P110.23

$$P(Y\le y \mid 0<X<\tfrac{1}{n}) = \frac{P(Y\le y,\ 0<X<\frac{1}{n})}{P(0<X<\frac{1}{n})}$$

$$= \frac{\int_{-\infty}^{y}\int_{0}^{\frac{1}{n}} f(x,y)\,d\sigma}{\int_{0}^{\frac{1}{n}}\left(\int_{-\infty}^{+\infty} f(x,y)\,dy\right)dx} = \begin{cases} 0 & y<0 \\ \frac{y(1+ny)}{n+1} & 0\le y<1 \\ 1 & y\ge 1 \end{cases}$$

P110.24

(1) $Z=XY$ $\therefore$ $P(Z\le z)=F(XY\le z)=\iint\limits_{XY\le z} f(x,y)\,d\sigma$

$$\therefore P(Z\le z)=\begin{cases} 1-e^{-z} & z>0 \\ 0 & \text{others} \end{cases}$$

(2) $f_X(x)=\int_{-\infty}^{+\infty} f(x,y)\,dy = \begin{cases} 10 & x\in(0.1,0.2) \\ 0 & \text{others} \end{cases}$

(3) $f_{Y|X}(y|x)=\dfrac{f(x,y)}{f_X(x)} = \begin{cases} xe^{-xy} & y>0,\ x\in(0.1,0.2) \\ 0 & \text{others} \end{cases}$

$\therefore P(Y\ge 1 \mid X=0.15)=\int_{1}^{+\infty} 0.15e^{-0.15y}\,dy = e^{-0.15}$

$P(Y\ge 1 \mid X=0.2) = e^{-0.2} < e^{-0.15}$ $\therefore$ 买0.15 买的多

P110.25

Z	0	1	2	3	4	5	6	7	8
P	0	0.02	0.06	0.13	0.19	0.24	0.19	0.12	0.05

M	0	1	2	3	4	5
P	0.0	0.04	0.16	0.28	0.24	0.28

N	0	1	2	3
P	0.28	0.3	0.25	0.17

P111. 26

$$P(Z=i) = P(X+Y=i) = \sum_{j=0}^{i} P(X=j)P(Y=i-j) = \sum_{j=0}^{i} p(j)q(i-j)$$

~~内~~补 1

$\cancel{X}$ $\cancel{P(X=0)=\frac{C_4^4}{C_7^4}}$ $\cancel{P(X=1)=\frac{C_3^1 \cdot C_4^3}{C_7^4}}$ $\cancel{P(X=2)=\frac{C_3^2 \cdot C_4^2}{C_7^4}}$ $\cancel{P(X=3)=\frac{C_4^1}{C_7^4}}$

$\cancel{P}$ $\cancel{=\frac{1}{35}}$ $\cancel{=\frac{12}{35}}$ $\cancel{=\frac{18}{35}}$ $\cancel{=\frac{4}{35}}$

$P(X=Y=1) = \frac{C_3^1 C_2^1}{C_7^4} = \frac{6}{35}$

$\therefore P(X=Y) = \frac{9}{35}$

$P(X=Y=2) = \frac{C_3^2 C_2^2}{C_7^4} = \frac{3}{35}$

补 2.

$X \sim B(2, 0.2)$ $Y \sim B(2, 0.5)$.

$P(X=0) = C_2^0 0.8^2 = 0.64$ $P(X=1) = C_2^1 0.2 \times 0.8 = 0.32$. $P(X=2) = 0.04$

$P(Y=0) = 0.25$ $P(Y=1) = 0.5$ $P(Y=2) = 0.25$

$\therefore P(X \leq Y) = \cancel{0.64 \times 0.25} + 0.64 \times 1 + 0.32 \times 0.75 + 0.04 \times 0.25$

$= 0.89$.

补 3

$P(X<0.5, Y<0.5) + P(X<0.5, Y>0.5) + P(X>0.5, Y<0.5)$

$= 1 - P(X>0.5, Y>0.5)$

$P(X>0.5, Y>0.5) = P(X>0.5)P(Y>0.5)$ X, Y 独立

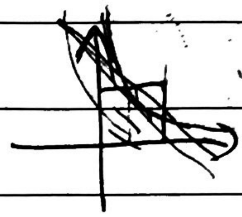
$= \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ $X \sim U(0,1)$

$\therefore P(\text{所求}) = \frac{5}{8}$ $Y \sim U(0,2)$.

补 4.

