The assignment aims to fit an appropriate ARIMA model for the daily historical Apple stock prices from February 1, 2002 to January 31, 2017 which is extracted from the Yahoo Finance website.

Raw Data Analysis

Mean and Conditional Variance

As the closing price reflects the last traded price of Apple stocks of the day, it can indicate the overall sentiment of investors towards the stock by the end of the trading session. Hence, Close Price of Apple stocks from the Yahoo Finance website is used.

```
file_path <- "C:/Users/mingy/Downloads/AAPL.csv"

data <- read.csv(file_path)

data <- data[, c("Date", "Close")]

data$Date <- as.Date(data$Date)

plot(data$Date, data$Close, type = "l", xlab = "Date", ylab = "Close Price"
, main = "Raw data of Close Apple Price Over Time")
```

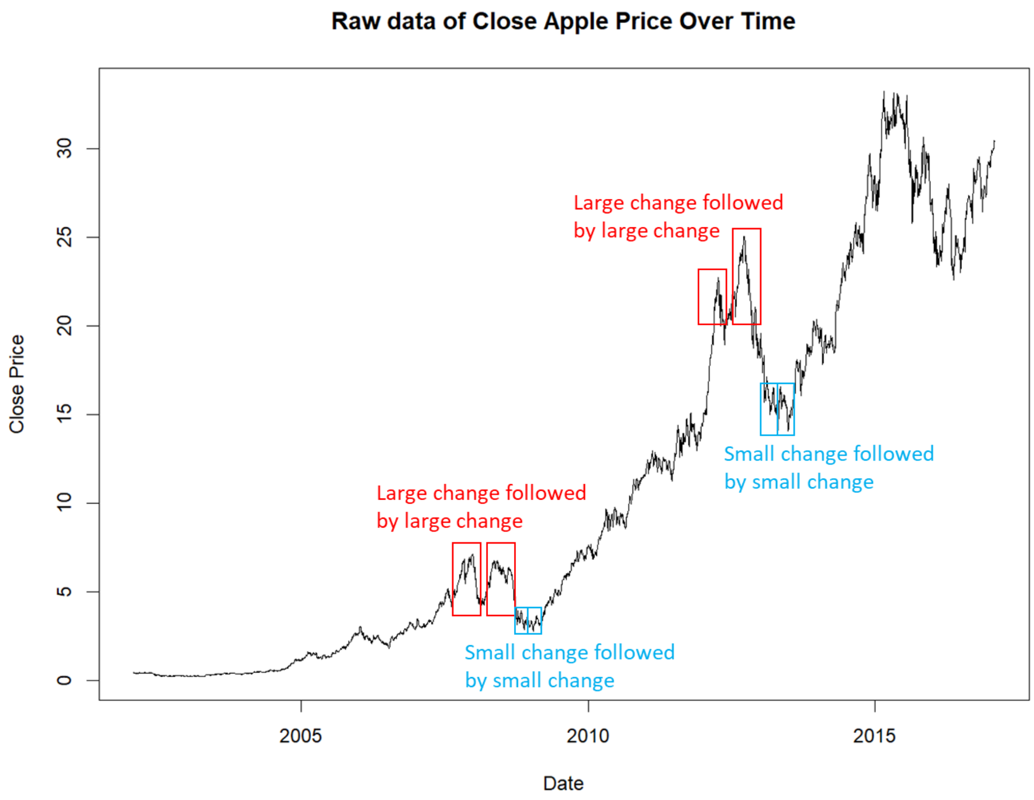

Fig 1: Plot of Close Apple price against time

Looking at the above figure, we see that there is a change in mean stock prices at different years. For example, the mean stock price at around 2008 is 5 USD, at around 2012 to 2013, it is 20 USD, and at around 2015 to 2016 the mean stock price is 30 USD. Hence, we conclude that the raw data is not stationary. We do also notice that there are patterns which implies volatility clustering, where there are large change in stock prices followed by large change, and small change in stock prices followed by small change. In year 2008 we notice 2 large changes and in year 2009 there are small changes.

From 2012 to 2013, there are 2 large changes followed by small changes in 2014. Hence, we conclude that the conditional variance of Apple stock prices is changing from 2002 to 2017.

Raw Data Transformation

Since the raw data is non-stationary, we conduct time differencing of the log transformation of the raw data to change the raw data into log returns.

```
r.cref=diff(log(data$Close))*100
plot(r.cref); abline(h=0)
ts.plot(r.cref)
```

