HW 2

Mingfeng Li
(ml4209)

Problem 1.

(a)

$$\hat{\pi}, \hat{\lambda}_{y,d} = \arg\max_{\pi, \lambda_{y,d}} \sum_{i=1}^{n} \ln p(y_i|\pi) + \sum_{d=1}^{D} \left(\ln p(\lambda_{y,d}) + \sum_{i=1}^{n} \ln p(x_{i,d} | \lambda_{y,d} \right)$$

$$\text{let } L = \sum_{i=1}^{n} \ln p(y_i|\pi) + \sum_{d=1}^{D} \left(\ln p(\lambda_{y,d}) + \sum_{i=1}^{n} \ln p(x_{i,d} | \lambda_{y,d}) \right)$$

$$= \sum_{i=1}^{n} \ln \pi^{y_i} (1-\pi)^{1-y_i} + \sum_{d=1}^{D} \left(\ln \frac{\lambda_{y,d} e^{-\lambda_{y,d}}}{\Gamma(2)} + \sum_{i=1}^{n} \ln \frac{\lambda_{y,d}^{x_i} e^{-\lambda_{y,d}}}{x_i!} \right)$$

$$\frac{\partial L}{\partial \hat{\pi}} = \sum_{i=1}^{n} \left(\frac{y_i}{\pi} - \frac{1-y_i}{1-\pi} \right) = 0$$

$$\sum_{i=1}^{n} \frac{y_i - \pi}{\pi(1-\pi)} = 0$$

$$n\pi = \sum_{i=1}^{n} y_i$$

$$\hat{\pi} = \frac{\sum_{i=1}^{n} y_i}{n}$$

(b) for $\hat{\lambda}_{0,1:D}$:

$$\frac{\partial L}{\partial \hat{\lambda}_0} = \left(\ln \lambda_0 e^{-\hat{\lambda}_0} + \sum_{i=1}^{n} \ln \frac{\lambda_0^{x_i} e^{-\lambda_0}}{x_i!} \right)' \quad \text{where } y_i = 0$$

$$= \frac{1-\lambda_0}{\lambda_0} + \frac{\sum x_{i,\, y_i=0}}{\lambda_0} - \sum \mathbb{1}[y_i=0] = 0$$

$$1 - \lambda_0 + \sum x_{i,\, y_i=0} = \lambda_0 \cdot \sum_{i=1}^{n} \mathbb{1}[y_i=0]$$

$$\hat{\lambda}_0 = \frac{1 + \sum x_i \cdot \mathbb{1}(y_i=0)}{1 + n_0}$$

Similarly,

$$\hat{\lambda}_1 = \frac{1 + \sum x_i \cdot \mathbb{1}(y_i=1)}{1 + n_1}$$

Overall,

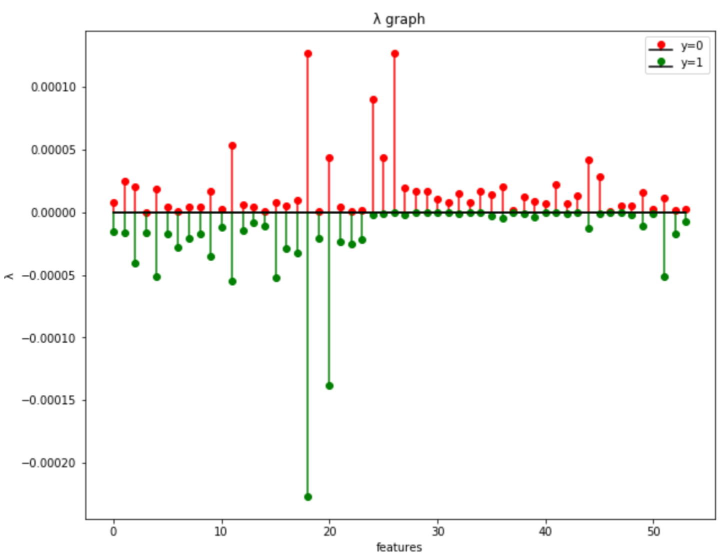
$$\hat{\lambda}_{y,d} = \frac{\sum x_i \cdot \mathbb{1}(y_i=y) + 1}{1 + \sum \mathbb{1}(y_i=y)}$$

Problem 2.

