P217. 4.9

(a) $x(n)=u(n)-u(n-6)$

$$X(\omega)=\sum_{n=0}^{5}e^{-j\omega n}=\frac{1-e^{-6j\omega}}{1-e^{-j\omega}}=(1-e^{-6j\omega})/(1-e^{-j\omega})$$

(b) $x(n)=2^n u(-n)$.

$$X(\omega)=\sum_{n=-\infty}^{+\infty}x(n)e^{-j\omega n}=\sum_{n=-\infty}^{0}2^n e^{-j\omega n}=\sum_{n=0}^{+\infty}2^{-n}e^{j\omega n}=\frac{1}{1-\frac{1}{2}e^{j\omega}}$$

(d) $x(n)=(\alpha^n\sin\omega_0 n)u(n)\quad |\alpha|<1$.

$$X(\omega)=\sum_{n=0}^{+\infty}\alpha^n\sin\omega_0 n\, e^{-j\omega n}=\sum_{n=0}^{+\infty}\alpha^n\frac{1}{2j}(e^{j\omega_0 n}-e^{-j\omega_0 n})e^{-j\omega n}$$

$$=\frac{1}{2j}\left[\frac{1}{1-\alpha e^{-j(\omega-\omega_0)}}-\frac{1}{1-\alpha e^{-j(\omega+\omega_0)}}\right]$$

P217. 4.10

(a) $X(\omega)=\begin{cases}0 & 0\le|\omega|\le\omega_0\\ 1 & \omega_0<|\omega|\le\pi\end{cases}$

$$x(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(\omega)e^{j\omega n}d\omega=\int_{-\pi}^{-\omega_0}e^{j\omega n}d\omega+\int_{\omega_0}^{\pi}e^{j\omega n}d\omega$$

$$=-\frac{\sin\omega_0 n}{n\pi}\ (n\neq0).\quad x(0)=-\frac{\omega_0}{\pi}+1$$

(b) $X(\omega)=\cos^2\omega=\frac{1}{2}(1+\cos2\omega)=\frac{1}{4}(2+e^{2j\omega}+e^{-2j\omega})$

$$x(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(\omega)e^{j\omega n}d\omega=\frac{1}{8\pi}[2\pi\cdot2\delta(n)+2\pi\cdot\delta(n+2)+2\pi\delta(n-2)]$$

$$=\frac{1}{4}[\delta(n+2)+2\delta(n)+\delta(n-2)]\ (n\neq0),\ x(0)=\frac{1}{2}$$

P218. 4.13

已知 $x(n)=\begin{cases}1 & |n|\le M\\ 0 & \text{o.w.}\end{cases}\iff X(\omega)=1+2\sum_{n=1}^{M}\cos\omega n.$

证明: ~~$x_1(n)=x(n)u(n)$. $X_1(\omega)=X(\omega)*\frac{1}{1-e^{-j\omega}}$~~

$X_1(\omega)=\sum_{n=0}^{M}e^{-j\omega n}=\frac{1-e^{-j\omega(M+1)}}{1-e^{-j\omega}}$, 等比求和. 同理证 $X_2(\omega)=\cdots$

$\therefore X_1(\omega)+X_2(\omega)=\frac{\sin(M+\frac{1}{2})\omega}{\sin(\frac{\omega}{2})}$, 用了 $\sin^2\omega=\frac{1}{2}(1-\cos2\omega)$ 与 $\sin\omega=2\sin\frac{\omega}{2}\cos\frac{\omega}{2}$.

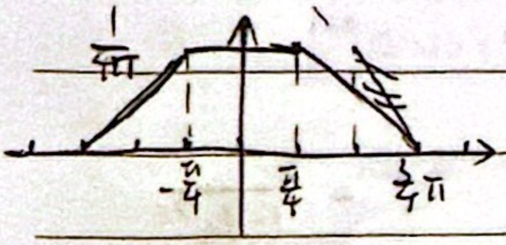
$x(n)=x_1(n)+x_2(n)\Rightarrow X(\omega)=X_1(\omega)+X_2(\omega)=\cdots$ 这里真写不下, 略去.

$\therefore 1+2\sum_{n=1}^{M}\cos\omega n=\frac{\sin(M+\frac{1}{2})\omega}{\sin(\frac{\omega}{2})}$. 实际上用 $\cos\omega n=(e^{j\omega n}+e^{-j\omega n})/2$ 也行.