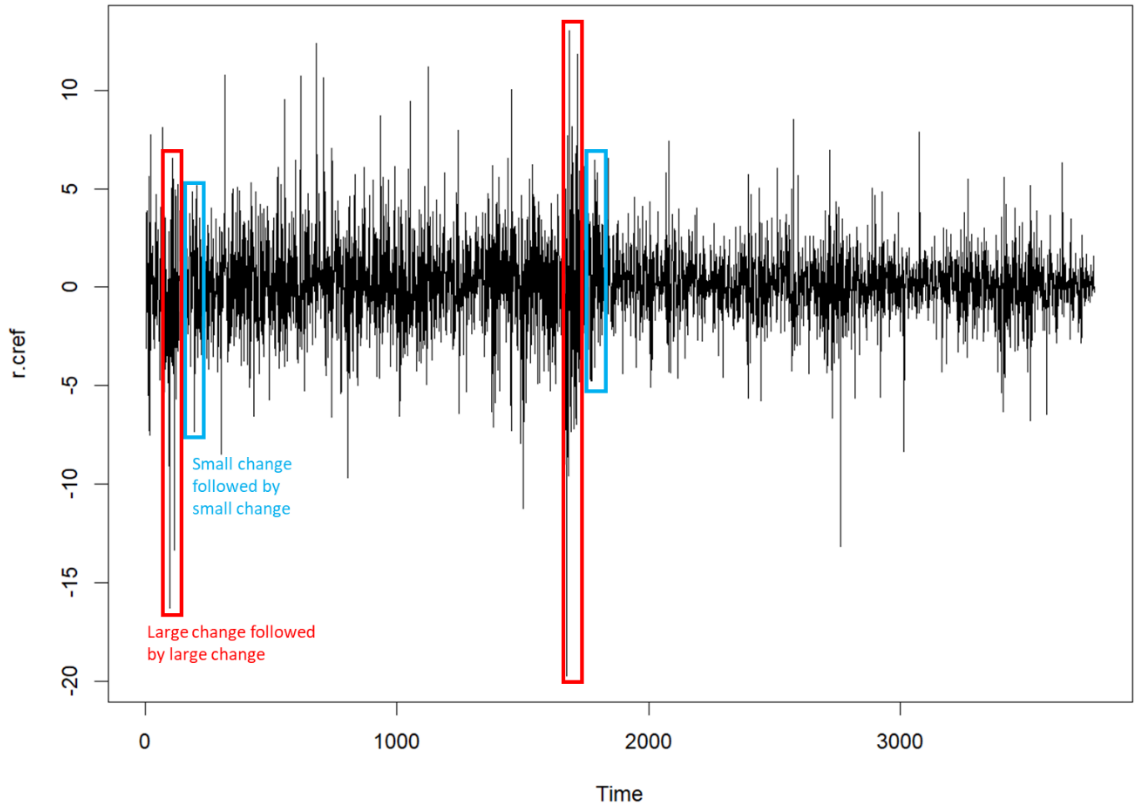

Fig 2: Log return of Close Apple Stock Price

From the figure above, visually speaking, the mean does not seem to be changing. Nevertheless, we still notice some difference in fluctuations of the data. For example, there is a big change in the data at around 100th time lag followed by another big change, and small change followed by small change in stock prices after that. The same pattern is observed for data at around time lag 1700. Hence, volatility clustering is still observed.

Augmented Dickey-Fuller (ADF) Test

Augmented Dickey-Fuller (ADF) Test is used to determine if the log return of the Apple Close stock prices contains a unit root, which indicates non-stationarity. The default null hypothesis of the ADF test is that the time series has a unit root which meant the data is not stationarity. The alternative hypothesis suggests that the data is stationary, lacking a unit root.

```
adf.test(r.cref)
```

```
        Augmented Dickey-Fuller Test

data:  r.cref
Dickey-Fuller = -14.928, Lag order = 15, p-value = 0.01
alternative hypothesis: stationary

Warning message:
In adf.test(r.cref) : p-value smaller than printed p-value
```

From the above return result in R Studio, we see that the p value is smaller than 0.05 which meant that the Apple stock after time differencing of log transformation to change the data to log return, the data is stationary.

Sample ACF and sample PACF

```
par(mfrow=c(1,2))
acf(r.cref)
pacf(r.cref)
```

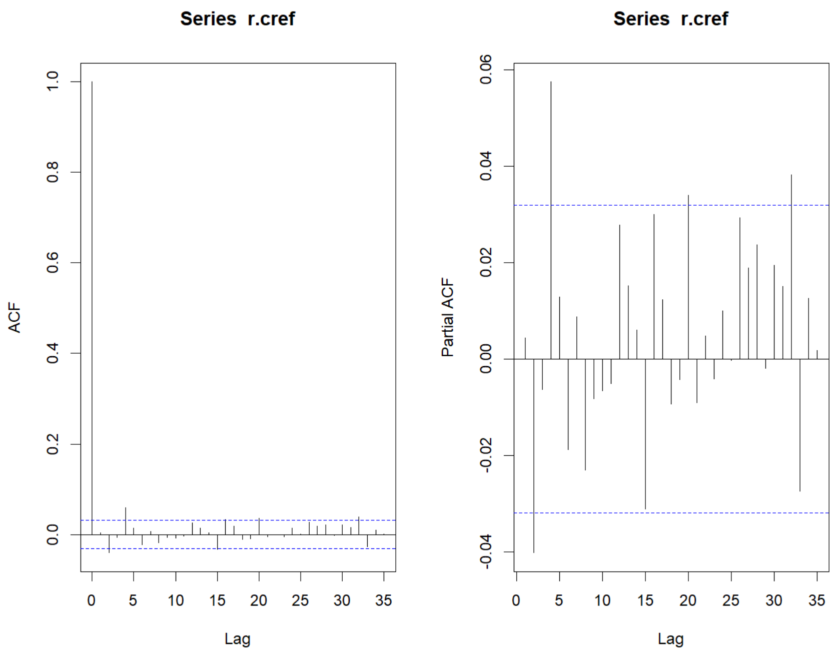

Fig 3: Sample ACF and sample PACF of log return of Apple stock prices

Looking at the diagram above, we see that some sample ACF values exceed the threshold boundary to consider the sample ACF as 0 such as at time lag 2, 4, 15, 16, 20 and 32. At large time lags like 20 and 32, the sample ACF might not really be exceeding the boundary. The boundary for considering sample ACF to be negligible is bigger as the time lag increases, and it is theoretically not a straight blue line as shown in fig 2. The below is the criteria for sample ACF to be negligible:

$$|r_h| \leq 1.96(\frac{1 + 2\sum_{j=1}^{h} r_j^2}{n})^{\frac{1}{2}} - - - \textit{equation } 1$$

where r_h is the sample ACF of the data at time lag h.

The below is the criteria used in R Studio for sample ACF to be negligible:

$$|r_{hh}| \leq 1.96(\frac{1}{n})^{\frac{1}{2}} - - - \textit{equation } 2$$

Hence, as the time lag increases, the boundary is supposed to expand as per equation 1 instead of staying as a horizontal line as per equation 2. With this, it is possible to conclude that the sample ACF cut off at around time lag 16.

For the sample PACF, it exceeds the boundary at time lag 2, 4, 20 and 32. Since the sample ACF and sample PACF are not 0, the data is correlated. Hence, we conclude that the financial data is not independent and is not white noise.

Since the sample ACF exceeds the boundary at time lag 4, an option might be MA(4) model. The sample PACF exceeds the boundary at time lag 4, AR(4) is an option model.

Sample ACF and sample PACF of absolute log return values

```
par(mfrow=c(1,2))
acf(abs(r.cref))
pacf(abs(r.cref))
```

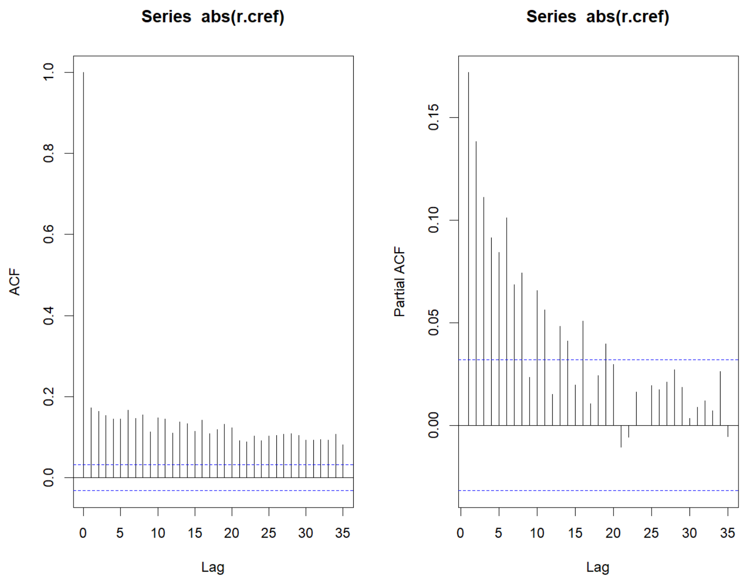

Fig 4: Sample ACF and sample PACF of absolute log return of Apple stock prices

The absolute value of log return of Apple stock prices after is plotted above as it gives a more accurate representation. Looking at the above figure, the sample ACF and sample PACF exceeds the boundary for many time lags. Hence, sample ACF and sample PACF are not negligible and the data points are not independent.

