P177. 1

(1) 全体CS本科生就业后工资

(2). 200名毕业生的工资

(3) 200

P177.2

(1) (2) (3) (7)

P178.5

(1) $P(\max\{X_1, X_2, X_3\}<5) = P(X_1<5)P(X_2<5)P(X_3<5)$

$= \Phi(\frac{5-2}{3})^3 = (0.8413)^3 = 0.595$

(2). $P(\cdots) = (\Phi(\frac{3.5-2}{3}) - \Phi(\frac{-2.5-2}{3})) + (\Phi(\frac{6.5-2}{3}) - \Phi(\frac{2-2}{3})) -$

$= 0.7873 \qquad - (\quad)(\quad)$

(3). $= E(X_1^2)E(X_2^2)E(X_3^2) = (E(X_1^2))^3 = 13^3 = 2197$

$E(X_1^2) = \sigma^2 + \mu^2 = 9+4 = 13$

(4) $D(X_1X_2X_3) = E(X_1^2X_2^2X_3^2) - E^2(X_1X_2X_3) = 2197-64$

$E(X_1X_2X_3) = \mu^3 = 8 \qquad = 2133$

$D(2X_1-3X_2-X_3) = 4\sigma^2 + 9\sigma^2 + \sigma^2 = \cancel{\sigma^2} = \cancel{-54}.\ 126$

$14\sigma^2$

P178. 6

~~$\mathrm{Cov}(X_i-\bar{X}, X_j-\bar{X}) = E[(X_i-\bar{X})(X_j-\bar{X})] - E(X_i-\bar{X})E(X_j-\bar{X})$~~

~~$\bar{X} = \frac{1}{n}\sum_{k=1}^{n} X_k$. $E(X_i-\bar{X}) = E(X_i) - E(\bar{X}) = \mu-\mu = 0 = E(X_j-\bar{X})$.~~

$E($~~$X_iX_j - \bar{X}(X_i+X_j) + \bar{X}^2) = \mu^2$~~

$P(|\bar{X}-\mu|<1) \geq 0.95$. $\bar{X} \sim N(\mu, \frac{36}{n})$

$\Rightarrow P(\frac{|\bar{X}-\mu|}{6/\sqrt{n}} < \frac{1}{6/\sqrt{n}}) \geq 0.95 \Rightarrow n \geq 139.$

$= 2\Phi(\frac{\sqrt{n}}{6}) - 1$

P178. 7.

$\mathrm{cov}(X_i-\bar{X}, X_j-\bar{X}) = \mathrm{cov}(X_i, X_j) - \mathrm{cov}(X_j, \bar{X}) - \mathrm{cov}(X_i, \bar{X}) + \mathrm{cov}(\bar{X}, \bar{X})$

$$\begin{aligned} i\neq j &= 0 + D(\bar{X}) - 2\,\mathrm{cov}(X_i, \bar{X}) \\ i=j &= D(X_i) + D(\bar{X}) - 2\,\mathrm{cov}(X_i,\bar{X}) \end{aligned} = \begin{cases} -\frac{\sigma^2}{n} \\ (1-\frac{1}{n})\sigma^2 \end{cases}$$

$D(X_i-\bar{X}) = D(X_j-\bar{X})$

$= D\left(-\frac{1}{n}\sum X_k + X_i\right) = \frac{1}{n^2} D\left((n-1)X_i - \sum_{k\neq i} X_k\right)$

$= \frac{(n-1)^2}{n^2} D(X_i) + \frac{n-1}{n^2}\sigma^2 = \frac{n-1}{n}\sigma^2$

$\therefore \rho_{(i)(j)} = \begin{cases} 1 & i=j \\ -\frac{1}{n-1} & i\neq j \end{cases}$ 说明 $i\neq j$ 时 $X_i-\bar{X}$ 与 $X_j-\bar{X}$ 部分负性相关.

P178. 9

(1) $\cancel{\sum_{i=1}^{10} X_i^2 \sim N(0, 0.25)}$ $X_i \sim N(0, 0.25) \Rightarrow 2X_i \sim N(0,1)$

$\therefore \sum_{i=1}^{10} 4X_i^2 \sim \chi^2(10)$ $P(\sum_{i=1}^{10} X_i^2 \geq 4) = P(4\sum_{i=1}^{10} X_i^2 \geq 16) = P(\chi^2(10) \geq 16)$

$\therefore p = 0.\ \ 16 \approx \chi^2_{0.05}(10)$.

