

5.1

$$G(s) = \frac{k}{s(s+1)} = \frac{2k}{s(s+2)}, \quad H(s) = 1, \quad \frac{c}{T} = \frac{c}{1+0.1H} = \frac{2k}{s^2+2s+2k}$$

~~$s_{1,2} = -1 \pm \sqrt{1-2k}$~~

$$\textcircled{1} k=2, \quad \frac{c}{T} = \frac{4}{s^2+2s+4} = \frac{4}{(s+1)^2+(\sqrt{3})^2} = \frac{4}{(s+1+\sqrt{3}i)(s+1-\sqrt{3}i)}$$

$$\therefore s_{1,2} = -1 \pm \sqrt{3}i = -\sigma \pm j\omega_d, \quad \sigma = 1, \omega_d = \sqrt{3}, \omega_n = 2$$

$$\therefore \xi = \frac{1}{2}$$

$$t_r = \frac{\pi - \beta}{\omega_d}, \quad \beta = \arctan \frac{\omega_d}{\sigma} = \arctan \frac{\sqrt{3}}{1} = \frac{\pi}{3}, \quad \therefore t_r = \frac{2\sqrt{3}}{3} \pi \text{ s}$$

$$M_p = \exp\left[-\pi \cdot \frac{\sigma}{\omega_d}\right] = e^{-\frac{\pi}{\sqrt{3}}}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\sqrt{3}}$$

$$\textcircled{2} k=8, \quad \frac{c}{T} = \frac{16}{s^2+2s+16} = \frac{16}{(s+1)^2+(\sqrt{15})^2} = \frac{16}{(s+1+\sqrt{15}i)(s+1-\sqrt{15}i)}$$

$$s_{1,2} = -1 \pm \sqrt{15}i, \quad \sigma = 1, \omega_d = \sqrt{15}, \omega_n = 4, \xi = \frac{1}{4}$$

$$\therefore t_r = \frac{\pi - \beta}{\omega_d}, \quad \tan \beta = \sqrt{15} \Rightarrow \beta = 75.5^\circ = 1.32 \text{ rad.}$$

$$= \frac{\pi - 1.32}{\sqrt{15}} = 0.47 \text{ s}$$

$$M_p = \exp\left[-\pi \cdot \frac{1}{\sqrt{15}}\right] = e^{-\frac{\pi}{\sqrt{15}}}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\sqrt{15}}$$

③ k 对动态性能影响。

$$t_r = \frac{\pi - \beta}{\omega_d}, \quad \omega_d = \sqrt{2k-1}, \quad \beta = \arctan \sqrt{2k-1}, \quad \therefore t_r \text{ 随 } k \text{ 单减}$$

$$M_p = e^{-\pi/\sqrt{2k-1}}, \quad M_p \text{ 随 } k \text{ 单增.}$$

$$t_p = \frac{\pi}{\sqrt{2k-1}} \text{ 随 } k \text{ 单减}$$



5.2.

$$G(s) = \frac{2k}{s(s+1)(s+4)}, \quad H(s) = 1, \quad \frac{C}{12} = \frac{2k}{s^3 + 5s^2 + 4s + 2k}.$$

$$\therefore s^3 \quad | \quad 4$$

$$s^2 \quad | \quad 5 \quad 2k \quad a_1 = \frac{20-2k}{5}, \quad a_2 = 0, \quad a_3 > 0$$

$$s^1 \quad | \quad a_1 \quad a_2 \quad a_3 = \frac{20k-5a_2}{a_1} = 2k \quad \therefore a_1, a_3 > 0$$

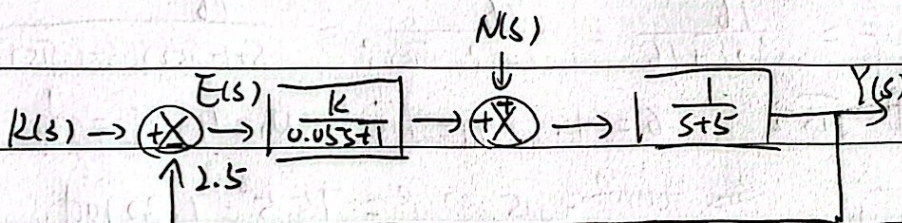
$$s^0 \quad | \quad a_3 \quad a_4$$

$$\therefore k \in (0, 10)$$

$$N=1, \text{ I型系统. } k_p = G(0) = \infty, \quad k_v = \lim_{s \rightarrow 0} sG(s) = \frac{k}{2}$$

$$k_a = \lim_{s \rightarrow 0} s^2 G(s) = 0$$

5.3.