Q-Q Plot and kurtosis

```
dev.off()
qqnorm(r.cref)
qqline(r.cref)
kurtosis(r.cref)
```

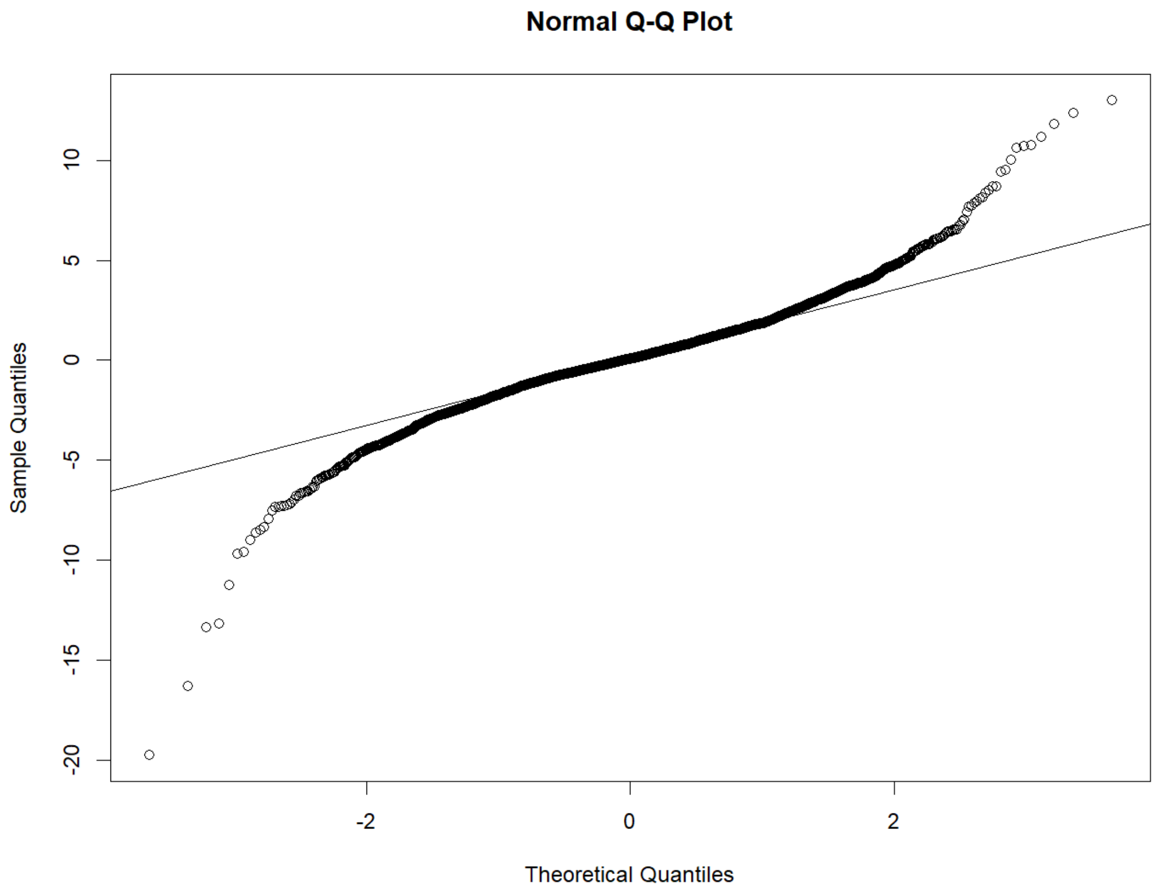

Fig 5: Q-Q Plot of the log return data of Close Apple stocks

Looking at the Q-Q Plot, the data points of the log return data of the Apple stocks does not lie completely on the theoretical quantile from standard normal distribution. Hence, the log return data of Apple stocks is not a normal distribution. We see a big deviation from upper tail and lower tail of the log return data of Apple stocks, a heavy-tailed distribution.

The thickness of the tail of a distribution relative to that of a normal distribution is often measured by the (excess) kurtosis, defined as $\frac{E(x-EX)^4}{\delta^4-3}$. A distribution with positive kurtosis is called a heavy-tailed distribution, whereas it is called light-tailed if its kurtosis is negative. The kurtosis can be estimated by the sample kurtosis below:

$$\frac{\sum_{i=1}^{n}(x_i - \bar{x})^4}{n\delta^4} - 3$$

The kurtosis of the Apple stocks after log time differencing is 8.437992. Heavy-tailed distribution is commonly observed in financial time series data.

2 Different Workflows in Generating Models

2.1 (Workflow 1) Apply GARCH(p,q) model on the Residuals of ARMA(p,q) model of the log return data

Given that the mean of the Apple Close stock prices change over the 15 years, using an ARMA(p,q) model can help to model the change in mean. Applying a GARCH model to the residuals of an ARIMA model allows you to further analyze the behavior of the residuals. If the residuals exhibit volatility clustering or other forms of heteroskedasticity, it suggests that the ARIMA model may not adequately capture all the dynamics of the data, and incorporating a GARCH model can help improve the overall model fit.

2.2 (Workflow 2) Apply GARCH(p,q) model on the log return data

The GARCH model can only model the change in conditional variance of the Apple Close stock prices over 15 years. We will explore the results from applying GARCH model on the log return data.

Workflow 1

EACF

Referring to Fig 3 in the previous section, the MA(4) and AR(4) are proposed option models. We will fit an ARMA(p,q) into the log return data of the Apple Close Stock Price from the option models and use the absolute value of the residual of the ARMA(p,q) model in the EACF.

```
fitARMA=arima(r.cref,order=c(4,0,4))

eacf(abs(fitARMA$residual))
```

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	x	x	x	x	x	x	x	x	x	x	x	x	x	x
1	x	o	o	o	o	o	o	x	x	o	o	x	o	o
2	x	x	o	o	o	o	o	o	x	o	x	x	o	o
3	x	x	x	o	o	o	o	o	o	o	o	o	o	o
4	x	x	x	x	o	o	o	o	o	o	o	o	o	o
5	x	x	x	x	x	o	o	o	o	o	o	o	o	o
6	x	x	x	x	x	o	o	o	o	o	o	o	o	o
7	x	x	x	x	x	o	o	o	o	o	o	o	o	o

Fig 6: EACF of the absolute value of the residual of ARMA(p,q) model

The EACF of the absolute value of the residual of the ARMA(p,q) model is plotted above as using the absolute values gives a more accurate deduction of the p and q values. Looking at the blue and green lines on the graph above, one possible GARCH model is GARCH(1,1). From the red and green lines, the other GARCH model is GARCH(2,2). Referring to the orange and green lines, the other GARCH model is GARCH(3,3).

