237, 6-4

$\cancel{(1)\ \omega=\sqrt{\frac{g}{L}}=0.1\ rad/s \quad f=\frac{\omega}{2\pi}=\frac{1}{20\pi}\ s^{-1},\ T=\frac{1}{f}=20\pi\ s}$

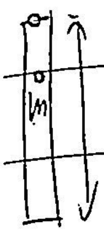
(1) $T=2\pi\sqrt{\frac{L}{g}}=\frac{\sqrt{10}}{5}\pi\ s \quad f=\frac{1}{T}=\frac{\sqrt{10}}{2\pi}\ s^{-1} \quad \omega=2\pi f=\sqrt{10}\ rad/s$

(2) $X=A\cos(\omega t+\varphi)$, $\omega=\sqrt{10}\ rad/s$.

$E=\frac{1}{2}mgLA^2=\frac{1}{2}mgL\theta^2+\frac{1}{2}m(\dot{\theta}L)^2 \Rightarrow A=0.088\ rad$.

$X(0)=-0.06 \Rightarrow \varphi=226.8^\circ \quad \therefore X=0.088\cos(3.13t+\frac{226.8}{180}\pi)\ rad$

238, 6-11



(1) $M=J\beta \Rightarrow J\frac{d^2\theta}{dt^2}+\frac{1}{2}mgL\theta=0,\ J=\frac{1}{3}mL^2$.

$\Rightarrow \omega^2=\frac{3g}{2L} \Rightarrow \omega=\sqrt{\frac{3g}{2L}},\ T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{2L}{3g}}$

(2) $J'=J_c+m(\frac{L}{2}-\frac{L}{3})^2=\frac{1}{9}mL^2$

$J'\frac{d^2\theta}{dt^2}+mg\cdot\frac{L}{6}\theta=0 \Rightarrow \omega^2=\frac{3g}{2L}=\omega$ ∴不变.

238, 6-12

弹簧串联后 $k_{eq}=\frac{k_1k_2}{k_1+k_2}$. ∴易知为简谐且 $\omega=\sqrt{\frac{k_{eq}}{m}}=\sqrt{\frac{k_1k_2}{(k_1+k_2)m}}$.

~~238, 6-18~~ 证明: $\begin{cases} k_{eq}\cdot X=k_1x_1 \cancel{+k_2x_2} \\ X=x_1+x_2 \\ k_1x_1=k_2x_2 \end{cases} \Rightarrow k_{eq}=\frac{k_1k_2}{k_1+k_2}$

238, 6-18

$-kX=(m_1+m_2)\frac{d^2x}{dt^2} \Rightarrow (m_1+m_2)\frac{d^2x}{dt^2}+kX=0 \Rightarrow X=A\cos(\omega t+\varphi)$.

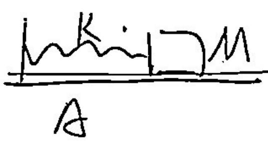
$v=\frac{dx}{dt}=A\omega\cos(\omega t+\frac{\pi}{2})$ 整体, $a=\frac{dv}{dt}=A\omega^2\cos(\omega t+\pi)$.

对m_2: $m_2g\mu=m_2\frac{d^2x}{dt^2}\geq m_2A\omega^2$, $\omega^2=\frac{k}{m_1+m_2}$

$\therefore k\leq\frac{(m_1+m_2)g\mu}{A}. \Rightarrow A\leq\frac{(m_1+m_2)g\mu}{k}$.

$\cancel{\frac{1}{2}V_m=}\ \frac{1}{2}(m+M)V_m^2=\frac{1}{2}kA^2 \Rightarrow V_m=Mg\sqrt{\frac{m_1+m_2}{k}}$

238.6-19 $\downarrow m$.



(1) $\frac{1}{2}kA^2 = \frac{1}{2}(m+M)V_m^2 = \frac{1}{2}MV_{m0}^2$

$\Rightarrow V_m^2 = \frac{M}{m+M}V_{m0}^2$，振幅不会变. $A'=A$

$X = A\cos(\omega t+\varphi)$, $V = A\omega\cos(\omega t+\frac{\pi}{2})$ ∴ $\omega' = \sqrt{\frac{M}{m+M}}\cdot\sqrt{\frac{k}{M}} = \sqrt{\frac{k}{m+M}}$.

(2) $\cancel{\frac{1}{2}MV_{m0}^2 = \frac{1}{2}}$ $MV_{m0} = (M+m)V_m' \Rightarrow V_m' = \frac{M}{M+m}V_{m0}$

∴ $\frac{1}{2}kA'^2 = \frac{1}{2}(M+m)V_m'^2 \Rightarrow A' = \sqrt{\frac{M}{M+m}}A$

$\omega' = \sqrt{\frac{M}{M+m}}\,\omega = \sqrt{\frac{k}{M+m}}$.

239.6-23

(1) $X = A_0e^{-\beta t}\cos(\sqrt{\omega_0^2-\beta^2}\,t+\varphi_0)$. ∵ $\frac{A_0e^{-\beta(t_0+T)}}{A_0e^{-\beta t_0}} = 90\%$ =

$T = \frac{2\pi}{\sqrt{\omega_0^2-\beta^2}}$

$T_0 = \frac{2\pi}{\omega_0}$

∴ $T = \cancel{\sqrt{1+\frac{\ln^2\frac{10}{9}}{4\pi^2}}}\sqrt{1+\frac{\ln^2\frac{10}{9}}{4\pi^2}}\,T_0$

$\beta = \frac{1}{T}\ln\frac{10}{9}$.

(2) 每经过一个周期 $A_{n+1} = \frac{9}{10}A_n$, $A_0 = 10cm$.

∴ $S \approx \sum_0^\infty 4A_n = 4\frac{10}{1-\frac{9}{10}} = 400cm = 4m$.