1) $k=40, N(s) = \frac{1}{s}, R(s)=0$, 求 $y_{ss}(\infty)$ 和 e_{ss}

$$Y(s) = \frac{1}{s+5} \frac{800}{s+20} E(s) + \frac{1}{s+5} \frac{1}{s}, \quad E(s) = -2.5Y(s).$$

$$\therefore Y(s) = \frac{\frac{1}{s}(s+20)}{2000 + (s+20)(s+5)}, \quad y(\infty) = \lim_{s \rightarrow 0} sY(s) = \frac{1}{105}$$

$$e_{ss} = 0 - y(\infty) = -\frac{1}{105}$$

2) $k=40, R(s) = \frac{1}{s}, N(s)=0$, 求 $y_{ss}(\infty)$ 和 e_{ss}

$$Y(s) = \frac{800}{s+20} \cdot \frac{1}{s+5}, \quad H(s) = 2.5.$$

$$Y(s) = \frac{1}{s} \cdot \frac{G(s)}{1+G(s)H(s)} = \frac{800}{2000 + (s+20)(s+5)} \cdot \frac{1}{s}, \quad y(\infty) = \frac{8}{21}$$

$$e_{ss} = 1 - y(\infty) = \frac{13}{21}$$

3)



3) $G(s) = \frac{20K}{s+20} \cdot \frac{1}{s} \cdot \frac{1}{s+5}$, $H(s) = 0.5$ 对 $R(s)$ 来说

$Y(s) = \frac{1}{s} \cdot \frac{20K}{s(s+20)(s+5)+50K} \Rightarrow Y(\infty) = 0.4$, $e_{ss} = 0.6$ 减小了.

对 $N(s)$ 来说, $G(s) = \frac{1}{s+5}$, $H(s) = 2.5 \frac{20K}{s+20} \cdot \frac{1}{s}$ 5K元关.

$Y(s) = \frac{s(s+20)}{s(s+20)(s+5)+50K} \cdot \frac{1}{s} \Rightarrow Y(\infty) = 0$, $e_{ss} = 0$, 减小了.

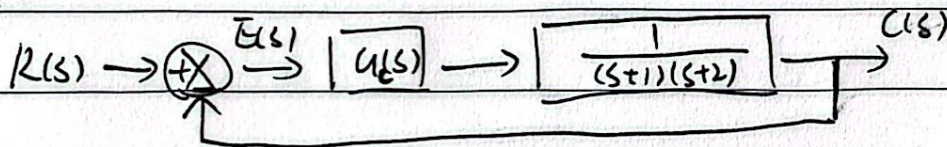
4) 对 $R(s)$ 来说结果不变.

对 $N(s)$ 来说, $G(s) = \frac{1}{s(s+5)}$, $H(s) = \frac{50K}{s+20}$

$Y(s) = \frac{s+20}{s(s+5)(s+20)+50K} \cdot \frac{1}{s} \Rightarrow Y(\infty) = \frac{2}{5K}$, $e_{ss} = -\frac{2}{5K}$

$K=40$ 时, $e_{ss} = -0.01$, 变大了.

5.4.



1) $G_c(s) = K_p$, $G(s) = \frac{G_c(s)}{(s+1)(s+2)}$, $H(s) = 1$.

$\frac{C}{R} = \frac{K_p}{s^2 + 3s + 2 + K_p}$

$s^2 \quad 1 \quad 2+K_p \quad \angle 2+K_p > 0 \Rightarrow K_p > -2$. $K_p > -2$ 时稳定.

$s^1 \quad 3 \quad 0$

$s^0 \quad 2+K_p$

$R(s) = \frac{1}{s} \Rightarrow C(s) = \frac{1}{s} \frac{K_p}{s^2 + 3s + 2 + K_p}$

$e_{ss}(\infty) = \lim_{s \rightarrow 0} s C(s) = \frac{K_p}{2+K_p}$, $e_{ss} = 1 - C_{ss} = \frac{2}{2+K_p}$.

2) $G_c(s) = 3(1+2s)$, $R(s) = \frac{1}{s}$ 求 e_{ss} .

$\frac{E}{R} = \frac{R-C}{R} = 1 - \frac{C}{R} = 1 - \frac{G}{1+GH} = 1 - \frac{G}{1+G} = \frac{1}{1+G}$

$G(s) = \frac{G_c(s)}{(s+1)(s+2)} = \frac{3(2s+1)}{(s+1)(s+2)}$, $E = \frac{1}{s} \frac{3(2s+1)}{(s+1)(s+2)} \cdot \frac{1}{1 + \frac{3(2s+1)}{(s+1)(s+2)}}$

$e(\infty) = \lim_{s \rightarrow 0} s E(s) = \frac{3}{5}$