Model Adequacy

GARCH(1,1)

```
g11residual=garch(fitARMA$residual,order=c(1,1))

summary(g11residual)
```

```
Call:
garch(x = fitARMA$residual, order = c(1, 1))

Model:
GARCH(1,1)

Residuals:
     Min       1Q   Median       3Q      Max 
-6.10594 -0.54380 -0.01044  0.58668  6.15833 

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)    
a0  0.051605    0.007374    6.998 2.59e-12 ***
a1  0.052369    0.003835   13.654  < 2e-16 ***
b1  0.938169    0.004594  204.215  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
	Jarque Bera Test

data:  Residuals
X-squared = 1963.2, df = 2, p-value < 2.2e-16


	Box-Ljung test

data:  Squared.Residuals
X-squared = 0.85034, df = 1, p-value = 0.3565
```

Looking at the Box-Ljung test, the p-value is 0.3565 which exceeds 0.05. Hence, the p-values from GARCH(1,1) is big and the GARCH(1,1) model is adequate. P-value from Jarque Bera Test being smaller than 2.2×10^{-16} meant that the log return of Close Apple stock price data is not normally distributed.

GARCH(2,2)

```
g22residual=garch(fitARMA$residual,order=c(2,2))

summary(g22residual)
```

```
Call:
garch(x = fitARMA$residual, order = c(2, 2))

Model:
GARCH(2,2)

Residuals:
     Min       1Q   Median       3Q      Max 
-8.63442 -0.49334 -0.00893  0.53114  5.16735 

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
```

```
a0 3.976e+00   2.272e-01   17.500  < 2e-16 ***
a1 1.292e-01   1.592e-02    8.115 4.44e-16 ***
a2 1.130e-01   1.936e-02    5.835 5.37e-09 ***
b1 3.327e-14   1.044e-01    0.000    1.000
b2 1.601e-02   7.403e-02    0.216    0.829
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 4312.4, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.22166, df = 1, p-value = 0.6378
```

Looking at the Box-Ljung test, the p-value is 0.6378 which exceeds 0.05. Hence, the p-values from GARCH(1,1) is big and the GARCH(2,2) model is adequate. P-value from Jarque Bera Test being smaller than 2.2×10^{-16} meant that the log return of Close Apple stock price data is not normally distributed.

GARCH(3,3)

```
g33residual=garch(fitARMA$residual,order=c(3,3))

summary(g33residual)
```

```
Call:
garch(x = fitARMA$residual, order = c(3, 3))

Model:
GARCH(3,3)

Residuals:
     Min       1Q   Median       3Q      Max
-8.75588 -0.49979 -0.00919  0.53645  5.44308

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
a0 3.478e+00   4.115e-01    8.453  < 2e-16 ***
a1 1.118e-01   1.588e-02    7.040 1.92e-12 ***
a2 9.677e-02   2.159e-02    4.482 7.38e-06 ***
a3 8.414e-02   1.952e-02    4.310 1.63e-05 ***
b1 2.217e-14   1.230e-01    0.000    1.000
b2 1.428e-02   1.096e-01    0.130    0.896
b3 2.997e-02   7.367e-02    0.407    0.684
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 4218.3, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.066538, df = 1, p-value = 0.7964
```

Looking at the Box-Ljung test, the p-value is 0.7964 which exceeds 0.05. Hence, the p-values from GARCH(1,1) is big and the GARCH(3,3) model is adequate. P-value from Jarque Bera Test being smaller than 2.2$\times$ 10^{-16} meant that the log return of Close Apple stock price data is not normally distributed.

Workflow 2

EACF

```
eacf(abs(r.cref))
```

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	x	x	x	x	x	x	x	x	x	x	x	x	x	x
1	x	o	o	o	o	o	o	x	x	o	o	x	o	o
2	x	x	o	o	o	o	o	o	x	o	o	x	o	o
3	x	x	x	o	o	o	o	o	o	o	o	o	o	o
4	x	x	o	o	o	o	o	o	o	o	o	o	o	o
5	x	x	x	x	x	o	o	o	o	o	o	o	o	o
6	x	x	x	x	x	x	o	o	o	o	o	o	o	o
7	x	x	x	x	x	x	x	o	o	o	o	o	o	o

Fig 7: EACF of the absolute value of the log return of Apple stock data

The EACF of the absolute value of the log return of Apple stock data is plotted above as using the absolute values gives a more accurate deduction of the p and q values. Looking at the blue and green lines, one possible GARCH model is GARCH(1,1). From the red and green lines, the other GARCH model is GARCH(2,2). Referring to the orange and green lines, the other GARCH model is GARCH(3,3).

Model Adequacy

GARCH(1,1)

```
g11=garch(r.cref,order=c(1,1))
summary(g11)
```

```
Call:
garch(x = r.cref, order = c(1, 1))

Model:
GARCH(1,1)

Residuals:
     Min       1Q   Median       3Q      Max
-5.98970 -0.48905  0.04531  0.63982  6.17971

Coefficient(s):
```

```
    Estimate  Std. Error  t value Pr(>|t|)    
a0  0.044480    0.006825    6.517 7.18e-11 ***
a1  0.047691    0.003472   13.735  < 2e-16 ***
b1  0.944122    0.004227  223.333  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
	Jarque Bera Test

data:  Residuals
X-squared = 1968, df = 2, p-value < 2.2e-16


	Box-Ljung test

data:  Squared.Residuals
X-squared = 1.1217, df = 1, p-value = 0.2896
```

Looking at the Box-Ljung test, the p-value is 0.2896 which exceeds 0.05. Hence, the p-values from GARCH(1,1) is big and the GARCH(1,1) model is adequate. P-value from Jarque Bera Test being smaller than 2.2×10^{-16} meant that the log return of Close Apple stock price data is not normally distributed.

