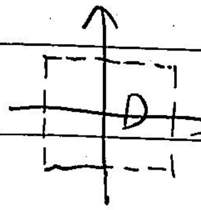
170. 8

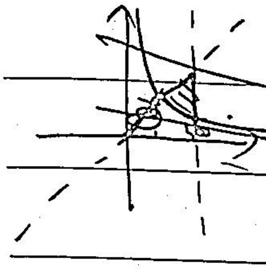
(1) 原式 $= \int_{-1}^{1} dx \int_{-1}^{1} (x^2+y^2)\,dy$



$$= \int_{-1}^{1} \left(x^2y+\frac{1}{3}y^3\right)\Big|_{-1}^{1} dx = 2\int_{-1}^{1}\left(x^2+\frac{1}{3}\right)dx$$

$$= 4\int_{0}^{1}\left(x^2+\frac{1}{3}\right)dx = 4\left(\frac{1}{3}x^3+\frac{1}{3}x\right)\Big|_{0}^{1} = \frac{8}{3}$$

(5)



$$\cancel{D=\{(x,y)\mid 0\le x\le 2,\ \ \overbrace{0\le y\le x\ (x\in[0,1])}^{D_1},\ \overbrace{0\le y\le \frac{1}{x},\ x\in[1,2]}^{D_2}}$$

$$\cancel{=\iint_{D_1}\frac{x^2}{y^2}d\sigma+\iint_{D_2}\frac{x^2}{y^2}d\sigma}$$

$$\cancel{=\int_0^1 dx\int_0^x \frac{x^2}{y^2}dy+\int_1^2 dx\int_0^{\frac{1}{x}}\frac{x^2}{y^2}dy}$$

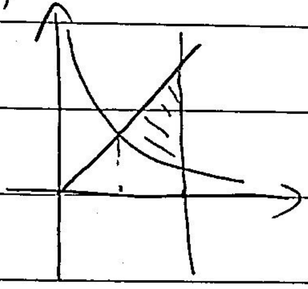
$=$

$$\cancel{D=\{(x,y)\mid 0\le y\le 1,\ \ y\le x\le 2\ (y\in[0,\tfrac{1}{2}]),\ \ y\le x\le}$$

$$\cancel{D=\{(x,y)\mid \tfrac{1}{2}\le y\le 2,\ \ \tfrac{1}{y}\le x\le 2,\ y\in[\tfrac{1}{2},1],\ \ y\le x\le 2\ \ y\in[1,2]\}}$$

$$\cancel{\text{原式}=\int_{\frac{1}{2}}^{1}dy\int_{\frac{1}{y}}^{2}\frac{x^2}{y^2}dx+\int_1^2 dy\int_y^2\frac{x^2}{y^2}dx}$$

(5).

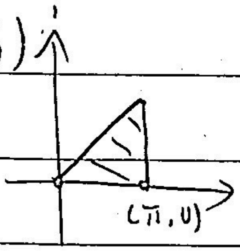


$$D=\{(x,y)\mid 1\le x\le 2,\ \ \frac{1}{x}\le y\le x\}$$

$$\iint_D \frac{x^2}{y^2}d\sigma = \int_1^2 dx\int_{\frac{1}{x}}^{x}\frac{x^2}{y^2}dy$$

$$=\int_1^2 x^3-x\,dx = \left(\frac{1}{4}x^4-\frac{1}{2}x^2\right)\Big|_1^2 = 2\frac{1}{4}$$

(6)



$$D=\{(x,y)\mid 0\le x\le \pi,\ \ 0\le y\le x\}$$

$$\iint_D x\cos(x+y)\,d\sigma = \int_0^{\pi}dx\int_0^x x\cos(x+y)\,dy$$

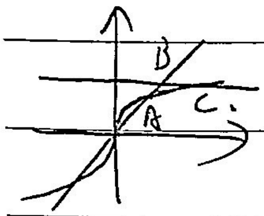
$$=\int_0^{\pi}(x\sin 2x - x\sin x)\,dx$$

$$\cancel{=\pi\int_0^{\frac{\pi}{2}}(\sin 2x-\sin x)\,dx}$$

$$=\left(-\frac{1}{2}\cos 2x+\cos x\right)x\Big|_0^{\pi}-\int_0^{\pi}\cos x-\frac{1}{2}\cos 2x\,dx$$

$$=-\frac{3}{2}\pi$$

(8)



