ds2_homework1

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We predict the sale price of a house based on various characteristics. The training data are in `housing train.csv`, and the test data are in `housing test.csv`. The response is in the column "Sale price", and other variables can be used as predictors. The variable definitions can be found in `dictionary.txt`.

```
house_train <- read_csv("./data/housing_training.csv")
```

```
## Rows: 1440 Columns: 26
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## chr  (4): Overall_Qual, Kitchen_Qual, Fireplace_Qu, Exter_Qual
## dbl (22): Gr_Liv_Area, First_Flr_SF, Second_Flr_SF, Total_Bsmt_SF, Low_Qual_...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
house_test <- read_csv("./data/housing_test.csv")
```

```
## Rows: 959 Columns: 26
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## chr  (4): Overall_Qual, Kitchen_Qual, Fireplace_Qu, Exter_Qual
## dbl (22): Gr_Liv_Area, First_Flr_SF, Second_Flr_SF, Total_Bsmt_SF, Low_Qual_...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
house_train <- na.omit(house_train) %>%
  mutate(across(where(is.character), as.factor))

house_test <- na.omit(house_test) %>%
  mutate(across(where(is.character), as.factor))
# str(house_train)
# str(house_test)
```

(a)

```
# cross validation setup
x <- model.matrix(Sale_Price ~ . -1, house_train)
y <- house_train$Sale_Price

set.seed(2025)
# cv.lasso <- cv.glmnet(x, y,
#                        alpha = 1)
# print(max(cv.lasso$lambda)) # 54596.94
```

```r
# print(min(cv.lasso$lambda)) # 38.51706

cv.lasso <- cv.glmnet(x, y,
                      alpha = 1,
                      lambda = exp(seq(9, 3, length = 100)))

predict(cv.lasso, s = cv.lasso$lambda.min, type = "coefficients")
```

```
## 41 x 1 sparse Matrix of class "dgCMatrix"
##                                        s1
## (Intercept)                 -4.855623e+06
## Gr_Liv_Area                  6.555839e+01
## First_Flr_SF                 7.966301e-01
## Second_Flr_SF                .
## Total_Bsmt_SF                3.538105e+01
## Low_Qual_Fin_SF             -4.112467e+01
## Wood_Deck_SF                 1.170219e+01
## Open_Porch_SF                1.557176e+01
## Bsmt_Unf_SF                 -2.089010e+01
## Mas_Vnr_Area                 1.080864e+01
## Garage_Cars                  4.113346e+03
## Garage_Area                  8.078203e+00
## Year_Built                   3.237171e+02
## TotRms_AbvGrd               -3.656728e+03
## Full_Bath                   -3.932055e+03
## Overall_QualAbove_Average    .
## Overall_QualAverage         -4.885004e+03
## Overall_QualBelow_Average   -1.252142e+04
## Overall_QualExcellent        7.498681e+04
## Overall_QualFair            -1.082769e+04
## Overall_QualGood             1.214899e+04
## Overall_QualVery_Excellent   1.347759e+05
## Overall_QualVery_Good        3.790737e+04
## Kitchen_QualFair            -2.521129e+04
## Kitchen_QualGood            -1.753427e+04
## Kitchen_QualTypical         -2.562010e+04
## Fireplaces                   1.069254e+04
## Fireplace_QuFair            -7.699186e+03
## Fireplace_QuGood             .
## Fireplace_QuNo_Fireplace     1.681203e+03
## Fireplace_QuPoor            -5.673994e+03
## Fireplace_QuTypical         -7.011896e+03
## Exter_QualFair              -3.416912e+04
## Exter_QualGood              -1.586478e+04
## Exter_QualTypical           -2.029687e+04
## Lot_Frontage                 1.001554e+02
## Lot_Area                     6.044026e-01
## Longitude                   -3.326185e+04
## Latitude                     5.577739e+04
## Misc_Val                     8.455682e-01
## Year_Sold                   -5.760325e+02
```

```r
X_test <- model.matrix(Sale_Price ~ . - 1, data = house_test)
y_test <- house_test$Sale_Price
```

```r
# Get predictions using the best lambda
predictions_min <- predict(cv.lasso, newx = X_test, s = cv.lasso$lambda.min)

# Calculate Mean Squared Error (MSE)
test_rmse_min <- sqrt(mean((predictions_min - y_test)^2))
```

Note: We predefined a range for tuning parameter based on the maximum and minimum lambda of the model with default(data-driven) tuning parameter.