GARCH(2,2)

```
g22=garch(r.cref,order=c(2,2))

summary(g22)
```

```
Call:
garch(x = r.cref, order = c(2, 2))

Model:
GARCH(2,2)

Residuals:
     Min       1Q   Median       3Q      Max 
-8.17054 -0.44393  0.04179  0.57264  5.21974 

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)    
a0 4.002e+00   2.183e-01   18.337  < 2e-16 ***
a1 1.284e-01   1.595e-02    8.050 8.88e-16 ***
a2 1.156e-01   1.736e-02    6.660 2.74e-11 ***
b1 2.336e-14   9.511e-02    0.000    1.000    
b2 1.732e-02   7.173e-02    0.241    0.809    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
	Jarque Bera Test

data:  Residuals
X-squared = 3906, df = 2, p-value < 2.2e-16


	Box-Ljung test

data:  Squared.Residuals
X-squared = 0.27963, df = 1, p-value = 0.5969
```

Looking at the Box-Ljung test, the p-value is 0.5969 which exceeds 0.05. Hence, the p-values from GARCH(2,2) is big and the GARCH(2,2) model is adequate. P-value from Jarque Bera Test being smaller than 2.2$\times 10^{-16}$ meant that the log return of Close Apple stock price data is not normally distributed.

GARCH(3,3)

```
g33=garch(r.cref,order=c(3,3))

summary(g33)
```

```
Call:
garch(x = r.cref, order = c(3, 3))

Model:
GARCH(3,3)

Residuals:
     Min       1Q   Median       3Q      Max 
-8.26727 -0.44816  0.04347  0.58591  5.47513 

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)    
a0 3.501e+00   3.850e-01    9.095  < 2e-16 ***
a1 1.124e-01   1.587e-02    7.079 1.45e-12 ***
a2 9.780e-02   2.096e-02    4.665 3.09e-06 ***
a3 8.654e-02   1.927e-02    4.491 7.08e-06 ***
b1 2.006e-14   1.211e-01    0.000    1.000    
b2 1.446e-02   1.171e-01    0.123    0.902    
b3 2.950e-02   7.912e-02    0.373    0.709    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
	Jarque Bera Test

data:  Residuals
X-squared = 3765.6, df = 2, p-value < 2.2e-16


	Box-Ljung test

data:  Squared.Residuals
X-squared = 0.069066, df = 1, p-value = 0.7927
```

Looking at the Box-Ljung test, the p-value is 0.7927 which exceeds 0.05. Hence, the p-values from GARCH(2,2) is big and the GARCH(2,2) model is adequate. P-value from Jarque Bera Test being smaller than 2.2$\times 10^{-16}$ meant that the log return of Close Apple stock price data is not normally distributed.

AIC

We compare the AIC of all GARCH models from workflow 1 and workflow 2 to determine which model is the best.

```
AIC(g11)

AIC(g22)

AIC(g33)

AIC(g11residual)

AIC(g22residual)

AIC(g33residual)
```

Table 1: AIC comparison of all GARCH models

Model	**AIC**
GARCH(1,1) on log return data	**16203.45**
GARCH(2,2) on log return data	16574.35
GARCH(3,3) on log return data	16504.82
GARCH(1,1) on residuals of ARMA(p,q) model	**16172.41**
GARCH(2,2) on residuals of ARMA(p,q) model	16541.62
GARCH(3,3) on residuals of ARMA(p,q) model	16473.7

From the above table, we see that both the GARCH(1,1) model on log return data as well as the GARCH(1,1) model on the residuals of ARMA(p,q) model gives low AIC values. We deduce that GARCH(1,1) on residuals of ARMA(p,q) model is the best model as it has the lowest AIC value of **16172.41**. It is expected that the GARCH(1,1) model on log return data gives a less favourable AIC value than the GARCH(1,1) model on residuals of ARMA(p,q) model as according to Fig 3 in the previous section, the model is not white noise as there are values of sample ACF and sample PACF exceeding the threshold boundaries.

Model Diagnostic

Before we accept GARCH(1,1) on residuals of ARMA(p,q) model as the best model, we need to check if the key model assumptions are violated. Standardized residuals are approximately independently and identically distributed if the model is correctly specified. The below is the definition of standardized residuals:

$$\widehat{\epsilon_t} = \frac{r_t}{\delta_t}$$

Where r_t is the log return, defined as $\frac{p_t - p_{t-1}}{p_{t-1}}$ and δ_t is the square root of conditional variance up to time point t-1. As in the case of model diagnostics for ARIMA models, the standardized residuals are very useful for checking the model specification.

```
plot(residuals(g11residual),type='h',ylab='Standardized Residuals')
win.graph(width=2.5,height=2.5,pointsize=8)
```

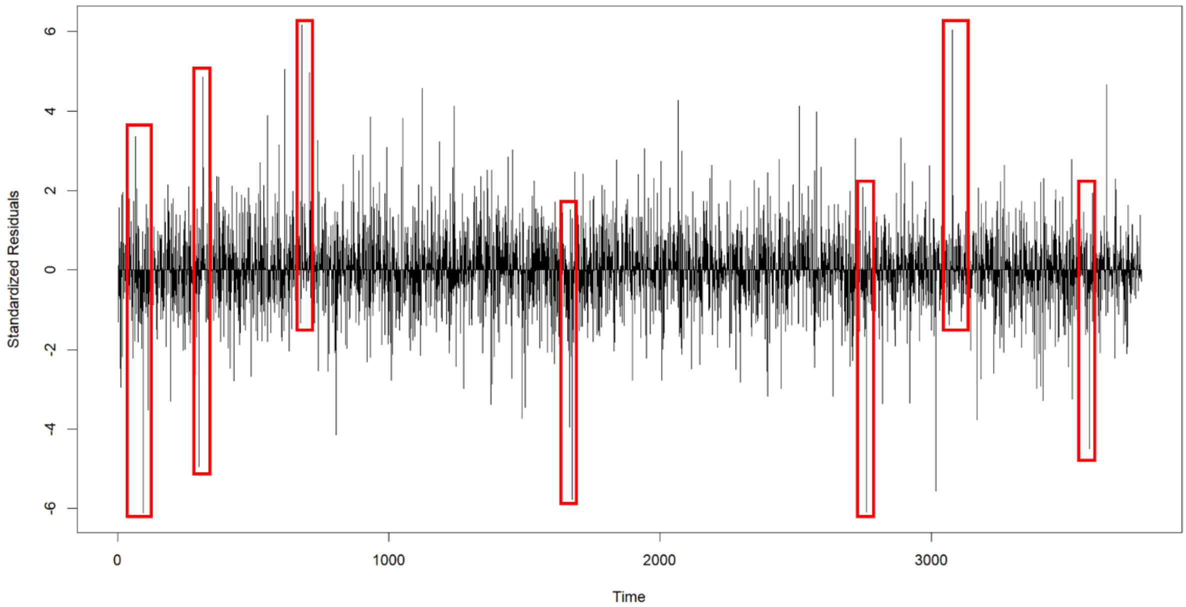

Fig 8: Residuals from the GARCH(1,1) on residuals of ARMA(p,q) model

Looking at the residual graph above, we see there are instances as shown in the red boxes where there is bigger magnitude in fluctuations compared to at other time lags. This meant that there is change in conditional variance and volatility in the residual of GARCH(1,1) model.

```
dev.off(); qqnorm(residuals(g11residual)); qqline(residuals(g11residual))
```