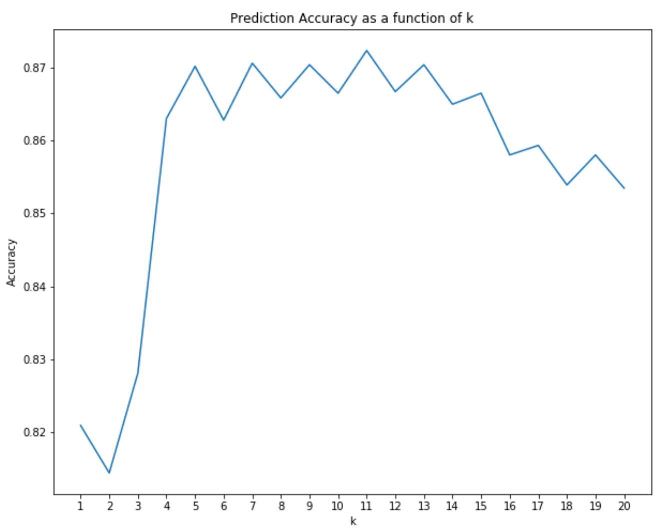
a)

Accuracy:	0.854	
	Predicted Negative	Predicted Positive
Negative Class	2230	557
Positive Class	114	1699

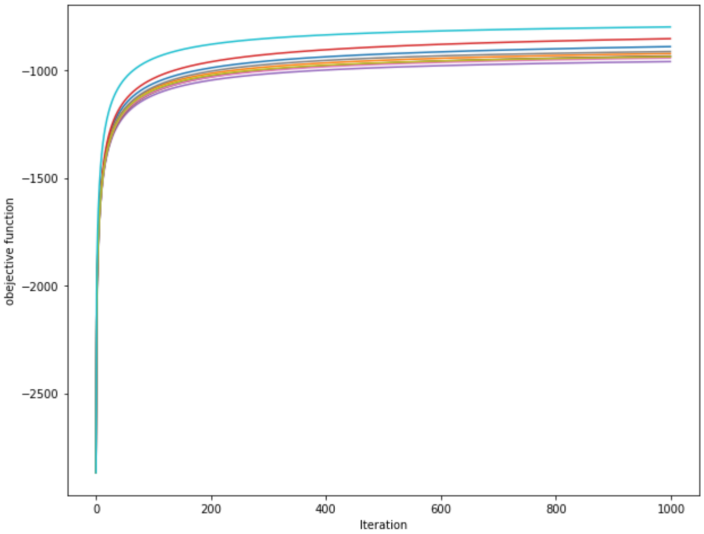
b)



c)



d)



e)

From Lectures, we know the first derivative of $\mathcal{L}$.

$$\nabla \mathcal{L} = \sum_{i=1}^{n} (1 - \sigma(y_i x_i, w)) y_i x_i$$

$$\nabla^2 \mathcal{L} = -\sum_{i=1}^{n} \frac{e^{y_i x_i^T w}}{(1 + e^{y_i x_i^T w})^2} x_i x_i^T$$

$$= -\sum_{i=1}^{n} \sigma_i (y_i w)[1 - \sigma_i (y_i w)] x_i x_i^T$$

plot the above equation into

$$\mathcal{L}(w) \approx \mathcal{L}'(w) \equiv \mathcal{L}(w_t) + (w - w_t)^T \nabla \mathcal{L}(w_t) + \frac{1}{2}(w - w_t)^T \mathcal{L}(w_t)(w - w_t)$$

Set $w_{t+1} = \text{argmax}_w \mathcal{L}'(w)$

Solve for w, $w_{t+1} = w_t - \nabla \mathcal{L}(w_t)[\nabla^2 \mathcal{L}(w_t)^{-1}]$

