```
In [1]: import torch import torch.nn as nn import copy import torchvision import torch.optim as optim import torchvision.transforms as transforms import sklearn from sklearn neighbors import KNeighborsClassifier import numpy as np import matplotlib.pyplot as plt from tqdm import tqdm
```

# **Exploring Deep Learning Through the Lense of Example Difficulty**

Much of this homework is inspired by the following paper: <a href="https://arxiv.org/abs/2106.09647">https://arxiv.org/abs/2106.09647</a> (<a href="https://arxiv.org/abs/2106.09647">https://arxiv.org/abs/2106.09647</a>)

Deep Learning Practioners have recognized that within the same task, particular examples in the test set can actually be harder to perform predictions on that others. Why is that? What kinds of things are easier to learn? We explore the notion of example difficulty, proposed by Baldock et. al. that will allow us to perform deeper investigations on the topic.

#### **Defining of Prediction Depth**

Consider a N-Layer neural network, with KNN Classifiers after each layer.

 $K_I(x)$  is the classification of the KNN after layer L

We will say that a prediction is made at depth L if L is the minimum value such that m > L implies  $K_m(x) = K_N(x)$ 

Essentially, we make a prediction at depth L if after that layer, the classifications stay consistent.

#### **Why Prediction Depth Matters**

Prediction depth can be viewed as a proxy for how hard a particular training example is. In this notebook we will explore the relation to what appears to be qualitatively difficult and prediction depth.

#### **Network Setup**

We will first train a ResNet-18. Once trained, we will pass in all the training data once more to get the intermediate representations after each layer. We will use these representations to train a KNN at each layer to classify data. We will then use the trained KNN classifiers on the evaluation/test data to determine prediction depth and accuracy.

Processing math: 100%

#### First Glance at the Data

Let's take a look at the data

```
In [2]: batch_size = 256
shapes = ['circle', 'square', 'rectangle', 'right_triangle', 'heart', 'ellipse']
```

Please download the data from the website and drag it into this folder.

We start with the standard dataloading pytorch definitions

```
In [3]: data = np.load('data.npy', allow_pickle=True).item()
    x_tensor = torch.FloatTensor(data['x'])
    y_tensor = torch.LongTensor(data['y'])
    dataset = torch.utils.data.TensorDataset(x_tensor, y_tensor)
    trainloader = torch.utils.data.DataLoader(dataset, batch_size=batch_size, num_worker

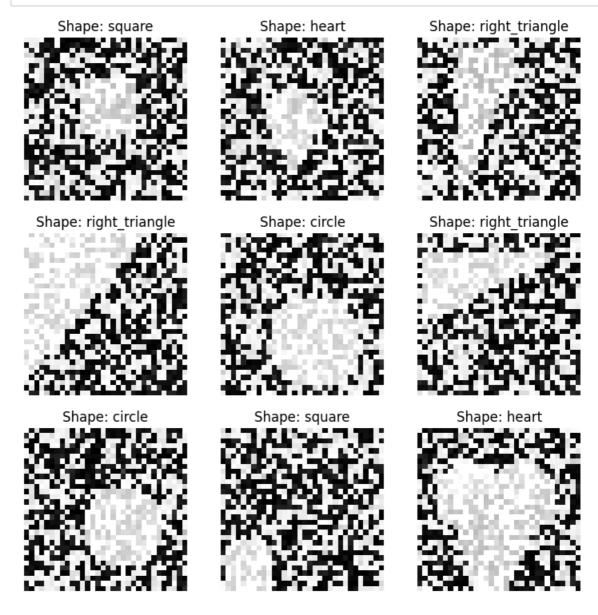
    test_data = np.load('test_data.npy', allow_pickle=True).item()
    x_tensor = torch.FloatTensor(test_data['x'])
    y_tensor = torch.LongTensor(test_data['y'])
    test_dataset = torch.utils.data.TensorDataset(x_tensor, y_tensor)
    testloader = torch.utils.data.DataLoader(test_dataset, batch_size=1, num_workers=2,
```

```
In [4]: random_indices = np.random.choice([i for i in range(6000)], 9, replace=False)

plt.figure(figsize=(9, 9))

for i, index in enumerate(random_indices, 1):
    x, y = test_data['x'][index], test_data['y'][index]

plt.subplot(3, 3, i) # 2 rows and 5 columns of subplots
    plt.imshow(x.reshape((32, 32, 1)), cmap='gray')
    plt.axis('off') # Turn off axis numbers and ticks
    plt.title(f'Shape: {shapes[y]}')
```



