

NOTE ON DUCROS' BOOK — CHAPTER 4

MINGCHEN XIA

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1. INTRODUCTION

These are a series of notes on the book [\[DucCurve\]](#) [\[Duc24\]](#).

2. NOTES

Let k be a non-Archimedean analytic field. Consider a k -analytic curve X .

Unlike Ducros' book, we assume that X is good.¹

4.1.1. Line 17, $\mathbb{P}_k^{1,\text{an}}$ should be \mathbb{P}_k^1 .

4.2.1. Line 4, $\varphi^{-1}(\varphi((x)))$ should be $\varphi^{-1}(\varphi(x))$.

4.2.3. Line 5, φ should be f .

4.2.4.1. Line 6, $= 0$ should be removed.

Line 8, X' should be X_0 .

4.2.5.1. The existence of function mentioned in the first paragraph is constructed in 3.5.9.

4.2.9. Line 3, $\mathbb{P}_{\mathcal{H}(x)}$ should be $\mathbb{P}_{\mathcal{H}(x)/k}$.

4.2.16. The reduction in the first paragraph of the proof is not quite correct, and is not what we need in the sequel. The correct version is the following:

Notons pour commencer que l'on peut, pour montrer 1), 2) i) et 2) ii), remplacer X par X_{red} et Y par $Y \times_X X_{\text{red}}$; cela permet de se ramener, pour montrer toutes les assertions, au cas où la courbe X est génériquement réduite, et l'on distingue alors deux cas selon la nature du point x .

4.2.16.1. Line 5, the second y should be x .

Line 5, U est une composante connexe de $\varphi^{-1}(x)$ should be V est une composante connexe de $\varphi^{-1}(U)$.

4.2.16.2. Line 26, $\varphi^{-1}U$ should be $\varphi^{-1}(U)$.

4.2.19. In iii), $X_{[23]}$ should be $X_{[2,3]}$.

The second part of iii) follows from the fact that $(\kappa(x), |\bullet|)$ is Henselian, a very general fact proved by Berkovich [\[Berk93\]](#), Theorem 2.3.3].

Line 8 in the proof, remove **est fini et**.

Line 17 in the proof, the left parenthesis should be larger.

Line -4 in the proof follows from 2.3.12.

¹This is proved in Ducros' book based on Temkin's goodness criterion. I cannot understand the proof of the latter as explained in my note on graded reductions.

4.2.19.2. Line 5, 4.2.9 should be 4.2.3.

Line 6, b should be a .

4.2.20. There is a serious issue here. The whole proposition only works if both germs (Y, y) and (X, x) are boundaryless.

The proof below implicitly assumed that y is of type 2. If y is of type 3, there is really nothing to prove in view of 4.2.19 iii).

The germ (X', x') is normal thanks to [\[stacks-project, Tag 034F\]](#). In particular, the reduction at the end of this part makes sense.

4.2.20.1. The second displayed formula follows from 2.3.14 ii).

4.3.3. In the statement of the theorem, p is the exponential characteristic of k .

In the third paragraph of the proof, add $\text{si } p \neq 1$ after $\text{par } p$.

4.3.4.1. Line 4, x_i should be x'_i .

4.3.5.1. In the third paragraph, the claim

$$\widetilde{\kappa_{k^a, r}} = \widetilde{k^a}$$

is obviously wrong.

The corrected version: comme $|\widehat{k^a}^\times|$ est divisible et comme $\widetilde{\kappa_{k^a, r_1}}$ est algébriquement clos (il est égal à $\widetilde{k^{a_1}}$),.

By la théorie de la ramification modérée, Ducros meant 2.3.39.

4.3.5.2. Line 1, 3) should be 1).

Line 7, $S(Z)\{x\}$ should be $S(Z) \setminus \{x\}$.

4.3.6. Line 1, en en should be en.

4.3.6.4. Line 1, b) should be b.

4.3.6.4. Line 8, $|\mathcal{O}_X(Z)^\times|$ should be $|\mathcal{O}_X(Z)^\times|_b$.

4.3.7. Line 2 of the proof, add et que X soit connexe after $\mathcal{O}_X(X)$.

Line 7 of the proof, $\text{br}(X_{\widehat{k^a}} y,)$ should be $\text{br}(X_{\widehat{k^a}}, y)$.

Line -1 of the proof, ∞ should be $\mathfrak{s}(Z)$.

In the proof, G is the absolute Galois group of k .

4.3.8. Line 5, $\widehat{k^a}$ should be k .

4.3.9. Line 1, there is a serious issue, $x \in X$ should be $x \in X_{[2,3]}$.

4.3.9.1. Line 18, Y^{an} should be $\partial^{\text{an}}(Y)$.

Line 19, X^{an} should be $\partial^{\text{an}}(X)$.

4.3.9.2. Line 7, (X, x) should be $(X_{\widehat{k^a}}, y)$.

4.3.10. Line 1, en en should be en.

4.3.10.4. Line -2, $|\mathcal{O}_X(b)_{\text{sep}}^\times|$ should be $|\mathcal{O}_X(b)_{\text{sep}}^\times|_b$.

4.3.11.1. Line 7, b should be y .

Line 8, a should be x ; le lemme should be la proposition; $S(V)$ should be $S^{\text{an}}(V)$.

Line 9, $S(V)$ should be $S^{\text{an}}(V)$.

Line -1, the first Z should be Z' .

4.4.3.1. Line 8, U should be $X \setminus \{x\}$.

Line 9, U should be Z .

4.4.5. Line 4, $H^1(\kappa(x), \mu_\ell)$ should be $H^1(\kappa(x), \mu_\ell)$.

4.4.5.3. Line 2, $H^1(X, x)_{\text{ét}}, \mu_\ell$ should be $H^1((X, x)_{\text{ét}}, \mu_\ell)$.

4.4.8.3. Line 10, H^1 should be H^1 .

In the displayed formula, $T^\ell - f(x)$ should be $(T^\ell - f(x))$.

4.4.10.4. Line 5, remove the first sentence.

4.4.14. Line 3, Y should be X .

Line 9, the formula should be $H^1((X, x)_{\text{ét}}, \mu_\ell) \sim H^1(\mathcal{H}(x), \mu_\ell)$.

4.4.23. Line 6, t should be T .

4.5.12. Line 1, $p: X \rightarrow X_{\widehat{k^a}}$ should be $p: X_{\widehat{k^a}} \rightarrow X$.

The finiteness of the fiber over $x \in X_{[0,2,3]}$ is due to the fact that x is Abhyankar. See 3.2.15.4.

REFERENCES

- [Berk93] V. G. Berkovich. Étale cohomology for non-Archimedean analytic spaces. *Inst. Hautes Études Sci. Publ. Math.* 78 (1993), 5–161 (1994). URL: http://www.numdam.org/item?id=PMIHES_1993__78__5_0.
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Mingchen Xia, CHALMERS TEKNISKA HÖGSKOLA AND INSTITUTE OF GEOMETRY AND PHYSICS, USTC

Email address, xiamingchen2008@gmail.com

Homepage, <https://mingchenxia.github.io/home/>.