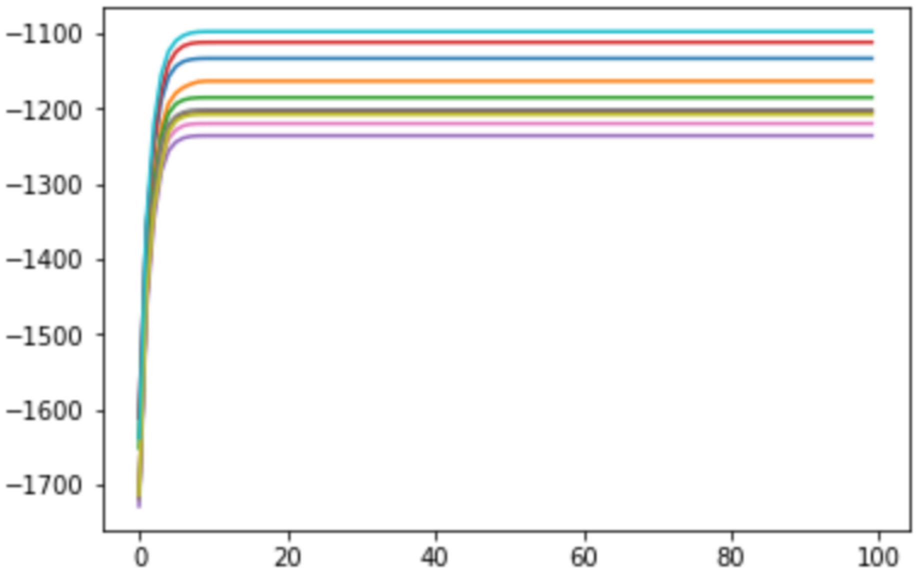
Process:

1. Set $w^{(1)} = \vec{0}$

2. For iteration $t = 1, 2, \cdots 100$ do

   - Update $w^{(t+1)} = w^{(t)} - \nabla \mathcal{L}(w_t)[\nabla^2 \mathcal{L}(w_t)^{-1}]$

We get:



f)

Accuracy: 0.889

	Predicted Negative	Predicted Positive
Negative Class	2501	286
Positive Class	222	1591

In [2]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [3]:

```
X = pd.read_csv('X.csv', header = None)
y = pd.read_csv('y.csv', header = None)
```

In [4]:

```
# creat 10 folds
from sklearn.model_selection import KFold
kf = KFold(n_splits =10)
kf.get_n_splits(X)
```

Out[4]:

```
10
```

2a naive bayes

In [5]:

```
from scipy.special import gamma
import math
tp, tn, fp, fn =0, 0, 0, 0
average_lam0 = 0
average_lam1 = 0

for train_index, test_index in kf.split(X):
    #print("train:", train_index, "Test", test_index)
    X_train, X_test = X.iloc[train_index,:], X.iloc[test_index,:]
    y_train, y_test = y.iloc[train_index,:], y.iloc[test_index,:]
    n = X_train.shape[0]
    columns = X_train.shape[1]
    pi = y_train[0].sum()/n
    n0 = n-y_train[0].sum()
    n1 = y_train[0].sum()
    sum0,sum1 = 0,0
    for i in range(n):
        if y_train.iloc[i,0] == 0:
            sum0 = sum0 + X_train.iloc[i,:]
        else:
            sum1 = sum1 + X_train.iloc[i,:]
    lamb0 = (1+sum0)/(1+n0)
    lamb1 = (1+sum1)/(1+n1)
    average_lam0 = average_lam0+lamb0
    average_lam1 = average_lam1+lamb1
    # predict y value
    y_pred = np.zeros((y_test.shape[0],))
    prior =[1-pi,pi]
    for i in range(y_test.shape[0]):
        p0 = prior[0]
        p1 = prior[1]
        for j in range(columns):
            p0=p0*lamb0[j]**X_test.iloc[i,j]*math.exp(-lamb0[j])/gamma(X_test.iloc[i,j]+1)
            p1=p1*lamb1[j]**X_test.iloc[i,j]*math.exp(-lamb1[j])/gamma(X_test.iloc[i,j]+1)
        y_pred[i] = int(p1 > p0)
        #print(int(p1 > p0))
        if y_pred[i] ==1 and y_test.iloc[i,0] == 1:
            tp +=1
        elif y_pred[i] ==1 and y_test.iloc[i,0] == 0:
            fp +=1
        elif y_pred[i] ==0 and y_test.iloc[i,0] == 0:
            tn +=1
        else:
```

```
            fn +=1
accuracy = (tp + tn)/4600
print ("accuracy:", accuracy)
print("true positive", tp)
print("true negative", tn)
print("false negative", fn)
print("false positive", fp)
```