# **Difficulty**

What kind of properties do you think will make an example from this dataset difficult?

## **Training a ResNet**

We begin by training a standard ResNet-18 to classify each example by its shape

```
In [5]: device = torch.device("cuda:0" if torch.cuda.is_available() else "cpu")
   [6]: transform = transforms.Compose([
             transforms. ToPILImage(),
                                                             # Convert arrays to PIL images
             transforms. Grayscale (num output channels=3),
                                                             # Convert grayscale to RGB
             transforms. Resize ((224, 224)),
                                                             # Resize all images to 224x224
             transforms. ToTensor(),
                                                          # Convert the images to PyTorch tens
         ])
         from copy import deepcopy
         resnet dataset = deepcopy(dataset)
         resnet dataset.transform = transform
         resnet_trainloader = torch.utils.data.DataLoader(resnet_dataset, batch_size=batch_si
         x_tensor = torch.FloatTensor(test_data['x'])
         y_tensor = torch.LongTensor(test_data['y'])
         test dataset = torch.utils.data.TensorDataset(x tensor, y tensor)
         resnet test dataset = deepcopy(test dataset)
         resnet_test_dataset.transform = transform
         resnet_testloader = torch.utils.data.DataLoader(resnet_test_dataset, batch_size=batch
In [7]: resnet = torchvision.models.resnet18()
         num ftrs = resnet.fc.in features
         resnet.fc = torch.nn.Linear(num_ftrs, 6)
         resnet = resnet. to (device)
In [8]: criterion = nn. CrossEntropyLoss()
         resnet optimizer = optim. Adam (resnet. parameters (), 1r=0.0001)
```

```
step = 0
  In [9]:
           resnet_losses = []
           for epoch in tqdm(range(10)): # loop over the dataset multiple times
               running loss = 0.0
               for i, data in enumerate (resnet trainloader, 0):
                   step += 1
                   # get the inputs; data is a list of [inputs, labels]
                   inputs, labels = inputs, labels = data[0].to(device), data[1].to(device)
                   inputs = inputs.unsqueeze(1)
                   inputs = inputs. repeat (1, 3, 1, 1)
                   inputs = inputs. to (device)
                   # zero the parameter gradients
                   resnet_optimizer.zero_grad()
                   # forward + backward + optimize
                   outputs = resnet(inputs)
                   loss = criterion(outputs, labels)
                   loss.backward()
                   resnet optimizer.step()
                   resnet losses.append(loss.item())
                   # print statistics
                   running loss += loss.item()
                   if i % 50 == 49: # print every 2000 mini-batches
                       print(f'[{epoch + 1}, {i + 1}] loss: {running_loss / 20:.3f}')
                       running loss = 0.0
           print('Finished Training')
                          | 0/10 [00:00<?, ?it/s]
             0%
           [1, 50] loss: 3.739
           [1, 100] loss: 3.017
            10%
                           1/10 [00:02<00:21, 2.38s/it]
           [2, 50] loss: 1.952
           [2, 100] loss: 1.850
            20%
                            2/10 [00:03<00:14, 1.81s/it]
           [3, 50] loss: 1.199
           [3, 100] loss: 1.169
            30%
                           3/10 [00:05<00:11, 1.65s/it]
           [4, 50] loss: 0.656
           [4, 100] loss: 0.668
                             4/10 [00:06<00:09, 1.58s/it]
            40%
           [5, 50] loss: 0.326
           [5, 100] loss: 0.280
            50% | 5/10 [00:08<00:07, 1.55s/it]
           [6, 50] loss: 0.133
Processing math: [60%100] loss: 0.108
```

