求证：$A_{m\times n}O_{n\times s}=O_{m\times s}$，$O_{s\times m}A_{m\times n}=O_{s\times n}$

证：$a_{ij}\cdot 0_{ij}=0$ $\therefore A_{m\times n}\cdot O_{m\times s}=O_{m\times s}$ 同理后者.

求证：$E_mA_{m\times n}=A_{m\times n}=A_{m\times n}\cdot E_n$

记 $E_m A_{m\times n}=C_{m\times n}$.

$\therefore c_{ij}=\sum_{k=1}^{n}(e_{ik}\cdot a_{kj})$ 又$\because e_{ij}=0$（$i\neq j$ 时）

$\therefore c_{ij}=e_{ii}\cdot a_{ij}=a_{ij}$ $\therefore C_{m\times n}=E_m\cdot A_{m\times n}$

同理后者

求证：$(kA)B=k(AB)=A(kB)$

第一层：$kA_{m\times n}=(ka_{ij})_{m\times n}$

$(kA_{m\times n})B_{n\times s}=(ka_{ij})_{m\times n}\cdot(b_{ij})_{n\times s}$

记 $(kA)B=C$ $\therefore c_{ij}=\sum_{k=1}^{n}(ka_{ik}b_{kj})=k(\sum_{k=1}^{n}(a_{ik}b_{kj}))$

记 $k(AB)=C'$ $\therefore c'_{ij}=k\sum_{k=1}^{n}(a_{ik}b_{kj})=c_{ij}$ $\therefore (kA)B=k(AB)$

记 $A\cdot(kB)=C''$ $\therefore c''_{ij}=\sum_{k=1}^{n}(a_{ik}\cdot kb_{kj})=k\sum_{k=1}^{n}(a_{ik}b_{kj})$

$\therefore$ 得证.

求证：$A(B+C)=AB+AC$.

记 $A_{m\times n}$，$B_{n\times s}$，$C_{n\times s}$ 之，$A(B+C)=D$

$\therefore d_{ij}=\sum_{k=1}^{n}(a_{ik}(b_{kj}+c_{kj}))=\sum_{k=1}^{n}a_{ik}b_{kj}+\sum_{k=1}^{n}a_{ik}c_{kj}$

记 $AB+AC=D'$ $\therefore d'_{ij}=\sum_{k=1}^{n}a_{ik}b_{kj}+\sum_{k=1}^{n}a_{ik}c_{kj}=d_{ij}$

$\therefore D'=D$ $\therefore A(B+C)=AB+AC$

同理 $(B+C)A=BA+CA$.

证：$A(BC)=(AB)C$.

记 $A_{m\times n}$, $B_{n\times s}$, $C_{s\times t}$, $A(BC)=D_{m\times t}$. $(AB)C=D'_{m\times t}$.

$BC=F_{n\times t}$, $AB=G_{m\times s}$

~~BC~~ $f_{ij}=\sum_{k=1}^{s}(b_{ik}\cdot c_{kj})$

$$d_{ij}=\sum_{p=1}^{n}(a_{ip}\cdot f_{pj})=\sum_{p=1}^{n}(a_{ip}(\sum_{k=1}^{s}b_{pk}\cdot c_{kj}))$$

同理，$d'_{ij}=\sum_{p=1}^{s}((\sum_{k=1}^{n}a_{ik}b_{kp})\cdot c_{pj}]=\sum_{p=1}^{s}((c_{pj}(\sum_{k=1}^{n}a_{ik}b_{kp}))$

对于 d_{ij}，制为 $n\times s$ 的矩阵．F.

$$\begin{pmatrix} a_{i1}b_{11}c_{1j} & a_{i1}b_{12}c_{2j} & \cdots & a_{i1}b_{1s}c_{sj} \\ a_{i2}b_{21}c_{1j} & a_{i2}b_{22}c_{2j} & & \vdots \\ \vdots & & & \vdots \\ a_{in}b_{n1}c_{1j} & - & - & a_{in}b_{ns}c_{sj} \end{pmatrix}$$

~~对于 d'_{ij}~~，d_{ij} = 表内所有元素之和.

