## Build Prediction Model

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## **Exploratory Data Analysis**

```
load("./data/dat1.RData")
load("./data/dat2.RData")
# no missing data
all(is.na(dat1))
## [1] FALSE
all(is.na(dat2))
## [1] FALSE
ifelse(all(names(dat1) == names(dat2)), "train and test data have same structure", "train and test data
## [1] "train and test data have same structure"
str(dat1)
## 'data.frame': 5000 obs. of 14 variables:
                : int 1 2 3 4 5 6 7 8 9 10 ...
## $ id
## $ age
                : num 50 71 58 63 56 59 67 62 60 64 ...
## $ gender
                : int 0 1 1 0 1 1 0 1 0 1 ...
                : Factor w/ 4 levels "1","2","3","4": 1 1 1 1 3 4 1 4 1 ...
## $ race
                : Factor w/ 3 levels "0","1","2": 1 1 2 1 1 1 1 1 1 1 ...
## $ smoking
## $ height
                : num 176 176 169 167 163 ...
## $ weight
                : num 68.3 69.6 76.9 90 83.9 86.8 91.4 87.7 85.7 76.6 ...
## $ bmi
                : num 22 22.6 27 32.1 31.7 30.8 29.7 28.1 29 31.5 ...
## $ diabetes : int 0 0 0 0 0 0 0 0 0 ...
## $ hypertension: num 0 1 0 1 0 1 1 0 0 1 ...
                : num 130 149 127 138 123 132 133 130 129 134 ...
## $ SBP
## $ LDL
                 : num 82 129 101 93 97 108 89 96 120 135 ...
                 : num 76 82 168 105 193 143 63 78 61 88 ...
## $ time
## $ log_antibody: num 10.65 9.89 10.9 9.91 9.56 ...
```

#### Univariate analysis (continuous & categorical)

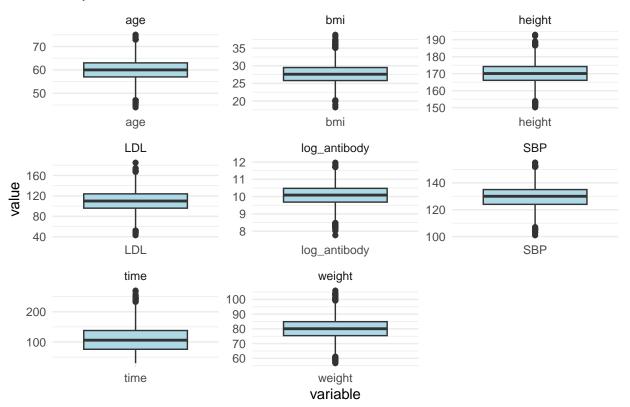
```
dat1 <- dat1 %>%
   select(-id)

dat2 <- dat2 %>%
   select(-id)

continuous_var <- dat1 %>%
```

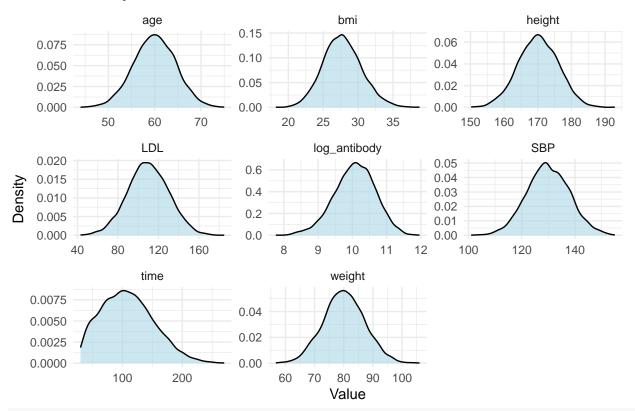
```
select(age, height, weight, bmi, SBP, LDL, time, log_antibody)
categorical_var <- dat1 %>%
 select(gender, race, smoking, diabetes, hypertension) %>%
 mutate(
    # Convert binary variables to factors with labels
   gender = factor(gender, levels = c(0, 1), labels = c("Female", "Male")),
   diabetes = factor(diabetes, levels = c(0, 1), labels = c("No", "Yes")),
   hypertension = factor(hypertension, levels = c(0, 1), labels = c("No", "Yes"))
# Continuous:
summary(continuous_var)
##
                       height
                                      weight
        age
                                                       bmi
## Min.
          :44.00
                   Min.
                         :150.2
                                  Min. : 56.70
                                                 Min.
                                                          :18.20
## 1st Qu.:57.00
                   1st Qu.:166.1
                                  1st Qu.: 75.40
                                                  1st Qu.:25.80
## Median :60.00
                  Median :170.1
                                  Median : 80.10
                                                  Median :27.60
## Mean
         :59.97
                  Mean :170.1
                                  Mean : 80.11
                                                 Mean
                                                        :27.74
## 3rd Qu.:63.00
                   3rd Qu.:174.2
                                  3rd Qu.: 84.90
                                                 3rd Qu.:29.50
## Max. :75.00
                  Max. :192.9
                                  Max. :106.00
                                                 Max. :38.80
                                                  log_antibody
##
        SBP
                       LDL
                                       time
## Min.
          :101.0
                         : 43.0
                                  Min. : 30.0
                                                 Min.
                                                       : 7.765
                   Min.
## 1st Qu.:124.0
                 1st Qu.: 96.0
                                  1st Qu.: 76.0
                                                 1st Qu.: 9.682
                 Median :110.0
## Median :130.0
                                  Median :106.0
                                                 Median :10.089
## Mean :129.9
                 Mean :109.9
                                  Mean :108.9
                                                 Mean :10.064
## 3rd Qu.:135.0
                   3rd Qu.:124.0
                                  3rd Qu.:138.0
                                                  3rd Qu.:10.478
## Max.
          :155.0 Max.
                         :185.0
                                  Max. :270.0
                                                 Max.
                                                        :11.961
# Boxplots
continuous_var_long <- continuous_var %>%
 tidyr::pivot_longer(cols = everything(), names_to = "variable", values_to = "value")
ggplot(continuous_var_long, aes(x = variable, y = value)) +
 geom_boxplot(fill = "lightblue") +
 facet_wrap(~variable, scales = "free", ncol = 3) +
 theme_minimal() +
 labs(title = "Boxplots of Continuous Variables")
```