## **Finished Training**

Now that we've finished training our ResNet, let's visualize the training curve to make sure we've trained to convergence. 10 epochs should be enough

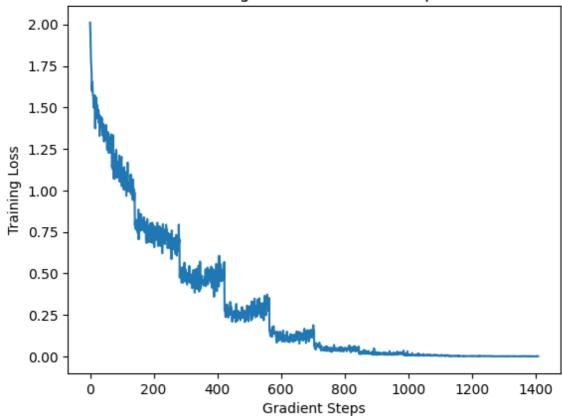
Note: Analyzing example difficulty without training to convergence would be faulty.

Processing math: 100%

```
In [10]: plt.plot([i for i in range(len(resnet_losses))], resnet_losses)
    plt.xlabel('Gradient Steps')
    plt.ylabel('Training Loss')
    plt.title('Training Loss vs Gradient Steps')
```

Out[10]: Text(0.5, 1.0, 'Training Loss vs Gradient Steps')





#### **Evaluation Set**

But did it actually learn? What is the evaluation accuracy?

```
In [11]: | resnet. eval()
          for epoch in range(1): # loop over the dataset multiple times
               total correct = 0
              with torch. no grad():
                  for i, data in tqdm(enumerate(resnet test dataset, 0)):
                      # get the inputs; data is a list of [inputs, labels]
                       inputs, labels = inputs, labels = data[0].to(device), data[1].to(device)
                       inputs = inputs.unsqueeze(0).unsqueeze(0)
                       inputs = inputs.repeat(1, 3, 1, 1)
                       inputs = inputs. to (device)
                       # forward + backward + optimize
                       outputs = resnet(inputs)
                       indices = torch.argmax(outputs, dim=1)
                       total correct += torch.sum(labels == indices)
          print(total correct)
          print('Finished Training')
          print(f'Accuracy: {total_correct/6000 * 100} %')
          6000it [00:09, 619.13it/s]
          tensor (4286, device='cuda:0')
          Finished Training
          Accuracy: 71.43333435058594 %
```

#### **Capturing Activations**

We will need to capture the activations to run KNN. We can do this in pytorch by attaching forward hooks. We need to this since we can't directly edit the model, as the code is abstracted away.

```
In [12]: activations = dict()
resnet_labels = []
```

```
In [13]: def forward hook(layer num, activations):
              def hook(module, input, output):
                  if layer num + 1 not in activations:
                      if layer num == 0:
                          activations[layer num] = [input[0]]
                      activations[layer num + 1] = [output]
                  else:
                      if layer_num == 0:
                          activations[layer_num].append(input[0])
                      activations[layer num + 1].append(output)
              return hook
In [14]: | 1ayer num = 0
          handles = []
          for layer in resnet.children():
              handles.append(layer.register forward hook(forward hook(layer num, activations))
              layer num += 1
In [15]:
          for epoch in tqdm(range(1)): # loop over the dataset multiple times
              running loss = 0.0
              for i, data in enumerate (resnet trainloader, 0):
                  step += 1
                  # get the inputs; data is a list of [inputs, labels]
                  inputs, labels = inputs, labels = data[0].to(device), data[1].to(device)
                  inputs = inputs.unsqueeze(1)
                  inputs = inputs. repeat (1, 3, 1, 1)
                  inputs = inputs. to (device)
                  resnet labels.append(labels)
                  # zero the parameter gradients
                  with torch. no grad():
                  # forward + backward + optimize
                      outputs = resnet(inputs)
```