(2). $\sum_{i=1}^{10}(X_i-\bar{X})^2 \geq 4.23 \Rightarrow \sum_{i=1}^{10}\left(\frac{X_i-\bar{X}}{\sigma}\right)^2 \geq 16.92$

$\sum_{i=1}^{10}\left(\frac{X_i-\bar{X}}{\sigma}\right)^2 \sim \chi^2(9)$ $\therefore$ $P(\chi^2(9) \geq 16.92) = 0.05$.

P178. 10

(1) $\sum_{i=5}^{9}(X_i-Y)^2/\sigma^2 \sim \chi^2(4)$ $\therefore c = \frac{1}{\sigma^2} = \frac{1}{4}$.

$aX_1^2 \sim \chi^2(1) \Rightarrow \sqrt{a}X_1 \sim N(0, 4a) \Rightarrow a = \frac{1}{4}$

$(X_2+X_3+X_4) \sim N(0,12)$ $\sqrt{b}(X_1+X_2+X_3) \sim N(0, 12b) \Rightarrow b = \frac{1}{12}$.

(2) $(X_1+X_2) \sim N(0,8)$ $\therefore \frac{1}{2\sqrt{2}}(X_1+X_2) \sim N(0,1)$

$\cancel{\sqrt{X_3^2+X_4^2+X_5^2}}$ $\frac{1}{4}X_i^2 \sim \chi^2(1)$ $\therefore \frac{1}{4}(X_3^2+X_4^2+X_5^2) \sim \chi^2(3)$.

$\therefore \dfrac{\frac{1}{2\sqrt{2}}(X_1+X_2)}{\sqrt{\frac{1}{4}(X_3^2+X_4^2+X_5^2)/3}} \sim t(3) \Rightarrow d = \frac{\sqrt{6}}{2}, n=3$

P179. 11

① $S=\frac{1}{\sqrt{n-1}}\sqrt{\sum_{i=1}^{n}(X_i-\bar{X}_n)^2}$，$X_{n+1}\sim N(\mu,\sigma^2)$，$\bar{X}\sim N(\mu,\frac{\sigma^2}{n})$.

$\because$ X_{n+1}，$\bar{X}$ 独立，$(X_{n+1}-\bar{X})\sim N(0,\frac{1+n}{n}\sigma^2)$.

$\therefore \frac{1}{\sigma}\sqrt{\frac{n}{n+1}}(X_{n+1}-\bar{X})\sim N(0,1)$.

$\frac{1}{\sigma}S=\frac{1}{\sqrt{n-1}}\sqrt{\sum(\frac{X_i-\bar{X}}{\sigma})^2}=\frac{1}{\sqrt{n-1}}\sqrt{\chi^2(n-1)}=\sqrt{\frac{\chi^2(n-1)}{n-1}}$.

$\therefore$ 原式 $\sim t(n-1)$

② $(\frac{1}{\sigma}(X_{n+1}-\bar{X})\sqrt{\frac{n}{n+1}})^2\sim\chi^2(1)=\frac{n}{n+1}(\frac{X_{n+1}-\bar{X}}{\sigma})^2$

$\frac{1}{\sigma^2}S^2=\frac{1}{n-1}\sum_{i=1}^{n}(\frac{X_i-\bar{X}}{\sigma})^2\sim\frac{1}{n-1}\chi^2(n-1)$.

$\therefore$ 原式 $\sim F(1,n-1)$.

P179. 12.

$X\sim\chi^2(12)$，$Y\sim\chi^2(12)$，X,Y 独立.

$F=\frac{X}{Y}$ $P(F>1)=P(X>Y)=\cancel{P(X-Y>0)}$

~~令 $X-Y=Z$，$Z\sim\chi^2($~~ $=1-P(X<Y)$.

X,Y 独立同分布 $\therefore P(X>Y)=P(X<Y)\Rightarrow P(F>1)=0.5$.