# Boxplots of Continuous Variables



```
ggplot(continuous_var_long, aes(x = value)) +
geom_density(fill = "lightblue", alpha = 0.6) +
facet_wrap(~variable, scales = "free", ncol = 3) +
theme_minimal() +
labs(title = "Density Plots of Continuous Variables", x = "Value", y = "Density")
```

#### **Density Plots of Continuous Variables**

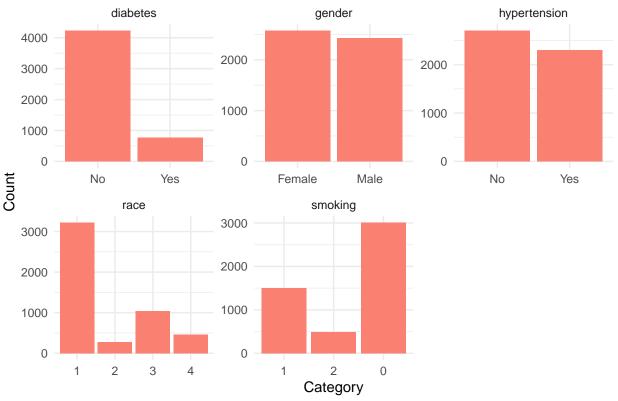


# # Categorical:

summary(continuous\_var)

```
##
                        height
                                         weight
                                                            bmi
         age
           :44.00
                            :150.2
                                     Min. : 56.70
                                                              :18.20
##
    Min.
                    Min.
                                                       Min.
                                     1st Qu.: 75.40
    1st Qu.:57.00
##
                    1st Qu.:166.1
                                                       1st Qu.:25.80
    Median :60.00
                    Median :170.1
                                     Median : 80.10
                                                       Median :27.60
##
    Mean
           :59.97
                    Mean
                           :170.1
                                     Mean
                                           : 80.11
                                                       Mean
                                                              :27.74
##
##
    3rd Qu.:63.00
                    3rd Qu.:174.2
                                     3rd Qu.: 84.90
                                                       3rd Qu.:29.50
           :75.00
                            :192.9
                                            :106.00
                                                              :38.80
##
    Max.
                    Max.
                                     Max.
                                                       Max.
         SBP
                         LDL
##
                                          time
                                                       log antibody
##
   Min.
           :101.0
                    Min.
                            : 43.0
                                     Min.
                                            : 30.0
                                                     Min.
                                                             : 7.765
                    1st Qu.: 96.0
##
    1st Qu.:124.0
                                     1st Qu.: 76.0
                                                      1st Qu.: 9.682
   Median :130.0
                    Median :110.0
                                     Median :106.0
                                                     Median :10.089
##
    Mean
           :129.9
                    Mean
                           :109.9
                                     Mean
                                            :108.9
                                                     Mean
                                                             :10.064
    3rd Qu.:135.0
                    3rd Qu.:124.0
                                     3rd Qu.:138.0
                                                     3rd Qu.:10.478
##
## Max.
           :155.0
                    Max.
                            :185.0
                                            :270.0
                                                             :11.961
                                     Max.
                                                     Max.
# bar plots
categorical_var_long <- categorical_var %>%
  tidyr::pivot_longer(cols = everything(), names_to = "variable", values_to = "value")
ggplot(categorical_var_long, aes(x = value)) +
  geom_bar(fill = "salmon") +
  facet_wrap(~variable, scales = "free", ncol = 3) +
  theme_minimal() +
  labs(title = "Bar Plots of Categorical Variables", x = "Category", y = "Count")
```

#### Bar Plots of Categorical Variables



According to the box plot for continuous variables:

- Age, BMI, and SBP appear reasonably normally distributed, with expected ranges for an adult population;
   LDL cholesterol and time since vaccination show a wider range, right-skewness and some outliers, which may impact linear models.
- log\_antibody (response) appears fairly symmetrical, which supports its use as a continuous response in linear or GAM models.
- Correlations and non-linear trends should be assessed in the next step to guide model form.

