P67.42

(1) $f_X(x)=\begin{cases}\frac{1}{5} & -2\le x\le 3\\ 0 & \text{others}\end{cases}$ $F_{Y_1}(y)=P(Y_1\le y)$

$y<-1$. $F_{Y_1}(y)=0$

$f_{Y_1}(y)=0\ (y\ne\pm1)$ $y\ge 1$ $F_{Y_1}(y)=1$

$-1\le y\le 1$ $F_{Y_1}(y)=P(X<0)=F_X(0)$

$\therefore F_{Y_1}(y)=\begin{cases}0 & y<-1\\ \frac{2}{5} & -1\le y<1\\ 1 & y\ge 1\end{cases}$

P	0.4	0.6
Y_1	-1	1

(2) $F_{Y_2}(y)=P(Y_2\le y)=P(X\le 2y-1)=F_X(2y-1)$. $Y_2\in[-\frac{1}{2},2]$.

$F_X(x)=\begin{cases}0 & x<-2\\ \frac{x+2}{5} & -2\le x<3\\ 1 & x\ge 3\end{cases}$ $\therefore$ $=\int_{-2}^{2y-1}\frac{1}{5}dx$

$\therefore \cancel{F_{Y_2}(y)=\begin{cases}0 & y<-\frac{1}{2}\\ \frac{4y+2}{5} & -\frac{1}{2}\le y<2\\ 1 & y\ge 2\end{cases}}$ $f_{Y_2}(y)=\begin{cases}0 & \text{others}\\ \frac{2}{5} & -\frac{1}{2}\le y\le 2.\end{cases}$

P67.43.

(1) $Y_1=X^2$. $F_{Y_1}(y)=P(Y_1\le y)=P(X^2\le y)=P(|X|\le\sqrt{y})=2\Phi(\sqrt{y})-$

$\therefore F_{Y_1}(y)=\begin{cases}0 & y<0\\ 2\Phi(\sqrt{y})-1 & 0\le y\end{cases}$ $\therefore f_{Y_1}(y)=\begin{cases}0 & y\le 0\\ \frac{1}{\sqrt{2\pi y}}e^{-\frac{y}{2}} & y>0.\end{cases}$

(2) $Y_2=e^{-X}$. $F_{Y_2}(y)=P(Y_2\le y)=P(e^{-X}\le y)=P(X\ge\ln\frac{1}{y})$

$Y_2>0$ $=1-P(X\le\ln\frac{1}{y})=1-\Phi(\ln\frac{1}{y})$.

$\therefore F_{Y_2}(y)=\begin{cases}0 & y\le 0\\ 1-\Phi(-\ln y) & y>0\end{cases}$ $\therefore f_{Y_2}(y)=\begin{cases}0 & y\le 0\\ \frac{1}{y}\frac{1}{\sqrt{2\pi}}e^{-\frac{\ln^2 y}{2}} & y>0.\end{cases}$

(3) $Y_3=\begin{cases}0 & X\le 0\\ 2X & X>0\end{cases}$ $F_{Y_3}(y)=P(Y_3\le y)=P(X\le 0)+P(2X\le$

$=\frac{1}{2}+\Phi(\frac{y}{2})-\frac{1}{2}$

$\therefore F_{Y_3}(y)=\begin{cases}0 & y<0\\ \Phi(\frac{y}{2}) & y\ge 0.\end{cases}$

p67. 44

$\cancel{P}\ F_Y(y) = P(Y\le y) = P(3-2X\le y) = P(X \ge \frac{3-y}{2})$

$= 1 - P(X \le \frac{3-y}{2})$

$\therefore F_Y(y) = 1 - F_X(\frac{3-y}{2}) + F_X(\frac{3-y}{2}) - F_X(\frac{3-y}{2}-0) = 1 - P(X<\frac{3-y}{2}) + P(X=\frac{3-y}{2})$