$X\sim U(0,1)$ $Y\sim U(0,1)$ $f(x,y)=\begin{cases}1 & x,y\in(0,1)\\ 0 & \text{others.}\end{cases}$

$M=XY$, $N=X+Y$



$\therefore P(M\ge \frac{3}{16}, N\le 1) = P(XY\ge\frac{3}{16}, X+Y\le 1)$

$=P(XY\ge \frac{3}{16}\mid X+Y\le1)\,P(X+Y\le1)$

$F_M(m)=\iint\limits_{XY\le m} f(x,y)\,d\sigma = \int_{-\infty}^{+\infty}dx\int_{-\infty}^{\frac{m}{x}} f(x,y)\,dy$

$=\begin{cases} m(1-\ln m) & 0\le m<1\\ 0 & m<0 \\ 1 & m\ge 1\end{cases}$

$F_N(n)=\iint\limits_{X+Y\le n} f(x,y)\,d\sigma$

$=\int_{-\infty}^{+\infty}dx\int_{-\infty}^{n-x}f(x,y)\,d\sigma = \begin{cases}0 & n<0\\ -\frac12 n^2+2n-1 & 0\le n<2 \\ 1 & n\ge 2.\end{cases}$

$\therefore$ ~~联合~~ $f_M(m)=-\ln m$, $f_N(n)=-n+2.$

几何概型: $P=\frac{\iint d\sigma}{1}=\frac14+\frac{3}{16}\ln 3$

用公式的做法再想想

补 5 $P(A)=\frac14$, $P(B)=\frac16$

$P(AB)=P(A)P(B|A)=\frac14\times\frac13=P(A|B)P(B) \Rightarrow P(B)=\frac16$

$\therefore P(X=0,Y=0)=P(\bar A\bar B)=\cancel{P(\bar A|\bar B)P(\bar B)}=P(\overline{A+B})=1-P(A+B)=\frac23$

X\Y	0	1	
0	$\frac23$	$\frac{1}{12}$	$\frac34$
1	$\frac16$	$\frac{1}{12}$	$\frac14$
	$\frac56$	$\frac16$	

其它问题

Z	0	1	2
P	$\frac23$	$\frac14$	$\frac{1}{12}$

补6.

$X \sim U(1,3) \quad Y \sim U(2,5)$

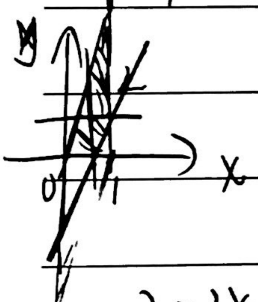
$Z = X+Y$.

$P(Z \le z) = P(X+Y \le z) = \iint_{x+y \le z} f(x,y)\,dx\,dy \quad 3 \le z \le 8$

$\therefore F_Z(z) = \begin{cases} \frac{1}{12}(z-3)^2 & 3 \le z < 5 \\ \frac{z}{3} - \frac{4}{3} & 5 \le z < 6 \\ \frac{2}{3} + \frac{1}{6}(-30 + 6z - \frac{1}{2}z^2) & 6 \le z \le 8. \end{cases}$

$\therefore f_Z(z) = \begin{cases} \frac{1}{6}(z-3) & 3 \le z < 5 \\ \frac{1}{3} & 5 \le z < 6 \\ \frac{4}{3} - \frac{1}{6}z & 6 \le z \le 8. \\ 0 & \text{others}. \end{cases}$

补7.



(1) $f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \begin{cases} 1 - \frac{y}{2} & 0 \le y \le 2 \\ 0 & \text{others} \end{cases}$

$f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \begin{cases} 2x & 0 \le x < 1 \\ 0 & \text{others}. \end{cases}$

(2). $Z = 2X - Y$, $\cancel{Z \in (-2,0)}$. $Z \in (0,2)$.

$P(Z \le z) = F(2X - Y \le z) = F(X \le \frac{Y+z}{2})$

$= \int_{-\infty}^{+\infty} dy \int_{-\infty}^{\frac{y+z}{2}} f(x,y)\,dx = \begin{cases} 0 & z \le 0 \\ 1 & z \ge 2. \\ 1 - \frac{1}{2}(2 - 2z + \frac{z^2}{2}) & 0 < z < 2. \end{cases}$

$\therefore f_Z(z) = \begin{cases} 1 - \frac{z}{2} & 0 \le z \le 2 \\ 0 & \text{others} \end{cases}$

(3) $P(Y \le \frac{1}{2} \mid X \le \frac{1}{2}) = \frac{P(Y \le \frac{1}{2}, X \le \frac{1}{2})}{P(X \le \frac{1}{2})} = \frac{3}{4}.$

P111. 29

$P(Z=0)=P(X=1,Y=0)+P(X=0,Y=1)=2p(1-p)$

$P(Z=1)=1-2p(1-p)$

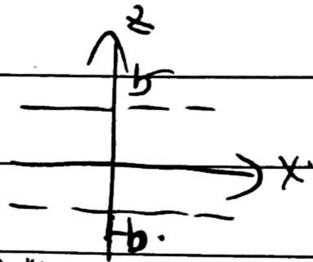
$P(Z=0,X=0)=P(Z=0|X=0)P(X=0)=(1-p)\cancel{(1-p)}p.$

