P64. 11

$X \sim B(15, 0.2)$ 二项分布.

$P(X=k) = C_{15}^{k}\, 0.2^{k}\, 0.8^{15-k}$.

(1) $P(X=3) = 0.25$.

(2) $P(X \geq 2) = 1 - P(X<2) = 1 - P(X=0) - P(X=1) = 0.833$

(3) $P(1 \leq X \leq 3) = P(X=1) + P(X=2) + P(X=3) = 0.613$

(4) $P(X \geq 5) = \sum_{i=5}^{15} C_{15}^{i}\, 0.2^{i}\, 0.8^{15-i} = 0.0608$

P64. 13.

$A=\{$带菌$\}$ $B=\{$阳$\}$ $C=\{(1)\}$.

$\therefore$ $P(A)=0.1$, $P(\bar{A})=0.9$ $P(B|A)=0.95$. $P(B|\bar{A})=0.01$

$P(\bar{B}|A)=0.05$ $P(\bar{B}|\bar{A})=0.99$.

(1) $P(C)=P(A)P(C|A)+P(\bar{A})P(C|\bar{A})$

$=0.1\times C_3^2\, 0.95^2\, 0.05+0.9\times C_3^2\, 0.01^2\times 0.99$

$=0.0138$.

(2). $P(A|C)=\dfrac{P(AC)}{P(C)}=\dfrac{0.1\times C_3^2\, 0.95^2\, 0.05}{0.0138}=0.9806$.

P14. 14

$(20+1)\cdot 0.3 = 6.3$ ∴最大的P为6点

$P(X=6) = C_{20}^{6}\, 0.3^{6} \cdot 0.7^{14} = 0.1916$

P64. 15

$p=\frac{1}{500}$. $X\sim B(500,\frac{1}{500})$

$P(X\geq 3)=1-P(X<3)=1-P(X=0)-P(X=1)-P(X=2)$

$=1-\sum_{k=0}^{2} C_{500}^{k}\left(\frac{1}{500}\right)^k\cdot\left(\frac{499}{500}\right)^{500-k}$

$\lambda=np=1$ $\quad =1-\sum_{k=0}^{2}e^{-1}\frac{1^k}{k!}=1-\frac{5}{2e}\approx 0.0803.$

P64. 16

设 $P(X=k)=e^{-3}\frac{3^k}{k!}$ 设存n件

$\therefore P(X\leq n)=\sum_{i=0}^{n}e^{-3}\frac{3^i}{i!}\geq 99.6\%$

查表, $n=8$.

P64 18

(1) $P(X=k)=e^{-10}\frac{10^k}{k!}$, $P(X>15)=1-P(X\le 15)$ $0.0487.$

$$=1-\sum_{k=0}^{15} e^{-10}\frac{10^k}{k!}=1-0.9513=\cancel{0.487}$$

(2) $P(X\le 0)=0.5$. $P(X\le 0)=P(X=0)=e^{-\lambda}\Rightarrow \lambda=\ln 2$.

$P(X\ge 2)=1-P(X=0)-P(X=1)=1-\frac{1}{2}\frac{\ln 2}{1}-\frac{1}{2}\frac{\ln^2 2}{2}$

$=1-\frac{1}{2}\ln 2-\frac{1}{4}\ln^2 2$

P65. 23

$F(-\infty)=0$, $F(2)=b+2a=1$. $F(0)=a\Rightarrow b=a$

$\therefore a=\frac{1}{3}=b$; $\cancel{F(x)=}$

P65. 26

$\cancel{F(x)=\int_{-\infty}^{x} f(x)dx=\begin{cases}0 & x<0\\ \frac{1}{2}ax^2 & 0\le x<1\\ \frac{1}{2}a+bx-\frac{1}{2}x^2 & 1\le x<2\\ 1 & x\ge 2\end{cases}}$ $\cancel{a=4}$ $b=\frac{1}{2}$.

(1)

$P(X<1)=\frac{1}{2}\Rightarrow P(X<1)=P(X<0)+P(0\le X<1)=0+\int_0^1 ax=\frac{1}{2}\Rightarrow a=1$.

$P(X\ge 1)=\frac{1}{2}\Rightarrow P(1\le X<2)+P(X\ge 2)=P(1\le X<2)=\int_1^2 (b-x)dx=\frac{1}{2}\Rightarrow b=2$.

(2) $f(x)=\begin{cases}x & 0\le x<1\\ 2-x & 1\le x<2\\ 0 & x<0,\ x\ge 2.\end{cases}$ $\therefore F(x)=\int_{-\infty}^{x} f(x)dx=\begin{cases}0, & x<0\\ \frac{1}{2}x^2 & 0\le x<1\\ \cancel{2x-\frac{1}{2}x^2-1} & 1\le x<2\\ 1 & x\ge 2.\end{cases}$

P65 27.

(1) $\int_{-\infty}^{+\infty} f(x) = \int_{-1}^{1} \frac{a}{\sqrt{1-x^2}}dx = a\arccos x\Big|_{-1}^{1} = \pi a = 1 \Rightarrow a = \frac{1}{\pi}$

(2) $P(-\frac{1}{2}<X<\frac{1}{2}) = F(\frac{1}{2}) - F(-\frac{1}{2}) = \frac{1}{3}$

(3) $F(x) = \begin{cases} 0 & x\le -1 \\ \frac{1}{\pi}\arcsin x + \frac{1}{2} & -1<x<1 \\ 1 & x\ge 1 \end{cases}$

P66. 32

$X\sim E(\frac{1}{50})$ ∴ $f(x) = \begin{cases} \frac{1}{50}e^{-\frac{1}{50}x} & x\ge 0 \\ 0 & x<0 \end{cases}$

(1) $F(x) = \begin{cases} 1-e^{-\frac{1}{50}x} & x\ge 0 \\ 0 & x<0 \end{cases}$ ∴ $P(X\le 2) = F(2) = 1-e^{-\frac{1}{25}}$

(2) $P(X\ge 10) = F(\infty)-F(10) = 1-F(10) = e^{-\frac{1}{5}}$

(3) $P(X\ge 20 \mid X\ge 10) = P(X\ge 20-10) = P(X\ge 10) = e^{-\frac{1}{5}}$

P67. 33

$T\sim E(0.5)$,

$F(T) = \begin{cases} 1-e^{-\frac{1}{2}T} & T\ge 0 \\ 0 & T<0 \end{cases}$

$P(T>10) = 1-P(T\le 10) = 1-F(10) = e^{-5}$

∴ 记有X人超10min ∴ $X\sim B(282, e^{-5})$.

$P(X\ge 2) = 1-P(X=1)-P(X=0)$

$= 1 - C_{282}^{1}\cdot e^{-5}\cdot(1-e^{-5})^{281} - C_{282}^{0}e^{-5\cdot 0}\cdot(1-e^{-5})^{282}$.

$\lambda = np = 282e^{-5}$ ∴ $P(X=1) = e^{-\lambda}\frac{\lambda}{1}$, $P(X=2) = e^{-\lambda}\frac{\lambda^2}{2}$.