对于 d'_{ij}，制为 ~~$s\times n$~~ $n\times s$ 的矩阵．G.

$$\begin{pmatrix} c_{1j}a_{i1}b_{11} & c_{2j}a_{i1}b_{12} & \cdots & c_{sj}a_{i1}b_{1s} \\ c_{1j}a_{i2}b_{21} & c_{2j}a_{i2}b_{22} & & \vdots \\ c_{1j}a_{in}b_{n1} & \cdots & & c_{sj}b_{ns}a_{in} \end{pmatrix}$$

~~f_{ij}~~ $=$ f_{pk} $=a_{ip}b_{pk}c_{kj}$, ~~$g_{kp}=a_{ip}b_{pk}c_{kj}=f_{pk}$~~

~~$g_{pk}=g_{kp}=G$ $g_{pk}=a_{ik}b_{kp}c_{pj}=f_{pk}$~~ $g_{pk}=c_{kj}a_{ip}b_{pk}$ $=f_{pk}$.

$\therefore F=G$ $\therefore d'_{ij}$ = 表内元素和 $=d_{ij}$

$\therefore A(BC)=(AB)C$

82.3

$$A+2B=\begin{pmatrix} x & 0 \\ 7 & y \end{pmatrix}+2\begin{pmatrix} u & 2v \\ y & 2 \end{pmatrix}=\begin{pmatrix} x+2u & 4v \\ 7+2y & y+4 \end{pmatrix}=\begin{pmatrix} 3 & -4 \\ x & v \end{pmatrix}$$

$$\therefore \begin{cases} x+2u=3 \\ 4v=-4 \\ 7+2y=x \\ y+4=v \end{cases} \Rightarrow \begin{cases} x=-3 \\ y=-5 \\ v=-1 \\ u=3 \end{cases}$$

82.5

(1) $$原式=\begin{pmatrix} 7 & 24 & 3 \\ 7 & -8 & 13 \\ 7 & 40 & -2 \end{pmatrix}$$

(2) $$原式=\begin{pmatrix} -8 \\ -2 \\ 10 \end{pmatrix}$$

(3) 原式 $=(2)=2$

(4) $$原式=\begin{pmatrix} 3 & -1 & 2 \\ -6 & 2 & -4 \\ 9 & -3 & 6 \end{pmatrix}$$

(5) $$原式=\begin{pmatrix} a_{11}x_1+a_{21}x_2+a_{31}x_3 & a_{12}x_1+a_{22}x_2+a_{32}x_3 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$=\left((a_{11}x_1+a_{21}x_2+a_{31}x_3)y_1+y_2(a_{12}x_1+a_{22}x_2+a_{32}x_3)\right)$$

82.6

证明：记 A, B 为 n 阶上三角阵. $\therefore a_{ij}=0$ $(i>j)$

$b_{ij}=0$ $(i>j)$.

记 $A_{n\times n}\cdot B_{n\times n}=C_{n\times n}$.

$\therefore c_{ij}=\sum_{k=1}^{n}(a_{ik}\cdot b_{kj})$

当 $i>j$ 时. 1) $k<i$ 时 $\therefore a_{ik}=0$ 2) $k\geq i$ 时 $\therefore b_{kj}=0$

$\therefore 0=a_{ik}\cdot b_{kj}$ 在 $i>j$ 时恒成立 $\therefore$ $c_{ij}=0$ 在 $i>j$ 时成立

$\therefore$ C_n 为上三角阵.

§2.7

(1)

$$AB=\begin{pmatrix}1&0&3\\0&2&1\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&2&1\\3&0&2\end{pmatrix}=\begin{pmatrix}10&0&6\\3&4&4\\3&0&2\end{pmatrix}$$

$$BA=\begin{pmatrix}1&0&0\\0&2&1\\3&0&2\end{pmatrix}\begin{pmatrix}1&0&3\\0&2&1\\0&0&1\end{pmatrix}=\begin{pmatrix}1&0&3\\0&4&3\\3&0&11\end{pmatrix}$$