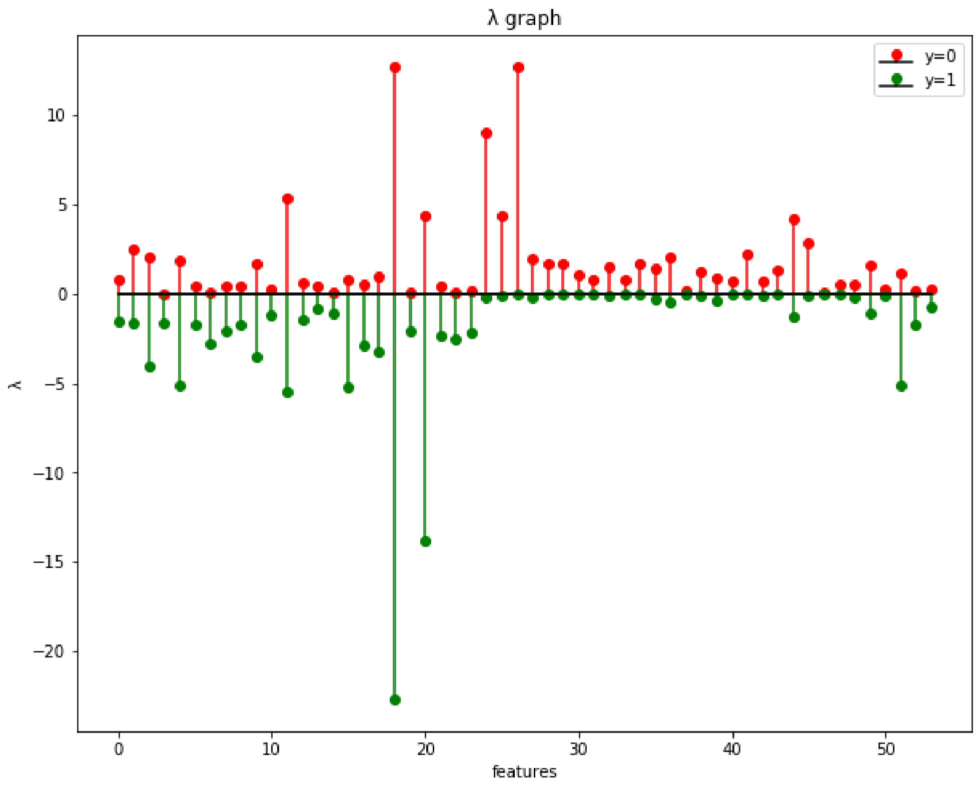
```
C:\Users\Mingfeng\Anaconda3\lib\site-packages\ipykernel_launcher.py:33: RuntimeWarning: overflow e
ncountered in double_scalars
C:\Users\Mingfeng\Anaconda3\lib\site-packages\ipykernel_launcher.py:33: RuntimeWarning: invalid
value encountered in double_scalars
```

```
accuracy: 0.8541304347826087
true positive 1699
true negative 2230
false negative 114
false positive 557
```

2b stem plot

In [6]:

```
plt.figure(figsize=(10, 8))
average_lam0 = average_lam0/10
average_lam1 = average_lam1/10
plt.stem(np.arange(54), average_lam0, 'r', markerfmt='ro', basefmt='black', label='y=0' )
plt.stem(np.arange(54), -average_lam1, 'g',markerfmt='go', basefmt='black', label='y=1')
plt.legend()
plt.title('λ graph')
plt.xlabel('features')
plt.ylabel('λ')
plt.show()
#plt.savefig('2b.png')
```



2c KNN

In [7]:

```
def KNN (X_train, X_test, y_train, y_test):
    accuracy = []
```

```
    error = []
    X_train=X_train.values
    X_test=X_test.values
    y_train = np.squeeze(np.asarray(y_train))
    y_test = np.squeeze(np.asarray(y_test))
    y_predict = np.zeros((y_test.shape[0],20))
    for i in range(X_test.shape[0]):
        dist = np.sum(np.absolute(X_train-X_test[i,:]),axis=1)
        for k in range(1,21):
            y_predict[i,k-1] = np.argmax(np.bincount(y_train[np.argpartition(dist, k-1)[:k]]))
    error = np.sum(np.absolute(y_predict-y_test.reshape(-1,1)),axis = 0)
    accuracy = 1-error/460
    return accuracy
```

