例3 线性FIR带通 $f_L=4kHz$, $f_{PL}=16kHz$, $f_{PH}=24kHz$, $f_H=36kHz$, $f_s=64kHz$

用黑人窗, 用奇数长度. $\omega_1=2\pi(f_{PL}-f_L)/f_s=0.5\pi$, $\omega_2=2\pi(f_H-f_{PH})/f_s=0.375\pi$

$\Delta\omega=\frac{5.5\times2\pi}{N}\le0.375\pi \Rightarrow N\ge40$, 取 $N=41$

1. 高通 IIR. $f_p=2000Hz$, $A_p=-3dB$, $f_s=8000Hz$, 1阶.

$\omega_p=\frac{2}{T}\tan(2\pi f_p/f_s/2)=2f_s\tan(\frac{2\pi f_p}{2f_s})=$ ~~$2f_s\tan\frac{\pi}{4}$~~ $\tan\frac{\pi}{4}=1$.

$=1.6\times10^4$ rad/s ~~$\omega_c=$~~ $\omega_c=1.6\times10^4$ rad/s

$\varepsilon^2=10^{-\frac{A_p}{10}}-1=1$ $H_p(s)=\frac{1}{s+1}$, $\Leftrightarrow$

$H_{HP}(s)=\frac{1}{s+1}\Big|_{s=\frac{\omega_c}{s}}=\frac{s}{\omega_c+s}$ $\Rightarrow$ $H(z)=H_{HP}(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}}$

$\Rightarrow\boxed{H(z)=\frac{z-1}{2z}}$

$\frac{\frac{2}{T}\frac{z-1}{z+1}}{\frac{2}{T}\frac{z-1}{z+1}+1}=\frac{z-1}{2z}$

2. 低通 $A_p=-3dB$, $f_p=1.5kHz$, $f_{st}=3kHz$, $A_{st}=-40dB$

$\varepsilon^2=10^{-\frac{A_p}{10}}-1=1$, $\nu_s=\frac{f_{st}}{f_p}=2$ rad/s

$n\ge\lg\frac{10^{-\frac{A_{st}}{10}}-1}{\varepsilon^2}\Big/2\lg\nu_s=6.6$ $\therefore\boxed{n=7}$ 7阶

3. $H(s)=\frac{50}{s+50}$, $f_s=100Hz$. IIR的 $H(z)$.

$h_a(t)=50e^{-50t}$, $h(n)=50e^{-50nT_s}$ $T_s=1/f_s=0.01s$

$h(n)=50e^{-\frac{n}{2}}$ $(n\ge0)$. $H(z)=\sum_{n=-\infty}^{+\infty}h(n)e^{-z}=\sum50e^{-\frac{n}{2}}z^{-n}$

$\Rightarrow H(z)=50\frac{1}{1-e^{-\frac{1}{2}}z^{-1}}$

4. 1阶 Butterworth Lowpass Filter, $H_a(s)=\frac{1}{s+1}$, $A_p=-3dB$, $\omega_c=0.2\pi$.

$\nu_c=\frac{2}{T}\tan\frac{\omega_c}{2}=\frac{2}{T}\tan\frac{\pi}{10}=0.65\frac{1}{T}$

$H_a(s)=H_a(\frac{s}{\nu_c})=\frac{\nu_c}{s+\nu_c}$ $\Rightarrow H(z)=H(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}}=\frac{0.65\frac{1}{T}}{\frac{2}{T}\frac{z-1}{z+1}+0.65\frac{1}{T}}$

$\Rightarrow H(\omega)=\frac{0.245(1+e^{-j\omega})}{1-0.509e^{-j\omega}}$ $=\frac{0.65(z+1)}{2.65z-1.35}$