P179. 13

(1) $\cancel{Y=(1+\frac{2X_2}{X_1-X_2})^2}$ $Y=(\frac{X_1+X_2}{\sqrt{2}})^2/(\frac{X_1-X_2}{\sqrt{2}})^2$，$(\frac{X_1+X_2}{\sqrt{2}})^2\sim\chi^2(1)$，$(\frac{X_1-X_2}{\sqrt{2}})^2\sim\chi^2(1)$

下证 (X_1+X_2) 与 (X_1-X_2) 独立. $\because (X_1+X_2),(X_1-X_2)\sim N(0,2)$.

$Cov(X_1+X_2,X_1-X_2)=D(X_1)-D(X_2)-Cov(X_1,X_2)+Cov(X_2,X_1)=0$

$\therefore \rho_{XY}=0\Leftrightarrow(X_1+X_2),(X_1-X_2)$ 独立. $\therefore Y\sim F(1,1)$

(2). $\sum_{i=2}^{n}X_i^2\sim\chi^2(n-1)$ $\therefore \frac{X_1^2}{\sum_{i=2}^{n}X_i^2/(n-1)}\sim F(1,n-1)$.

补1.

(1) $X_{(16)} = X_{max}$ $P(\max\{X_i\} > 10) = 1 - P(\max\{X_i\} \le 10)$.

$P(\max\{X_i\} \le 10) = \prod_{i=1}^{16} P(X_i \le 10)$, $P(X_i \le 10) = \Phi\left(\frac{10-8}{2}\right) = \Phi(1)$

$\therefore P(X_{(16)} > 10) = 1 - (0.8413)^{16}$ $= 0.8413$

(2). $P(X_{(1)} > 5) = P(\min\{X_i\} > 5) = (1 - P(X_i \le 5))^{16}$

$P(X_i \le 5) = \Phi\left(\frac{5-8}{2}\right) = \Phi(-1.5) = 1 - \Phi(1.5) = 0.0668$

$\therefore P(X_{(1)} > 5) = (0.9332)^{16}$

补2:

$\sum_{i=1}^{n}(X_i + X_{n+i} - 2\overline{X})^2 = \sum_{i=1}^{n}\{(X_i - \overline{X}) + (X_{n+i} - \overline{X})\}^2 = \sum_{i=1}^{n}\{(X_i-\overline{X})^2 + (X_{n+i} - \overline{X})^2 + 2(X_i - \overline{X})(X_{n+i} - \overline{X})$

$X_i \sim N(\mu, \sigma^2)$, $\overline{X} \sim N(\mu, \frac{\sigma^2}{2n})$, X_i 独立.

$\sum_{i=1}^{n}(X_i^2 + X_{n+i}^2 + 4\overline{X}^2 + 2X_iX_{n+i} - 4X_i\overline{X} - 4X_{n+i}\overline{X})$

$= (\sum_{i=1}^{2n} X_i^2) + \cancel{4}\overline{X}^2 + (2\sum_{i=1}^{n} X_iX_{n+i}) - 4\overline{X}\sum_{i=1}^{2n} X_i = \sum_{i=1}^{2n} X_i^2 + 2\sum_{i=1}^{n} X_iX_{n+i}$

$\therefore E(Y) = E(\sum_{i=1}^{2n} X_i^2) + 2E(\sum_{i=1}^{n} X_iX_{n+i})$, ~~$\sum_{i=1}^{2n} X_i^2 \sim \chi^2(2n)$~~

$= E(\sum X_i^2) + 2\sum_{i=1}^{n} E(X_i)E(X_{n+i}) = E(\sum X_i^2) + 2 \cdot \sum_{i=1}^{n} \mu^2 = E(\sum X_i^2) + 2n\mu^2$

$E(\sum X_i^2) = 2nE(X_i^2) = 2n(\mu^2 + \sigma^2)$ $\therefore$ 原式 $= 2n\sigma^2 + 4n\mu^2$

补3.