$= 1 - F_X(\frac{3-y}{2}-0)$

p68. 46

$X = \frac{1}{\tan\theta}$ $\therefore$ $\cancel{\theta x}$

$F(x) = P(X\le x) = P(\frac{1}{\tan\theta}\le x)$

$f_\Theta(\theta) = \begin{cases}\frac{1}{\pi} & 0\le\theta\le\pi \\ 0 & \text{others}\end{cases}$ $\therefore F_\Theta(\theta) = \begin{cases}0 & \theta<0 \\ \frac{\theta}{\pi} & 0\le\theta<\pi \\ 1 & \theta\ge\pi\end{cases}$

$x\le 0$ 时. $\cancel{P(\frac{1}{\tan\theta}\le x) = P(\tan\theta\ge\frac{1}{x})} = P(\arctan\frac{1}{x}\le\theta)$

$\cancel{= 1 - P(\theta\le\arctan\frac{1}{x}) = 1 - F(\arctan\frac{1}{x})}$

$P(\frac{1}{\tan\theta}\le x) = P(\pi+\arctan\frac{1}{x}\le\theta) = 1 - F(\pi+\arctan\frac{1}{x})$

$= 1 - \frac{\pi+\arctan\frac{1}{x}}{\pi} = -\frac{1}{\pi}\arctan\frac{1}{x}$

$x>0$ 时 $P(\frac{1}{\tan\theta}\le x) = \cancel{P(\frac{\pi}{2}\le\theta)}\ P(\arctan\frac{1}{x}<\theta\le\frac{\pi}{2}) + P(\frac{\pi}{2}<\theta<\pi)$

$= 1 - \frac{1}{\pi}\arctan\frac{1}{x}$

$\therefore f(x) = \frac{1}{\pi}\frac{1}{1+x^2}$ $x\in R$

补1.

$P(x < X \le x+\Delta x) = k\Delta x = F(x+\Delta x) - F(x)$

$\therefore k \cdot a = F(a) - F(0) = 1$. $F(0) = 0$ $\therefore k = \frac{1}{a}$

$$\therefore F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{a} & 0 \le x < a \\ 1 & x \ge a \end{cases}$$

补2.

设 X 为感冒次数 $\therefore X \sim P(5)$，$P(X=k) = e^{-\lambda}\frac{\lambda^k}{k!}$

设：$A = \{$两次感冒$\}$，$B = \{$有效$\}$.

$\therefore P(B) = \cancel{75\%}\ \frac{3}{4}$，$P(\bar{B}) = \frac{1}{4}$

$P(X=k \mid B) = e^{-3}\frac{3^k}{k!}$，$P(X=k \mid \bar{B}) = e^{-5}\frac{5^k}{k!}$

$$P(B \mid A) = \frac{P(AB)}{P(A)} = \cancel{\frac{P(A|B)P(B)+P(A|\bar{B})P(\bar{B})}{P(A)}}$$

$$= \frac{P(A|B)P(B)}{P(A|B)P(B)+P(A|\bar{B})P(\bar{B})} = \frac{27e^2}{25+27e^2}$$

补3.

$P(|X-\mu| < \sigma) = 2\Phi(1) - 1$，为定值 $\therefore$ 不变.

~~随 $\sigma \uparrow$，$\Phi(\sigma)\uparrow$ $\therefore$ 变大~~

补4.

$X \sim E(\lambda)$，$\cancel{P(X\le}$ $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$ $F(x) = \begin{cases} 0 & x < 0 \\ 1-e^{-\lambda x} & x \ge 0 \end{cases}$

$Y = \begin{cases} 2 & X \ge 2 \\ X & X \le 2 \end{cases}$ $\therefore Y \in [0, 2]$

$P(Y \le y) = P(X \le y) = F(y) = 1 - e^{-\lambda y}$ $y \in [0,2]$

$$\therefore F_Y(y) = \begin{cases} 0 & y < 0 \\ 1-e^{-\lambda y} & 0 \le y < 2 \\ 1 & y \ge 2 \end{cases}$$ $\therefore Y$ 是非连续型 --

补5.

$V \sim N(120, 2^2)$ 记事 $A=\{$落在118, 122之外$\}$.