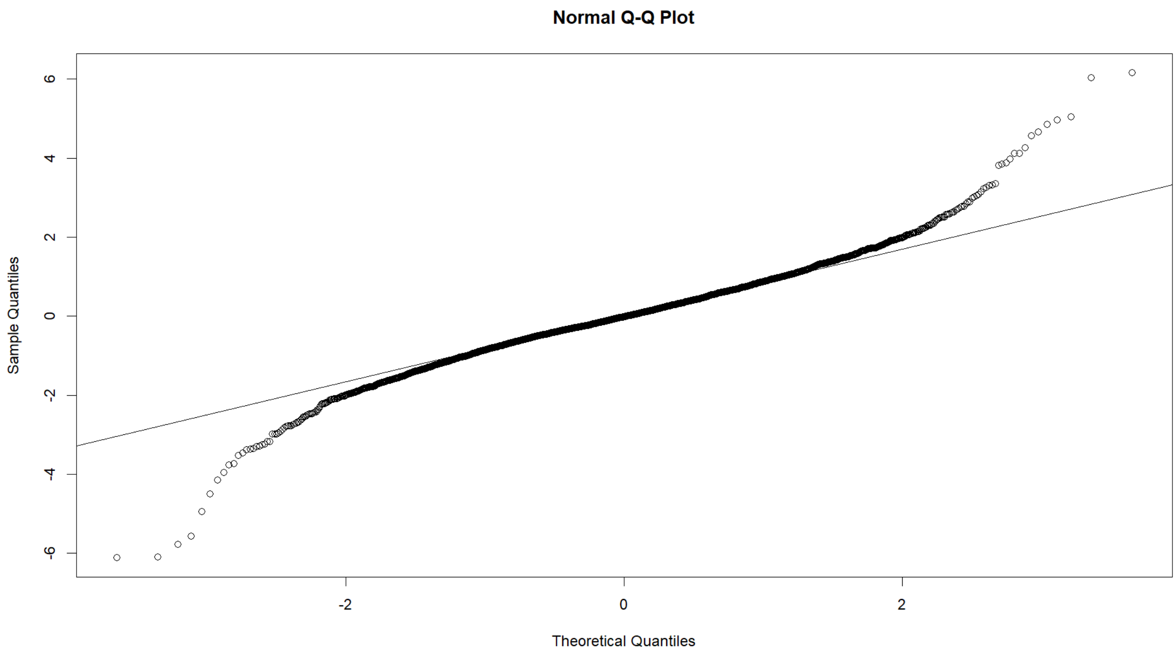

Fig 9: Q-Q plot of residuals of GARCH(1,1) model on residuals of ARMA(p,q) model

Looking at the Q-Q Plot, we see that the residuals of the GARCH(1,1) model does not lie completely on the theoretical quantile from standard normal distribution and we see a big deviation from upper tail and lower tail of the residuals of the GARCH(1,1) model. Hence, the residuals of the GARCH(1,1) model is not normally distributed. Moreover, looking at the p-value of the Jarque-Bera test from previous sections, the p-value is less than 2.2×10^{-16} for GARCH(1,1) model. Hence, the normality assumption is rejected.

```
acf(residuals(g11residual)^2,na.action=na.omit)
```

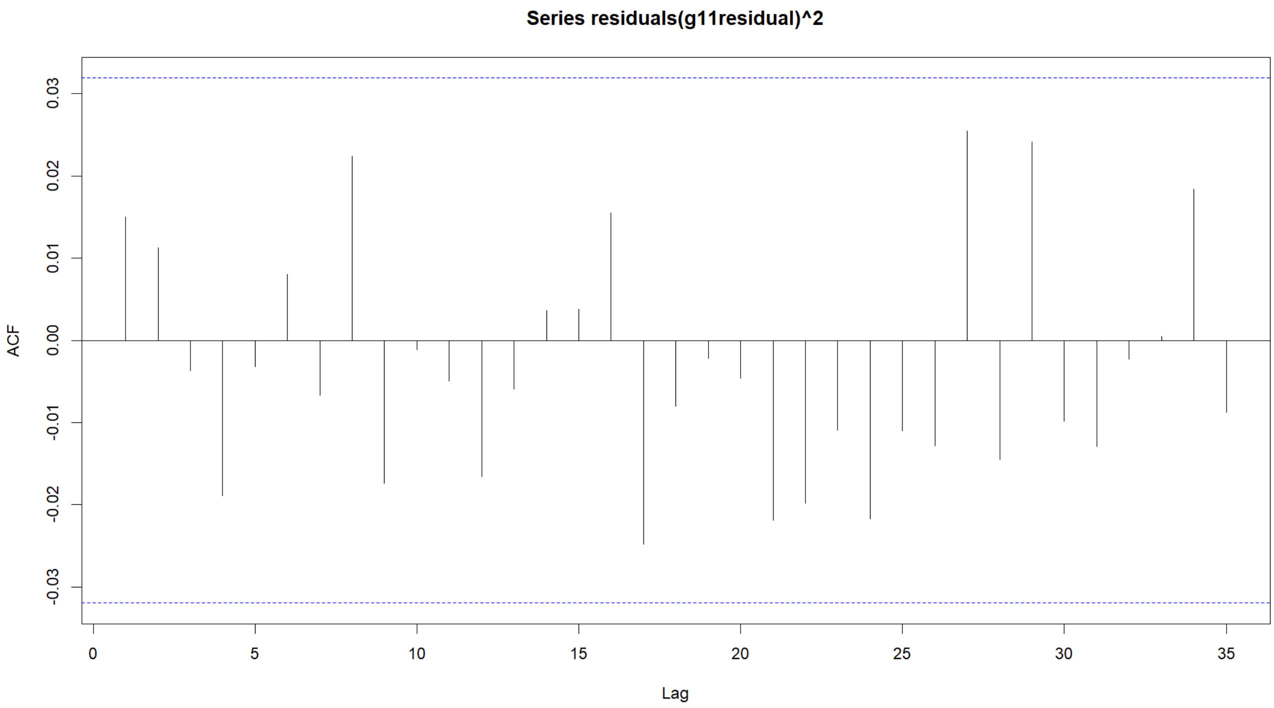

Fig 10: Sample ACF of Squared Residuals from the GARCH(1,1) Model

```
gBox(g11residual,method='squared')
```

Looking at the above graph, we conclude that the squared residuals are serially uncorrelated as none of the sample ACF value exceeds the blue dotted line boundary.