According to the bar plot for categorical variables:

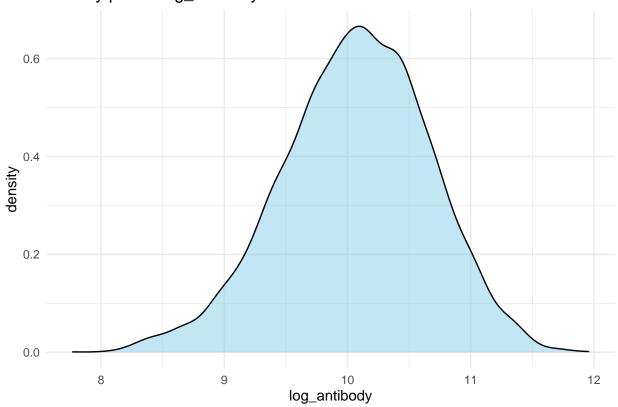
- Gender is fairly balanced between Female and Male;
- Race is skewed, with a majority of participants identifying as White (Category 1). Other racial/ethnic groups are underrepresented;
- Smoking status shows that the majority are never smokers (Category 0), with fewer current and former smokers;
- A large proportion of participants do not have diabetes;
- A moderate split exists for hypertension, which may contribute meaningfully to clinical outcome variation
- Demographically, the population is balanced by gender but skewed by race and smoking status.

Overall, we believe the response variable <code>log\_antibody</code> is well-behaved, and further correlation analysis(eg. bivariate) is needed.

```
ggplot(dat1, aes(x = log_antibody)) +
  geom_density(fill = "skyblue", alpha = 0.5) +
```

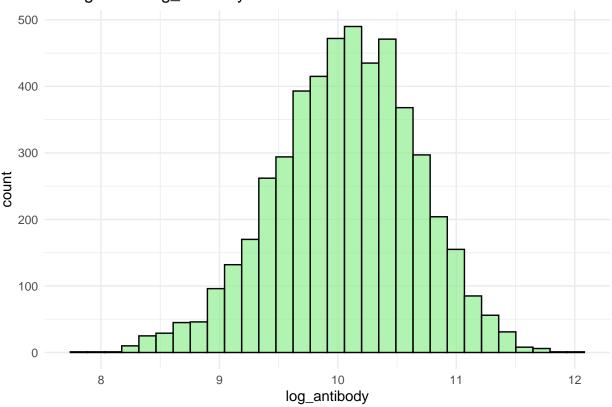
```
ggtitle("Density plot of log_antibody") +
xlab("log_antibody") +
theme_minimal()
```

# Density plot of log\_antibody



```
ggplot(dat1, aes(x = log_antibody)) +
  geom_histogram(bins = 30, fill = "lightgreen", color = "black", alpha = 0.7) +
  ggtitle("Histogram of log_antibody") +
  xlab("log_antibody") +
  theme_minimal()
```

## Histogram of log\_antibody

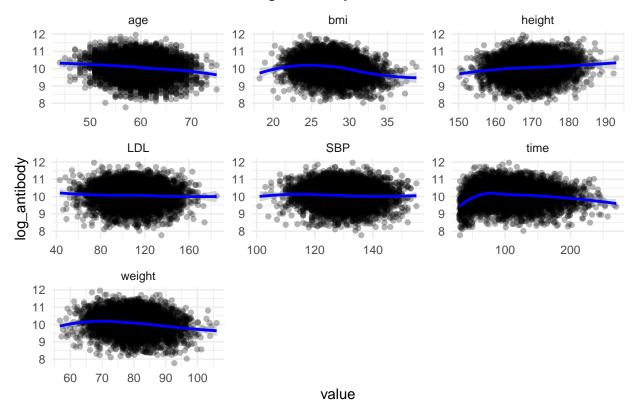


```
# continuous variable
continuous_var_long <- dat1 %>%
    select(age, height, weight, bmi, SBP, LDL, time, log_antibody) %>%
    tidyr::pivot_longer(cols = -log_antibody, names_to = "variable", values_to = "value")

# Scatterplots with smoothing lines
ggplot(continuous_var_long, aes(x = value, y = log_antibody)) +
    geom_point(alpha = 0.3) +
    geom_smooth(method = "loess", color = "blue") +
    facet_wrap(~variable, scales = "free", ncol = 3) +
    theme_minimal() +
    labs(title = "Continuous Predictors vs. log_antibody")
```

