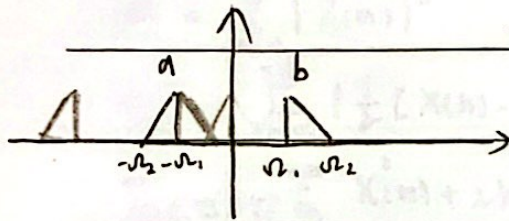随堂练习.

(1) $T = \frac{2\pi}{\Omega_1} \Rightarrow \Omega_s = \frac{2\pi}{T} = \Omega_1$, 于 $\hat{X}_a(j\Omega) = \frac{1}{T}\sum X_a(j\Omega - jk\Omega_1)$, $k \in \mathbb{Z}$

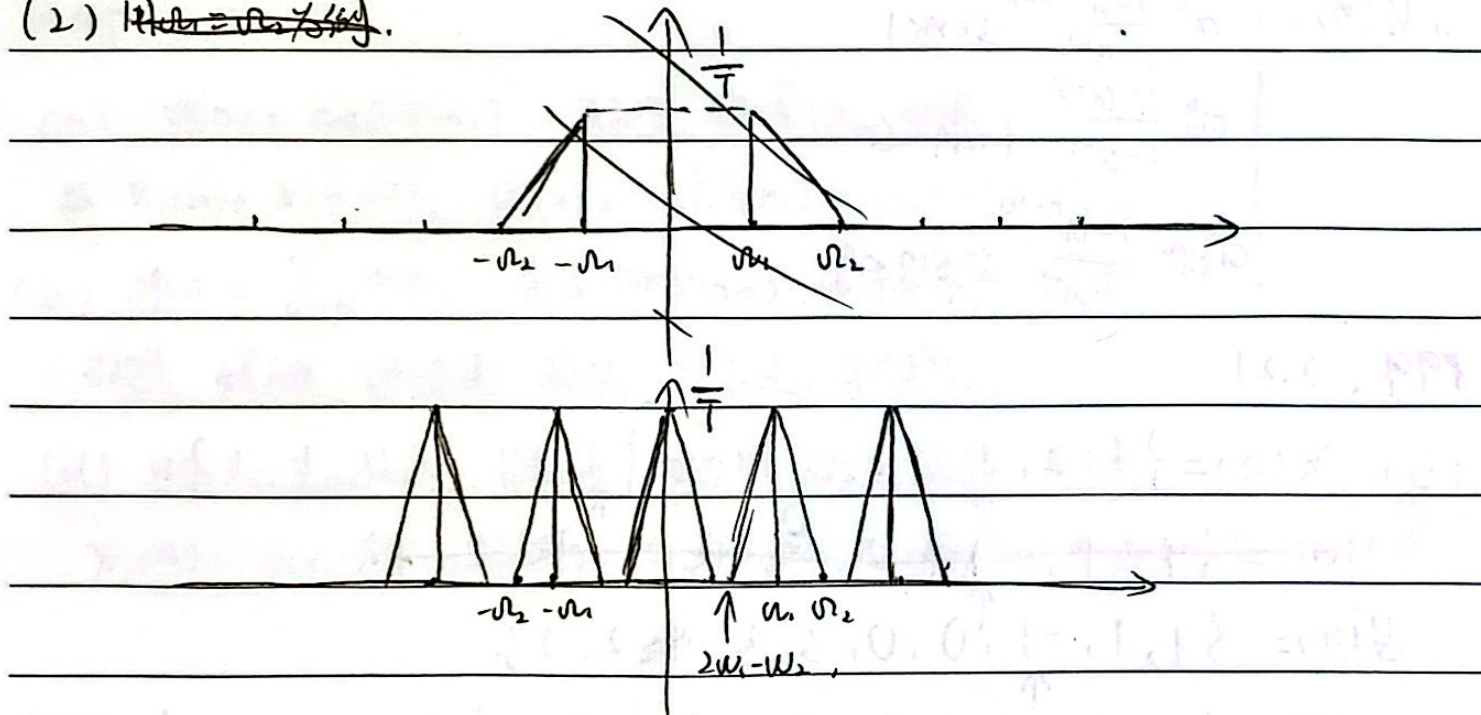$-2\Omega_1 < -\Omega_2 \Rightarrow \Omega_1 > \frac{1}{2}\Omega_2$

