49.1

(1) $|a|=3$, $a^0=\frac{1}{3}a$

(2) $\overrightarrow{AB}=(1,-1,-4)$ $|\overrightarrow{AB}|=3\sqrt{2}$. $\overrightarrow{AB}^0=\frac{1}{3\sqrt{2}}\overrightarrow{AB}$

49.2

(1) $a+b=(1,-1,6)=i-j+6k$

(2) $-\frac{1}{2}b=(1,-\frac{1}{2},0)=i-\frac{1}{2}j+0k$

(3) $\frac{1}{3}a-b=(3,-\frac{5}{3},2)=3i-\frac{5}{3}j+2k$

49.8

(1) $a\cdot b=1$, $\angle(\widehat{a,b})=\arccos\frac{a\cdot b}{|a|\cdot|b|}=\arccos\frac{1}{14}$

(2) $a=(2,3,-1)$ $b=(1,-3,-7)$ $a\cdot b=0$, $(\widehat{a,b})=\frac{\pi}{2}$

49.9

$\sqrt{|a+b|^2}=\sqrt{a^2+2ab+b^2}=\sqrt{25+64+40}=\sqrt{129}$

$\sqrt{|a-b|^2}=\sqrt{a^2-2ab+b^2}=\sqrt{25+64-40}=7$

49.10

(1) $a\cdot b=|a||b|\cdot\cos(\widehat{a,b})=3\times4\times(-\frac{1}{2})=-6$

(2) $(3a-2b)(a+2b)=3a^2-4b^2+4ab=27-64-24=-61$

49.11

$\begin{cases}(a+3b)(7a-5b)=0\\(a-4b)(7a-2b)=0\end{cases}\Rightarrow\begin{cases}7a^2+16ab-15b^2=0\\7a^2-30ab+8b^2=0\end{cases}$ 令 $\frac{|a|}{|b|}=t$, $\cos\theta=m$.

$\therefore\begin{cases}7t^2+16mt-15=0\\7t^2-30mt+8=0\end{cases}\Rightarrow\begin{cases}t=1\\m=\frac{1}{2}\end{cases}$ $\therefore$ $\theta=\frac{\pi}{3}$

49.12

$2a-3b=(-2,1,-3)$ $\therefore$ $|2a-3b|=\sqrt{14}$.

$\therefore$ $(\cos\alpha, \cos\beta, \cos\gamma)=\left(-\frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}\right)$

49.13

(1) $a^0=\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)$. $(\vec{b})_a = |b|\cos\theta = \frac{3}{\sqrt{5}}$

$\cos\theta = \frac{a\cdot b}{|a||b|} = \sqrt{\frac{3}{5}}$ $(\vec{b})_a a^0 = \left(\frac{6}{5}, \frac{3}{5}, 0\right)$

(2) $a=(1,0,1)$ $b=(-1,-1,0)$, $a^0=\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

$\cos\theta = \frac{a\cdot b}{|a||b|} = -\frac{1}{2}$ $\therefore (\vec{b})_a = |b|\cos\theta = -\frac{\sqrt{2}}{2}$

$(\vec{b})_a a^0 = \left(-\frac{1}{2}, 0, -\frac{1}{2}\right)$

49.4

$\vec{a}=(\alpha,5,-1)$ $\vec{b}=(3,1,y)$ ∴ $\frac{\alpha}{3}=\frac{-1}{y}=5 \Rightarrow \alpha=15,\ y=-\frac{1}{5}$

49.5

$A(1,1,1)$ $B(1,2,0)$ ∴ $\vec{OC}=\frac{1}{3}\vec{OA}+\frac{2}{3}\vec{OB}$ 或

∴ $C(1,\frac{5}{3},\frac{1}{3})$ 或 $C(1,\frac{4}{3},\frac{2}{3})$ $\vec{OC}=\frac{2}{3}\vec{OA}+\frac{1}{3}\vec{OB}$

