HW 2

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Problem 1

(a)

$$\hat{\pi}, \hat{\lambda}_{y,d} = \arg\max_{\pi, \lambda_{y,d}} \sum_{i=1}^{n} \ln p(y_i|\pi) + \sum_{d=1}^{D} \left(\ln p(\lambda_{y,d}) + \sum_{i=1}^{n} \ln p(x_{i,d}|\lambda_{y,d}) \right)$$

$$\text{let } L = \sum_{i=1}^{n} \ln p(y_i|\pi) + \sum_{d=1}^{D} \left(\ln p(\lambda_{y,d}) + \sum_{i=1}^{n} \ln p(x_{i,d}|\lambda_{y,d}) \right)$$

$$= \sum_{i=1}^{n} \ln \pi^{y_i}(1-\pi)^{1-y_i} + \sum_{d=1}^{D} \left(\ln \frac{\lambda_{y,d} e^{-\lambda_{y,d}}}{\Gamma(2)} + \sum_{i=1}^{n} \ln \frac{\lambda_{y,d}^{x_i} e^{-\lambda_{y,d}}}{x_i!} \right)$$

$$\frac{\partial L}{\partial \hat{\pi}} = \sum_{i=1}^{n} \left(\frac{y_i}{\pi} - \frac{1-y_i}{1-\pi} \right) = 0$$

$$\sum_{i=1}^{n} \frac{y_i - \pi}{\pi(1-\pi)} = 0$$

$$n\pi = \sum_{i=1}^{n} y_i$$

$$\hat{\pi} = \frac{\sum_{i=1}^{n} y_i}{n}$$

(b) for $\hat{\lambda}_{0,1:D}$:

$$\frac{\partial L}{\partial \hat{\lambda}_0} = \left(\ln \lambda_0 e^{-\hat{\lambda}_0} + \sum_{i=1}^{n} \ln \frac{\lambda_0^{x_i} e^{-\lambda_0}}{x_i!} \right)' \quad \text{where } y_i = 0$$

$$= \frac{1-\lambda_0}{\lambda_0} + \frac{\sum x_{i, y_i=0}}{\lambda_0} - \sum \mathbb{I}[y_i=0] = 0$$

$$1 - \lambda_0 + \sum x_{i, y_i=0} = \lambda_0 \cdot \sum_{i=1}^{n} \mathbb{I}[y_i=0]$$

$$\hat{\lambda}_0 = \frac{1 + \sum x_i \cdot \mathbb{I}(y_i=0)}{1 + n_0}$$

Similarly,

$$\hat{\lambda}_1 = \frac{1 + \sum x_i \cdot \mathbb{I}(y_i=1)}{1 + n_1}$$

Overall,

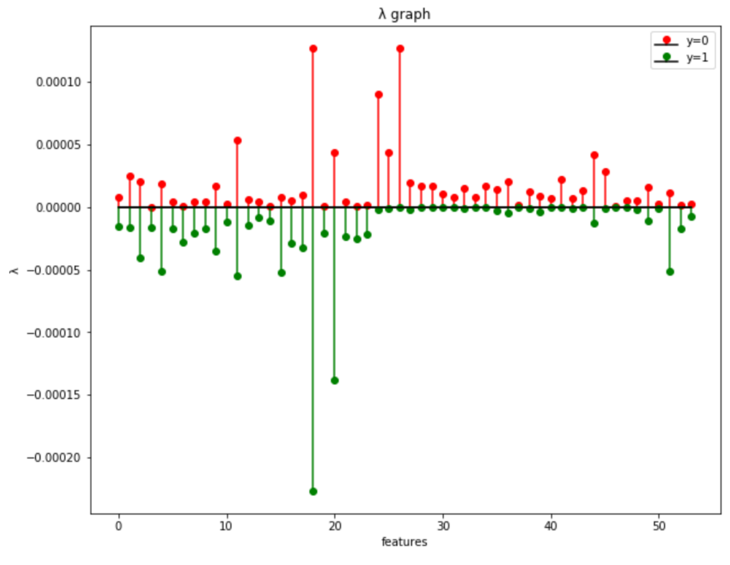
$$\hat{\lambda}_{y,d} = \frac{\sum x_i \cdot \mathbb{I}(y_i=y) + 1}{1 + \sum \mathbb{I}(y_i=y)}$$

Problem 2.

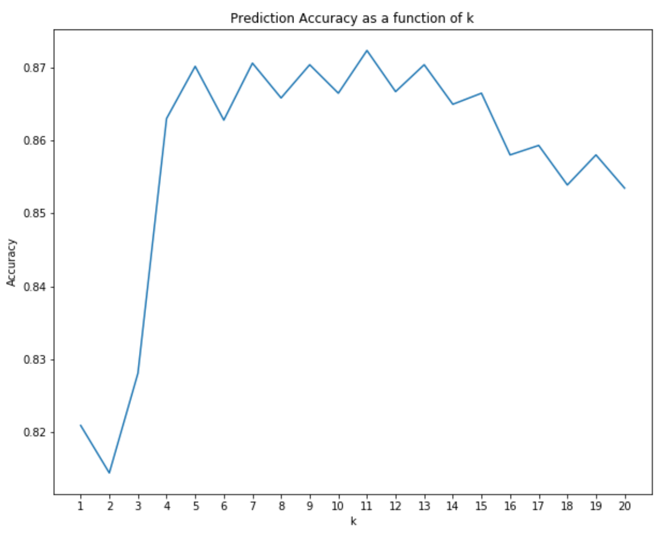
a)