P220. 4.19

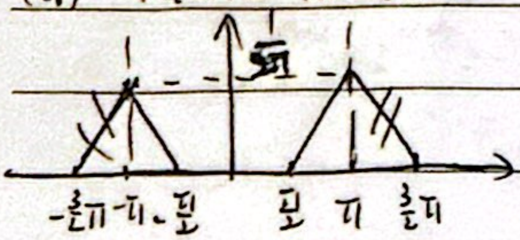
(c) $x_1(n) = x(n)\cos(\frac{\pi n}{4})$, $X_1(\omega) = \frac{1}{4\pi} X(\omega) * [\delta(\omega-\frac{\pi}{4}) + \delta(\omega+\frac{\pi}{4})]$

$$= \frac{1}{4\pi}[X(\omega-\frac{\pi}{4}) + X(\omega+\frac{\pi}{4})]$$



(d). $x_4(n) = x(n)\cos\pi n \Rightarrow X_4(\omega) = \frac{1}{4\pi}[X(\omega-\pi) + X(\omega+\pi)]$

$$= \frac{1}{2\pi} X(\omega-\pi).$$



P220. 4.22

(a) $x(2n+1) = x_1$ $\quad X_1(\omega) = \sum_{n=-\infty}^{+\infty} x(2n+1) e^{-j\omega n} \overset{2n+1=N}{=\!=} \sum_{N=-\infty}^{+\infty} x(N) e^{-j\frac{\omega}{2}(N-1)}$

$$= \sum_{N=-\infty}^{+\infty} x(N) e^{-\frac{\omega}{2}Nj} \cdot e^{j\frac{\omega}{2}} = e^{j\frac{\omega}{2}} X(\frac{\omega}{2})$$

$$= e^{j\frac{\omega}{2}} \frac{1}{1-ae^{-j\frac{\omega}{2}}}$$

(f) $x(n) * x(-n) = x_2$, $\cancel{X_2(\omega) = \sum_{n=-\infty}^{+\infty}}$

由 $F[x(n) * x(-n)] = X(\omega) \cdot X(-\omega) = \frac{1}{1+a^2-2a\cos\omega}$

P267. 5.3

(a) $h(n) = (\frac{1}{2})^n u(n)$. 求 $|H|$, $\angle H$.

$\cancel{H(\omega)=}$ $H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \Rightarrow H(\omega) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} = \frac{1}{1-\frac{1}{2}\cos\omega + \frac{1}{2}j\sin\omega}$

∴ $|H(\omega)| = (\frac{5}{4} - \cos\omega)^{-\frac{1}{2}}$

$\angle H = -\tan^{-1}\frac{\sin\omega}{2-\cos\omega}$

P267. 5.4

(a) $y(n)=\frac{1}{2}[x(n)+x(n-1)] \Rightarrow Y(z)=\frac{1}{2}[X(z)+z^{-1}X(z)] \Rightarrow H(z)=\frac{1}{2}(1+z^{-1})$

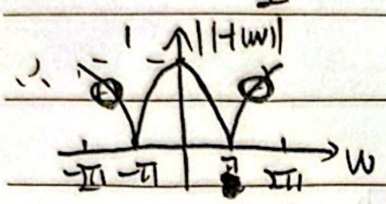
$H(w)=\frac{1}{2}(1+e^{-jw})=\frac{1}{2}(1+\cos w-j\sin w)$

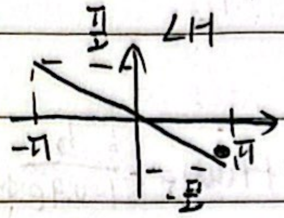
$|H(w)|=\cancel{\frac{1}{2}\sqrt{\ }}\ \cancel{1+\cos w}\ |\cos\frac{w}{2}|$.

$\angle H(w)=-\tan^{-1}\frac{\sin w}{1+\cos w}=\cancel{-w}$, $\frac{\sin w}{1+\cos w}=\frac{2\sin\frac{w}{2}\cos\frac{w}{2}}{2\cos^2\frac{w}{2}}=\tan\frac{w}{2}$.

$=-\frac{w}{2}$

∴

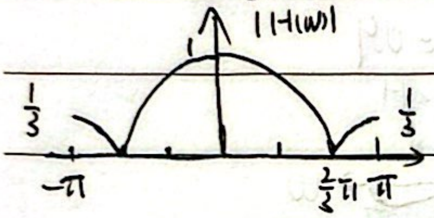


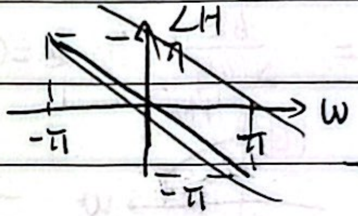


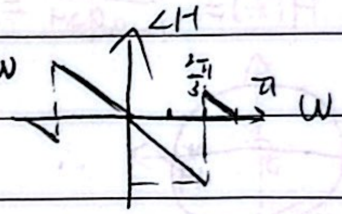
(g) $y(n)=\frac{1}{3}[x(n)+x(n-1)+x(n-2)] \Rightarrow H(z)=\frac{1}{3}(1+z^{-1}+z^{-2})$

$H(w)=\frac{1}{3}(1+e^{-jw}+e^{-2jw})=\frac{1}{3}[(1+\cos w+\cos 2w)-j(\sin w+\sin 2w)]$