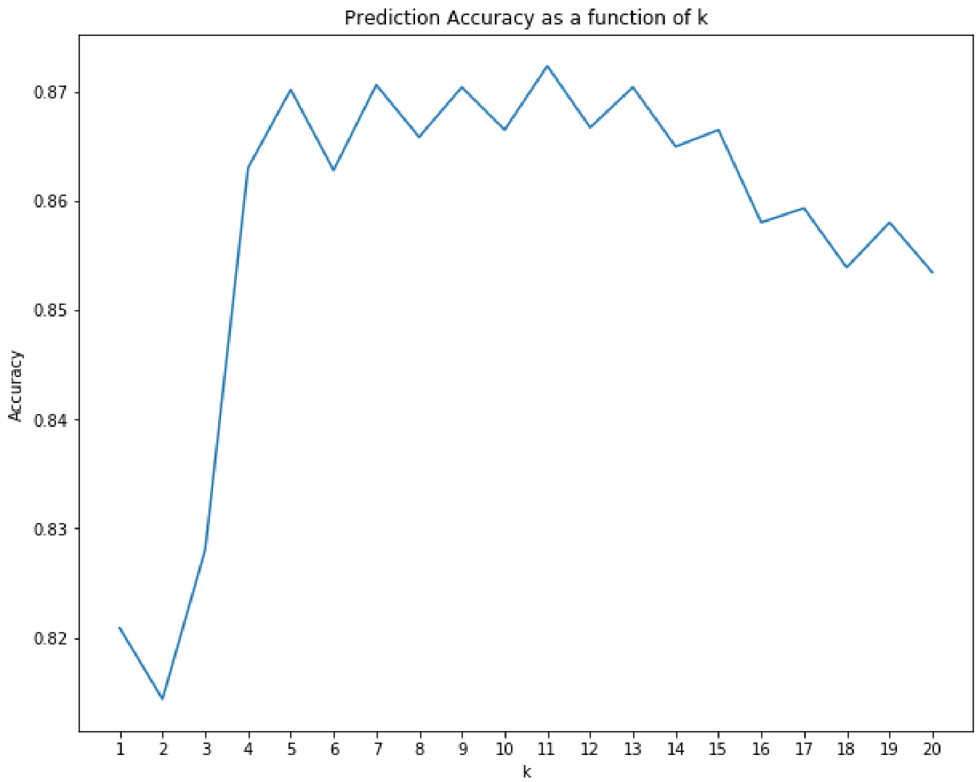
In [8]:

```
%%time
ave_accuracy =0
for train_index, test_index in kf.split(X):
    X_train, X_test = X.iloc[train_index,:], X.iloc[test_index,:]
    y_train, y_test = y.iloc[train_index,:], y.iloc[test_index,:]
    accuracy_list = KNN(X_train, X_test, y_train, y_test)
    ave_accuracy = ave_accuracy+np.array(accuracy_list)
ave_accuracy =ave_accuracy/10
```

```
Wall time: 17.9 s
```

In [9]:

```
plt.figure(figsize=(10, 8))
plt.plot(np.arange(1,21), ave_accuracy)
plt.xticks(range(1,21))
plt.xlabel('k')
plt.ylabel('Accuracy')
plt.title('Prediction Accuracy as a function of k')
plt.savefig('2c.png')
```



