113.21

(2) $\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$

$= -\frac{1}{\sqrt{1-(x-y)^2}} \cdot 3 + \frac{1}{\sqrt{1-(x-y)^2}} 8t = \frac{8t-3}{\sqrt{1-(3t-4t^2)^2}}$

(4) $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$

$= \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{1}{t} + \frac{\partial f}{\partial z}\frac{1}{(\ln^2 t)}$

113.22

(3) $\cancel{\frac{\partial z}{\partial x}} =$ 令 $2x+y=u$ $\therefore$ $z = u^u$.

$\frac{\partial z}{\partial x} = \frac{dz}{du}\frac{\partial u}{\partial x} = 2u^u(\ln u+1) = 2(2x+y)^{2x+y}(\ln(2x+y)+1)$

$\frac{\partial z}{\partial y} = \frac{dz}{du}\frac{\partial u}{\partial x} = u^u(\ln u+1) = (2x+y)^{2x+y}(\ln(2x+y)+1)$.

(4) $x^y = u$ $\therefore$ $z = f(x,u) = x^u$

$\cancel{\frac{\partial u}{\partial z} = \frac{\partial u}{\partial}}$ $\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} = ux^{u-1} + yx^u \ln x$ yx^{y-1}.

$= \cancel{x^y x^{x^y} + yx^{x^y}\ln x = yx^{x^y}(\ln x+1)}$

$= \cancel{x^y x^{x^y-1} + yx^{x^y}\ln x}$.

$x^{x^y+y-1}(1+y\ln x)$.

$\frac{\partial z}{\partial y} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y} = \cancel{x^{xy+1}\ln x}$.

$= x^{x^y+y}\ln^2 x$.

1~~13.23~~

~~(1) 令 $x^2-y^2=u$ $\therefore$ $z=yf(u)$, $u=u(x,y)$~~.

$\cancel{\frac{\partial z}{\partial x} = \frac{\partial f}{\partial u}\frac{df}{du} \cdot \frac{\partial u}{\partial x} =}$

~~(1) 令 $u=f(x^2-y^2)$ $\therefore$ $u=u(x,y)$. $z=yu=g(y,u)$~~

$\therefore \cancel{\frac{\partial z}{\partial x} = \frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial x} = y}$.

113.23.

(3) $\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} \cdot \frac{\partial z}{\partial f} = y f'(\frac{y}{x}) \cdot -\frac{y}{x^2} = -\frac{y^2}{x^2} f'(\frac{y}{x})$

$\frac{\partial z}{\partial y} = \frac{\partial z(y,f)}{\partial y} + \frac{\partial z}{\partial f}\frac{\partial f}{\partial y} = f(\frac{y}{x}) + \frac{y}{x} f'(\frac{y}{x})$

(4) $\frac{\partial z}{\partial t} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial ts}\frac{\partial ts}{\partial t} + \frac{\partial f}{\partial tsr}\frac{\partial tsr}{\partial t}$

$= \frac{\partial f}{\partial t} + \frac{\partial f}{\partial ts}\cdot s + \frac{\partial f}{\partial tsr}\cdot sr$

$\frac{\partial z}{\partial s} = \frac{\partial f}{\partial ts}\cdot\frac{\partial ts}{\partial s} + \frac{\partial f}{\partial tsr}\cdot\frac{\partial tsr}{\partial s} = t\frac{\partial f}{\partial ts} + tr\frac{\partial f}{\partial tsr}$

$\frac{\partial z}{\partial r} = \frac{\partial f}{\partial tsr}\cdot\frac{\partial trs}{\partial r} = ts\frac{\partial f}{\partial tsr}$.

113.24

(1) $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial f}\cdot\frac{df}{d\cancel{x}} = y\,2x f'(x^2-y^2)$

$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial y} + \frac{\partial z}{\partial f}\cdot\frac{\partial f}{\partial y} = f(x^2-y^2) - 2y^2 f'(x^2-y^2)$

$\therefore \frac{1}{x}\frac{\partial z}{\partial x} + \frac{1}{y}\frac{\partial z}{\partial y} = 2yf'(x^2-y^2) + \frac{1}{y}f(x^2-y^2) - 2yf'(x^2-y^2) = \frac{1}{y}f(x^2-y^2)$

$= \frac{z}{y^2}$.

(2).

