

ds2_homework1

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We predict the sale price of a house based on various characteristics. The training data are in `housing_train.csv`, and the test data are in `housing_test.csv`. The response is in the column "Sale price", and other variables can be used as predictors. The variable definitions can be found in `dictionary.txt`.

```
house_train <- read_csv("./data/housing_training.csv")

## Rows: 1440 Columns: 26
## -- Column specification -----
## Delimiter: ","
## chr (4): Overall_Qual, Kitchen_Qual, Fireplace_Qu, Exter_Qual
## dbl (22): Gr_Liv_Area, First_Flr_SF, Second_Flr_SF, Total_Bsmt_SF, Low_Qual_...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
house_test <- read_csv("./data/housing_test.csv")

## Rows: 959 Columns: 26
## -- Column specification -----
## Delimiter: ","
## chr (4): Overall_Qual, Kitchen_Qual, Fireplace_Qu, Exter_Qual
## dbl (22): Gr_Liv_Area, First_Flr_SF, Second_Flr_SF, Total_Bsmt_SF, Low_Qual_...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
house_train <- na.omit(house_train) %>%
  mutate(across(where(is.character), as.factor))

house_test <- na.omit(house_test) %>%
  mutate(across(where(is.character), as.factor))
# str(house_train)
# str(house_test)
```

(a)

```
# cross validation setup
x <- model.matrix(Sale_Price ~ . -1, house_train)
y <- house_train$Sale_Price

set.seed(2025)
# cv.lasso <- cv.glmnet(x, y,
#                       alpha = 1)
# print(max(cv.lasso$lambda)) # 54596.94
```

```

# print(min(cv.lasso$lambda)) # 38.51706

cv.lasso <- cv.glmnet(x, y,
                      alpha = 1,
                      lambda = exp(seq(9, 3, length = 100)))

predict(cv.lasso, s = cv.lasso$lambda.min, type = "coefficients")

## 41 x 1 sparse Matrix of class "dgCMatrix"
##                               s1
## (Intercept)                  -4.855623e+06
## Gr_Liv_Area                   6.555839e+01
## First_Flr_SF                  7.966301e-01
## Second_Flr_SF                 .
## Total_Bsmt_SF                 3.538105e+01
## Low_Qual_Fin_SF              -4.112467e+01
## Wood_Deck_SF                 1.170219e+01
## Open_Porch_SF                1.557176e+01
## Bsmt_Unf_SF                  -2.089010e+01
## Mas_Vnr_Area                 1.080864e+01
## Garage_Cars                   4.113346e+03
## Garage_Area                   8.078203e+00
## Year_Built                    3.237171e+02
## TotRms_AbvGrd                -3.656728e+03
## Full_Bath                    -3.932055e+03
## Overall_QualAbove_Average     .
## Overall_QualAverage          -4.885004e+03
## Overall_QualBelow_Average    -1.252142e+04
## Overall_QualExcellent        7.498681e+04
## Overall_QualFair             -1.082769e+04
## Overall_QualGood             1.214899e+04
## Overall_QualVery_Excellent   1.347759e+05
## Overall_QualVery_Good        3.790737e+04
## Kitchen_QualFair            -2.521129e+04
## Kitchen_QualGood            -1.753427e+04
## Kitchen_QualTypical         -2.562010e+04
## Fireplaces                   1.069254e+04
## Fireplace_QuFair            -7.699186e+03
## Fireplace_QuGood            .
## Fireplace_QuNo_Fireplace     1.681203e+03
## Fireplace_QuPoor            -5.673994e+03
## Fireplace_QuTypical         -7.011896e+03
## Exter_QualFair              -3.416912e+04
## Exter_QualGood              -1.586478e+04
## Exter_QualTypical           -2.029687e+04
## Lot_Frontage                 1.001554e+02
## Lot_Area                     6.044026e-01
## Longitude                   -3.326185e+04
## Latitude                     5.577739e+04
## Misc_Val                     8.455682e-01
## Year_Sold                    -5.760325e+02

X_test <- model.matrix(Sale_Price ~ . - 1, data = house_test)
y_test <- house_test$Sale_Price

```

```
# Get predictions using the best lambda
predictions_min <- predict(cv.lasso, newx = X_test, s = cv.lasso$lambda.min)

# Calculate Mean Squared Error (MSE)
test_rmse_min <- sqrt(mean((predictions_min - y_test)^2))
```

Note: We predefined a range for tuning parameter based on the maximum and minimum lambda of the model with default(data-driven) tuning parameter.