$\therefore P(A) = 1 - P(118 \le V \le 122) = 1 - P(122) + P(118) = 1 - \Phi(\frac{122-120}{2}) + \Phi(\frac{118-120}{2})$

$= 2(1-\Phi(1)) = \cancel{0.32}\ 0.3128$

设 $Y=\{Y$次在$A\}$. $\therefore Y \sim B(5, P(A))$

$\therefore P(Y=2) = C_5^2 P(A)^2 (1-P(A))^3 = \cancel{0.22}\ \cancel{0.32}\ 0.318$

P106 3.

(1) $P(X=i, Y=j) = \begin{cases} \frac{1}{n}\frac{1}{n-1} & i \ne j \\ 0 & i=j \end{cases}$

(2) $n=3$. $\therefore P(X=i, Y=j) = \frac{1}{6}$ $i \ne j$.

X\Y	1	2	3
1	0	$\frac{1}{6}$	$\frac{1}{6}$
2	$\frac{1}{6}$	0	$\frac{1}{6}$
3	$\frac{1}{6}$	$\frac{1}{6}$	0

P107. 4

$f_X(x) = \begin{cases} \frac{1}{3} & -1 \le x \le 2 \\ 0 & \text{others.} \end{cases}$

$\{(Y_1, Y_2)\} = \{(0,1), (1,1), (1,-1), (2,-1)\}$.

$P(Y_1=0, Y_2=1) = P(X<0) = P(X \le 0) = \int_{-1}^{0} \frac{1}{3} dx = \frac{1}{3}$

$P(Y_1=1, Y_2=1) = P(X=0) = 0$

$P(Y_1=1, Y_2=-1) = P(0<X<1) = \int_0^1 \frac{1}{3} dx = \frac{1}{3}$

$P(Y_1=2, Y_2=-1) = P(X \ge 1) = \int_1^2 \frac{1}{3} dx = \frac{1}{3}$

$P(Y_1=0) = \frac{1}{3}$, $P(Y_1=1) = \frac{1}{3}$, $P(Y_1=2) = \frac{1}{3}$

$P(Y_2=1) = \frac{1}{3}$, $P(Y_2=-1) = \frac{2}{3}$

P107.5

(1) $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\,d\sigma = k\left(\int_0^{+\infty} e^{-2x}dx\right)\left(\int_0^{+\infty} e^{-4y}dy\right) = 1$

$\Rightarrow k = 8$

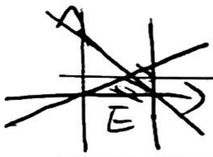
(2) $P(0\le X\le 2, 0\le Y\le 1) = \int_0^2\int_0^1 8e^{-2x}e^{-4y}\,d\sigma = (1-e^{-4})^2$

(3) $P(X+Y<1) = \int_0^{1}\int_0^{1-x} 8e^{-2x}e^{-4y}\,dy\,dx = 1+e^{-4}-2e^{-2}.$

(4) $F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(x,y)\,d\sigma = 8\left(\int_0^x e^{-2x}dx\right)\left(\int_0^y e^{-4y}dy\right)$

$\therefore F(x,y) = \begin{cases} (1-e^{-2x})(1-e^{-4y}), & x>0, y>0 \\ 0 & \text{others.} \end{cases}$

P107.7



(1) $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\,d\sigma = \int_0^2 dx\int_0^{\frac{x}{2}} kx\,dy = 1$

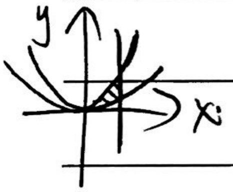
$\Rightarrow k = \frac{3}{4}$

(2) $P(X+Y\le 2) = \iint_E f(x,y)\,d\sigma = \int_0^{\frac{2}{3}}dy\int_{2y}^{2-y}\frac{3}{4}x\,dx = \frac{5}{9}.$

(3) $f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \int_0^{\frac{x}{2}}\frac{3}{4}x\,dy = \frac{3}{8}x^2 \quad \therefore f_X(x) = \begin{cases} \frac{3}{8}x^2 & 0<x<2 \\ 0 & \text{others} \end{cases}$

$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \int_{2y}^{2}\frac{3}{4}x\,dx = \frac{3}{2}(1-y^2) \quad \therefore f_Y(y) = \begin{cases} \frac{3}{2}(1-y^2) & 0<y<1 \\ 0 & \text{others} \end{cases}$

P107.9.