2d Logistics Regression steepest ascent

In [10]:

```
ite = 1000
```

```
step = 0.01/4600
```

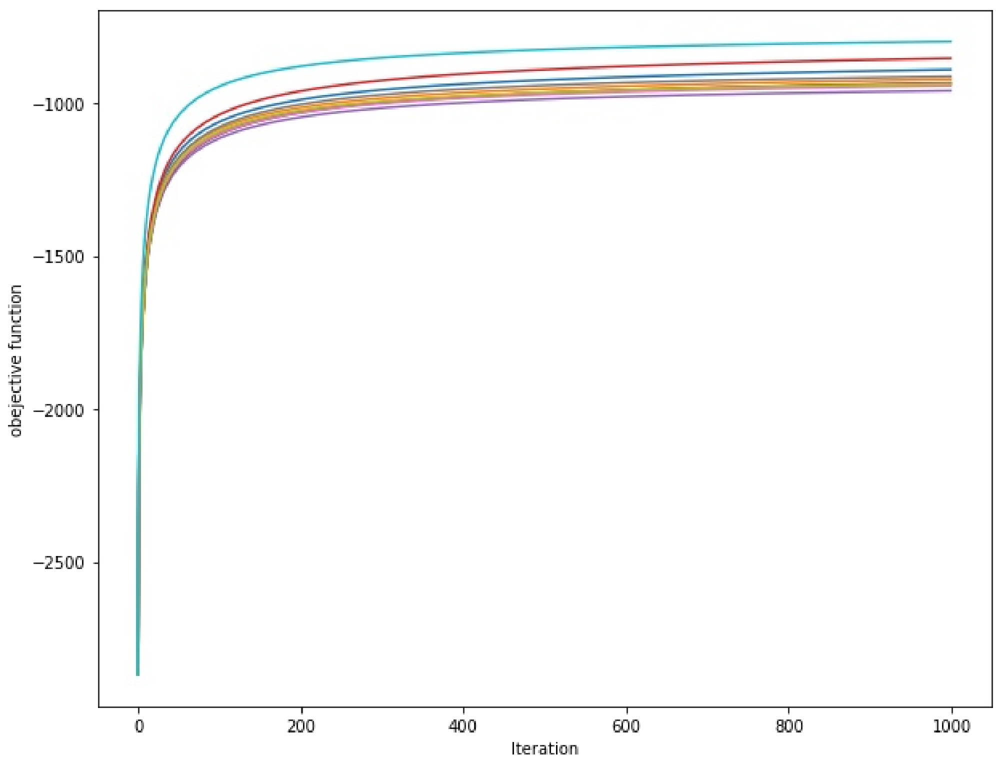
In [11]:

```
def logit_r(X_train, X_test, y_train, y_test):
    y_train2 =y_train.copy()
    y_test2 =y_test.copy()
    y_train2[y_train2 ==0]=-1
    y_test2[y_test2 ==0]=-1
    X_train2 = np.full((X_train.shape[0],X_train.shape[1]+1),1)
    X_train2[:,:-1] = X_train.values
    X_test2 = np.full((X_test.shape[0],X_test.shape[1]+1),1)
    X_test2[:,:-1] = X_test.values
    y_train2 = np.squeeze(np.asarray(y_train2))
    y_test2 = np.squeeze(np.asarray(y_test2))
    w = np.zeros((X_train2.shape[1],))
    delta =np.zeros((X_train2.shape[1],))
    #w =np.zeros((1000,))
    L =np.zeros((1000,))
    for t in range(ite):
        log_odd = np.exp(X_train2*y_train2.reshape(-1,1)@w)
        obs_p = log_odd/(1+log_odd)
        delta = np.sum(X_train2*y_train2.reshape(-1,1)*(1-obs_p).reshape(-1,1), axis =0)
        w = w+step*delta
        L[t] = np.sum(np.log(obs_p))
    return w, L, X_test2, y_test2
```

In [12]:

```
%%time
plt.figure(figsize=(10, 8))
plt.xlabel('Iteration')
plt.ylabel('obejective function')
for train_index, test_index in kf.split(X):
    X_train, X_test = X.iloc[train_index,:], X.iloc[test_index,:]
    y_train, y_test = y.iloc[train_index,:], y.iloc[test_index,:]
    w, L, X_test2, y_test2 = logit_r(X_train, X_test, y_train, y_test)
    plt.plot(np.arange(1000), L)
plt.savefig('2d.png')
```

Wall time: 1min 24s



In [323]:

```
plt.savefig('2d.png')
```

```
<Figure size 432x288 with 0 Axes>
```

2e Newton's method

In [19]:

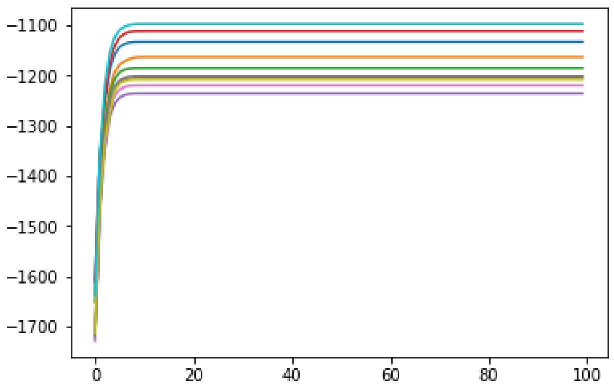
```
def newton_m(X_train, X_test, y_train, y_test):
    y_train2 =y_train.copy()
    y_test2 =y_test.copy()
    y_train2[y_train2 ==0]=-1
    y_test2[y_test2 ==0]=-1
    X_train2 = np.full((X_train.shape[0],X_train.shape[1]+1),1)
    X_train2[:,:-1] = X_train.values
    X_test2 = np.full((X_test.shape[0],X_test.shape[1]+1),1)
    X_test2[:,:-1] = X_test.values
    y_train2 = np.squeeze(np.asarray(y_train2))
    y_test2 = np.squeeze(np.asarray(y_test2))

    w = np.zeros((X_train2.shape[1],))
    sigmoid = np.exp((y_train2 * X_train2.T).T @ w)/(1 + np.exp((y_train2 * X_train2.T).T @ w))
    l= np.sum(np.log(sigmoid))
    L =np.zeros((100,))
    for t in range(100):
        gradient = np.sum((1-sigmoid).reshape(-1,1) *(y_train2 * X_train2.T).T, axis=0)
        diagonal = np.diag(sigmoid * (1-sigmoid).reshape(-1,1))
        hessian = - X_train2.T * diagonal @ X_train2 - (10**(-2))*np.identity(55)
        w = w - (gradient.T @ np.linalg.inv(hessian)).T
        sigmoid = np.exp((y_train2 * X_train2.T).T @ w)/(1 + np.exp((y_train2 * X_train2.T).T @ w))
        l += - (gradient.T @ np.linalg.inv(hessian)) @ gradient + 0.5 * (gradient.T @ np.linalg.inv(
hessian)) @ hessian @ (gradient.T @ np.linalg.inv(hessian)).T
        L[t] =l
    return w, L
```

In [20]:

```
%%time
for train_index, test_index in kf.split(X):
    X_train, X_test = X.iloc[train_index,:], X.iloc[test_index,:]
    y_train, y_test = y.iloc[train_index,:], y.iloc[test_index,:]
    w,L= newton_m(X_train, X_test, y_train, y_test)
    plt.plot(np.arange(100), L)
```

```
Wall time: 2min 22s
```



2f

In [29]:

```
%%time
tp, tn, fp, fn =0, 0, 0, 0
for train_index, test_index in kf.split(X):
    X_train, X_test = X.iloc[train_index,:], X.iloc[test_index,:]
```

```python
    y_train, y_test = y.iloc[train_index,:], y.iloc[test_index,:]
    w,L, X_test2, y_test2= logit_r(X_train, X_test, y_train, y_test)
    y_pred = w @ X_test2.T
    y_pred[np.where(y_pred >=0)]=1
    y_pred[np.where(y_pred <0)]=0
    fp = fp + np.sum((y_pred - y_test2)==2,axis=0)
    fn = fn + np.sum((y_pred - y_test2)==-1,axis=0)
    tp = tp + np.sum((y_pred - y_test2)==0,axis=0)
    tn = tn + np.sum((y_pred - y_test2)==1,axis=0)
accuracy = (tp + tn)/4600
```

```
Wall time: 1min 47s
```

In [30]:

```python
df = pd.DataFrame([[tn,fp],[fn,tp]], index=['Negative Class','Positive Class'],columns=['Predicted Negative','Predicted Positive'])
print(df)
print(accuracy)
```

```
                Predicted Negative  Predicted Positive
Negative Class                2501                 286
Positive Class                 222                1591
0.8895652173913043
```