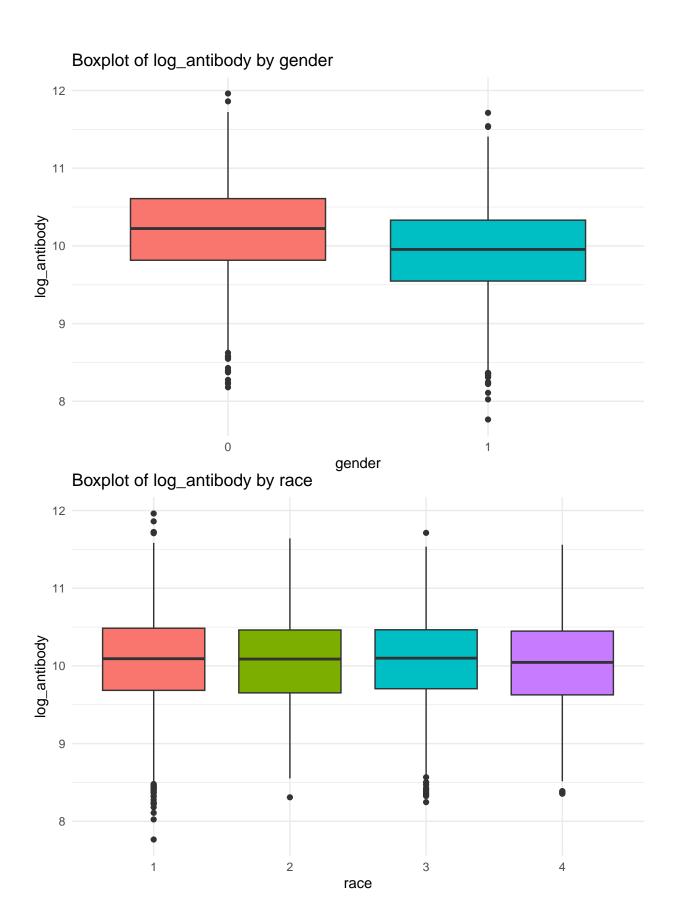
## `geom\_smooth()` using formula = 'y ~ x'

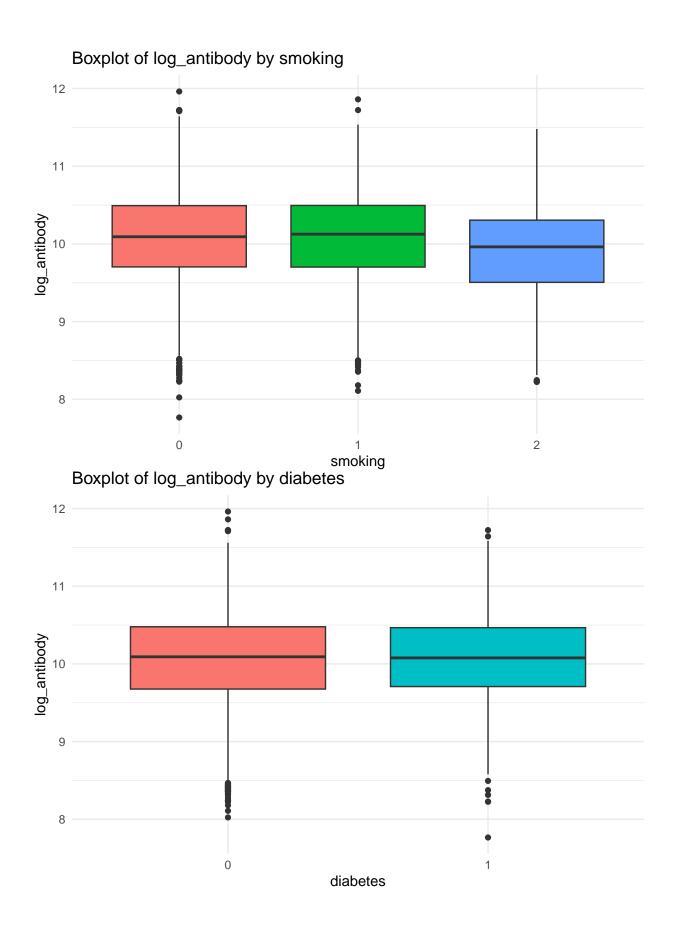
### Continuous Predictors vs. log\_antibody