100% | 1/1 [00:00<00:00, 1.87it/s]

#### Training KNN Classifiers and Removing Hooks

Let's train the classifiers with the activations that we've collected

#### **Collecting Test Set Activations**

Now we want to check the predictions of the test set examples. Using the activations and trained KNN's we can predict the output at each layer in the ResNet to determine things like prediction depth

```
[17]: test activations = dict()
       test resnet labels = []
       1 \text{ayer num} = 0
       for layer in resnet.children():
           layer.register forward hook(forward hook(layer num, test activations))
           layer num += 1
           test resnet labels = []
       for epoch in tqdm(range(1)): # loop over the dataset multiple times
           running loss = 0.0
           for i, data in enumerate (resnet testloader, 0):
               # get the inputs; data is a list of [inputs, labels]
               inputs, labels = inputs, labels = data[0].to(device), data[1].to(device)
               inputs = inputs.unsqueeze(1)
               inputs = inputs. repeat (1, 3, 1, 1)
               inputs = inputs. to (device)
               test resnet labels.append(labels)
               # zero the parameter gradients
               with torch.no grad():
               # forward + backward + optimize
                   outputs = resnet(inputs)
               if i == 0:
                   correct = torch.argmax(outputs, dim=1) == labels
               else:
                   correct = torch.cat((correct, torch.argmax(outputs, dim=1) == labels))
```

100% | 1/1 [00:00<00:00, 3.37it/s]

```
[18]: for layer in test_activations:
           test activations[layer] = torch.cat(test activations[layer], dim=0)
           test activations[layer] = torch.flatten(test activations[layer], start dim=1)
       test resnet labels = torch.cat(test resnet labels, dim=0)
       knn outputs = [knn.predict(test activations[i].cpu().numpy()) for i, knn in tqdm(en
       lit [00:04, 4.83s/it]OpenBLAS Warning: Detect OpenMP Loop and this applicati
       on may hang. Please rebuild the library with USE OPENMP=1 option.
       OpenBLAS Warning: Detect OpenMP Loop and this application may hang. Please re
       build the library with USE OPENMP=1 option.
       OpenBLAS Warning: Detect OpenMP Loop and this application may hang. Please re
       build the library with USE OPENMP=1 option.
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       build the library with USE OPENMP=1 option.
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       build the library with USE OPENMP=1 option.
       OpenBLAS Warning: Detect OpenMP Loop and this application may hang. Please re
       build the library with USE OPENMP=1 option.
       OpenBLAS Warning: Detect OpenMP Loop and this application may hang. Please re
       build the library with USE OPENMP=1 option.
       OpenBLAS Warning: Detect OpenMP Loop and this application may hang. Please re
       build the library with USE OPENMP=1 option.
       OpenBLAS Warning: Detect OpenMP Loop and this application may hang. Please re
```

#### **Finding Prediction depths**

We will need a function to find the prediction depth.

build the library with USE\_OPENMP=1 option.

```
In [19]: def find_constant_index(row):
    """
    Input: [Knn(L) for L in 1...N]
    Output: Prediction depth
    """
    # Start from the end of the row
    value = row[-1]
    for i in range(len(row)-2, -1, -1): # iterate backwards
        if row[i] != value:
            return i+1
    return 0
```