∴ $|H(w)|=\frac{1}{3}|1+2\cos w|$, $\cancel{\angle H(w)=-w}$ $\angle H(w)=\begin{cases}-w & 0<w<\frac{2}{3}\pi \\ \pi-w & \frac{\pi}{3}<w<\pi.\end{cases}$







P268. 5.5

(a) $y(n)=x(n)+x(n-10) \Rightarrow H(z)=1+z^{-10} \Rightarrow H(w)=1+e^{-10jw}$

$|H(w)|=2|\cos 5w|$ $\angle H(w)=-5w$

(b) 1. $x(n)=\cos\frac{\pi}{10}n+3\sin(\frac{\pi}{3}n+\frac{\pi}{10})=\frac{1}{2}[e^{j\frac{\pi}{10}n}+e^{-j\frac{\pi}{10}n}]+\frac{3}{2j}[e^{j(\frac{\pi}{3}n+\frac{\pi}{10})}-e^{-j(\frac{\pi}{3}n+\frac{\pi}{10})}]$

※已知 e^{jwn} 经过LTI系统，输出 $y(n)=H(w)*e^{jwn}$.

∴ $y(n)=H(\frac{\pi}{10})\cos\frac{\pi}{10}n+3\sin(\frac{\pi}{3}n+\frac{\pi}{10})H(\frac{\pi}{3})$ $\cancel{3(\frac{1}{2}+\frac{\sqrt{3}}{2})}$

$=3(\frac{1}{2}+\frac{\sqrt{3}}{2}j)\sin(\frac{\pi}{3}n+\frac{\pi}{10})$.

P268. 5.10

(a) $y(n) = \frac{1}{2}[x(n)+x(n-1)] \Rightarrow H(\omega) = \frac{1}{2}(1+e^{-j\omega})$.

$|H(\omega)| = |\cos\frac{\omega}{2}|$, $\angle H(\omega) = \cancel{-\omega} -\frac{\omega}{2}$

(b) $y(n) = \frac{1}{2}[x(n-1) - x(n)] \Rightarrow H(\omega) = \frac{1}{2}(e^{-j\omega}-1)$

$|H(\omega)| = |\sin\frac{\omega}{2}|$, $\angle H(\omega) = \frac{\pi}{2}-\frac{\omega}{2}$

P269. 5.12

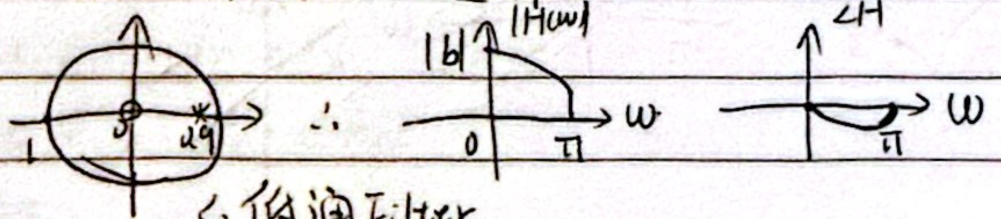
$y(n) = 0.9\, y(n-1) + b\,x(n) \Rightarrow H(\omega) = \frac{b}{1-0.9e^{-j\omega}}$

(a) $\cancel{H(\omega)=}$ $|H(\omega)| = \frac{b}{\sqrt{1.81-1.8\cos\omega}}$ $\quad |H(0)| = \frac{b}{\sqrt{0.01}} = 1 \Rightarrow b = \sqrt{0.01} = 0.1$.

~~(b) $|H(\omega)| = \sqrt{\ }$~~

(b) $|H(\omega)| = (181 - 180\cos\omega)^{-\frac{1}{2}} = 2^{-\frac{1}{2}} \Rightarrow \omega = \pm\cos^{-1}\frac{179}{180} = \pm 0.105$

(c) $H(z) = \frac{b}{1-0.9z^{-1}} = \frac{bz}{z-0.9}$ $\quad z=0,\ p=0.9$



∴ 低通 Filter

附加. II型 Filter 的群时延.

$H(\omega) = \sum_{n=0}^{N} h(n)e^{-j\omega n} = \sum_{n=0}^{\frac{N-1}{2}} h(n)e^{-j\omega n} + \sum_{n=\frac{N+1}{2}}^{N} h(n)e^{-j\omega n}$

令 $n'=N-n$ ∴ $\sum_{n=\frac{N+1}{2}}^{N} h(n)e^{-j\omega n} = \sum_{n'=0}^{\frac{N-1}{2}} h(n')e^{-j\omega(N-n')}$

$= e^{-j\frac{N}{2}\omega} \sum_{n=0}^{\frac{N-1}{2}} h(n)e^{-j\omega(\frac{N}{2}-n)}$

∴ $H(\omega) = e^{-j\frac{N}{2}\omega} \sum_{n=0}^{\frac{N-1}{2}} 2h(n)\cos(n-\frac{N}{2})\omega$ 由于 $h(n) \in R$

∴ $\angle H(\omega) = -\frac{N}{2}\omega \Rightarrow \tau(\omega) = \frac{N}{2}$