The selected tuning parameter λ is 52.9684774, and the test error(RMSE) is 2.1005705×10^4 .

```
# get prediction w/ lambda + 1se
predictions_1se <- predict(cv.lasso, newx = X_test, s = cv.lasso$lambda.1se)

# get number of predictors
coef_1se_lasso <- predict(cv.lasso, s = cv.lasso$lambda.1se, type = "coefficients")
num_predictors_1se_lasso <- sum(coef_1se_lasso[-1] != 0)

# Calculate Mean Squared Error (MSE)
test_rmse_1se <- sqrt(mean((predictions_1se - y_test)^2))
```

When applied 1SE rule(i.e. minimum lambda + 1 standard error), the model has number of predictors = 29 ; test error(RMSE) = 2.052481×10^4 , which is acceptable.

(b)

```
ctrl1 <- trainControl(method = "cv", number = 10, selectionFunction = "best")
ctrl2 <- trainControl(method = "cv", number = 10, selectionFunction = "oneSE")

set.seed(2025)

enet.fit_caret_bestPar <- train(Sale_Price ~ ., data = house_train,
                              method = "glmnet",
                              tuneGrid = expand.grid(alpha = seq(0, 1, length = 21),
                                                    lambda = exp(seq(9, 3, length = 100))),
                              trControl = ctrl1)

enet.fit_caret_1sePar <- train(Sale_Price ~ ., data = house_train,
                              method = "glmnet",
                              tuneGrid = expand.grid(alpha = seq(0, 1, length = 21),
                                                    lambda = exp(seq(9, 3, length = 100))),
                              trControl = ctrl2)

# cat("best lambda by caret:", enet.fit_glm$lambda.min) # 469.4281
# cat("1se lambda by caret:", enet.fit_caret$bestTune$lambda) # 529.9206

enet_pred_best <- predict(enet.fit_caret_bestPar, newdata = house_test,
                          s = enet.fit_caret_bestPar$bestTune)
enet_pred_1se <- predict(enet.fit_caret_1sePar, newdata = house_test,
                         s = enet.fit_caret_1sePar$bestTune)

test_rmse_enet_min <- sqrt(mean((y_test - enet_pred_best)^2))
```

```
test_rmse_enet_1se <- sqrt(mean((y_test - enet_pred_1se)^2))
```

The selected tuning parameter are $\lambda = 635.5847584$, $\alpha = 0.05$, and its RMSE is 2.0929442×10^4 .

1SE Rule can be applied to λ only, not to α . We need to use `caret` elastic net model to find the best tune α and then use `glmnet` to apply the 1SE Rule while keeping α fixed.

When 1SE Rule applied, the tuning parameters are $\alpha = 0$, $\lambda = 8103.0839276$, and test RMSE is 2.0687914×10^4 . This result suggests that cross validation process might find the most optimal model is ridge regression for given dataset, which deserves further investigation on the reason behind it.

(c)

```
set.seed(2025)
pls.fit_caret <- train(Sale_Price ~ ., data = house_train,
                      method = "pls",
                      tuneGrid = data.frame(ncomp = 1:29),
                      trControl = ctrl1,
                      preprocess = c("center", "scale"))

best_ncomp_caret_pls <- pls.fit_caret$bestTune$ncomp

pred_pls <- predict(pls.fit_caret, newdata = house_test)
test_rmse_pls <- sqrt(mean((y_test - pred_pls)^2))
```

The test RMSE using PLS model is 2.124327×10^4 , and there are 11 components selected by the best PLS model.

(d)

```
error_table <- data.frame(
  Model = c("LASSO", "Elastic Net", "PLS"),
  Test_RMSE = c(test_rmse_min, test_rmse_enet_min, test_rmse_pls)
)

print(error_table)
```