OpenBLAS Warning: Detect OpenMP Loop and this application may hang. Please re

## **Preparations for Analysis**

We need a few things before we conduct some analysis

Predictions[i][j] = a numpy array containing the knn outputs of data point i at layer i

indices[i] = prediction depth of data point i

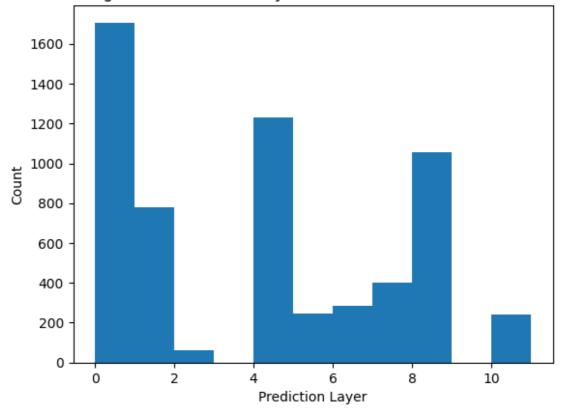
```
In [20]: predictions = np.array(knn_outputs)
    indices = np.apply_along_axis(find_constant_index, axis=0, arr=predictions)
    correct = correct.cpu().numpy()
    prediction_layer_list = []
    for num in range(11): # Numbers 0-9
        temp_indices = np.where(indices == num)[0]
        prediction_layer_list.append(temp_indices.tolist())
    total_accuracy_list = {}
    for i, layer in enumerate(prediction_layer_list):
        if layer != []:
            total_accuracy_list[i] = (np.sum(correct[layer])/len(layer), len(layer)/6000
        else:
            total_accuracy_list[i] = None
```

#### **Visualizing the Histogram of Prediction Layers**

The below visualization shows how many of each data point had prediction layer 0, for instance. If there were 500 examples that had prediction layer 1, this means that the KNN outputs do not change after layer 1 for 500 images. This could be interpreted as there was enough information at layer 1 to determine the class of the image with high confidence, and that the extra computation of the resnet was not necessary

```
In [24]: plt.hist(indices, bins=[i for i in range(12)], weights=[1 for _ in range(6000)])
    plt.xlabel('Prediction Layer')
    plt.ylabel('Count')
    plt.title('Histogram of Prediction Layers for a ResNet-18 on the Dataset')
    plt.show()
```

#### Histogram of Prediction Layers for a ResNet-18 on the Dataset

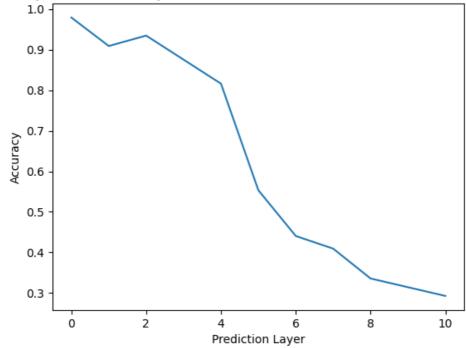


#### Visualizing the Output Accuracy vs Prediction Layer

The below visualization shows the average accuracy of ResNet classification on points that exited at layer L. Notice that test examples that had lower prediction layer generally had higher accuracy from the ResNet. Note that the accuracy is from the predictions at the end of the ResNet, not the KNN classifiers. Prediction layer is still determined by the outputs of the KNN classifiers

```
In [25]: plt.plot([i for i in range(11) if total_accuracy_list[i] is not None], [ total_accurate_plt.xlabel('Prediction Layer') plt.ylabel('Accuracy') plt.title('Accuracy vs Prediction Layer of an Resnet18 with KNN Classifiers on the D plt.show()
```

Accuracy vs Prediction Layer of an Resnet18 with KNN Classifiers on the Dataset



### Visualizing Easy and Hard Examples

Let's try to find some patterns in what might make an easy example different than a hard example

```
In [26]: easiest_examples = prediction_layer_list[0]
hardest_examples = prediction_layer_list[10]
```