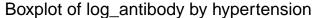


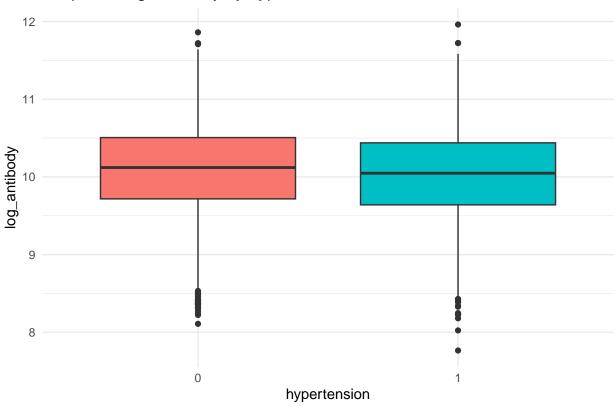
Using LOESS method, we observe linearity between predictors and the response. The plot shows that bmi, time, and weight has clear non linear trend against resopnse log\_antibody, indicating potential need to use GAM or non linear model.

```
# categorical data
categorical_name <- c("gender", "race", "smoking", "diabetes", "hypertension")</pre>
dat1[categorical_name] <- lapply(dat1[categorical_name], factor)</pre>
for (name in categorical_name) {
  p <- ggplot(dat1, aes_string(x = name, y = "log_antibody", fill = name)) +</pre>
    geom_boxplot() +
    ggtitle(paste("Boxplot of log_antibody by", name)) +
    theme_minimal() +
    theme(legend.position = "none")
  print(p)
}
## Warning: `aes_string()` was deprecated in ggplot2 3.0.0.
## i Please use tidy evaluation idioms with `aes()`.
## i See also `vignette("ggplot2-in-packages")` for more information.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```





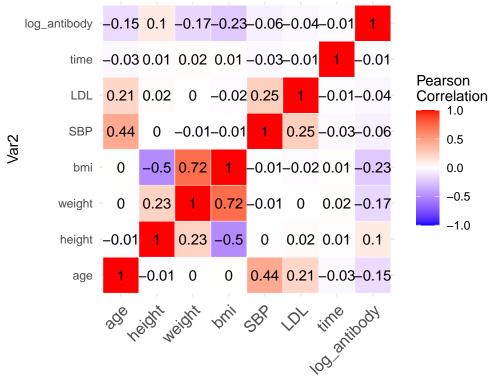




#### Correlation Analysis

```
continous_name <- c("age", "height", "weight", "bmi", "SBP", "LDL", "time", "log_antibody")
dat_cont <- dat1[ , continous_name]</pre>
# coefficient matrix
cor_matrix <- cor(dat_cont, use = "complete.obs", method = "pearson")</pre>
print(round(cor_matrix, 2))
##
                 age height weight
                                    bmi
                                           SBP
                                                 LDL time log_antibody
## age
                1.00 -0.01
                              0.00 0.00 0.44 0.21 -0.03
                                                                  -0.15
## height
               -0.01
                       1.00
                              0.23 -0.50 0.00 0.02 0.01
                                                                   0.10
                0.00
                       0.23
                              1.00 0.72 -0.01 0.00 0.02
                                                                   -0.17
## weight
## bmi
                0.00 -0.50
                              0.72 1.00 -0.01 -0.02 0.01
                                                                   -0.23
## SBP
                0.44
                       0.00 -0.01 -0.01 1.00 0.25 -0.03
                                                                  -0.06
## LDL
                0.21
                       0.02
                              0.00 -0.02 0.25 1.00 -0.01
                                                                   -0.04
               -0.03
                              0.02 0.01 -0.03 -0.01 1.00
                                                                   -0.01
## time
                       0.01
## log_antibody -0.15
                             -0.17 -0.23 -0.06 -0.04 -0.01
                                                                   1.00
                       0.10
cor_melt <- melt(cor_matrix)</pre>
ggplot(cor_melt, aes(Var1, Var2, fill = value)) +
  geom_tile(color = "white") +
  scale_fill_gradient2(low = "blue", high = "red", mid = "white",
                      midpoint = 0, limit = c(-1, 1), space = "Lab",
                      name = "Pearson\nCorrelation") +
```

#### Correlation Heatmap of Continuous Variables



#### Var1

```
for (i in 1:(length(categorical_name)-1)) {
  for (j in (i+1):length(categorical_name)) {
    var1 <- categorical_name[i]</pre>
    var2 <- categorical_name[j]</pre>
    cat("\nContingency Table:", var1, "vs", var2, "\n")
    tab <- table(dat1[[var1]], dat1[[var2]])</pre>
    print(tab)
    cat("Chi-squared Test:\n")
    print(chisq.test(tab))
 }
}
##
## Contingency Table: gender vs race
##
##
          1
                     3
                          4
                  542
##
     0 1642 151
                        238
     1 1579 127 494
## Chi-squared Test:
##
```