(2) $$A^2-B^2=\begin{pmatrix}1&0&3\\0&2&1\\0&0&1\end{pmatrix}^2-\begin{pmatrix}1&0&0\\0&2&1\\3&0&2\end{pmatrix}^2$$

$$=\begin{pmatrix}1&0&6\\0&4&3\\0&0&1\end{pmatrix}-\begin{pmatrix}1&0&0\\3&4&4\\9&0&4\end{pmatrix}=\begin{pmatrix}0&0&6\\-3&0&-1\\-9&0&-3\end{pmatrix}$$

$$(A+B)(A-B)=A^2-AB+BA-B^2$$

$$=\begin{pmatrix}1&0&6\\0&4&3\\0&0&1\end{pmatrix}-\begin{pmatrix}10&0&6\\3&4&4\\3&0&2\end{pmatrix}+\begin{pmatrix}1&0&3\\0&4&3\\3&0&11\end{pmatrix}-\begin{pmatrix}1&0&0\\3&4&4\\9&0&4\end{pmatrix}$$

$$=\begin{pmatrix}-9&0&3\\-6&0&-2\\-9&0&6\end{pmatrix}$$

§2.9.

(1) 记 $A=\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}$ ∴ $\begin{pmatrix}2&1\\0&1\end{pmatrix}A=\begin{pmatrix}2a_{11}+a_{21}&2a_{12}+a_{22}\\a_{21}&a_{22}\end{pmatrix}$

$$A\begin{pmatrix}2&1\\0&1\end{pmatrix}=\begin{pmatrix}2a_{11}&a_{11}+a_{12}\\2a_{21}&a_{21}+a_{22}\end{pmatrix}=\begin{pmatrix}2a_{11}+a_{21}&2a_{12}+a_{22}\\a_{21}&a_{22}\end{pmatrix}$$

$$\Rightarrow\begin{cases}a_{11}=a_{11}\\a_{12}=a_{11}-a_{22}\\a_{21}=0\\a_{22}=a_{22}\end{cases}\quad\therefore A\begin{pmatrix}a_{11}&a_{11}-a_{22}\\0&a_{22}\end{pmatrix}\quad a_{11},a_{22}\in R.$$

82.4

(2) ~~$A=($~~ $A=(a_{ij})_4=\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$

$\therefore A\cdot\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}=\begin{pmatrix} a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ 0 & 0 & 0 & 0 \end{pmatrix}$

~~$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\cdot A=($~~

$A\cdot\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}=\begin{pmatrix} 0 & a_{11} & a_{12} & a_{13} \\ 0 & a_{21} & a_{22} & a_{23} \\ 0 & a_{31} & a_{32} & a_{33} \\ 0 & a_{41} & a_{42} & a_{43} \end{pmatrix}$

$=\begin{pmatrix} a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 对照时看漏了一行，下次横着写

$\therefore A=\begin{pmatrix} a_{44} & a_{44} & a_{13} & a_{14} \\ 0 & a_{44} & a_{44} & a_{13} \\ 0 & 0 & a_{44} & a_{44} \\ 0 & 0 & 0 & a_{44} \end{pmatrix} \cancel{=\begin{pmatrix} x & x & y & z \\ 0 & x & x & y \\ 0 & 0 & x & x \\ 0 & 0 & 0 & x \end{pmatrix}}$

$A=\begin{pmatrix} a & b & c & d \\ 0 & a & b & c \\ 0 & 0 & a & b \\ 0 & 0 & 0 & a \end{pmatrix}$

$a,b,c,d\in R$

82.10

$A_2=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$A_2^2=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}=\begin{pmatrix} a_{11}^2+a_{12}a_{21} & a_{11}a_{12}+a_{12}a_{22} \\ a_{11}a_{21}+a_{22}a_{21} & a_{12}a_{21}+a_{22}^2 \end{pmatrix}$

$\therefore \begin{cases} a_{11}^2+a_{12}a_{21}=0 \\ (a_{11}+a_{22})a_{12}=0 \\ a_{21}(a_{11}+a_{22})=0 \\ a_{22}^2+a_{12}a_{21}=0 \end{cases}$

若 $a_{12}, a_{21}=0$，则 $A=O_2$ 不行.

$\therefore a_{11}=-a_{22}=x \quad \therefore a_{12}a_{21}=-x^2$

$\therefore A=\begin{pmatrix} a_{11} & \frac{-a_{11}^2}{a_{21}} \\ a_{21} & -a_{11} \end{pmatrix}$

当 $a_{11}=1$，$a_{21}=1$ 时

$A=\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$

~~设 $A=(0\ 0$~~