$\Omega_2 - 2\Omega_1 < \Omega_1 - \Omega_2 \Rightarrow 3\Omega_1 > 2\Omega_2 \Rightarrow 即\,\Omega_1 > \frac{2}{3}\Omega_2$.
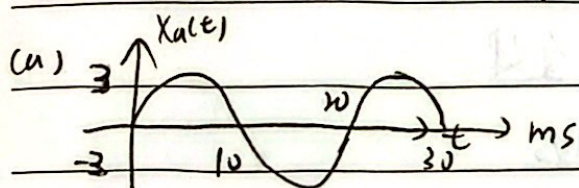
b part 左移 $2\Omega_1$ 和 a part 右移 Ω_1 不能重合.

∴ B

(2) 用 $\Omega_s = \Omega_2$ 为例.

$\frac{1}{T}$

$-\Omega_2 \quad -\Omega_1 \qquad \Omega_1 \quad \Omega_2$

$\frac{1}{T}$

$-\Omega_2 \quad -\Omega_1 \qquad \Omega_1 \quad \Omega_2$

$2\Omega_1 - \Omega_2$

P27. 15.

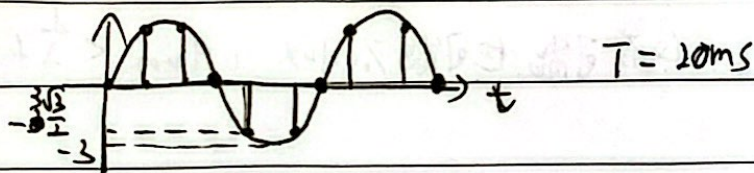$x_a(t) = 3\sin(100\pi t)$. $T = \dfrac{2\pi}{100\pi} = 20ms$.

(a)

(b) . $x(n) = x_a(nT) = 3\sin(100\pi nT)$

$x(n) = x(n+n') = 3\sin(100\pi nT + 100\pi n'T)$ $T = 300^{-1}$ ∴ $\dfrac{100\pi n'}{300} = 2\pi k$

$\Rightarrow n' = 6$ 最小   ∴ $x(n) = x(n+6)$

$T' = 6T \Rightarrow f' = \dfrac{1}{6T} = 50Hz$

(c) $n = 6T/T = 6$ 个

$T = 20ms$

(d) $x(n) = 3\sin(100\pi n / F_s)$ $\dfrac{100\pi n}{F_s} = \dfrac{\pi}{2} + \pi k$ ⇒ $F_s = \dfrac{200n}{2k+1} \in \mathbb{Z}^+$