```
## Pearson's Chi-squared test
##
## data: tab
## X-squared = 1.5264, df = 3, p-value = 0.6762
##
## Contingency Table: gender vs smoking
##
##
          0
               1
##
     0 1554 759
                  260
    1 1456 745 226
## Chi-squared Test:
##
## Pearson's Chi-squared test
##
## data: tab
## X-squared = 1.4376, df = 2, p-value = 0.4873
##
##
## Contingency Table: gender vs diabetes
##
##
##
    0 2167 406
    1 2061 366
##
## Chi-squared Test:
## Pearson's Chi-squared test with Yates' continuity correction
##
## data: tab
## X-squared = 0.41526, df = 1, p-value = 0.5193
##
##
## Contingency Table: gender vs hypertension
##
          0
##
    0 1419 1154
##
##
     1 1283 1144
## Chi-squared Test:
##
## Pearson's Chi-squared test with Yates' continuity correction
##
## data: tab
## X-squared = 2.5366, df = 1, p-value = 0.1112
##
##
## Contingency Table: race vs smoking
##
##
                    2
          0
               1
     1 1981 956
##
                  284
     2 149
##
             89
                   40
##
     3 605 325
                  106
     4 275 134
##
## Chi-squared Test:
##
```

```
## Pearson's Chi-squared test
##
## data: tab
## X-squared = 16.7, df = 6, p-value = 0.01045
##
## Contingency Table: race vs diabetes
##
##
         0
              1
##
     1 2725
            496
##
    2 240
             38
##
    3 879 157
    4 384
##
             81
## Chi-squared Test:
##
## Pearson's Chi-squared test
##
## data: tab
## X-squared = 2.132, df = 3, p-value = 0.5455
##
## Contingency Table: race vs hypertension
##
##
         0
    1 1741 1480
##
##
    2 152 126
##
    3 566 470
##
    4 243 222
## Chi-squared Test:
##
## Pearson's Chi-squared test
##
## data: tab
## X-squared = 0.78675, df = 3, p-value = 0.8526
##
##
## Contingency Table: smoking vs diabetes
##
##
         0
##
    0 2556 454
##
    1 1259 245
##
    2 413
            73
## Chi-squared Test:
##
## Pearson's Chi-squared test
##
## data: tab
## X-squared = 1.1913, df = 2, p-value = 0.5512
##
##
## Contingency Table: smoking vs hypertension
##
##
         0
## 0 1621 1389
```

```
##
     1 814 690
##
    2 267 219
## Chi-squared Test:
##
## Pearson's Chi-squared test
##
## data: tab
## X-squared = 0.204, df = 2, p-value = 0.903
##
##
## Contingency Table: diabetes vs hypertension
##
##
         0
               1
##
    0 2284 1944
##
    1 418 354
## Chi-squared Test:
##
## Pearson's Chi-squared test with Yates' continuity correction
##
## data: tab
## X-squared = 0.00059731, df = 1, p-value = 0.9805
Model Selection
# VIF
lm_full <- lm(log_antibody ~ age + gender + smoking + height + weight + bmi +</pre>
                diabetes + hypertension + SBP + LDL + time, data = dat1)
vif(lm_full)
                      GVIF Df GVIF^(1/(2*Df))
##
## age
                 1.258104 1
                                    1.121652
                                    1.001493
## gender
                1.002988 1
## smoking
                 1.002682 2
                                    1.000670
               107.111548 1
## height
                                    10.349471
## weight
               169.112707 1
                                    13.004334
## bmi
               213.764468 1
                                  14.620686
               1.001898 1
                                    1.000949
## diabetes
## hypertension 2.791341 1
                                    1.670731
## SBP
                 3.070211 1
                                    1.752202
## LDL
                 1.085268 1
                                    1.041762
                 1.002242 1
                                    1.001120
x <- model.matrix(log_antibody ~ ., data = dat1)[, -1]</pre>
y <- dat1$log_antibody
lasso_cv <- cv.glmnet(x, y, alpha = 1, standardize = TRUE)</pre>
lasso cv$lambda.min
## [1] 2.082471e-05
lasso_coef <- coef(lasso_cv, s = "lambda.1se")</pre>
print(lasso_coef)
```