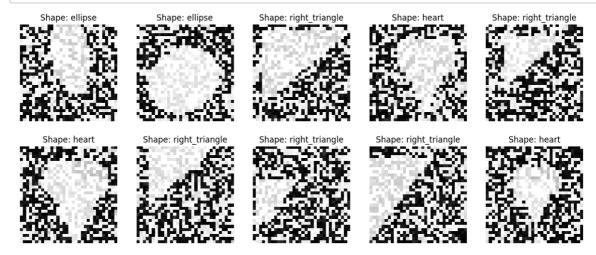
## **Easy Examples**

```
In [27]: from itertools import islice
    random_indices = np.random.choice(easiest_examples, 10, replace=False)

plt.figure(figsize=(15, 6))

for i, index in enumerate(random_indices, 1):
    x, y = test_data['x'][index], test_data['y'][index]

    plt.subplot(2, 5, i) # 2 rows and 5 columns of subplots
    plt.imshow(x.reshape((32, 32, 1)), cmap='gray')
    plt.axis('off') # Turn off axis numbers and ticks
    plt.title(f'Shape: {shapes[y]}')
```



#### **Hard Examples**

```
In [28]: random_indices = np.random.choice(hardest_examples, 10, replace=False)

plt.figure(figsize=(15, 6))

for i, index in enumerate(random_indices, 1):
    x, y = test_data['x'][index], test_data['y'][index]

plt.subplot(2, 5, i) # 2 rows and 5 columns of subplots
    plt.imshow(x.reshape((32, 32, 1)), cmap='gray')
    plt.axis('off') # Turn off axis numbers and ticks
    plt.title(f'Shape: {shapes[y]}')
```



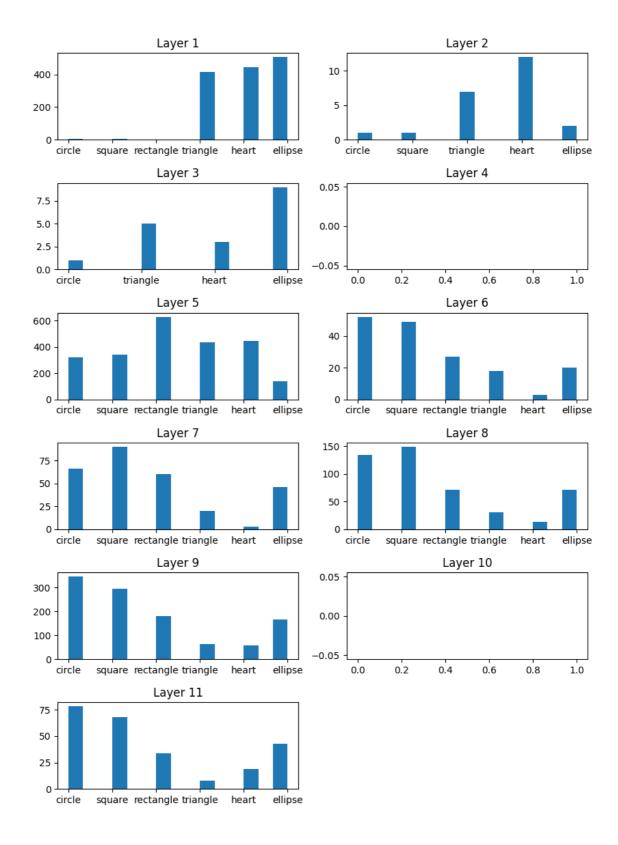
#### Can you spot a difference

What would make these hard vs easy?

```
In [29]: prediction_shapes = []
    for layer in prediction_layer_list:
        prediction_shapes.append([])
        for index in layer:
            prediction_shapes[-1].append(test_data['y'][index])
```

