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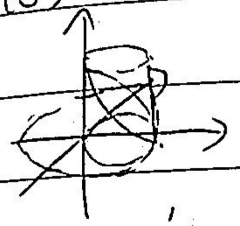
(3) $S:\ x^2+y^2+z^2-a^2=0. \Rightarrow \cancel{y=\pm\sqrt{a^2-x^2-z^2}}\quad z=\pm\sqrt{a^2-x^2-y^2}.$



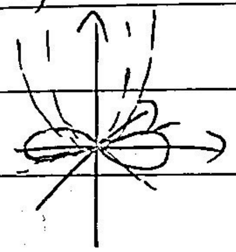
$$\begin{cases} x^2+y^2+z^2=a^2 \\ x^2+y^2=ax \end{cases} \Rightarrow \begin{matrix} x^2+y^2=ax \\ \cancel{z^2\le ax\le a^2}. \end{matrix} \leftarrow D_{xy}$$

$\therefore\ \cancel{A_s=2\iint_{D_{xy}}\sqrt{1+y_z^2+y_x^2}\,d\sigma = 2a\iint_{D_{zx}}\frac{1}{\sqrt{a^2-x^2-z^2}}\,d\sigma}.$

$A_s=2a\iint_{D_{xy}}\frac{1}{\sqrt{a^2-x^2-y^2}}\,d\sigma.\qquad D_{r\theta}=\{(r,\theta)\mid -\frac{\pi}{2}\le\theta\le\frac{\pi}{2},\ 0\le r\le a\cos\theta\}.$

$=2a\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\theta\int_0^{a\cos\theta}\frac{r}{\sqrt{a^2-r^2}}\,dr=\cancel{\frac{2a^4}{3}(\pi-\frac{4}{3})}.\ 2a^2(\pi-2)$

(4).



$S:\ 2z=x^2+y^2 \Rightarrow z-\frac{1}{2}(x^2+y^2)=0$

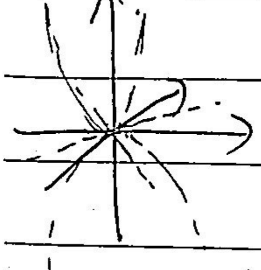
$$\begin{cases} z-\frac{1}{2}(x^2+y^2)=0 \\ (x^2+y^2)^2=x^2-y^2 \end{cases} \Rightarrow D_{xy}=(x^2+y^2)^2-(x^2-y^2)$$

$\therefore D_{r\theta}=\{(r,\theta)\mid -\frac{\pi}{4}\le\theta\le\frac{\pi}{4},\ 0\le r\le\sqrt{\cos2\theta}\}.$

$A_s=2\iint_{D_{r\theta}}\sqrt{1+z_x^2+z_y^2}\,d\sigma=2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}d\theta\int_0^{\sqrt{\cos2\theta}}\sqrt{1+r^2}\,r\,dr$

$=\cancel{\frac{40}{9}-\frac{2}{3}\pi}.\ \frac{20}{9}-\frac{\pi}{3}$

(5). $S:\ z=y^2-x^2.$



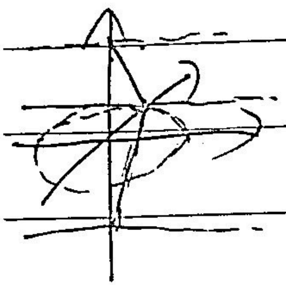
$$\begin{cases} z=y^2-x^2 \\ x^2+y^2=1 \end{cases} \Rightarrow D_{xy_1}=x^2+y^2=1\quad D_{r\theta 1}=\{(r,\theta)\mid 0\le\theta\le2\pi,\ 0\le r\le1$$

$\therefore\ A_{s_1}=\int_0^{2\pi}d\theta\int_0^1\sqrt{1+4r^2}\,r\,dr=\frac{\pi}{6}(5^{\frac{3}{2}}-1).$

$A_{s_2}=\int_0^{2\pi}d\theta\int_0^2\sqrt{1+4r^2}\,r\,dr=\frac{\pi}{6}(17\sqrt{17}-1)$

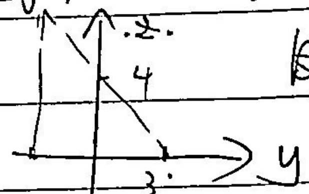
$\therefore A_s=A_{s_2}-A_{s_1}=\frac{\pi}{6}(17\sqrt{17}-5\sqrt{5})$

32.(7)

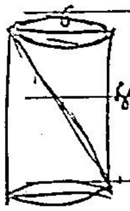


$S: x^2+y^2=9 \Rightarrow \cancel{y=\sqrt{9-x^2}}\ x=\sqrt{9-y^2}.$

$\begin{cases} x^2+y^2=9. \\ 4y+3z=12 \end{cases} \Rightarrow$



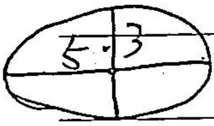
$D_{yz}=\{(y,z) \mid -3\le y\le 3,\ 0\le z\le 4-\frac{4}{3}y\}.$