若独立，则 $P(Z=0,X=0)=P(Z=0)\cdot P(X=0) \Rightarrow p=\frac{1}{2}$

P111. 31

$\cancel{f_Z(z)=\int_{-\infty}^{+\infty} f_X(x)\cdot f_Y(z-x)\,dx}$

$f_X(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$, $f_Y(y)=\begin{cases}\frac{1}{2b} & -b<y<b \\ 0 & \text{others.}\end{cases}$



$\therefore -b<z-x<b \Rightarrow x-b<z<b+x$

$F_Z(z)=P(X+Y\le z)=P(X\le z-Y)$

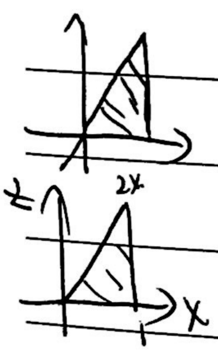
$=\iint_{x+y\le z} f_X(x)f_Y(y)\,d\sigma=\int_{-\infty}^{+\infty}dx\int_{-\infty}^{z-x}f_X(x)f_Y(y)\,dy$

$z-b<x<z+b.$

$f_Z(z)=\int_{-\infty}^{+\infty}f_X(x)f_Y(z-x)\,dx=\frac{1}{2b}\left[\Phi\left(\frac{z+b-\mu}{\sigma}\right)-\Phi\left(\frac{z-b-\mu}{\sigma}\right)\right]$

P112. 32.

$f(x,y)=\begin{cases}1 & 0<x<1,\ 0<y<2x \\ 0 & \text{o.w}\end{cases}$



$f_Z(z)=\frac{d}{dz}\cancel{\int_{-\infty}^{+\infty}}P(2X-Y\le z)=P\left(X\le\frac{Y+z}{2}\right)$

$=\frac{d}{dz}\iint_{2x-y\le z}f(x,y)\,d\sigma=\int_{-\infty}^{+\infty}f(x,2x-z)\,dx$

$=\begin{cases}\int_{\frac{z}{2}}^{1}1\,dx & 0\le z<2 \\ 0 & z<0 \\ 0 & z\ge 2\end{cases}=(1-\frac{z}{2})\ 0\le z\le 2.$

P112. 34.

$P(Z\le z)=P(\frac{X}{Y}\le z)=\iint_{\frac{X}{Y}\le z} f(x,y)\,d\sigma$

$=\int_0^{+\infty}dy\int_{-\infty}^{yz} f(x,y)\,dx+\int_{-\infty}^{0}dy\int_{yz}^{+\infty} f(x,y)\,dx$

$\therefore f_Z(z)=\int_{-\infty}^{+\infty} f(yz,y)|y|\,dy=\begin{cases}\int_{\frac{10}{z}}^{\infty}\frac{1}{z^2}\frac{100}{y^3}dy & 0<z<1\\ 0 & z<0\\ \int_{10}^{\infty}\frac{1}{z^2}\frac{100}{y^3}dy & z>1\end{cases}$

$\therefore f_Z(z)=\begin{cases}\frac{1}{2} & 0<z<1\\ 0 & z<0\\ \frac{1}{2}z^{-2} & z>1\end{cases}$

P112. 35

$f_X(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}}$ $f_Y(y)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{y^2}{2\sigma^2}}$

$Z=\sqrt{X^2+Y^2}$. 易知 $f_Z(z)=0$ $(z<0)$

$P(Z\le z)=P(X^2+Y^2\le z^2)=\iint_{x^2+y^2\le z^2} f_X(x)f_Y(y)\,d\sigma$

$=\int_0^{2\pi}d\theta\int_0^{z} f_X(r\cos\theta)f_Y(r\sin\theta)\cdot r\,dr$

$\therefore f_Z(z)=\int_0^{2\pi} f_X(z\cos\theta)f_Y(z\sin\theta)\,z\,d\theta$

$=\begin{cases}\frac{z}{\sigma^2}e^{-\frac{z^2}{2\sigma^2}} & z\ge 0\\ 0 & z<0\end{cases}$

P112. 36

$f_{X_i}(x)=\begin{cases}\frac{x}{4}e^{-\frac{x^2}{8}} & x\ge 0\\ 0 & x<0\end{cases}$

(1) $F_Z(z)=\{F_{X_i}(z)\}^n=\begin{cases}(1-e^{-\frac{z^2}{8}})^5 & z\ge 0\\ 0 & z<0\end{cases}$

(2) $1-F_Z(4)=1-(1-e^{-2})^5=0.5167$