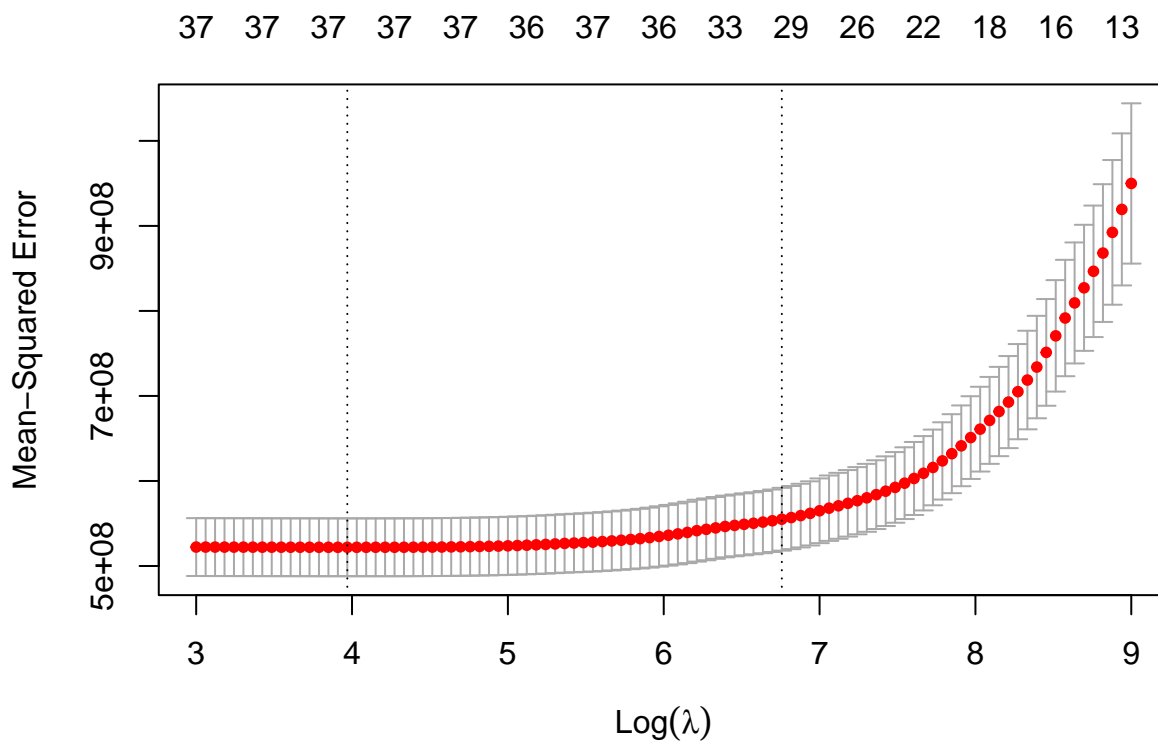
```
##           Model Test_RMSE
## 1          LASSO  21005.70
## 2 Elastic Net  20929.44
## 3           PLS  21243.27
```

(e)

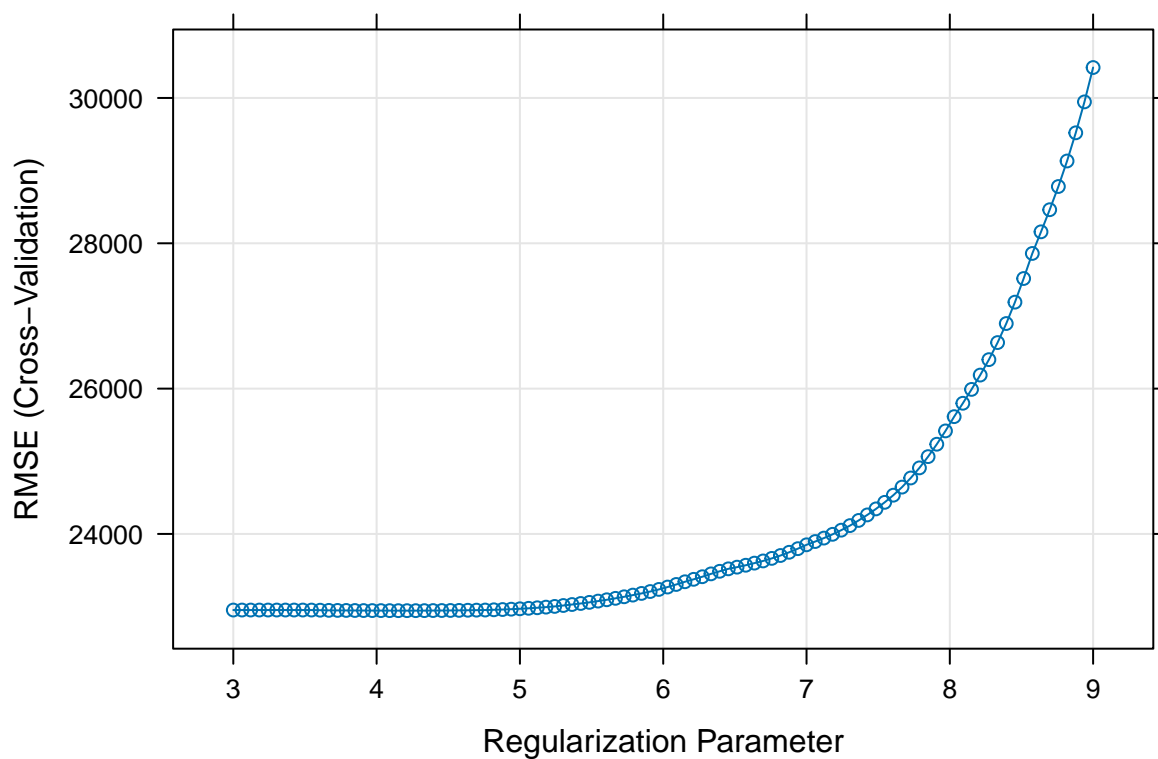
```
set.seed(2025)
lasso.fit_caret <- train(Sale_Price ~ ., data = house_train,
                        method = "glmnet",
                        tuneGrid = expand.grid(alpha = 1,
                                                lambda = exp(seq(9, 3, length = 100))),
                        trControl = ctrl1)

lambda_glm <- cv.lasso$lambda.min
```

```
lambda_caret <- lasso.fit_caret$bestTune$lambda
plot(cv.lasso)
```



```
plot(lasso.fit_caret, xTrans = log)
```