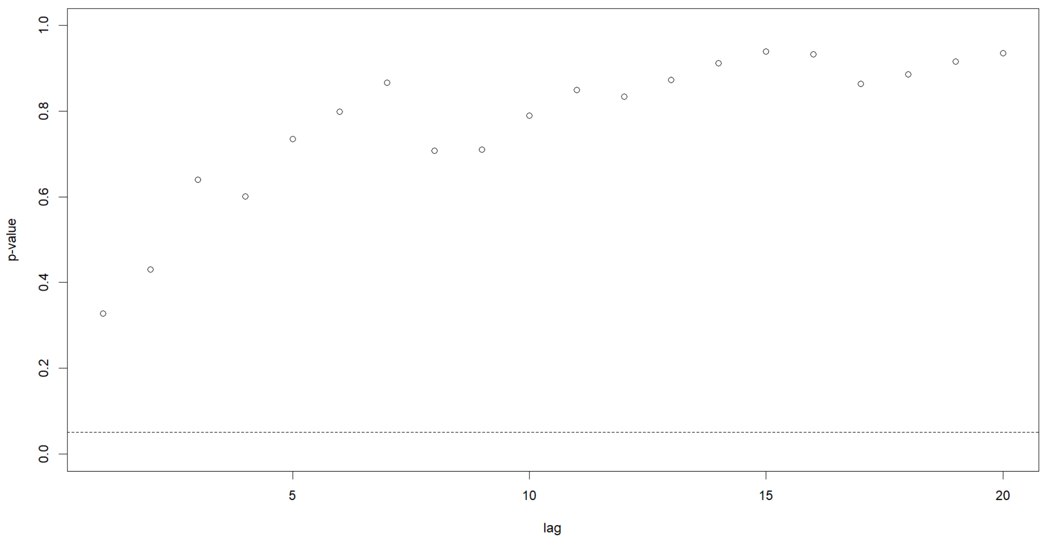

Fig 11: Generalized Portmanteau Test p-Values for the Squared Standardized Residuals for the GARCH(1,1) Model

From the above plot, displays the p-values of the generalized portmanteau tests with the squared standardized residuals from the GARCH(1,1) model of the log return of Apply stock prices for time lag 1 to 20. All p-values are higher than 0.05 which is above the black dotted line, suggesting that the squared residuals are uncorrelated over time, and hence the standardized residuals may be independent.

We deduce that the GARCH(1,1) model assumption is supported by data.

Fitting GARCH(1,1) to the log return data

```
plot(r.cref, type = 'l', , xlab = "Date", ylab = "Log return of Apple Close Stock Price", main = "Comparing GARCH(1,1) model with log return of Apple Close Stock Price")

lines(r.cref- g11residual$residuals,type = 'l', lty=2, col='red')
```

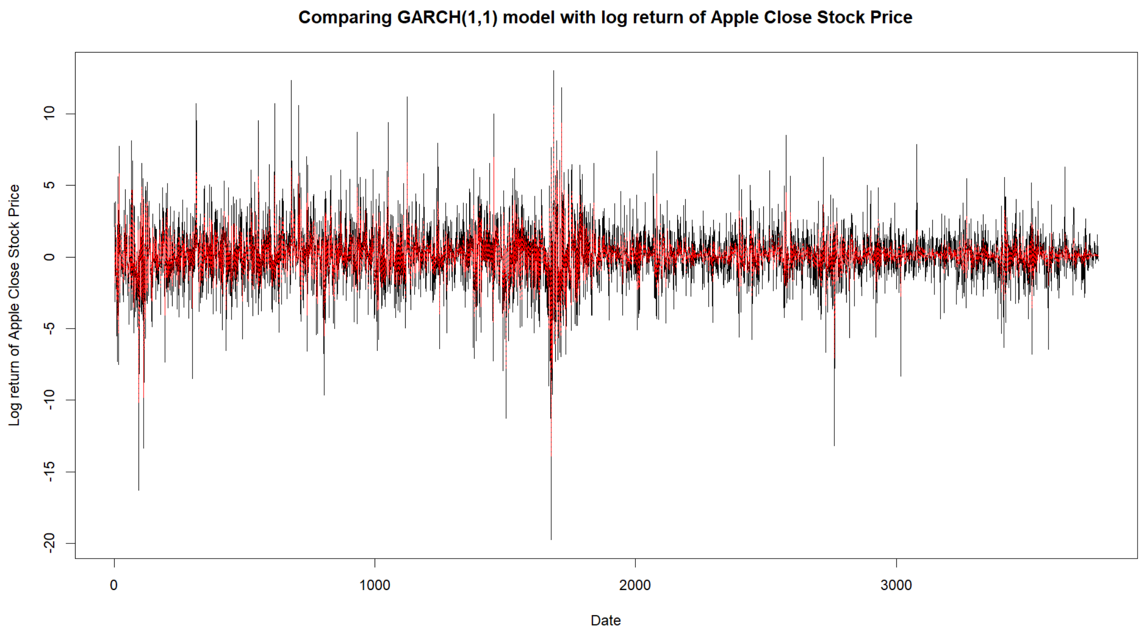

Fig 12: Fitting GARCH(1,1) model onto the log return Close Apple stock prices

Looking at the figure above, we see that the fitted GARCH(1,1) model does model the log return of Close Apple Stock Prices well as it captured the key high fluctuations of the stock prices.