49.6

$\vec{r}=\frac{n}{m+n}\vec{r_1}+\frac{m}{m+n}\vec{r_2}$ 若 $A\overset{m}{\text{——}}\cdot\overset{n}{\text{——}}B$

$\vec{r}=\frac{m}{m+n}\vec{r_1}+\frac{n}{m+n}\vec{r_2}$ 若 $A\overset{n}{\text{——}}\cdot\overset{m}{\text{——}}B$

49.14

$\vec{AB}=(4,-4,7)$ $\vec{AB}=\vec{OB}-\vec{OA}=(2,-1,7)-\vec{OA}$

∴ $A(-2,3,0)$

49.16

~~(1) a·b=0 ∴ a⊥b sinθ=1 a×b=~~

$a=i-k,\ b=j$ ∴ $\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & 0\end{vmatrix}=i+k=(1,0,1)$

(3) $a\times b=\begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 3 & -1 & 7\end{vmatrix}=13i-10j-7k=(13,-10,-7)$

49.17

$\vec{OA}=(\pm\frac{1}{\sqrt3},\pm\frac{1}{\sqrt3},\pm\frac{1}{\sqrt3})$ $B(x,y,z)$

∴ $\vec{MN}=\vec{MB} \Rightarrow$ ~~B(−3, 3, 2)~~ $B(-3,7,0)$

∴ $\vec{OA}\times\vec{OB}=\begin{vmatrix} i & j & k \\ \frac{1}{\sqrt3} & \frac{1}{\sqrt3} & \frac{1}{\sqrt3} \\ -3 & 7 & 0\end{vmatrix}=\cancel{\frac{10}{\sqrt3}(j+k)}\ (\mp\frac{7}{\sqrt3},\ \mp\frac{3}{\sqrt3},\ \pm\frac{10}{\sqrt3})$

So.18

(1) $c\cdot d = 2ka^2 + b^2 + (k+4)ab = 2ka^2 + b^2 = 2k+4 = 0 \Rightarrow k=-2$

(2) $|\vec{c}\times\vec{d}| = |(2a+b)\times(ka+b)| = |(k-2)|a||b|| = 6$

$\Rightarrow k = -1$ 或 5.

So.19

$[(a+b)\times(b+c)] = (\vec{a}\times\vec{b} + \vec{b}\times\vec{b} + \vec{a}\times\vec{c} + \vec{b}\times\vec{c})$

$= (\vec{a}\times\vec{b} + \vec{a}\times\vec{c} + \vec{b}\times\vec{c}\cancel{+\vec{c}\times\vec{c}) = (\vec{a}+\vec{c})\times(\vec{b}+\vec{c})\cdot(\vec{c}+\vec{a}})$

原式 $= (a\times b + a\times c + b\times c)\cdot(c+a) = (a\times b)\cdot c + a\times b\cdot a + a\times c\cdot a + a\times c\cdot c$

$= 2+0+0+0+0+(b\times c)\cdot a + (b\times c)\cdot c + (b\times c)\cdot a$

$= 2 + (a\times b)\cdot c = 4$

So.20

$\vec{a} = (2,5,0)$ $\vec{b} = (0,3,3)$, $\vec{c} = (0,2,-5)$

$\therefore V = |[a,b,c]| = \left|\begin{vmatrix} 0 & 2 & -5 \\ 2 & 5 & 0 \\ 0 & 3 & 3 \end{vmatrix}\right| = 42.$

So.21

$\vec{OA} = (5,2,0)$ $\vec{OB} = (2,5,0)$, $\vec{OC} = (1,2,4)$

$\therefore V = \frac{1}{6}|[\vec{OA},\vec{OB},\vec{OC}]| = \frac{1}{6}\left|\begin{vmatrix} 1 & 2 & 4 \\ 5 & 2 & 0 \\ 2 & 5 & 0 \end{vmatrix}\right| = 14$