$\cancel{A_{S_1}=\iint_{D_{yz}}\sqrt{1+(\frac{\partial x}{\partial z})^2}\,d\sigma=\int_{-3}^{3}dy\int_0^{4-\frac{4}{3}y}\frac{3}{\sqrt{9-y^2}}\,dz}$

$\therefore A_{S_1}=\frac{1}{2}\times 2\pi\times 3\times 8=24\pi.$

$A_{S_2}=\pi\times 3\times 5=15\pi \quad \therefore S=2\times(24+15)\pi=78\pi.$



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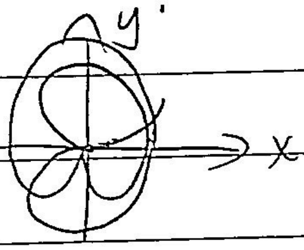
(3).



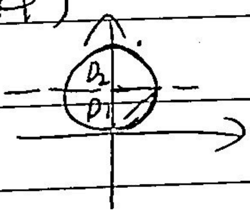
$\vec{r_c}=\frac{\sum \Delta m_i \vec{r_i}}{M} \qquad D_{r\theta}=\{(r,\theta) \mid 0\le\theta\le 2\pi,\ 0\le r\le 1+\sin\theta\}$

$\Rightarrow \bar{x}=\cancel{\frac{1}{m}\iint_D x\mu(x,y)\,d\sigma}=0.$

$\bar{y}=\frac{1}{m}\iint_D y\mu(x,y)\,d\sigma=\frac{5}{6}$

$\therefore \cancel{\vec{r_c}=(0,0)}. \qquad m=\iint_D \mu(x,y)\,d\sigma=2A_D=3\pi. \quad \vec{r_c}=(0,\frac{5}{6}).$

(4).



$M=\iint_{D_1}d\sigma+\iint_{D_2}(2y-1)\,d\sigma$

$D_{r\theta_1}=\{(r,\theta) \mid 0\le\theta\le\frac{\pi}{4},\ 0\le r\le\sin\theta\}. \quad D_{r\theta_2}=\{(r,\theta) \mid \frac{\pi}{4}\le\theta\le\frac{\pi}{2},\ 0\le r\le\frac{1}{\sin\theta}\}$

$\therefore D_{r\theta_3}=\{(r,\theta) \mid \frac{\pi}{4}\le\theta\le\frac{\pi}{2},\ \frac{1}{\sin\theta}\le r\le\sin\theta\}.$

$\therefore M=\frac{4}{3}+\pi$

$\bar{x}=\frac{1}{m}(\iint_{D_1}x\mu(x,y)\,d\sigma+\iint_{D_2}x\mu(x,y)\,d\sigma)=0. \qquad \vec{r_c}=(0,\frac{15\pi+16}{12\pi+16}).$

$\bar{y}=\cancel{\frac{1}{m}(\iint_{D_1}y\mu(x,y)\,d\sigma+\iint_{D_2}y\mu(x,y)\,d\sigma)}=\frac{15\pi+16}{12\pi+16}$

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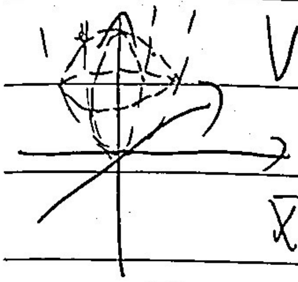
(1)

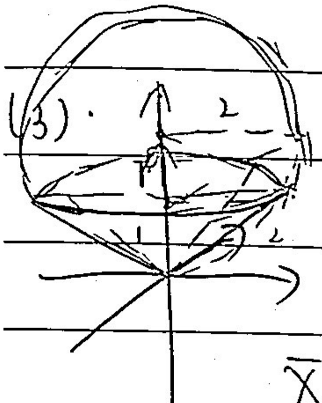


$V = \iint_D d\sigma \int_{x^2+y^2}^{36-3x^2-3y^2} dz = \int_0^{2\pi} d\theta \int_0^3 (36-4r^2) r\,dr$

$= 162\pi.$

$\bar{x} = \bar{y} = 0$. $\bar{z} = \frac{1}{V} \iiint_\Omega z\, dV = \frac{57}{\pi}$. $\therefore (0, 0, \frac{57}{\pi})$.

(3).



$\Omega_{\rho\varphi\theta} = \{(\rho, \varphi, \theta) \mid 0 \le \theta \le 2\pi, 0 \le \varphi \le \frac{\pi}{3}, 0 \le \rho \le 4\cos\varphi\}$

$\therefore V = \int_0^{2\pi} d\theta \int_0^{\frac{\pi}{3}} d\varphi \int_0^{4\cos\varphi} \rho^2 \sin\varphi\, d\rho$

$= 10\pi$.

$\bar{x} = \bar{y} = 0$, $\bar{z} = \frac{1}{V} \iiint_\Omega z\, dV = \frac{21}{10}$. $\vec{r} = (0, 0, \frac{21}{10})$