NOTE ON DUCROS' BOOK — CHAPTER 4

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Contents

1. Introduction

These are a series of notes on the book [Duc24].

2. Notes

Let k be a non-Archimedean analytic field. Consider a k-analytic curve X.
Unlike Ducros' book, we assume that X is good.[1]

4.1.1. Line 17, $\mathbb{P}_k^{1,\mathrm{an}}$ should be $\mathbb{P}_k^1$.

4.2.1. Line 4, $\varphi^{-1}(\varphi((x))$ should be $\varphi^{-1}(\varphi(x))$.

4.2.3. Line 5, φ should be f.

4.2.4.1. Line 6, $=0$ should be removed.
Line 8, X' should be X_0.

4.2.5.1. The existence of function mentioned in the first paragraph is constructed in 3.5.9.

4.2.9. Line 3, $\mathbb{P}_{\mathscr{H}(x)}$ should be $\mathbb{P}_{\mathscr{H}(x)/k}$.

4.2.16. The reduction in the first paragraph of the proof is not quite correct, and is not what we need in the sequel. The correct version is the following:

Notons pour commencer que l'on peut, pour montrer 1), 2) i) et 2) ii), remplacer X par X_{red} et Y par $Y \times_X X_{\mathrm{red}}$; cela permet de se ramener, pour montrer toutes les assertions, au cas où la courbe X est génériquement réduite, et l'on distingue alors deux cas selon la nature du point x.

4.2.16.1. Line 5, the second y should be x.
Line 5, U est une composante connexe de $\varphi^{-1}(x)$ should be V est une composante connexe de $\varphi^{-1}(U)$.

4.2.16.2. Line 26, $\varphi^{-1}U$ should be $\varphi^{-1}(U)$.

4.2.19. In iii), $X_{[23]}$ should be $X_{[2,3]}$.
The second part of iii) follows from the fact that $(\kappa(x), |\bullet|)$ is Henselian, a very general fact proved by Berkovich [Ber93, Theorem 2.3.3].
Line 8 in the proof, remove est fini et.
Line 17 in the proof, the left parenthesis should be larger.
Line -4 in the proof follows from 2.3.12.

[1]This is proved in Ducros' book based on Temkin's goodness criterion. I cannot understand the proof of the latter as explained in my note on graded reductions.

4.2.19.2. Line 5, 4.2.9 should be 4.2.3.
Line 6, b should be a.

4.2.20. There is a serious issue here. The whole proposition only works if both germs (Y,y) and (X,x) are boundaryless.

The proof below implicitly assumed that y is of type 2. If y is of type 3, there is really nothing to prove in view of 4.2.19 iii).

The germ (X',x') is normal thanks to [Stacks, Tag 034F]. In particular, the reduction at the end of this part makes sense.

4.2.20.1. The second displayed formula follows from 2.3.14 ii).

4.3.3. In the statement of the theorem, p is the exponential characteristic of k.
In the third paragraph of the proof, add si $p\neq 1$ after par p.

4.3.4.1. Line 4, x_i should be x_i'.

4.3.5.1. In the third paragraph, the claim

$$\widetilde{\kappa_{\widehat{k^a},r}}=\widetilde{k^a}$$

is obviously wrong.

The corrected version: comme $|\widehat{k^a}^{\times}|$ est divisible et comme $\widetilde{\kappa_{\widehat{k^a},r_1}}$ est algébriquement clos (il est égal à $\widetilde{k^a}_1$),.

By la théorie de la ramification modérée, Ducros meant 2.3.39.

4.3.5.2. Line 1, 3) should be 1).
Line 7, $\mathrm{S}(Z)\{x\}$ should be $\mathrm{S}(Z)\setminus\{x\}$.

4.3.6. Line 1, en en should be en.

4.3.6.4. Line 1, b) should be b.

4.3.6.4. Line 8, $|\mathcal{O}_X(Z)^\times|$ should be $|\mathcal{O}_X(Z)^\times|_b$.

4.3.7. Line 2 of the proof, add et que X soit connexe after $\mathcal{O}_X(X)$.
Line 7 of the proof, $\mathrm{br}(X_{\widehat{k^a}}y,)$ should be $\mathrm{br}(X_{\widehat{k^a}},y)$.
Line -1 of the proof, ∞ should be $\mathfrak{s}(Z)$.
In the proof, G is the absolute Galois group of k.

4.3.8. Line 5, $\widehat{k^a}$ should be k.

4.3.9. Line 1, there is a serious issue, $x\in X$ should be $x\in X_{[2,3]}$.

4.3.9.1. Line 18, Y^{an} should be $\partial^{\mathrm{an}}(Y)$.
Line 19, X^{an} should be $\partial^{\mathrm{an}}(X)$.

4.3.9.2. Line 7, (X,x) should be $(X_{\widehat{k^a}},y)$.

4.3.10. Line 1, en en should be en.

4.3.10.4. Line -2, $|\mathcal{O}_X(b)^\times_{\mathrm{sep}}|$ should be $|\mathcal{O}_X(b)^\times_{\mathrm{sep}}|_b$.

4.3.11.1. Line 7, b should be y.
Line 8, a should be x.
Line 8, le lemma should be la proposition.

4.4.3.1. Line 8, U should be $X\setminus\{x\}$.
Line 9, U should be Z.

4.4.5. Line 4, $\mathrm{H}^1(\kappa(x),\mu_\ell))$ should be $\mathrm{H}^1(\kappa(x),\mu_\ell)$.

4.4.5.3. Line 2, $\mathrm{H}^1(X,x)_{\text{ét}},\mu_\ell)$ should be $\mathrm{H}^1((X,x)_{\text{ét}},\mu_\ell)$.

4.4.8.3. Line 10, H^1 should be H^1.
In the displayed formula, $T^\ell - f(x)$ should be $(T^\ell - f(x))$.

4.4.10.4. Line 5, remove the first sentence.

4.4.14. Line 3, Y should be X.
Line 9, the formula should be $\mathrm{H}^1((X,x)_{\text{ét}}, \mu_\ell) \sim \mathrm{H}^1(\mathscr{H}(x), \mu_\ell)$.

4.4.23. Line 6, t should be T.

4.5.12. Line 1, $p\colon X \to X_{\widehat{k^a}}$ should be $p\colon X_{\widehat{k^a}} \to X$.
The finiteness of the fiber over $x \in X_{[0,2,3]}$ is due to the fact that x is Abhyankar. See 3.2.15.4.

References

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