By `glmnet`, the tuning parameter $\lambda = 52.9684774$; by `caret`, $\lambda = 67.499569$. Even though we specify

using `glmnet` method in `caret` model fitting, there is still difference in selection of “best” λ . However, we consider this as an acceptable difference especially when considering within the exponential grid ($\log(\lambda_{glm}) = 3.969697$; $\log(\lambda_{caret}) = 4.2121212$). The source of such difference might be from how each function sets up cross-validation or potentially different default error metrics or tie-breaking decisions.

```
set.seed(2025)
resamp <- resamples(list(
  Lasso = lasso.fit_caret,
  ElasticNet = enet.fit_caret_bestPar,
  PartialLeastSq = pls.fit_caret
))

summary(resamp)
```

```
##
## Call:
## summary.resamples(object = resamp)
##
## Models: Lasso, ElasticNet, PartialLeastSq
## Number of resamples: 10
##
## MAE
##           Min. 1st Qu.  Median    Mean 3rd Qu.    Max. NA's
## Lasso       14741.81 15558.94 16287.66 16679.12 17493.05 19165.85    0
## ElasticNet   14793.66 15493.40 16258.31 16648.24 17495.94 19152.10    0
## PartialLeastSq 14848.76 15656.74 16411.28 16743.46 17517.21 19194.05    0
##
## RMSE
##           Min. 1st Qu.  Median    Mean 3rd Qu.    Max. NA's
## Lasso       19791.29 21882.13 22646.63 22944.34 23780.51 27025.10    0
## ElasticNet   19835.20 21896.69 22623.14 22939.56 23780.08 27076.72    0
## PartialLeastSq 19837.01 21953.53 22716.43 22945.08 23772.46 26935.47    0
##
## Rsquared
##           Min. 1st Qu.  Median    Mean 3rd Qu.    Max. NA's
## Lasso       0.8774196 0.8893621 0.9002852 0.9021467 0.9163729 0.9249267    0
## ElasticNet   0.8778045 0.8894517 0.9004570 0.9021908 0.9159685 0.9255766    0
## PartialLeastSq 0.8774820 0.8893300 0.8997448 0.9020588 0.9164506 0.9243377    0
```

