

NOTE ON DUCROS' BOOK — CHAPTER 4

MINGCHEN XIA

CONTENTS

1. Introduction	1
2. Notes	1
References	3

1. INTRODUCTION

These are a series of notes on the book [\[DucCurve\]](#) [\[Duc24\]](#).

2. NOTES

Let k be a non-Archimedean analytic field. Consider a k -analytic curve X .

Unlike Ducros' book, we assume that X is good.¹

4.1.1. Line 17, $\mathbb{P}_k^{1,\text{an}}$ should be \mathbb{P}_k^1 .

4.2.1. Line 4, $\varphi^{-1}(\varphi((x)))$ should be $\varphi^{-1}(\varphi(x))$.

4.2.3. Line 5, φ should be f .

4.2.4.1. Line 6, $= 0$ should be removed.

Line 8, X' should be X_0 .

4.2.5.1. The existence of function mentioned in the first paragraph is constructed in 3.5.9.

4.2.9. Line 3, $\mathbb{P}_{\mathcal{H}(x)}$ should be $\mathbb{P}_{\mathcal{H}(x)/k}$.

4.2.16. The reduction in the first paragraph of the proof is not quite correct, and is not what we need in the sequel. The correct version is the following:

Notons pour commencer que l'on peut, pour montrer 1), 2) i) et 2) ii), remplacer X par X_{red} et Y par $Y \times_X X_{\text{red}}$; cela permet de se ramener, pour montrer toutes les assertions, au cas où la courbe X est génériquement réduite, et l'on distingue alors deux cas selon la nature du point x .

4.2.16.1. Line 5, the second y should be x .

Line 5, U est une composante connexe de $\varphi^{-1}(x)$ should be V est une composante connexe de $\varphi^{-1}(U)$.

4.2.16.2. Line 26, $\varphi^{-1}U$ should be $\varphi^{-1}(U)$.

4.2.19. In iii), $X_{[23]}$ should be $X_{[2,3]}$.

The second part of iii) follows from the fact that $(\kappa(x), |\bullet|)$ is Henselian, a very general fact proved by Berkovich [\[Berk93\]](#), Theorem 2.3.3].

Line 8 in the proof, remove **est fini et**.

Line 17 in the proof, the left parenthesis should be larger.

Line -4 in the proof follows from 2.3.12.

¹This is proved in Ducros' book based on Temkin's goodness criterion. I cannot understand the proof of the latter as explained in my note on graded reductions.

4.2.19.2. Line 5, **4.2.9** should be **4.2.3**.

Line 6, **b** should be **a**.

4.2.20. There is a serious issue here. **The whole proposition only works if both germs (Y, y) and (X, x) are boundaryless.**

The proof below implicitly assumed that y is of type 2. If y is of type 3, there is really nothing to prove in view of 4.2.19 iii).

The germ (X', x') is normal thanks to [\[stacks-project, Tag 034F\]](#). In particular, the reduction at the end of this part makes sense.

4.2.20.1. The second displayed formula follows from 2.3.14 ii).

4.3.3. In the statement of the theorem, p is the exponential characteristic of k .

In the third paragraph of the proof, add **si $p \neq 1$** after **par p** .

4.3.4.1. Line 4, **x_i** should be **x'_i** .

4.3.5.2. Line 1, **3)** should be **1)**.

4.3.6.4. Line 8, **$|\mathcal{O}_X(Z)^\times|$** should be **$|\mathcal{O}_X(Z)^\times|_b$** .

4.3.9.1. Line 18, **Y^{an}** should be **$S^{\text{an}}(Y)$** .

Line 19, **X^{an}** should be **$S^{\text{an}}(X)$** .

4.3.11.1. Line 7, **b** should be **y** .

Line 8, **a** should be **x** .

Line 8, **le lemma** should be **la proposition**.

4.4.3.1. Line 8, **U** should be **$X \setminus \{x\}$** .

Line 9, **U** should be **Z** .

4.4.5. Line 4, **$H^1(\kappa(x), \mu_\ell)$** should be **$H^1(\kappa(x), \mu_\ell)$** .

4.4.5.3. Line 2, **$H^1(X, x)_{\text{ét}}, \mu_\ell$** should be **$H^1((X, x)_{\text{ét}}, \mu_\ell)$** .

4.4.8.3. Line 10, **H^1** should be **H^1** .

In the displayed formula, **$T^\ell - f(x)$** should be **$(T^\ell - f(x))$** .

4.4.10.4. Line 5, remove the first sentence.

4.4.14. Line 3, **Y** should be **X** .

Line 9, the formula should be **$H^1((X, x)_{\text{ét}}, \mu_\ell) \sim H^1(\mathcal{H}(x), \mu_\ell)$** .

4.4.23. Line 6, **t** should be **T** .

4.5.4. Line 6, **coronaire** should be **une couronne virtuelle**.

4.5.12. Line 1, **$p: X \rightarrow X_{\widehat{k^a}}$** should be **$p: X_{\widehat{k^a}} \rightarrow X$** .

The finiteness of the fiber over $x \in X_{[0,2,3]}$ is due to the fact that x is Abhyankar. See 3.2.15.4.

REFERENCES

- [Berk93] V. G. Berkovich. Étale cohomology for non-Archimedean analytic spaces. *Inst. Hautes Études Sci. Publ. Math.* 78 (1993), 5–161 (1994). URL: http://www.numdam.org/item?id=PMIHES_1993__78__5_0.
- [Duc24] A. Ducros. La structure des courbes analytiques. 2024. arXiv: [2405.10619](https://arxiv.org/abs/2405.10619) [math.AG].
- [Stacks] T. Stacks Project Authors. Stacks Project. <http://stacks.math.columbia.edu>. 2020.

Mingchen Xia, CHALMERS TEKNISKA HÖGSKOLA AND INSTITUTE OF GEOMETRY AND PHYSICS, USTC

Email address, xiamingchen2008@gmail.com

Homepage, <https://mingchenxia.github.io/home/>.