交点: $A(1,1)\quad B(2,2).\quad C(8,2).$

$D: \{(x,y) \mid 1 \le y \le 2,\ y \le x \le y^3\}.$

$\iint_D \sin(\frac{x}{y})d\sigma = \int_1^2 dy \int_y^{y^3} \sin\frac{x}{y}\, dx$

$= \int_1^2 (-y\cos y^2 + y\cos 1)dy = \frac{3}{2}\cos 1 - \frac{1}{2}\sin 4 + \frac{1}{2}\sin 1$

170.9

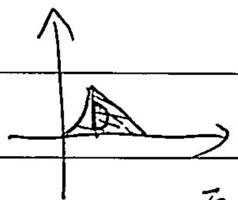
$\iint_D f(x)g(y)\, d\sigma = \int_a^b dx \int_c^d f(x)g(y)\, dy = \int_a^b (f(x)(G(d)-G(c)))dx$

$= [G(d)-G(c)][F(b)-F(a)]$

$= (\int_a^b f(x)dx)\cdot(\int_c^d g(y)\,dy)$

170.10

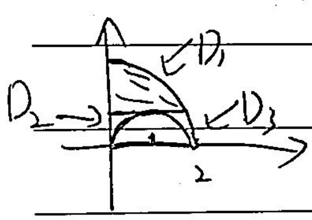
(2) $\int_0^1 dx \int_0^{x^2} f(x,y)dy + \int_1^2 dx \int_0^{2-x} f(x,y)dy$



$D = \{(x,y) \mid 0 \le y \le 1,\ \sqrt{y} \le x \le 2-y\}.$

$\therefore$ 原式 $= \int_0^1 dy \int_{\sqrt{y}}^{2-y} f(x,y)\, dx.$

(4) $\int_0^2 dx \int_{\sqrt{2x-x^2}}^{\sqrt{4-x^2}} f(x,y)dy.$



$D_1 = \{(x,y) \mid 1 \le y \le 2,\ 0 \le x \le \sqrt{4-y^2}.\}.$

$D_2 = \{(x,y) \mid 0 \le y \le 1,\ 0 \le x \le 1-\sqrt{1-y^2}\}.$

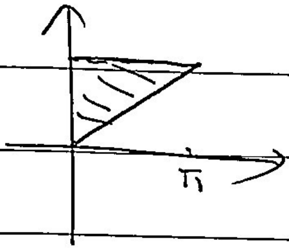
$D_3 = \{(x,y) \mid 0 \le y \le 1,\ 1+\sqrt{1-y^2} \le x \le \sqrt{4-y^2}\}.$

$\therefore$ 原式 $= \int_1^2 dy \int_0^{\sqrt{4-y^2}} f(x,y)dx + \int_0^1 dy \int_0^{1-\sqrt{1-y^2}} f(x,y)dx$

$+ \int_0^1 dy \int_{1+\sqrt{1-y^2}}^{\sqrt{4-y^2}} f(x,y)dx.$

170. 11

(2).

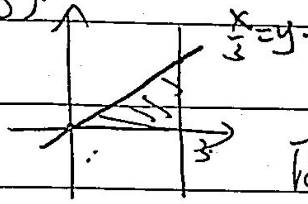


$\cancel{\int_0^{\pi} d}$ $D=\{(x,y) \mid 0\le y\le \pi,\ 0\le x\le y\}$.

