NOTE ON DUCROS' BOOK — CHAPTER 4

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Contents

1. Introduction

These are a series of notes on the book DucCurve [Duc24].
This note contains a very incomplete erratum.

2. Notes

Let k be a non-Archimedean analytic field. Consider a k-analytic curve X.
Unlike Ducros' book, we assume that X is good.[1]

4.1.1. Line 17, $\mathbb{P}_k^{1,\mathrm{an}}$ should be $\mathbb{P}_k^1$.

4.2.1. Line 4, $\varphi^{-1}(\varphi((x))$ should be $\varphi^{-1}(\varphi(x))$.

4.2.3. Line 5, φ should be f.

4.2.4.1. Line 6, $=0$ should be removed.
Line 8, X' should be X_0.

4.2.5.1. The existence of function mentioned in the first paragraph is constructed in 3.5.9.

4.2.9. Line 3, $\mathbb{P}_{\mathscr{H}(x)}$ should be $\mathbb{P}_{\mathscr{H}(x)/k}$.

4.2.16. The reduction in the first paragraph of the proof is not quite correct, and is not what we need in the sequel. The correct version is the following:

Notons pour commencer que l'on peut, pour montrer 1), 2) i) et 2) ii), remplacer X par X_{red} et Y par $Y \times_X X_{\mathrm{red}}$; cela permet de se ramener, pour montrer toutes les assertions, au cas où la courbe X est génériquement réduite, et l'on distingue alors deux cas selon la nature du point x.

4.2.16.1. Line 5, the second y should be x.
Line 5, U est une composante connexe de $\varphi^{-1}(x)$ should be V est une composante connexe de $\varphi^{-1}(U)$.

4.2.16.2. Line 26, $\varphi^{-1}U$ should be $\varphi^{-1}(U)$.

4.2.19. In iii), $X_{[23]}$ should be $X_{[2,3]}$.
The second part of iii) follows from the fact that $(\kappa(x), |\bullet|)$ is Henselian, a very general fact proved by Berkovich Berk93 [Ber93, Theorem 2.3.3].
Line 8 in the proof, remove est fini et.
Line 17 in the proof, the left parenthesis should be larger.
Line -4 in the proof follows from 2.3.12.

[1] This is proved in Ducros' book based on Temkin's goodness criterion. I cannot understand the proof of the latter as explained in my note on graded reductions.

4.2.19.2. Line 5, 4.2.9 should be 4.2.3.
Line 6, b should be a.

4.2.20. There is a serious issue here. The whole proposition only works if both germs (Y, y) and (X, x) are boundaryless.

The proof below implicitly assumed that y is of type 2. If y is of type 3, there is really nothing to prove in view of 4.2.19 iii).

The germ (X', x') is normal thanks to [stacks-project Stacks, Tag 034F]. In particular, the reduction at the end of this part makes sense.

4.2.20.1. The second displayed formula follows from 2.3.14 ii).

4.3.3. In the statement of the theorem, p is the exponential characteristic of k.
In the third paragraph of the proof, add si $p \neq 1$ after par p.

4.3.4.1. Line 4, x_i should be x'_i.

4.3.5.1. In the third paragraph, the claim

$$\widetilde{\kappa_{\widehat{k^a},r}} = \widetilde{k^a}$$

is obviously wrong.

The corrected version: comme $|\widehat{k^a}^\times|$ est divisible et comme $\widetilde{\kappa_{\widehat{k^a},r_1}}$ est algébriquement clos (il est égal à $\widetilde{k^a}_1$),.

By la théorie de la ramification modérée, Ducros meant 2.3.39.

4.3.5.2. Line 1, 3) should be 1).
Line 7, $\mathrm{S}(Z)\{x\}$ should be $\mathrm{S}(Z) \setminus \{x\}$.

4.3.6. Line 1, en en should be en.

4.3.6.4. Line 1, b) should be b.

4.3.6.4. Line 8, $|\mathscr{O}_X(Z)^\times|$ should be $|\mathscr{O}_X(Z)^\times|_b$.

4.3.7. Line 2 of the proof, add et que X soit connexe after $\mathscr{O}_X(X)$.
Line 7 of the proof, $\mathrm{br}(X_{\widehat{k^a}} y,)$ should be $\mathrm{br}(X_{\widehat{k^a}}, y)$.
Line -1 of the proof, ∞ should be $\mathfrak{s}(Z)$.
In the proof, G is the absolute Galois group of k.