```
## 16 x 1 sparse Matrix of class "dgCMatrix"
##
## (Intercept)
                  11.92665650
                  -0.01217839
## age
## gender1
                  -0.22108381
## race2
## race3
## race4
## smoking1
## smoking2
                  -0.07608186
## height
## weight
## bmi
                  -0.03667170
## diabetes1
## hypertension1
## SBP
## LDL
## time
plot(lasso_cv)
                  15
                       15
                           15
                                15 15 14 12 9 8 7 5
                                                                            4 3 2
             15
     0.36
Mean-Squared Error
     0.34
     0.32
     0.30
                   -10
                                   -8
                                                   -6
                                                                                   -2
                                              Log(\lambda)
ridge_cv <- cv.glmnet(x, y, alpha = 0, standardize = TRUE)</pre>
ridge_cv$lambda.min
## [1] 0.01435366
ridge_coef <- coef(ridge_cv, s = "lambda.min")</pre>
print(ridge_coef)
```

## 16 x 1 sparse Matrix of class "dgCMatrix"

s1

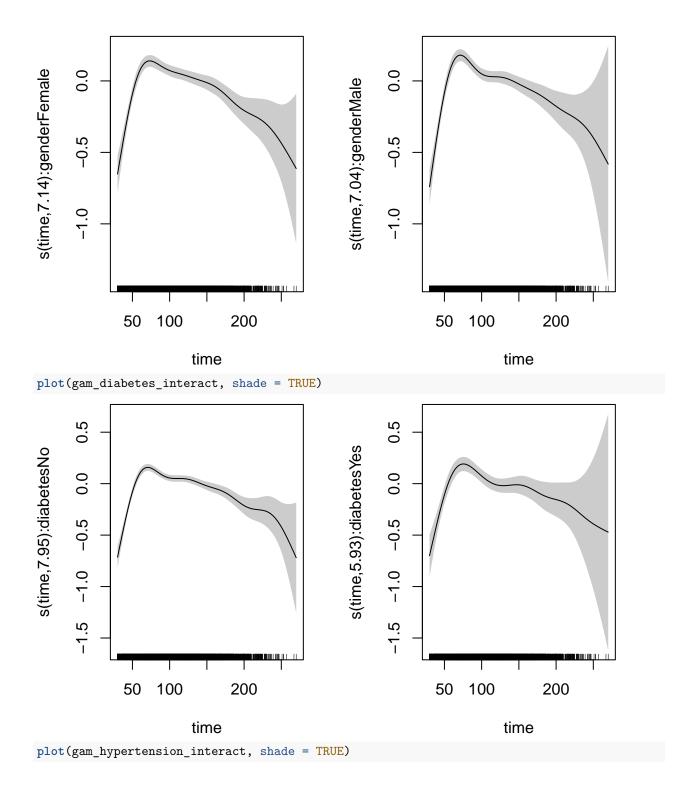
##

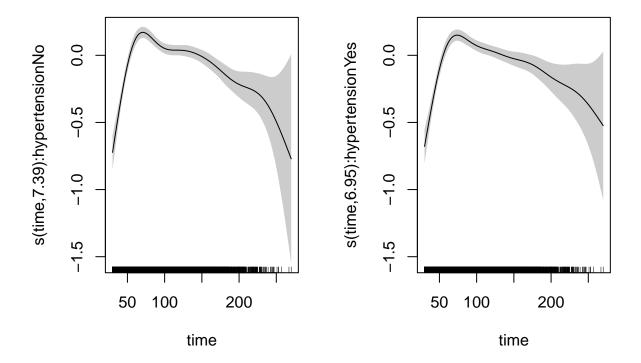
```
## (Intercept)
                12.7384560516
## age
                -0.0197679168
## gender1
               -0.2880871615
## race2
               -0.0038318346
## race3
                -0.0066654722
## race4
               -0.0417690245
## smoking1
               0.0242172365
               -0.1847225211
## smoking2
## height
                -0.0001949552
               -0.0009185524
## weight
## bmi
                -0.0473178893
## diabetes1
                0.0113069924
## hypertension1 -0.0166641972
## SBP
               0.0010847735
## LDL
               -0.0001614795
## time
                -0.0002807285
```

#### Interaction Analysis

```
dat1_ageGroup <- dat1 %>%
 mutate(age_group = ntile(age, 3)) %>%
 mutate(age_group = factor(age_group, labels = c("Young", "Middle", "Older"))) %>%
 mutate(
    gender = factor(gender, labels = c("Female", "Male")),
    diabetes = factor(diabetes, labels = c("No", "Yes")),
    hypertension = factor(hypertension, labels = c("No", "Yes"))
  )
library(mgcv)
gam_age_interact <- gam(</pre>
 log_antibody ~ s(time, by = age_group) + age_group + gender + bmi + SBP + LDL +
    race + smoking + diabetes + hypertension,
 data = dat1_ageGroup,
  method = "GCV.Cp"
gam_gender_interact <- gam(</pre>
 log antibody ~ s(time, by = gender) + gender + age + bmi + SBP + LDL +
   race + smoking + diabetes + hypertension,
 data = dat1 ageGroup,
 method = "GCV.Cp"
gam_diabetes_interact <- gam(</pre>
 log_antibody ~ s(time, by = diabetes) + diabetes + age + bmi + SBP + LDL +
    race + gender + smoking + hypertension,
 data = dat1_ageGroup,
 method = "GCV.Cp"
gam_hypertension_interact <- gam(</pre>
  log_antibody ~ s(time, by = hypertension) + diabetes + age + bmi + SBP + LDL +
   race + gender + smoking + hypertension,
```