In last question, we found the Elastic Net slightly outperform the other two model by measuring RMSE. Since we just fit the lasso model w/ `caret`, we can directly compare how well three model predicts on test data. And the result aligns w/ our belief that Elastic Net still perform better than other two models overall.

```
set.seed(2025)
train_id_list <- lasso.fit_caret$control$index

dat_dummy <- data.frame(Sale_Price = y, x)
M <- 10
lambda.grid <- exp(seq(9, 3, length = 100))
rmse <- rmse_caret <- matrix(NA, ncol = 100, nrow = M)

for (m in 1:M)
{
  tsdata <- dat_dummy[train_id_list[[m]],]
  vsdata <- dat_dummy[-train_id_list[[m]],]
```

```

x1 <- as.matrix(tsdata[,-1])
y1 <- tsdata[,1]
x2 <- as.matrix(vsdata[,-1])
y2 <- vsdata[,1]

fit <- glmnet(x1, y1, alpha = 1,
              lambda = lambda.grid)

# caret implementation did not specify lambda
# the default grid of lambda is different from lambda.grid
fit_caret <- glmnet(x1, y1, alpha = 1)

pred <- predict(fit, newx = x2, s = lambda.grid)
pred_caret <- predict(fit_caret, newx = x2, s = lambda.grid)

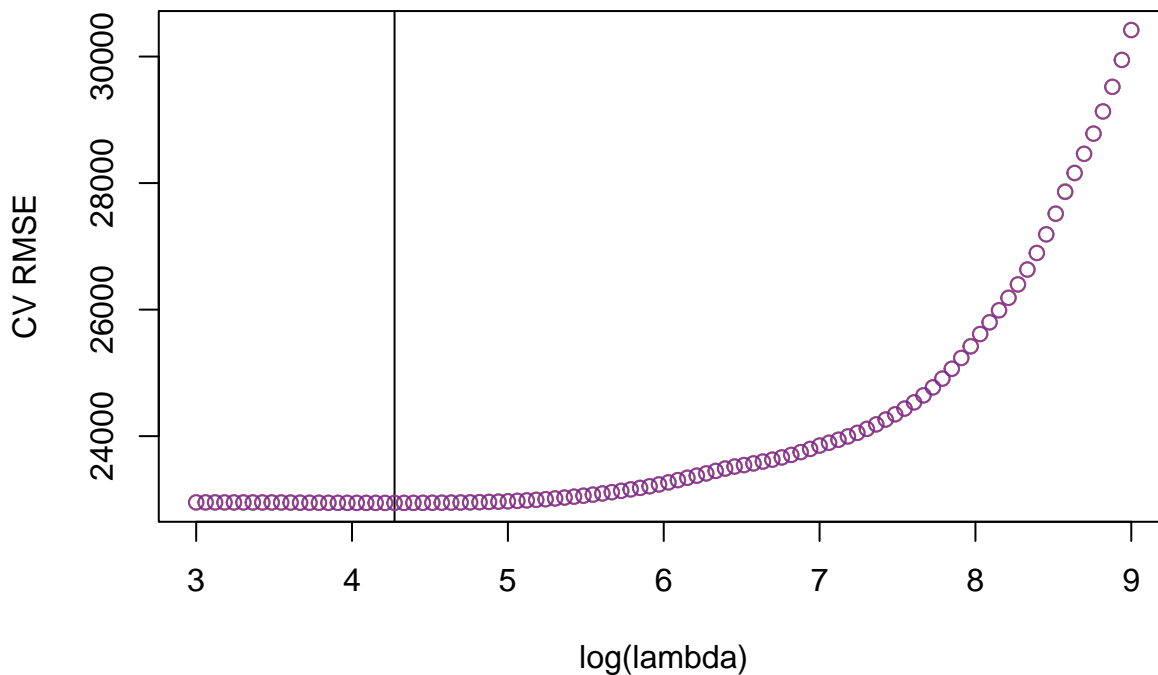
rmse[m,] <- sqrt(colMeans((y2 - pred)^2))
rmse_caret[m,] <- sqrt(colMeans((y2 - pred_caret)^2))
}

# curve from glmnet (correct)
plot(log(lambda.grid), colMeans(rmse), col = 3, xlab = "log(lambda)", ylab = "CV RMSE")
abline(v = log(lambda.grid[which.min(colMeans(rmse))]))

# caret results
points(log(lasso.fit_caret$results$lambda), lasso.fit_caret$results$RMSE, col = 2)

# try to reproduce caret results from scratch
points(log(lambda.grid), colMeans(rmse_caret), col = rgb(0,0,1,alpha = 0.3))

```



```

# selected lambda
lambda.grid[which.min(colMeans(rmse))]

```

```
## [1] 71.71696
```

```
# the corresponding CV RMSE  
min(colMeans(rmse))
```

```
## [1] 22945.54
```

As being aware of a small implementation error of caret model, we adjust the algorithm of caret. And the selection of lambda is still in an acceptable range comparing to glmnet.