

# NOTE ON DUCROS' BOOK — CHAPTER 4

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## 1. INTRODUCTION

These are a series of notes on the book [\[DucCurve\]](#) [\[Duc24\]](#).

## 2. NOTES

Let  $k$  be a non-Archimedean analytic field. Consider a  $k$ -analytic curve  $X$ .

Unlike Ducros' book, we assume that  $X$  is good.<sup>1</sup>

4.1.1. Line 17,  $\mathbb{P}_k^{1,\text{an}}$  should be  $\mathbb{P}_k^1$ .

4.2.1. Line 4,  $\varphi^{-1}(\varphi((x)))$  should be  $\varphi^{-1}(\varphi(x))$ .

4.2.3. Line 5,  $\varphi$  should be  $f$ .

4.2.4.1. Line 6,  $= 0$  should be removed.

Line 8,  $X'$  should be  $X_0$ .

4.2.5.1. The existence of function mentioned in the first paragraph is constructed in 3.5.9.

4.2.9. Line 3,  $\mathbb{P}_{\mathcal{H}(x)}$  should be  $\mathbb{P}_{\mathcal{H}(x)/k}$ .

4.2.16. The reduction in the first paragraph of the proof is not quite correct, and is not what we need in the sequel. The correct version is the following:

Notons pour commencer que l'on peut, pour montrer 1), 2) i) et 2) ii), remplacer  $X$  par  $X_{\text{red}}$  et  $Y$  par  $Y \times_X X_{\text{red}}$ ; cela permet de se ramener, pour montrer toutes les assertions, au cas où la courbe  $X$  est génériquement réduite, et l'on distingue alors deux cas selon la nature du point  $x$ .

4.2.16.1. Line 5, the second  $y$  should be  $x$ .

Line 5,  $U$  est une composante connexe de  $\varphi^{-1}(x)$  should be  $V$  est une composante connexe de  $\varphi^{-1}(U)$ .

4.2.16.2. Line 26,  $\varphi^{-1}U$  should be  $\varphi^{-1}(U)$ .

4.3.4.1. Line 4,  $x_i$  should be  $x'_i$ .

4.3.5.2. Line 1, 3) should be 1).

4.3.6.4. Line 8,  $|\mathcal{O}_X(Z)^\times|$  should be  $|\mathcal{O}_X(Z)^\times|_b$ .

4.3.9.1. Line 18,  $Y^{\text{an}}$  should be  $S^{\text{an}}(Y)$ .

Line 19,  $X^{\text{an}}$  should be  $S^{\text{an}}(X)$ .

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<sup>1</sup>This is proved in Ducros' book based on Temkin's goodness criterion. I cannot understand the proof of the latter as explained in my note on graded reductions.

**4.3.11.1.** Line 7,  $b$  should be  $y$ .

Line 8,  $a$  should be  $x$ .

Line 8, **le lemma** should be **la proposition**.

**4.4.3.1.** Line 8,  $U$  should be  $X \setminus \{x\}$ .

Line 9,  $U$  should be  $Z$ .

**4.4.5.** Line 4,  $H^1(\kappa(x), \mu_\ell)$  should be  $H^1(\kappa(x), \mu_\ell)$ .

**4.4.5.3.** Line 2,  $H^1(X, x)_{\text{ét}}, \mu_\ell$  should be  $H^1((X, x)_{\text{ét}}, \mu_\ell)$ .

**4.4.8.3.** Line 10,  $H^1$  should be  $H^1$ .

In the displayed formula,  $T^\ell - f(x)$  should be  $(T^\ell - f(x))$ .

**4.4.10.4.** Line 5, remove the first sentence.

**4.4.14.** Line 3,  $Y$  should be  $X$ .

Line 9, the formula should be  $H^1((X, x)_{\text{ét}}, \mu_\ell) \sim H^1(\mathcal{H}(x), \mu_\ell)$ .

**4.4.23.** Line 6,  $t$  should be  $T$ .

**4.5.4.** Line 6, **coronaire** should be **une couronne virtuelle**.

**4.5.12.** Line 1,  $p: X \rightarrow X_{\widehat{k^a}}$  should be  $p: X_{\widehat{k^a}} \rightarrow X$ .

The finiteness of the fiber over  $x \in X_{[0,2,3]}$  is due to the fact that  $x$  is Abhyankar. See 3.2.15.4.

## REFERENCES

- [Duc24] A. Ducros. La structure des courbes analytiques. 2024. arXiv: [2405.10619 \[math.AG\]](#).  
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