Sep. 12^{th}, 2016. Discussion Session.

- Remarks

① Don't believe your "Common Knowledges" studied from Calculus.
② Use theorems, Axioms and definitions ...
... to prove stuff.

#29, where
$0_V = 1$, not $\emptyset$.

- A re-cover of ~~Theo~~ Theorem 1.3.

Given a V, where $(V, +, \cdot_{\mathbb{R}})$,
a x, which $x \in V$.

① Prove that $(-1)\cdot_{\mathbb{R}} x = -x$.

Multiplication of -1 to x. => Negative of x.

pf) By suggestion from remark ②.

$$x + (-1)\cdot_{\mathbb{R}} x \overset{Ax.10}{=} \underline{1\cdot_{\mathbb{R}} x} + (-1)\cdot_{\mathbb{R}} x \qquad 1x = x$$

$$\overset{Ax.9}{=} \underbrace{(1+(-1))}_{0_{\mathbb{R}}}\cdot_{\mathbb{R}} x$$

$$= 0\cdot_{\mathbb{R}} x = \emptyset \quad \text{(Proven in Theorem 1.3 [a])}.$$

Therefore, by definition of the ~~zero~~ $-x$ and uniqueness of $-x$, $(-1)\cdot_{\mathbb{R}} x = -x$.

(NO COMMON KNOWLEDGE USED IN THIS CASE, ONLY Axioms 10 AND 9, AND THEOREM 1.3 [a])

- $a\cdot_{\mathbb{R}}(-x) = (-a)\cdot_{\mathbb{R}} x = -(a\cdot_{\mathbb{R}} x)$.

All three sides of the equation has different meanings, however, their products serve the same role in the linear space V.

- Example.

$V_3 = \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$

Is $(V_3, +, \cdot\mathbb{R})$ linear space?

- Proving Axiom 3. Communicativity.

Let $(x_1, y_1, z_1), (x_2, y_2, z_2) \in V_3$.

$(x_1, \cancel{x}y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$.

$\Downarrow$

Of each coordinates were made up of real numbers.

Therefore $(x_1 + x_2, y_1 + y_2, z_1 + z_2)$

$= (x_2 + x_1, y_2 + y_1, z_2 + z_1)$.

Thus $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_2, y_2, z_2) + (x_1, y_1, z_1)$.

$\boxed{\text{Proving Axiom 3}}$ The law of communicativity holds.

- Subspace of V.

Given a subspace S $(S, +, \cdot\mathbb{R})$

$S \subseteq V$ is a subspace of V.

iff. $(S, +, \cdot\mathbb{R})$ satisfies Axiom 1-10.

(inherits operations from the original space).

- Theorem 1.4.

One would only need to prove Axiom 1-2 to state subjectivity of a subspace to its superset.

$S = \{(x, x, z) \mid x, z \in \mathbb{R}\}$

$= \{(x, y, z) \mid x = y, x, y, z \in \mathbb{R}\}$.

Is $(S, +, \cdot\mathbb{R})$ a subspace to V_3?

① Is $S \subseteq V_3$? If not, nothing to prove... WHY?
Yes.

② Axiom 1.

Let $(x_1, x_1, z_1), (x_2, x_2, z_2) \in S$.

$(x_1 + x_2, x_1 + x_2, z_1 + z_2) \in S$.

$\Downarrow$

WHY? As $x_1 + x_2 = x_1 + x_2$, as $x = y$.
Axiom 1 holds.

③ Axiom 2 can be proven similarly.

- How to construct a subspace of V that contains *finitely* many vectors such that

$v_1, v_2, v_3, \ldots, v_n \in V$.

- Notation

$L(A)$ L: subspace ...
(A): constructed by A which contains finitely many vectors included in A.

Example: $A = \{(1,1,1), (2,0,1)\}$.

$\{(1,1,1), (2,0,1), (3,1,2), (2,2,2), (5,1,3), (-100,-100,-100), \ldots\}$

$\Rightarrow$ By Axioms 1 and 2.

or expressed as...

$\{a\cdot(1,1,1), b\cdot(2,0,1)\}$ for real numbers a, b. Which forms $L(A)$.

a, b are constants.

[a, b] are independent from each other.

Linear Combination of finitely many vector components

Exercise 1.5. #28.

(All vectors in V_n that are linear combinations of A and B.) Determine whether this describes a linear space.

$V_n = \{(x_1, x_2, \dots, x_n) \mid x_i \in \mathbb{R}\}$

Same for $\forall i = 1, 2, \dots, n$.

Is $(V, +, \cdot \mathbb{R})$ linear space?

$\Downarrow$

$V = \{aA + bB\}$. where a, b are constants and are both $\mathbb{R}$.

① Axiom ~~Ax~~ 1:

Let $a_1A + b_1B$, and $a_2A + b_2B \in V$.

$(a_1A + b_1B) + (\cancel{b}\, a_2A + b_2B)$

$\overset{Ax.3-4}{=} (a_1A + a_2A) + (b_1B + b_2B)$

(Therefore) $\overset{Ax.9.}{=} (a_1 + a_2)A + (b_1 + b_2)B. \in V.$

Rearrangements were made under the condition that this is a combination made from V_n.

Axiom 1 proven.

② Axiom 2 can be proven similarly.

V is a subset of V_n, therefore with Axiom 1 and 2 proven, Axioms 3-10 are automatically proven under condition of Theorem 1.4.

Therefore the hypothesis of #28 is proven true.

Exercise 1.5. #29.

Proving Axiom 4.

$x, y, z \in V$.

"normal" multiplication by definition.

$(x \text{ "+" } y) \text{ "+" } z = (x \cdot y) \text{ "+" } z$.

Therefore $(x \cdot y) \cdot z = x \cdot (y \cdot z)$.

$$\Downarrow \qquad \Downarrow$$

$x \text{ "+" } (y \cdot z) = x \text{ "+" } (y \text{ "+" } z)$.

Axiom 4 holds for #29.

Exercise 1.10 must be attempted prior to Wednesday, and will be discussed on Wednesday prior to submission.