4.3.8. Line 5, $\widehat{k^a}$ should be k.

4.3.9. Line 1, there is a serious issue, $x \in X$ should be $x \in X_{[2,3]}$.

4.3.9.1. Line 18, Y^{an} should be $\partial^{\mathrm{an}}(Y)$.
Line 19, X^{an} should be $\partial^{\mathrm{an}}(X)$.

4.3.9.2. Line 7, (X, x) should be $(X_{\widehat{k^a}}, y)$.

4.3.10. Line 1, en en should be en.

4.3.10.4. Line -2, $|\mathscr{O}_X(b)^\times_{\mathrm{sep}}|$ should be $|\mathscr{O}_X(b)^\times_{\mathrm{sep}}|_b$.

4.3.11.1. Line 7, b should be y.
Line 8, a should be x; le lemme should be la proposition; $\mathrm{S}(V)$ should be $\mathrm{S}^{\mathrm{an}}(V)$.
Line 9, $\mathrm{S}(V)$ should be $\mathrm{S}^{\mathrm{an}}(V)$.
Line -1, the first Z should be Z'.

4.3.13.2. Line 14, $\tilde{k}$ should be $\widetilde{\widehat{k^a}}_1$.
Line 2 of the third paragraph, remove).
Line 5 of the third paragraph, $\widetilde{\mathscr{H}(y)}$ should be $\widetilde{\mathscr{H}(y)}_1$.
Line 6 of the third paragraph, $\widetilde{\mathscr{H}(x)}$ should be $\widetilde{\mathscr{H}(x)}_1$.

4.3.14. Line 2, remove dont on note d le rang.
Line 1 of the third paragraph in the proof, $\eta_{\widehat{k^a}}$ should be z.

4.3.16. Here one has to apply 3.2.12.3, which says that

$$[\mathfrak{s}(x) : k] < \infty.$$

4.4.3.1. Line 8, U should be $X \setminus \{x\}$.
Line 9, U should be Z.

4.4.5. Line 4, $\mathrm{H}^1(\kappa(x), \mu_\ell))$ should be $\mathrm{H}^1(\kappa(x), \mu_\ell)$.

4.4.5.3. Line 2, $\mathrm{H}^1(X, x)_{\text{ét}}, \mu_\ell)$ should be $\mathrm{H}^1((X, x)_{\text{ét}}, \mu_\ell)$.

4.4.8.3. Line 10, H^1 should be H^1.
In the displayed formula, $T^\ell - f(x)$ should be $(T^\ell - f(x))$.

4.4.10.4. Line 5, remove the first sentence.

4.4.14. Line 3, Y should be X.
Line 9, the formula should be $\mathrm{H}^1((X, x)_{\text{ét}}, \mu_\ell) \sim \mathrm{H}^1(\mathscr{H}(x), \mu_\ell)$.

4.4.20. In the second paragraph of the proof, the order defined by η is introduced in 1.3.14. Note that η is the maximal element.
Line 3 of the third paragraph of the proof, $\widehat{k^a}$ should be F.
Line 2 of the seventh paragraph of the proof, $\widetilde{\kappa(t')}$ should be $\widetilde{\kappa(t')}_1$; $\widetilde{\kappa(t)}$ should be $\widetilde{\kappa(t)}_1$.

4.4.23. Line 6, t should be T.

4.5.3. The proof based on 4.4.17 is not correct, since the latter only works for proper curves.
After the mentioned reductions, we need to apply 4.1.2 to further reduce to the case where $X = \mathcal{X}^{\mathrm{an}}$ for some smooth, projective curve. Then 4.4.17 is applicable.
In fact, since X is compact, it suffices to prove the assertion locally. So we can take a single point $x \in X$ and apply 4.1.2 to $S = \{x\}$.

4.5.12. Line 1, $p\colon X \to X_{\widehat{k^a}}$ should be $p\colon X_{\widehat{k^a}} \to X$.
The finiteness of the fiber over $x \in X_{[0,2,3]}$ is due to the fact that x is Abhyankar. See 3.2.15.4.

4.5.21. Line 4, $X_{[23]}$ should be $X_{[2,3]}$.

References

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