168.1

(2) $z=\sqrt{1-x^2-y^2}$ $\quad \iint_D z\,d\sigma = \frac{2}{3}\pi$

(3) $z=\sqrt{9-y^2}$ $\quad \iint_D z\,d\sigma = h\cdot S = 4\times\frac{1}{4}\pi\cdot 9 = \cancel{18\pi}\,9\pi$

169.2

(3) $\rho = k d = k\sqrt{x^2+y^2+z^2}$ $\quad m=\iint_D k\sqrt{x^2+y^2+z^2}\,d\sigma$, $D=\{(x,y,z)\mid x^2+y^2+z^2\le R^2\}$.

$\cancel{m=\lim_{\lambda\to0}\sum_{i=1}^{n}\rho(x_i,y_i)\Delta\sigma = \iint_D k\sqrt{x^2+y^2}\,d\sigma \quad D=\{(x,y)\mid x^2+y^2\le R^2\}}$.

169.3

(1) (a) $0\le x,y\le 1$ $\quad 0\le x+y\le 1$ $\quad \therefore (x+y)^2\ge(x+y)^3$

$\therefore$ $\cancel{D}$ $I_1\ge I_2$

(b) $5\ge x+y\ge 1$ $\quad \therefore (x+y)^3\ge(x+y)^2$ $\quad \therefore I_2\ge I_1$.

(3) $\sin^2(x+y)\le(x+y)^2$ $\quad \therefore I_1\le I_2$

169.4

(1) $xy(x+y)\in[0,2]$ $\quad A_D=1$ $\quad \therefore I\in[0,2]$

(3) $\frac{1}{\ln(4+x+y)}\in[\frac{1}{\ln 16},\frac{1}{\ln 4}]$ $\quad A_D=32$ $\quad \therefore I\in[\frac{32}{\ln 16},\frac{32}{\ln 4}]$

169.5

$\iint_{D_r} f(x,y)\,d\sigma = f(\xi,\eta)A_D = f(\xi,\eta)\cdot\pi r^2$ $\quad \xi,\eta\in D$.

$\lim_{r\to0^+}\frac{1}{\pi r^2}\iint_{D_r} f(x,y)\,d\sigma = f(\xi,\eta) = \cancel{f(0,0)}\,f(x_0,y_0)$