原式 $=\int_0^{\pi} dy \int_0^{y} \frac{\sin y}{y} dx$

$=\int_0^{\pi} \sin y\, dy = -\cos y\big|_0^{\pi} = 2$

(3).

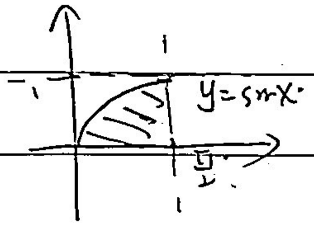


$D=\{(x,y) \mid 0\le x\le 3,\ 0\le y\le \frac{x}{3}\}$.

原式 $=\int_0^3 dx \int_0^{\frac{x}{3}} e^{x^2} dy$

$=\int_0^3 \frac{1}{3} e^{x^2} x\, dx = \frac{1}{6}\int_0^3 e^{x^2} dx^2 = \frac{1}{6}(e^9-1)$.

(5).



$D=\{(x,y) \mid 0\le x\le \frac{\pi}{2},\ 0\le y\le \sin x\}$.

原式 $=\int_0^{\frac{\pi}{2}} dx \int_0^{\sin x} \cos x\sqrt{1+\cos^2 x}\, dy$.

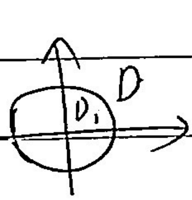
$=\int_0^{\frac{\pi}{2}} \sin x\cos x\sqrt{1+\cos^2 x}\, dx = -\int_0^{\frac{\pi}{2}} \cos x\sqrt{1+\cos^2 x}\, d\cos x$

$\overset{\cos x=u}{=\!=} -\int_1^0 u\sqrt{1+u^2}\, du = +\frac{1}{2}\int_0^1 \sqrt{1+u^2}\, du^2 \overset{u^2=t}{=\!=} +\frac{1}{2}\int_0^1 \sqrt{1+t}\, dt$

$= +\frac{1}{2}\times\frac{2}{3}(1+t)^{\frac{3}{2}}\Big|_0^1 = \frac{1}{3}(2^{\frac{3}{2}}-1)$.

171. 12.

(1)



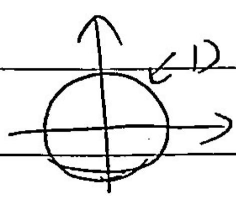
原式 $=4\iint_{D_1} xy\, d\sigma$

$D_1=\{(x,y) \mid 0\le x\le R,\ 0\le y\le \sqrt{R^2-x^2}\}$.

原式 $=4\int_0^R dx \int_0^{\sqrt{R^2-x^2}} xy\, dy = 2\int_0^R R^2x-x^3\, dx$

$=\frac{1}{2}R^4$.

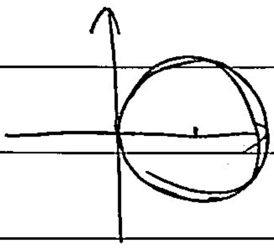
(2)、



$\iint_D y^3 d\sigma = 0$ $\iint_D x^2 \tan x \, d\sigma = 0$

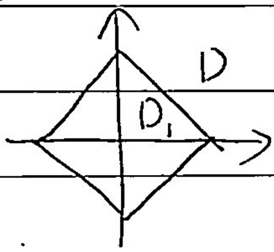
$\therefore$ 原式 $= 4 A_D = 4\times\pi\times 4 = 16\pi$

(3)



原式 $= 0$

(4)



原式 $= 4\iint_{D_1}(|x|+|y|)d\sigma = 4\iint_{D_1}(x+y)d\sigma$.

$D_1 = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le 1-x\}$.

原式 $= 4\int_0^1 dx \int_0^x (x+y)dy = 4\int_0^1 (xy+\frac{1}{2}y^2)\Big|_0^{x+1} dx$

~~$= \int_0^1 \frac{3}{2}x^2 dx = \frac{1}{2}x^3\Big|_0^1 = \frac{1}{2}$~~

$= \frac{4}{3}$