235.31

(1) $\Phi = \iint_{\Sigma} \vec{A}\cdot d\vec{S} = -\iint_{D_{xy}} \cancel{xy}\,(xz, xy, yz)(z_x, z_y, -1)\,d\sigma$

$D_{xy}: \{ 0\le x\le 1,\ 0\le y\le 1-x \}$ $\therefore$ 原式 $= \iint_{D_{xy}} (\cancel{xy} + x + y - x^2 - \cancel{2}xy - y^2)\,d\sigma$

$= \int_0^1 dx \cdot \int_0^{1-x} \cdots dy = \frac{1}{8}$

(2) $\Phi = \iint_{\Sigma^+} (x^3, y^3, z^3)\,d\vec{S}$

$= \iiint_{\Omega} (3x^2+3y^2+3z^2)\,dV = 3\int_0^{2\pi} d\theta \int_0^{\pi} d\varphi \int_0^{R} \rho^2 \rho^2 \sin\varphi\, d\rho$

$= \frac{12}{5}\pi R^5$

236.32.

(1) $\operatorname{div}\vec{A} = 4 - 2x + 2z = 4 - 2 + 6 = 8$

(2) $\vec{A} = (x^2yz,\ xy^2z,\ xyz^2)$ $\operatorname{div}\vec{A} = \cancel{8}\ 2xyz + 2xyz + 2xyz = 6xyz = 36$

(3) $u\vec{A} = (x^3yz^4, -x^2y^3z^3, 2x^4y^2z^3)$

$\operatorname{div}\vec{A} = (3x^2yz^4\cancel{,} -3x^2y^2z^3 + 6x^4y^2z^2\cancel{)} = 3x^2yz^4 - 3x^2y^2z^3 + 6x^4y^2z^2$

~~236.33~~

$\cancel{\operatorname{div}A = 6x^2yz + y^2z^y - 2x^2yz - x^2z^x - 2x^2yz^{z} - x^y y^x \triangleq F(x,y,z)}$.

$\cancel{dF = 2\{2yz\,dx + x^2(y\,dz + z\,dy)\} + y^{z+1}z^{y-1}\,dz + z^{y+1}y^{z-1}\,dy}$

$\cancel{- x^{z+1}z^{x-1}\,dz - z^{x+1}x^{z-1}\,dx - x^{y+1}y^{x-1}\,dy - y^{x+1}x^{y-1}\,dx}$

$\cancel{= (4yz - z^{x+1}x^{z-1} - y^{x+1}x^{y-1})\,dx + (2x^2z + z^{y+1}y^{z-1} - x^{y+1}y^{x-1})\,dy}$

$\cancel{+ (2x^2y + y^{z+1}z^{y-1} - x^{z+1}z^{x-1})\,dz}$.

$\cancel{\vec{l_0} = (\frac{2}{3}, \frac{2}{3}, -1)}$

$\cancel{\therefore \frac{\partial F}{\partial \vec{l_0}}\Big|_M = \frac{14}{3} \quad (\frac{\partial F}{\partial \vec{l_0}}\Big|_M)_{max} = \sqrt{62}}$

236.33.

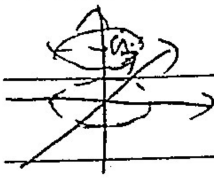
$\operatorname{div}A = 6x^2yz - 2x^2yz - 2x^2yz = 2x^2yz. = F.$

$dF = 4yz\,dx + 2x^2z\,dy + 2x^2y\,dz$, $\vec{l_0} = (\frac{2}{3}, \frac{2}{3}, -\frac{1}{3})$.

$\frac{\partial F}{\partial \vec{l_0}}\Big|_M = \frac{22}{3}$, $(\frac{\partial F}{\partial \vec{l_0}}\Big|_M)_{max} = |\nabla \vec{F}| = 2\sqrt{41}$.

236.36

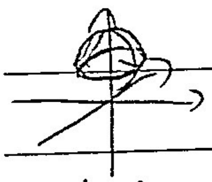
(1) $z=a,\ x=\cos\theta,\ y=\sin\theta.$ ~~θ∈[0~~ $\theta: 0\to 2\pi.$



$=\int_0^{2\pi}(-y,x,a)(x'(\theta),y'(\theta),0)\,d\theta$

$=2\pi$

(2) $z=1,\ x=\cos\theta,\ y=\sin\theta,\ \theta: 0\to 2\pi$



$=\int_0^{2\pi}(\cos\theta\sin\theta,\ \cos\theta+\sin^2\theta,\ z)(-\sin\theta,\cos\theta,0)\,d\theta$

$=\int_0^{2\pi}\cos^3\theta\,d\theta = 4\int_0^{\frac{\pi}{2}}\cos^3\theta\,d\theta = \pi$

236.37

(1) $\mathrm{rot}\vec{A}=\nabla\times\vec{A}=(xz-xy,\ xy-yz,\ yz-xz)=(-1,-3,4)$

(3) $\mathrm{rot}\vec{A}=\nabla\times\vec{A}=(0,\ -x\sin z,\ \frac{y}{x})$