```
data = dat1_ageGroup,
   method = "GCV.Cp"
)
# Plotting smooth terms
par(mfrow = c(1, 3))
plot(gam_age_interact, shade = TRUE)
                                                  0.5
                                                                                              0.5
     0.0
                                                  0.0
                                                                                              0.0
                                            s(time, 6.94):age_groupMiddle
s(time,6.32):age_groupYoung
                                                                                         s(time, 6.88):age_groupOlder
     -0.5
                                                  -0.5
                                                                                              -0.5
     -1.0
                                                  1.0
                                                                                              -1.0
     -1.5
                                                  -1.5
                            200
            50 100
                                                         50 100
                                                                        200
                                                                                                     50 100
                                                                                                                     200
                      time
                                                                  time
                                                                                                               time
par(mfrow = c(1, 2))
plot(gam_gender_interact, shade = TRUE)
```





## **Model Training**

```
ctrl1 <- trainControl(method = "cv", number = 10)</pre>
train_y <- dat1$log_antibody</pre>
train_x <- dat1[, -which(names(dat1) == "log_antibody")]</pre>
set.seed(2)
gam.fit <- train(train_x, train_y,</pre>
                  method = "gam",
                  # tuneGrid = data.frame(method = "GCV.Cp", select = c(TRUE,FALSE)),
                  trControl = ctrl1)
gam.fit$bestTune
     select method
       TRUE GCV.Cp
gam.fit$finalModel
##
## Family: gaussian
## Link function: identity
##
## .outcome ~ gender + diabetes + hypertension + smoking + race +
       s(age) + s(SBP) + s(LDL) + s(bmi) + s(time) + s(height) +
##
##
       s(weight)
## Estimated degrees of freedom:
## 0.991 0.000 0.000 4.179 7.892 1.234 0.000
## total = 23.3
```

```
##
## GCV score: 0.2786734
mars_grid <- expand.grid(degree = 1:3,</pre>
                           nprune = 2:15)
set.seed(2)
mars.fit <- train(train_x, train_y,</pre>
                    method = "earth",
                    tuneGrid = mars_grid,
                    trControl = ctrl1)
ggplot(mars.fit)
   0.58 -
   0.57 -
RMSE (Cross-Validation)
                                                                               Product Degree
   0.56 -
                                                                                   2
   0.55 -
  0.54 -
   0.53 -
                    4
                                                         12
                                      8
                                      #Terms
mars.fit$bestTune
     nprune degree
## 8
           9
coef(mars.fit$finalModel)
## (Intercept) h(27.8-bmi)
                                  h(time-57)
                                                h(57-time)
                                                                  gender1
                                                                              h(age-59)
## 10.847446930 -0.061997354 -0.002254182 -0.033529326 -0.296290451 -0.022957648
                      smoking2 h(bmi-23.7)
      h(59-age)
   0.016138468 -0.205126851 -0.084380175
resamp <- resamples(list(mars = mars.fit, gam = gam.fit))</pre>
summary(resamp)
```

##

```
## Call:
## summary.resamples(object = resamp)
## Models: mars, gam
## Number of resamples: 10
##
## MAE
##
             Min.
                    1st Qu.
                                Median
                                            Mean
                                                   3rd Qu.
## mars 0.4120189 0.4180233 0.4203065 0.4224208 0.4285348 0.4360995
                                                                         0
   gam 0.4127242 0.4190074 0.4202804 0.4224455 0.4273258 0.4352565
##
## RMSE
                                Median
##
             Min.
                    1st Qu.
                                            Mean
                                                   3rd Qu.
                                                                 Max. NA's
## mars 0.5066327 0.5230870 0.5316602 0.5282995 0.5354905 0.5457286
## gam 0.5091877 0.5223781 0.5306669 0.5279212 0.5336806 0.5451253
                                                                         0
##
## Rsquared
##
             Min.
                    1st Qu.
                                Median
                                            Mean
                                                   3rd Qu.
                                                                 Max. NA's
## mars 0.1766328 0.1941155 0.2028183 0.2159220 0.2369173 0.2730827
                                                                         0
## gam 0.1795042 0.1955026 0.2071224 0.2170568 0.2376473 0.2735385
bwplot(resamp, metric = "RMSE")
mars
gam
                               0.52
                                                 0.53
              0.51
                                                                  0.54
                                         RMSE
mars.pred <- predict(mars.fit, newdata = dat2)</pre>
# test RMSE
mars_test_rmse = sqrt(mean((mars.pred - dat2[, "log_antibody"])^2))
mars_test_rmse
```

```
gam.pred <- predict(gam.fit, newdata = dat2)
# test RMSE
gam_test_rmse = sqrt(mean((gam.pred - dat2[, "log_antibody"])^2))
gam_test_rmse</pre>
```

## [1] 0.5700836