## What kinds of Shapes Exit at each Layer?

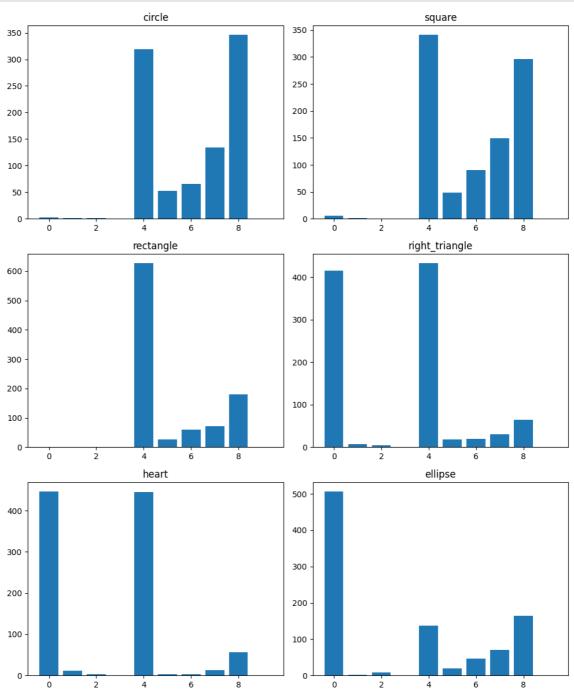
At each prediction layer, there may be different classes of shapes that are more common to appear. The following visualization shows at each layer, what is the distribution and count of the classes of shapes that will have prediction layer L



# What is the empirical prediction layer distribution for each Shape?

Each shape may have a different distribution of layers that they exit on. For instance, one might think that triangles are harder to classify, and therefore more of the distribution mass would be towards the later layers. We aim to show, for each shape, the distribution of what prediction layers the shape generally tended to

```
In [103]: frequency = []
           data = [np.array(d) for d in data]
           for i in range (6):
                frequency.append(dict())
                for j in range (10):
                    frequency[-1][j] = np. sum(data[j] == i)
   [104]: | fig, axs = plt.subplots(3, 2, figsize=(10, 12))
           for i, ax in enumerate(axs.flatten()):
                categories, counts = zip(*frequency[i].items())
                ax.bar(categories, counts)
                ax. set_title(shapes[i])
           plt. tight_layout()
           plt.show()
                                                                          square
                                 circle
                                                       350
            350
```



# **Examining Layers**

```
Input Layer
            Conv2d(3, 64, kernel size=(7, 7), stride=(2, 2), padding=(3, 3), bias=False)
            BatchNorm2d(64, eps=1e-05, momentum=0.1, affine=True, track_running_stats=True)
            ReLU(inplace=True)
            MaxPool2d(kernel size=3, stride=2, padding=1, dilation=1, ceil mode=False)
            Sequential (
              (0): BasicBlock(
                (conv1): Conv2d(64, 64, kernel size=(3, 3), stride=(1, 1), padding=(1, 1), bi
            as=False)
                (bn1): BatchNorm2d(64, eps=1e-05, momentum=0.1, affine=True, track running st
            ats=True)
                (relu): ReLU(inplace=True)
                (conv2): Conv2d(64, 64, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1), bi
                (bn2): BatchNorm2d(64, eps=1e-05, momentum=0.1, affine=True, track running st
            ats=True)
              (1): BasicBlock(
                (conv1): Conv2d(64, 64, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1), bi
            as=False)
                (bn1): BatchNorm2d(64, eps=1e-05, momentum=0.1, affine=True, track running st
            ats=True)
                (relu): ReLU(inplace=True)
                (conv2): Conv2d(64, 64, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1), bi
            as=False)
                (bn2): BatchNorm2d(64, eps=1e-05, momentum=0.1, affine=True, track running st
            ats=True)
            Sequential (
              (0): BasicBlock(
                (conv1): Conv2d(64, 128, kernel size=(3, 3), stride=(2, 2), padding=(1, 1), b
            ias=False)
                (bn1): BatchNorm2d(128, eps=1e-05, momentum=0.1, affine=True, track running s
            tats=True)
                (relu): ReLU(inplace=True)
                (conv2): Conv2d(128, 128, kernel size=(3, 3), stride=(1, 1), padding=(1, 1),
            bias=False)
                (bn2): BatchNorm2d(128, eps=1e-05, momentum=0.1, affine=True, track running s
            tats=True)
                (downsample): Sequential(
                  (0): Conv2d(64, 128, kernel_size=(1, 1), stride=(2, 2), bias=False)
                  (1): BatchNorm2d(128, eps=1e-05, momentum=0.1, affine=True, track running s
            tats=True)
                )
             )
              (1): BasicBlock(
                (conv1): Conv2d(128, 128, kernel size=(3, 3), stride=(1, 1), padding=(1, 1),
           bias=False)
                (bn1): BatchNorm2d(128, eps=1e-05, momentum=0.1, affine=True, track running s
            tats=True)
                (relu): ReLU(inplace=True)
                (conv2): Conv2d(128, 128, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1),
Processing math: hoio = False)
```

