116.44

(1) $\vec{r}=(a\sin^2 t,\ b\sin t\cos t,\ c\cos^2 t)$

$\vec{r}'=(2a\sin t\cos t,\ b\cos 2t,\ -c\sin 2t)$. $t=\frac{\pi}{4}$

$\Rightarrow \vec{r}'=(a, 0, -c)$. $\vec{r}=(\frac{1}{2}a, \frac{1}{2}b, \frac{1}{2}c)$.

$\therefore$ l: $\frac{x-\frac{1}{2}a}{a}=\frac{y-\frac{1}{2}b}{0}=\frac{z-\frac{1}{2}c}{-c}$

π: $a(x-\frac{1}{2}a)-c(z-\frac{1}{2}c)=0$

(3) $\begin{cases} x^2+y^2+z^2-3x=0 \\ 2x-3y+5z-4=0 \end{cases}$ 两边 d.

$\Rightarrow \begin{cases} 2xdx+2ydy+2zdz-3dx=0 \\ 2dx-3dy+5dz=0 \end{cases} \Rightarrow \begin{cases} dy=\frac{9}{16}dx \\ dz=-\frac{1}{16}dx \end{cases}$

$\Rightarrow$ l: $\frac{x-1}{dx}=\frac{y-1}{dy}=\frac{z-1}{dz} \Rightarrow \frac{x-1}{1}=\frac{16}{9}(y-1)=-16(z-1)$.

π: $(x-1)+\frac{9}{16}(y-1)-\frac{1}{16}(z-1)=0$.

117.46

(1) $z-\arctan\frac{y}{x}=0=F(x,y,z)$

$\Rightarrow F_x=\frac{y}{x^2+y^2}=\frac{1}{2}$, $F_y=-\frac{x}{x^2+y^2}=-\frac{1}{2}$, $F_z=1$.

$\therefore$ l: $2(x-1)=-2(y-1)=z-\frac{\pi}{4}$.

π: $\frac{1}{2}(x-1)-\frac{1}{2}(y-1)+z-\frac{\pi}{4}=0$

(3) $e^{\frac{x}{z}}+e^{\frac{y}{z}}-4=0=F(x,y,z)$ $(\ln 2, \ln 2, 1)$.

$\Rightarrow F_x=\frac{1}{z}e^{\frac{x}{z}}$, $F_y=\frac{1}{z}e^{\frac{y}{z}}$, $F_z=-\frac{x}{z^2}e^{\frac{x}{z}}-\frac{y}{z^2}e^{\frac{y}{z}}$

$=2$, $=2$, $=-4\ln 2$.

$\therefore$ l: $\frac{x-\ln 2}{2}=\frac{y-\ln 2}{2}=\frac{z-1}{-4\ln 2}$

π: $2(x-\ln 2)+2(y-\ln 2)-4\ln 2(z-1)=0$

117.47

设 π: $(4+\lambda)x+(1+\lambda)y-(1+\lambda)z-3=0$. 与 (x,y,z) 相切

$F(x,y,z)=3x^2+y^2-z^2-3=0$. $F_x=6x$ $F_y=2y$ $F_z=2z$

$\therefore \begin{cases} (4+\lambda,\ 1+\lambda,\ -1-\lambda) \parallel (6x,\ 2y,\ -2z) \\ (4+\lambda)x+(1+\lambda)y-(1+\lambda)z-3=0 \\ 3x^2+y^2-z^2-3=0 \end{cases}$

$\Rightarrow$ $\lambda=-1$ 或 -7.

$\therefore$ $3x-3=0 \Rightarrow x=1$

或 $\cancel{\frac{18}{7}x-\frac{3}{7}y+\frac{3}{7}z-3=0 \Rightarrow 6x-y+z-7=0}$ $x+2y-2z+1=0$.

117.48

(1) $x^2+y^2+z^2-14=0$

$F_x=2x=-2$, $F_y=2y=-4$, $F_z=2z=6$. $\therefore \vec{s_1}=(-2,-4,6)$

$3x^2+y^2+z^2-16=0$

$F_x=6x=-6$, $F_y=2y=-4$, $F_z=2z=6$ $\therefore \vec{s_2}=(-6,-4,6)$

$\therefore$ $\cos\theta=\left|\frac{\vec{s_1}\cdot\vec{s_2}}{|\vec{s_1}||\vec{s_2}|}\right|=\frac{8}{\sqrt{77}}$ $\therefore \theta=\arccos\frac{8}{\sqrt{77}}$

(2) 证明:

交点 $\begin{cases} x^2+y^2+z^2=ax \\ x^2+y^2+z^2=by \end{cases}$ $\Rightarrow$ $\cancel{y=\frac{b}{a}x}$ $\begin{cases} y=\frac{a}{b}x \\ z^2=ax-(1+\frac{a^2}{b^2})x^2. \end{cases}$

两边d $\Rightarrow \begin{cases} 2xdx+2ydy+2zdz-adx=0 \\ 2xdx+(2y-b)dy+2zdz=0. \end{cases}$

$\vec{s_1}=(2x-a,\ 2y,\ 2z)$ $\vec{s_2}=(2x,\ 2y-b,\ 2z)$.

$\therefore \vec{s_1}\cdot\vec{s_2}=0$ $\therefore$ 正交.

117.49

(2) 证明: $F(x,y,z) = x^{\frac{2}{3}} + y^{\frac{2}{3}} + z^{\frac{2}{3}} - a^{\frac{2}{3}} = 0$.

$F_x = \frac{2}{3}x^{-\frac{1}{3}}$ $F_y = \frac{2}{3}y^{-\frac{1}{3}}$ $F_z = \frac{2}{3}z^{-\frac{1}{3}}$

$\therefore \frac{2}{3}x_0^{-\frac{1}{3}}(x-x_0) + \frac{2}{3}y_0^{-\frac{1}{3}}(y-y_0) + \frac{2}{3}z_0^{-\frac{1}{3}}(z-z_0) = 0$

交 $(0, 0, a^{\frac{2}{3}}z_0^{\frac{1}{3}})$ $(a^{\frac{2}{3}}x_0^{\frac{1}{3}}, 0, 0)$ $(0, a^{\frac{2}{3}}y_0^{\frac{1}{3}}, 0)$

$\therefore \cancel{a^{\frac{2}{3}}(z_0)^{\frac{2}{3}}}$ $a^{\frac{4}{3}}(z_0^{\frac{2}{3}} + x_0^{\frac{2}{3}} + y_0^{\frac{2}{3}}) = a^2$.

(3) $z = xf(\frac{y}{x}) \Rightarrow F(x,y,z) = xf(\frac{y}{x}) - z \equiv 0$

两边d $f(\frac{y}{x})dx + x\,df(\frac{y}{x}) - dz = 0$

$\Rightarrow \cancel{f(\frac{y}{x})dx + x(df}).\ df(\frac{y}{x}) = f'(\frac{y}{x}) \cdot \frac{xdy - \cancel{xdy}\,ydx}{x^2}$

$\Rightarrow F_x = f - f'\frac{y}{x}$ $F_y = f'$. $F_z = -1$.

$\therefore (f - f'\frac{y_0}{x_0})(x-x_0) + f'(y-y_0) - (z-z_0) = 0$

$\Rightarrow f\cdot(x-x_0) - f'\frac{x}{x_0}y_0 + f'y_0 + f'y - f'y_0 - z + z_0$

$\Rightarrow \cancel{-x_0 f} - f'\frac{x}{x_0}y_0 + f'y = 0$

$\Rightarrow y = \frac{y_0}{x_0}x$ 恒过 $(x_0, y_0, 0)$