$ 当 $n>[G]+1$ 时

$\forall G>0$，$\exists N=[G]+1\in N_+$，当 $n>N$ 时. $\frac{n^2+1}{3n-1}>G$

$\therefore \lim\limits_{n\to\infty}\frac{n^2+1}{3n-1}=+\infty$.

§3.12

(1) $\lim\limits_{n\to\infty} \dfrac{1+\frac{1}{2}+\cdots+\frac{1}{2^n}}{1+\frac{1}{3}+\cdots+\frac{1}{3^n}} = \dfrac{\lim\limits_{n\to\infty} 1+\frac{1}{2}+\cdots+\frac{1}{2^n}}{\lim\limits_{n\to\infty} 1+\frac{1}{3}+\cdots+\frac{1}{3^n}} = \dfrac{\frac{1}{1-\frac{1}{2}}}{\frac{1}{1-\frac{1}{3}}} = \dfrac{2}{\frac{3}{2}} = \dfrac{4}{3}$

(2) $\lim\limits_{n\to\infty} \dfrac{1^2+2^2+\cdots+n^2}{n^3} = \lim\limits_{n\to\infty} \dfrac{\frac{1}{6}n(n+1)(2n+1)}{n^3} = \lim\limits_{n\to\infty} \dfrac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$

$= \dfrac{1}{6}\lim\limits_{n\to\infty} \cancel{}\left(1+\frac{1}{n}\right)\cdot \lim\limits_{n\to\infty}\left(2+\frac{1}{n}\right) = \dfrac{1}{3}$

(5) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{1}{i(i+2)} = \lim\limits_{n\to\infty}\dfrac{1}{2}\left(\dfrac{1}{1}+\dfrac{1}{2}-\dfrac{1}{i+1}-\dfrac{1}{i+2}\right) = \cancel{\frac{2}{5}}\ \dfrac{3}{4}$

(7) $\left(1-\dfrac{1}{n^2}\right) = \dfrac{n^2-1}{n^2} = \dfrac{(n+1)(n-1)}{n^2}$

∴ 原式 $= \dfrac{1}{2}\times\dfrac{n+1}{n}$ ∴ $\lim\limits_{n\to\infty}$ 原式 $= \cancel{1}\ \dfrac{1}{2}$