$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} + \cancel{\frac{\partial f}{\partial x}}\frac{\partial u}{\partial f}\left(\frac{\partial f}{\partial \frac{z}{x}}\cdot\frac{\partial \frac{z}{x}}{\partial x} + \frac{\partial f}{\partial \frac{y}{x}}\cdot\frac{\partial \frac{y}{x}}{\partial x}\right)$

$= kx^{k-1}f - x^k\left(\frac{z}{x^2}\frac{\partial f}{\partial \frac{z}{x}} + \frac{y}{x^2}\frac{\partial f}{\partial \frac{y}{x}}\right)$.

$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial f}\cdot\frac{\partial f}{\partial \frac{y}{x}}\frac{\partial \frac{y}{x}}{\partial y} = x^k\cdot\frac{\partial f}{\partial \frac{y}{x}}\cdot\frac{1}{x}$

$\frac{\partial u}{\partial z} = x^k\frac{\partial f}{\partial \frac{z}{x}}\cdot\frac{1}{x}$ $\quad\therefore$ 左边 $= kx^k f - x^{k-1}\left(z\frac{\partial f}{\partial \frac{z}{x}} + y\frac{\partial f}{\partial \frac{y}{x}}\right)$

$+ x^{k-1}\left(z\frac{\partial f}{\partial \frac{z}{x}} + y\frac{\partial f}{\partial \frac{y}{x}}\right)$

$= kx^k f = ku$.

113.25.

$x = x(r,\theta)$, $y = y(r,\theta)$. $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

$\therefore \cancel{\frac{\partial u}{\partial x}} \frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r}$

$= \frac{\partial u}{\partial x}\cdot\cos\theta + \frac{\partial u}{\partial y}\cdot\sin\theta$.

$\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial y}\frac{\partial y}{\partial \theta} = \frac{\partial v}{\partial y}\cdot r\cos\theta$ $\therefore \frac{1}{r}\frac{\partial v}{\partial \theta} = \cancel{\frac{\partial u}{\partial x}}\frac{\partial u}{\partial r}$.

$+ \frac{\partial v}{\partial x}\frac{\partial x}{\partial \theta} - \frac{\partial v}{\partial x}r\sin\theta$.

$\cancel{\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta} = \frac{\partial u}{\partial y} r\sin\theta}$

$\cancel{\frac{\partial v}{\partial r} = \frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial r} = \frac{\partial v}{\partial x}\cos\theta}$.

$x = r\cos\theta$

$y = r\sin\theta$.

$$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial v}{\partial x}\cos\theta + \frac{\partial v}{\partial y}\sin\theta.$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta} = -\frac{\partial u}{\partial x} r\sin\theta + r\frac{\partial u}{\partial y}\cos\theta.$$

2. $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$

113.26.

$\cancel{(1)\ \frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{x+y}{x^2+y^2}}$.

(1) $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial x} = \frac{x}{x^2+y^2}\frac{\partial z}{\partial u} - \frac{y}{x^2+y^2}\frac{\partial z}{\partial v}$.

$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y} = \frac{y}{x^2+y^2}\frac{\partial z}{\partial u} + \frac{x}{x^2+y^2}\frac{\partial z}{\partial v}$.

代入，得 $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial v}$

(2) $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial x} = 2x\frac{\partial z}{\partial u} - \frac{1}{x^2}\frac{\partial z}{\partial v}$

$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y} = 2y\frac{\partial z}{\partial u} - \frac{1}{y^2}\frac{\partial z}{\partial v}$ X.

$\therefore$ 左边 $= \frac{x}{y^2}\frac{\partial z}{\partial v} - \frac{y}{x^2}\frac{\partial z}{\partial v} = \frac{x^3-y^3}{x^2y^2}\frac{\partial z}{\partial v} = (y-x)z$

$\Rightarrow \frac{x^2+xy+y^2}{x^2y^2}\frac{\partial z}{\partial v} = -z$

(2) $z = e^{w+x+y}$. $\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial w}\left(\frac{\partial w}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial x}\right)$.

记 $z = f(w, x, y)$

$\frac{\partial z}{\partial y} = \frac{\partial f}{\partial y} + \frac{\partial f}{\partial w}\left(\frac{\partial w}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial y}\right)$

$\therefore \frac{x}{y^2}\frac{\partial w}{\partial v} = \frac{y}{x^2}\frac{\partial w}{\partial v}$

$\therefore \begin{cases} u = 2x^2 \ ? \\ v = \frac{2}{x} \ ? \\ w = \ln z - 2x \end{cases}$

$\Rightarrow x^3 = y^3 \Rightarrow x = y$

故 $\frac{\partial w}{\partial v} = 0$