Forecast

```
additional_forecast <- c(
  30.47, 30.52, 30.57, 30.63, 30.66, 30.73, 30.77, 30.82, 30.88, 30.93,
  30.98, 31.03, 31.08, 31.14, 31.19, 31.24, 31.29, 31.34, 31.40, 31.45,
  31.50, 31.55, 31.61, 31.66, 31.71, 31.76, 31.82, 31.87, 31.92, 31.98,
  32.03, 32.08, 32.14, 32.19, 32.24, 32.30, 32.35, 32.41, 32.46, 32.51,
  32.57, 32.62, 32.68, 32.73, 32.79, 32.84, 32.90, 32.95, 33.01, 33.06,
  33.12, 33.17, 33.23, 33.28, 33.34, 33.39, 33.45, 33.51, 33.56, 33.62,
```

```
  33.67, 33.73, 33.79, 33.84, 33.90, 33.96, 34.01, 34.07, 34.13, 34.18,
  34.24, 34.30, 34.36, 34.41, 34.47, 34.53, 34.59, 34.64, 34.70, 34.76,
  34.82, 34.88, 34.93, 34.99, 35.05, 35.11, 35.17, 35.23, 35.29, 35.35)
appended_data <- data.frame(
  Date = seq(as.Date("1980-05-03"), by = "day", length.out = 90),
  Close = additional_forecast)
plot(data$Date, data$Close, type = "l", xlab = "Date", ylab = "Close Price"
, ylim=c(0,36),main = "90 days forecast of Raw data of Close Apple Price Ov
er Time")
lines(appended_data$Date, appended_data$Close, type = 'l', col = 'red')
```

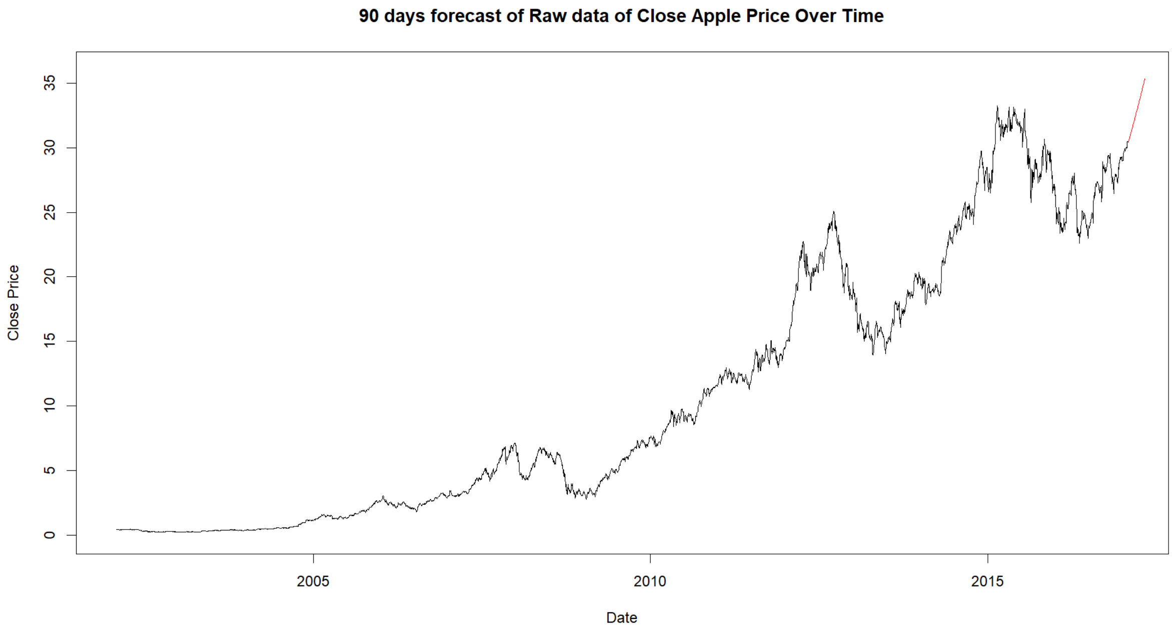

Fig 13: 90 days forecast of the Apple Close price

The figure above forecasts the next 90 days of the Apple close price. An increasing trend is seen.

Appendix:

```
> ugarchforecast(fitORspec = g11_fit ,n.ahead=90)

*------------------------------------*
*       GARCH Model Forecast         *
*------------------------------------*
Model: sGARCH
Horizon: 90
Roll Steps: 0
Out of Sample: 0

0-roll forecast [T0=1980-05-03]:
     Series  Sigma
T+1    30.47 0.1760
T+2    30.52 0.1759
T+3    30.57 0.1758
T+4    30.63 0.1758
```

T+5	30.66	0.1757
T+6	30.73	0.1756
T+7	30.77	0.1755
T+8	30.82	0.1754
T+9	30.88	0.1753
T+10	30.93	0.1753
T+11	30.98	0.1752
T+12	31.03	0.1751
T+13	31.08	0.1750
T+14	31.14	0.1749
T+15	31.19	0.1748
T+16	31.24	0.1748
T+17	31.29	0.1747
T+18	31.34	0.1746
T+19	31.40	0.1745
T+20	31.45	0.1744
T+21	31.50	0.1743
T+22	31.55	0.1743
T+23	31.61	0.1742
T+24	31.66	0.1741
T+25	31.71	0.1740
T+26	31.76	0.1739
T+27	31.82	0.1738
T+28	31.87	0.1737
T+29	31.92	0.1737
T+30	31.98	0.1736
T+31	32.03	0.1735
T+32	32.08	0.1734
T+33	32.14	0.1733
T+34	32.19	0.1732
T+35	32.24	0.1732
T+36	32.30	0.1731
T+37	32.35	0.1730
T+38	32.41	0.1729
T+39	32.46	0.1728
T+40	32.51	0.1728
T+41	32.57	0.1727
T+42	32.62	0.1726
T+43	32.68	0.1725
T+44	32.73	0.1724
T+45	32.79	0.1723
T+46	32.84	0.1723
T+47	32.90	0.1722
T+48	32.95	0.1721
T+49	33.01	0.1720
T+50	33.06	0.1719
T+51	33.12	0.1718
T+52	33.17	0.1718
T+53	33.23	0.1717
T+54	33.28	0.1716
T+55	33.34	0.1715
T+56	33.39	0.1714
T+57	33.45	0.1713
T+58	33.51	0.1713
T+59	33.56	0.1712
T+60	33.62	0.1711
T+61	33.67	0.1710
T+62	33.73	0.1709
T+63	33.79	0.1709
T+64	33.84	0.1708
T+65	33.90	0.1707
T+66	33.96	0.1706
T+67	34.01	0.1705
T+68	34.07	0.1704
T+69	34.13	0.1704
T+70	34.18	0.1703
T+71	34.24	0.1702
T+72	34.30	0.1701
T+73	34.36	0.1700

T+74	34.41	0.1700
T+75	34.47	0.1699
T+76	34.53	0.1698
T+77	34.59	0.1697
T+78	34.64	0.1696
T+79	34.70	0.1695
T+80	34.76	0.1695
T+81	34.82	0.1694
T+82	34.88	0.1693
T+83	34.93	0.1692
T+84	34.99	0.1691
T+85	35.05	0.1691
T+86	35.11	0.1690
T+87	35.17	0.1689
T+88	35.23	0.1688
T+89	35.29	0.1687
T+90	35.35	0.1687