(1) $f(x,y) = \begin{cases} \frac{1}{A_G} & (x,y)\in G \\ 0 & \text{others.} \end{cases}$

$A_G = \int_0^1 x^2 - \frac{x^2}{2}\,dy = \frac{1}{6} \quad \therefore f(x,y) = \begin{cases} 6 & (x,y)\in G \\ 0 & \text{others.} \end{cases}$

(2) $f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \int_{\frac{x^2}{2}}^{x^2} 6\,dy = 3x^2 \quad \therefore f_X(x) = \begin{cases} 3x^2 & x\in(0,1) \\ 0 & \text{others.} \end{cases}$

$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx$

$y\le\frac{1}{2},\ f_Y(y) = \int_{\sqrt{y}}^{\sqrt{2y}} 6\,dx = 6(\sqrt{2y}-\sqrt{y})$

$\frac{1}{2}<y\le 1,\ f_Y(y) = \int_{\sqrt{y}}^{1} 6\,dx = 6-6\sqrt{y}.$

$\therefore f_Y(y) = \begin{cases} 6\sqrt{2y}-6\sqrt{y} & 0<y\le\frac{1}{2} \\ 6(1-\sqrt{y}) & \frac{1}{2}<y\le 1 \\ 0 & \text{others.} \end{cases}$

补. $X=\{$点数为偶$\}$. $Y=\{$点数为6$\}$. $X\sim B(3,\frac{1}{2})$ $Y\sim B(X,\frac{1}{3})$.

(1) $\cancel{P}(X,Y)=\{(0,0),(1,0),(1,1),(2,0),(2,1),(2,2),(3,0),(3,1),(3,2),(3,3)\}$

$P(X=0,Y=0)=P(X=0)P(Y=0|X=0)=\frac{1}{8}$

$P(X=1,Y=0)=P(X=1)P(Y=0|X=1)=C_3^1\frac{1}{2}\cdot\frac{1}{2}^2\cdot C_1^0\frac{2}{3}=\frac{1}{4}$

$P(X=1,Y=1)=P(X=1)P(Y=1|X=1)=C_3^1\frac{1}{2}\frac{1}{2}^2\cdot C_1^1\frac{1}{3}=\frac{1}{8}$

$P(X=2,Y=0)=P(X=2)P(Y=0|X=2)=C_3^2\frac{1}{2}^3C_2^0\frac{2}{3}^2=\frac{1}{6}$

$P(X=2,Y=1)=P(X=2)P(Y=1|X=2)=C_3^2\frac{1}{2}^3C_2^1\frac{1}{3}\frac{2}{3}=\frac{1}{6}$

$P(X=2,Y=2)=C_3^2\frac{1}{2}^3\cdot C_2^2(\frac{1}{3})^2=\cancel{\frac{1}{8}}\frac{1}{24}$ $P(X=3,Y=0)=\frac{1}{2}^3\cdot\frac{2}{3}^3=\frac{1}{27}$

$P(X=3,Y=1)=\frac{1}{2}^3\cdot C_3^1\frac{1}{3}\cdot\frac{2}{3}^2=\frac{1}{18}$, $P(X=3,Y=2)=\frac{1}{2}^3C_3^2\frac{1}{3}^2\frac{2}{3}=\frac{1}{36}$

$P(X=3,Y=3)=\frac{1}{2}^3\cdot\frac{1}{3}^3=\frac{1}{216}$

X\Y	0	1	2	3
0	$\frac{1}{8}$	0	0	0
1	$\frac{1}{4}$	$\frac{1}{8}$	0	0
2	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{27}$	0
3	$\frac{1}{27}$	$\frac{1}{18}$	$\frac{1}{36}$	$\frac{1}{216}$

(2). $P(Y=0)=0.58$. $P(Y=1)=0.35$.

$P(Y=2)=0.06$ $P(Y=3)=0.0046$

(3) $P(X>Y)=\frac{1}{4}+\frac{1}{6}+\frac{1}{6}+\frac{1}{27}+\frac{1}{18}+\frac{1}{36}=0.704$