$\vec{AB} = (-3,3,0)$ $\vec{AC} = (-4,0,4)$

$S = \frac{1}{2}|\vec{AB}\times\vec{AC}| = \left|\begin{vmatrix} i & j & k \\ -3 & 3 & 0 \\ -4 & 0 & 4 \end{vmatrix}\right| = |12i+12j+12k| = 12\sqrt{3}\times\frac{1}{2} = 6\sqrt{3}$

$V = \frac{1}{3}Sd \Rightarrow d = \cancel{\frac{7}{6}\sqrt{3}}\ \frac{7}{3}\sqrt{3}.$

50.24

(2) $\vec{n}=(3,-6,2)$ 设 $3x-6y+2z+D=0$ 代入P

$\therefore 3x-6y+2z-49=0$

(4) 设 $Ax+Cz+D=0$ 代入 $(1,-5,1)$, $(3,2,-2)$

$\therefore \begin{cases} A+C+D=0 \\ 3A-2C+D=0 \end{cases} \Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 3 & -2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 \\ 0 & -5 & -2 \end{pmatrix}$

$\therefore \begin{cases} A=-\frac{3}{5}D \\ C=-\frac{2}{5}D \\ D=D \end{cases} \quad \therefore -\frac{3}{5}x-\frac{2}{5}z+1=0$

(6) 设 $x+5y-z+D=0$ 代入 $(1,-3,2)$

$\Rightarrow x+5y-z+16=0$

(8) 设 $2x+y+2z+D=0$ 截距为 $(-\frac{D}{2},0,0)$ $(0,-D,0)$ $(0,0,-\frac{D}{2})$

$\therefore V=\frac{1}{3}\times\frac{1}{2}\times\left|\left(-\frac{D}{2}\times(-D)\right)\times\left(-\frac{D}{2}\right)\right|=1 \Rightarrow D=\pm2\sqrt[3]{3}$

$\therefore 2x+y+2z\pm2\sqrt[3]{3}=0$

50.25

(2) $\vec{S}=\vec{n}=(1,5,-1)$ ∴ $\frac{x-1}{1}=\frac{y+3}{5}=\frac{z-2}{-1}$

(4) $\vec{S}=(m,n,0)$ 过 $(0,0,2)$, $(1,-3,2)$

∴ $x-1=\frac{y+3}{-3}=\frac{z-2}{0}$? 这么写?

(6) $\overrightarrow{M_1M_2}=(1,-4,4)$ $\overrightarrow{M_1M_3}=(4,-1,2)$

∴ $S\perp\overrightarrow{M_1M_2}$, $S\perp\overrightarrow{M_1M_3}$ ⇒ $S=M_1M_2\times M_1M_3$

∴ $S=(4,6,5)\times -1$ 取 $S=(4,6,5)$

∴ $\frac{x-1}{4}=\frac{y-2}{6}=\frac{z-3}{5}$

51.26

(1) $\vec{n_1}=(2,-4,1)$ $\vec{n_2}=(3,-1,-2)$ ∴ $S=n_1\times n_2=(9,7,10)$

取 $x=0$. ∴ $(0,1,4)$在 l 上. ∴ $\frac{x}{9}=\frac{y-1}{7}=\frac{z-4}{10}$

51.27

(2) $d=\frac{|56-35|}{\sqrt{9+4+36}}=3$

51.28

(2) $\vec{n_1}=(-8,-6,2)$ $\vec{n_2}=(4,3,-1)$

∴ $\cos\theta=\left|\frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right|=1$ ∴ $\theta=0$

51.29

(2). $M_1(1,0,1)$ $M_2(0,-2,-2)$ $\overrightarrow{M_1M_2}=(-1,-2,-3)$

$\vec{S_1}=(2,1,4)$, $\vec{S_2}=(1,2,3)$ ∴ $[\overrightarrow{M_1M_2},\vec{S_1},\vec{S_2}]=0$ ∴ 共面.