(bn2): BatchNorm2d(128, eps=1e-05, momentum=0.1, affine=True, track running s

```
tats=True)
              )
            Sequential (
              (0): BasicBlock(
                (conv1): Conv2d(128, 256, kernel size=(3, 3), stride=(2, 2), padding=(1, 1),
            bias=False)
                (bn1): BatchNorm2d(256, eps=1e-05, momentum=0.1, affine=True, track running s
            tats=True)
                (relu): ReLU(inplace=True)
                (conv2): Conv2d(256, 256, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1),
                (bn2): BatchNorm2d(256, eps=1e-05, momentum=0.1, affine=True, track running s
            tats=True)
                (downsample): Sequential(
                  (0): Conv2d(128, 256, kernel size=(1, 1), stride=(2, 2), bias=False)
                  (1): BatchNorm2d(256, eps=1e-05, momentum=0.1, affine=True, track_running_s
            tats=True)
                )
              )
              (1): BasicBlock(
                (conv1): Conv2d(256, 256, kernel size=(3, 3), stride=(1, 1), padding=(1, 1),
            bias=False)
                (bn1): BatchNorm2d(256, eps=1e-05, momentum=0.1, affine=True, track running s
            tats=True)
                (relu): ReLU(inplace=True)
                (conv2): Conv2d(256, 256, kernel size=(3, 3), stride=(1, 1), padding=(1, 1),
            bias=False)
                (bn2): BatchNorm2d(256, eps=1e-05, momentum=0.1, affine=True, track running s
            tats=True)
              )
            )
            Sequential (
              (0): BasicBlock(
                (conv1): Conv2d(256, 512, kernel size=(3, 3), stride=(2, 2), padding=(1, 1),
            bias=False)
                (bn1): BatchNorm2d(512, eps=1e-05, momentum=0.1, affine=True, track running s
            tats=True)
                (relu): ReLU(inplace=True)
                (conv2): Conv2d(512, 512, kernel size=(3, 3), stride=(1, 1), padding=(1, 1),
            bias=False)
                (bn2): BatchNorm2d(512, eps=1e-05, momentum=0.1, affine=True, track running s
            tats=True)
                (downsample): Sequential(
                  (0): Conv2d(256, 512, kernel size=(1, 1), stride=(2, 2), bias=False)
                  (1): BatchNorm2d(512, eps=1e-05, momentum=0.1, affine=True, track running s
            tats=True)
                )
              (1): BasicBlock(
                (conv1): Conv2d(512, 512, kernel size=(3, 3), stride=(1, 1), padding=(1, 1),
            bias=False)
                (bn1): BatchNorm2d(512, eps=1e-05, momentum=0.1, affine=True, track running s
            tats=True)
                (relu): ReLU(inplace=True)
                (conv2): Conv2d(512, 512, kernel size=(3, 3), stride=(1, 1), padding=(1, 1),
            bias=False)
                -{bn2): BatchNorm2d(512, eps=1e-05, momentum=0.1, affine=True, track running s
Processing math: 100% True)
```

#### **Patterns**

What kinds of patterns do you notice? Based on the composition of the layers, does it make sense?

## **Concluding Thoughts**

From what you witnessed in this homework, what can you say about example difficulty? How can we come up with better metrics of example difficulty? Why does it even matter? What are some possible applications of this line of work? In the next section of the homework, we will answer some of these questions.

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