F_s $F_{s\,min}$, $n=1$, $100 = 4 \times 25$, $2k+1 = 25 \Rightarrow k = 12$

$F_s = 8Hz$

如果要能重建 $2\pi F_s \geq 200\pi \Rightarrow F_s \geq 100$ ∴ $F_s = 200Hz$

P27. 1.7

(a) $f_N = 10kHz$, $f_s \geq 2f_N = 20kHz$

(b) $f_s < 2f_1$ ∴ 会混叠，无法唯一重建。

(c) $\omega_s = 16\pi k\ rad/s$ $\omega_1 = 10\pi k\ rad/s$

混叠. $\omega: 6\pi k \sim 10\pi k$, $f: 3k\sim 5k$ 混叠.
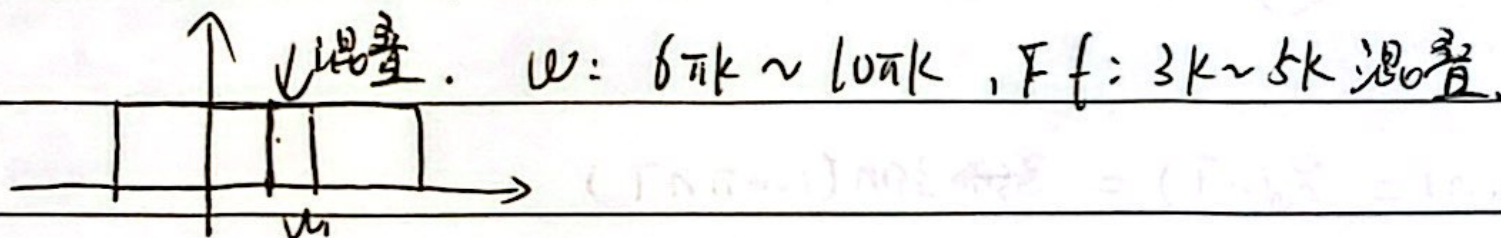
(c) 同理. $1k \sim 9k$ 混叠.

P27. 1.8

(1) $2\omega_n = 2\times 2\pi f = 400\pi\ rad/s$ 为 n率

(2) $f_{max} = \frac{1}{2}f_s = 125Hz$. 仅有可能. 也可能不可以 ($f_{max} < \frac{1}{2}f_s$)

P27. 1.9.

(a) $\omega_{max} = 720\pi$ rad/s, $\omega_N = \omega_{max}$ $\Rightarrow$ $2\omega_N = 1440\pi$ rad/s 名 h率.

(b) $f = \frac{1}{2} f_s = 300 Hz$

(c) $\overline{X}_{c(j\omega)} = (1200-720)\pi \sim 720\pi$ rad/s 的混叠.

$\sin(480\pi t)$ 可以采到, $\sin(720\pi t)$ 变成 $\sin(480\pi t)$.

$x(n) = \sin(0.8\pi n) + 3\sin(1.2\pi n) = \sin(0.8\pi n) - 3\sin(0.8\pi n) = -2\sin(0.8\pi n)$

$\omega' = 5$, $T' = nT = \frac{1}{120} s$, $\omega' = \frac{2\pi}{T'} = \frac{240\pi}{?}$ rad/s

(d) $y_a(t) = x(f_s t) = -2\sin(480\pi t)$

补1.

$\mathscr{F}[f(t)\cos(50t)] = F(j\omega) * \pi[\delta(\omega+50) + \delta(\omega-50)] = \pi[\overline{F}(j(\omega+50)) + F(j(\omega-50))]$

$\omega_s' = \omega_s + 50 = \times 150$ rad/s, 故 $\overline{F_s} = \omega_{采} = 2\omega_s' = 300$ rad.

补2.

(1) $f_{s\,min} = 2 f_{max} = 2\times 10^4$ Hz, $T_{max} = 10^{-4} s$

(2) $\omega = \frac{1}{2}\omega_s = \frac{1}{2} 2\pi f_s = 10^4\pi$ rad/s

补3. $x(t)$: $f_m = 300 Hz$, $y(t)$: $f_m = 300 Hz$

(1) $f = 2 f_{my} = 600 Hz$ (2) $F(j\omega) = 2 \overline{X}(\frac{1}{2}j\omega) \cdot 2X(2j\omega)$, $f = 4 f_{max}$

(3) $F(j\omega) = \frac{1}{2} Y(\frac{1}{2}j\omega)$, $\Rightarrow$ $f = 2\times \frac{1}{2} f_{mx} = 100 Hz$. $\frac{2}{?} = 400 Hz$

$f = 2\times 2 f_{my} = 1200 Hz$

(4) $F(j\omega) = \frac{1}{2\pi}[X(j\omega) * 3Y(3j\omega)] = \frac{3}{2\pi}\int_{-\omega_x}^{\omega_x} X(j\omega) \cdot Y(j(\omega-3j\omega)) d\omega$

∵ 当 ω最大有 $\frac{1}{3}\omega_y + \omega_x$ ∴ $f = 2\times(\frac{1}{3} f_{my} + f_{mx}) = 400 Hz$