不平行 ∴ 相交. 交点为 $(1,0,1)$. $\cos\theta=\left|\frac{S_1\cdot S_2}{|S_1||S_2|}\right|=\frac{16}{\sqrt{[illegible]}}$

(4) $M_1(1,2,0)$ $S_1=(1,-1,3)$ ∴ $[\overrightarrow{M_1M_2},\vec{S_1},\vec{S_2}]\neq 0$

$M_2(2,1,4)$ $S_2=(-1,2,1)$ ∴ 异面

51.30

(2) $S=(3,-2,7)$ $n=(3,-2,7)$ $\therefore S \parallel n$ $\therefore l \perp \pi$, $\theta=\frac{\pi}{2}$

联立交点为 $(\frac{3}{2}, -1, \frac{7}{2})$

(4) $S=(3,1,2)$ $n=(2,3,3)$ $\therefore$ 不平行，不垂直.

$\theta = \arcsin\left|\frac{s\cdot n}{|s||n|}\right| = \arcsin\frac{15}{2\sqrt{77}}$

51.31

(1) $M_0(5,0,1)$ $\vec{s}=(-1,3,2)$ $M(1,0,-1)$.

$\therefore d\cdot|\vec{s}| = |\overrightarrow{M_0M}\times\vec{s}| \Rightarrow d=2\sqrt{5}$.

(3) ~~M_0~~ $\vec{n}=(1,2,3)$ $\therefore l$: $x-3=\frac{y+1}{2}=\frac{z+1}{3}$

$\therefore$ 联立交点为 $(\frac{37}{7}, \frac{45}{7}, \frac{41}{7})$

51.32

证明: $\vec{s}_1=(1,1,2)$, $\vec{s}_2=(1,3,4)$, $M_1(-1,0,1)$ $M_2(0,-1,2)$

$\therefore \overrightarrow{M_1M_2}=(1,-1,1)$ $\therefore [\overrightarrow{M_1M_2}, \vec{s}_1, \vec{s}_2] \neq 0$

设公垂线 l: $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ $\therefore s=(m,n,p)$

$s\perp s_1$, $s\perp s_2$ $\Rightarrow s=\vec{s}_1\times\vec{s}_2=(-2,-2,2)$

$[\vec{s}, \overrightarrow{M_0M_1}, \vec{s}_1]=0 \Rightarrow \begin{cases} y_0-x_0-1=0 \\ 5y+9-7x-2z=0 \end{cases}$ ~~$\Rightarrow \begin{cases} x_0= \\ y_0= \\ z_0= \end{cases}$~~ $\Rightarrow \begin{cases} x_0=0 \\ y_0=1 \\ z_0=7 \end{cases}$

$[\vec{s}, \overrightarrow{M_0M_2}, \vec{s}_2]=0 \Rightarrow$

$\therefore l$: $\frac{x}{-2}=\frac{y-1}{-2}=\frac{z-7}{2}$ 这是对的

$d=|(\overrightarrow{M_1M_2})_{\vec{s}}| = \left|\frac{\overrightarrow{M_1M_2}\cdot\vec{s}}{|\vec{s}|}\right| = \frac{\sqrt{3}}{3}$

交点: l 与 l_1: $(\frac{4}{3}, \frac{7}{3}, \frac{17}{3})$ l 与 l_2: $(1,2,6)$

52.33

$$\begin{cases} x+y-z-1=0 \\ x-y+z+1=0 \end{cases} \Rightarrow \begin{matrix} \vec{n}_1=(1,1,-1) \\ \vec{n}_2=(1,-1,1) \end{matrix} \quad \therefore s=(0,-2,-2).$$

取 $y=0$ $\therefore$ $(0,0,-1)$. $\therefore$ $\frac{y}{2}=\frac{z-1}{2}$, $x=0$

$M_0(0,0,-1)$ $\quad \vec{n}=(1,1,1)$ $\quad M(x,y,z)$.

$$\therefore \begin{cases} [\overrightarrow{M_0M}, \vec{n}, \vec{s}]=0 \\ x+y+z=0 \end{cases} \Rightarrow l: \begin{cases} z+1-y=0 \\ x+y+z=0 \end{cases}.$$

对的

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