$X_i \sim N(\mu, \sigma^2)$, $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, $S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X-\bar{X})^2$

$t_c = C\frac{X_{n+1}-\bar{X}_n}{S_n}$, $\bar{X}_n \sim N(\mu, \frac{\sigma^2}{n})$

$\therefore (X_{n+1}-\bar{X}_n) \sim N(0, \frac{1+n}{n}\sigma^2)$ $\therefore \sqrt{\frac{n}{(1+n)}}\frac{1}{\sigma}(X_{n+1}-\bar{X}_n) \sim N(0,1)$

$S_n = \frac{1}{\sqrt{n-1}}\sqrt{\sum_{i=1}^{n}(X_i-\bar{X})^2}$, $\sum_{i=1}^{n}(\frac{X_i-\bar{X}}{\sigma})^2 \sim \chi^2(n-1)$.

$\therefore \frac{1}{\sigma}S_n = \sqrt{\sum_{i=1}^{n}(\frac{X_i-\bar{X}}{\sigma})^2/(n-1)} = \sqrt{\chi^2(n-1)/n-1}$, $\sqrt{\frac{n}{1+n}}\cdot\frac{1}{\sigma}(X_{n+1}-\bar{X}_n) \sim N(0,1)$.

$\therefore \sqrt{\frac{n}{n+1}}\frac{X_{n+1}-\bar{X}}{S_n} \sim t(n-1)$. $\therefore C = \sqrt{\frac{n}{n+1}}$

自由度为 $n-1$

补4.

$F(x)$ 严格增，连续 $\therefore F^{-1}(x)$ 存在，且严格增，连续.

记 $T = -2\sum_{i=1}^{n}\ln F(X_i) \sim \chi^2(2n)$

令 $Y_i = F(X_i)$ $\therefore P(Y \le y) = P(F(X) \le y) = P(X \le F^{-1}(y))$

$= F(F^{-1}(y)) = y$, $0 \le Y \le 1$.

$\therefore$ 易 ~~Y在~~ $Y \sim U(0,1)$ $\therefore \ln Y_i < 0$.

~~$F(\ln Y)$ $F(\ln y) = P(\ln Y \le$~~ $z = \ln Y$, $F_Z(z) = P(\ln Y \le z) = P(Y \le e^z)$

$\therefore P(Y \le e^z) = \begin{cases} 1 & z \ge 0 \\ \int_{-\infty}^{e^z} f(y)dy = \int_0^{e^z} 1 dy = e^z & z < 0 \end{cases}$

$\therefore F(z) = \begin{cases} e^z & z < 0 \\ 1 & z \ge 0 \end{cases}$ $f(z) = \begin{cases} e^z & z < 0 \\ 0 & z \ge 0. \end{cases}$

$f(-2z) = \begin{cases} \frac{1}{2}e^{-\frac{z}{2}} & z \ge 0 \\ 0 & z \le 0. \end{cases}$ $\therefore -2z \sim E(\frac{1}{2}) = \chi^2(2)$.

$\therefore \sum_{i=1}^{n}(-2z_i) \sim \chi^2(2n)$.

补5.

$X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$, X, Y 独立, $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, $\bar{Y} = \frac{1}{m}\sum_{i=1}^{m} Y_i$

求 $a\bar{X} + b\bar{Y}$, $\frac{1}{\sigma_1^2}\sum_{i=1}^{n}(X_i - \bar{X})^2 + \frac{1}{\sigma_2^2}\sum_{i=1}^{m}(Y_i - \bar{Y})^2$ 的分布.

$\bar{X} \sim N(\mu_1, \frac{\sigma_1^2}{n})$, $\bar{Y} \sim N(\mu_2, \frac{\sigma_2^2}{m})$, $\bar{X}, \bar{Y}$ 独立

$\therefore (a\bar{X} + b\bar{Y}) \sim N(a\mu_1 + b\mu_2, \frac{a^2}{n}\sigma_1^2 + \frac{b^2}{m}\sigma_2^2)$

$\sum_{i=1}^{n}\left(\frac{X_i - \bar{X}}{\sigma_1}\right)^2 \sim \chi^2(n-1)$, $\sum_{i=1}^{m}\left(\frac{Y_i - \bar{Y}}{\sigma_2}\right)^2 \sim \chi^2(m-1)$

$\therefore$ 原式 $\sim \chi^2(m+n-2)$