Accuracy:	0.854	
	Predicted Negative	Predicted Positive
Negative Class	2230	557
Positive Class	114	1699

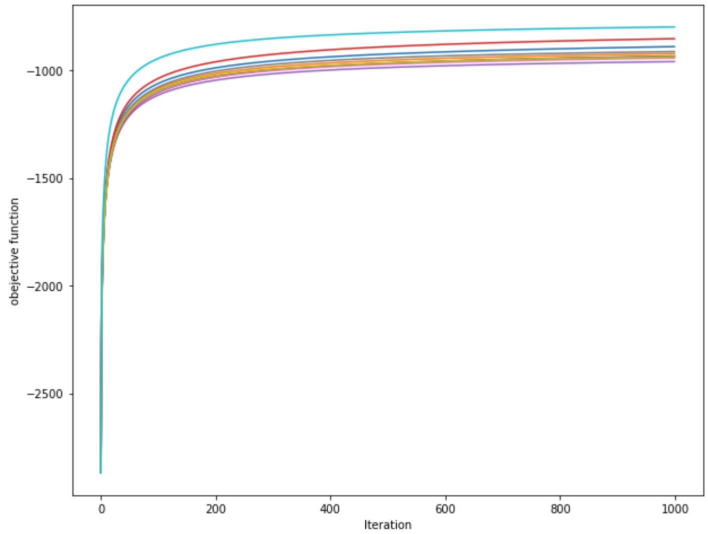
b)



c)



d)



e)

From Lectures, we know the first derivative of $\mathcal{L}$.

$$\nabla \mathcal{L} = \sum_{i=1}^{n} (1-\sigma(y_i x_i, w))\, y_i x_i$$

$$\nabla^2 \mathcal{L} = -\sum_{i=1}^{n} \frac{e^{y_i x_i^T w}}{(1+e^{y_i x_i^T w})^2} x_i x_i^T$$

$$= -\sum_{i=1}^{n} \sigma_i(y_i w)[1-\sigma_i(y_i w)]\, x_i x_i^T$$

plot the above equation into

$$\mathcal{L}(w) \approx \mathcal{L}'(w) \equiv \mathcal{L}(w_t) + (w-w_t)^T \nabla \mathcal{L}(w_t) + \frac{1}{2}(w-w_t)^T \mathcal{L}(w_t)(w-w_t)$$

Set $w_{t+1} = \arg\max_w \mathcal{L}'(w)$

Solve for w, $w_{t+1} = w_t - \nabla \mathcal{L}(w_t)[\nabla^2 \mathcal{L}(w_t)^{-1}]$

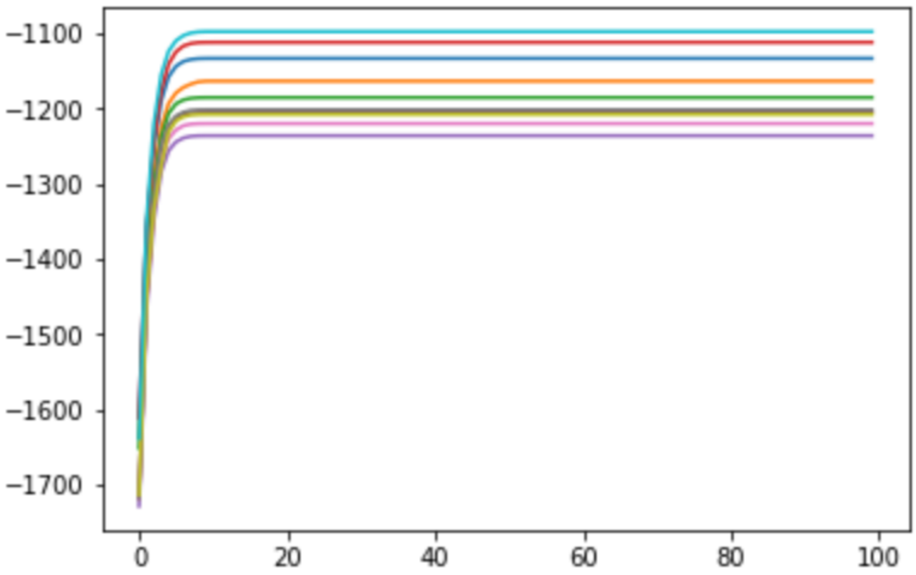
Process:

1. Set $w^{(1)} = \vec{0}$

2. For iteration $t = 1, 2, \cdots 100$ do

   - Update $w^{(t+1)} = w^{(t)} - \nabla \mathcal{L}(w_t)[\nabla^2 \mathcal{L}(w_t)^{-1}]$

We get:



f)

Accuracy: 0.889

	Predicted Negative	Predicted Positive
Negative Class	2501	286
Positive Class	222	1591