

# NOTE ON DUCROS' BOOK — CHAPTER 4

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## 1. INTRODUCTION

These are a series of notes on the book [\[DucCurve\]](#) [\[Duc24\]](#).

## 2. NOTES

Let  $k$  be a non-Archimedean analytic field. Consider a  $k$ -analytic curve  $X$ .

Unlike Ducros' book, we assume that  $X$  is good.<sup>1</sup>

4.1.1. Line 17,  $\mathbb{P}_k^{1,\text{an}}$  should be  $\mathbb{P}_k^1$ .

4.2.1. Line 4,  $\varphi^{-1}(\varphi((x)))$  should be  $\varphi^{-1}(\varphi(x))$ .

4.2.3. Line 5,  $\varphi$  should be  $f$ .

4.2.4.1. Line 6,  $= 0$  should be removed.

Line 8,  $X'$  should be  $X_0$ .

4.2.5.1. The existence of function mentioned in the first paragraph is constructed in 3.5.9.

4.2.9. Line 3,  $\mathbb{P}_{\mathcal{H}(x)}$  should be  $\mathbb{P}_{\mathcal{H}(x)/k}$ .

4.2.16. The reduction in the first paragraph of the proof is not quite correct, and is not what we need in the sequel. The correct version is the following:

Notons pour commencer que l'on peut, pour montrer 1), 2) i) et 2) ii), remplacer  $X$  par  $X_{\text{red}}$  et  $Y$  par  $Y \times_X X_{\text{red}}$ ; cela permet de se ramener, pour montrer toutes les assertions, au cas où la courbe  $X$  est génériquement réduite, et l'on distingue alors deux cas selon la nature du point  $x$ .

4.2.16.1. Line 5, the second  $y$  should be  $x$ .

Line 5,  $U$  est une composante connexe de  $\varphi^{-1}(x)$  should be  $V$  est une composante connexe de  $\varphi^{-1}(U)$ .

4.2.16.2. Line 26,  $\varphi^{-1}U$  should be  $\varphi^{-1}(U)$ .

4.2.19. In iii),  $X_{[23]}$  should be  $X_{[2,3]}$ .

The second part of iii) follows from the fact that  $(\kappa(x), |\bullet|)$  is Henselian, a very general fact proved by Berkovich [\[Berk93\]](#), Theorem 2.3.3].

Line 8 in the proof, remove **est fini et**.

Line 17 in the proof, the left parenthesis should be larger.

Line -4 in the proof follows from 2.3.12.

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<sup>1</sup>This is proved in Ducros' book based on Temkin's goodness criterion. I cannot understand the proof of the latter as explained in my note on graded reductions.

**4.2.19.2.** Line 5, **4.2.9** should be **4.2.3**.

Line 6, **b** should be **a**.

**4.3.4.1.** Line 4,  $x_i$  should be  $x'_i$ .

**4.3.5.2.** Line 1, **3)** should be **1)**.

**4.3.6.4.** Line 8,  $|\mathcal{O}_X(Z)^\times|$  should be  $|\mathcal{O}_X(Z)^\times|_b$ .

**4.3.9.1.** Line 18,  $Y^{\text{an}}$  should be  $S^{\text{an}}(Y)$ .

Line 19,  $X^{\text{an}}$  should be  $S^{\text{an}}(X)$ .

**4.3.11.1.** Line 7, **b** should be **y**.

Line 8, **a** should be **x**.

Line 8, **le lemma** should be **la proposition**.

**4.4.3.1.** Line 8, **U** should be  $X \setminus \{x\}$ .

Line 9, **U** should be **Z**.

**4.4.5.** Line 4,  $H^1(\kappa(x), \mu_\ell)$  should be  $H^1(\kappa(x), \mu_\ell)$ .

**4.4.5.3.** Line 2,  $H^1(X, x)_{\text{ét}}, \mu_\ell$  should be  $H^1((X, x)_{\text{ét}}, \mu_\ell)$ .

**4.4.8.3.** Line 10,  $H^1$  should be  $H^1$ .

In the displayed formula,  $T^\ell - f(x)$  should be  $(T^\ell - f(x))$ .

**4.4.10.4.** Line 5, remove the first sentence.

**4.4.14.** Line 3, **Y** should be **X**.

Line 9, the formula should be  $H^1((X, x)_{\text{ét}}, \mu_\ell) \sim H^1(\mathcal{H}(x), \mu_\ell)$ .

**4.4.23.** Line 6, **t** should be **T**.

**4.5.4.** Line 6, **coronaire** should be **une couronne virtuelle**.

**4.5.12.** Line 1,  $p: X \rightarrow X_{\widehat{k^a}}$  should be  $p: X_{\widehat{k^a}} \rightarrow X$ .

The finiteness of the fiber over  $x \in X_{[0,2,3]}$  is due to the fact that  $x$  is Abhyankar. See 3.2.15.4.

## REFERENCES

Berk93

- [Ber93] V. G. Berkovich. Étale cohomology for non-Archimedean analytic spaces. *Inst. Hautes Études Sci. Publ. Math.* 78 (1993), 5–161 (1994). URL: [http://www.numdam.org/item?id=PMIHES\\_1993\\_\\_78\\_\\_5\\_0](http://www.numdam.org/item?id=PMIHES_1993__78__5_0).

DucCurve

- [Duc24] A. Ducros. La structure des courbes analytiques. 2024. arXiv: [2405.10619](https://arxiv.org/abs/2405.10619) [math.AG].

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