The selected tuning parameter λ is 52.9684774, and the test error(RMSE) is 2.1005705×10^4.

```r
# get prediction w/ lambda + 1se
predictions_1se <- predict(cv.lasso, newx = X_test, s = cv.lasso$lambda.1se)

# get number of predictors
coef_1se_lasso <- predict(cv.lasso, s = cv.lasso$lambda.1se, type = "coefficients")
num_predictors_1se_lasso <- sum(coef_1se_lasso[-1] != 0)

# Calculate Mean Squared Error (MSE)
test_rmse_1se <- sqrt(mean((predictions_1se - y_test)^2))
```

When applied 1SE rule(i.e. minimum lambda + 1 standard error), the model has number of predictors = 29 ; test error(RMSE) = 2.052481×10^4, which is acceptable.

(b)

```r
ctrl1 <- trainControl(method = "cv", number = 10, selectionFunction = "best")
ctrl2 <- trainControl(method = "cv", number = 10, selectionFunction = "oneSE")

set.seed(2025)

enet.fit_caret_bestPar <- train(Sale_Price ~ ., data = house_train,
                           method = "glmnet",
                           tuneGrid = expand.grid(alpha = seq(0, 1, length = 21),
                                                  lambda = exp(seq(9, 3, length = 100))),
                           trControl = ctrl1)



enet.fit_caret_1sePar <- train(Sale_Price ~ ., data = house_train,
                           method = "glmnet",
                           tuneGrid = expand.grid(alpha = seq(0, 1, length = 21),
                                                  lambda = exp(seq(9, 3, length = 100))),
                           trControl = ctrl2)

# cat("best lambda by caret:", enet.fit_glm$lambda.min) # 469.4281
# cat("1se lambda by caret:", enet.fit_caret$bestTune$lambda) # 529.9206

enet_pred_best <- predict(enet.fit_caret_bestPar, newdata = house_test,
                          s = enet.fit_caret_bestPar$bestTune)
enet_pred_1se <- predict(enet.fit_caret_1sePar, newdata = house_test,
                         s = enet.fit_caret_1sePar$bestTune)

test_rmse_enet_min <- sqrt(mean((y_test - enet_pred_best)^2))
```

```r
test_rmse_enet_1se <- sqrt(mean((y_test - enet_pred_1se)^2))
```

The selected tuning parameter are $\lambda = 635.5847584$, $\alpha = 0.05$, and its RMSE is 2.0929442×10^4.

1SE Rule can be applied to λ only, not to α. We need to use `caret` elastic net model to find the best tune α and then use `glmnet` to apply the 1SE Rule while keeping α fixed.

When 1SE Rule applied, the tuning parameters are $\alpha = 0$, $\lambda = 8103.0839276$, and test RMSE is 2.0687914×10^4. This result suggests that cross validation process might find the most optimal model is ridge regression for given dataset, which deserves further investigation on the reason behind it.

(c)

```r
set.seed(2025)
pls.fit_caret <- train(Sale_Price ~ ., data = house_train,
                   method = "pls",
                   tuneGrid = data.frame(ncomp = 1:29),
                   trControl = ctrl1,
                   preProcess = c("center", "scale"))


best_ncomp_caret_pls <- pls.fit_caret$bestTune$ncomp

pred_pls <- predict(pls.fit_caret, newdata = house_test)
test_rmse_pls <- sqrt(mean((y_test - pred_pls)^2))
```

The test RMSE using PLS model is 2.124327×10^4, and there are 11 components selected by the best PLS model.

(d)

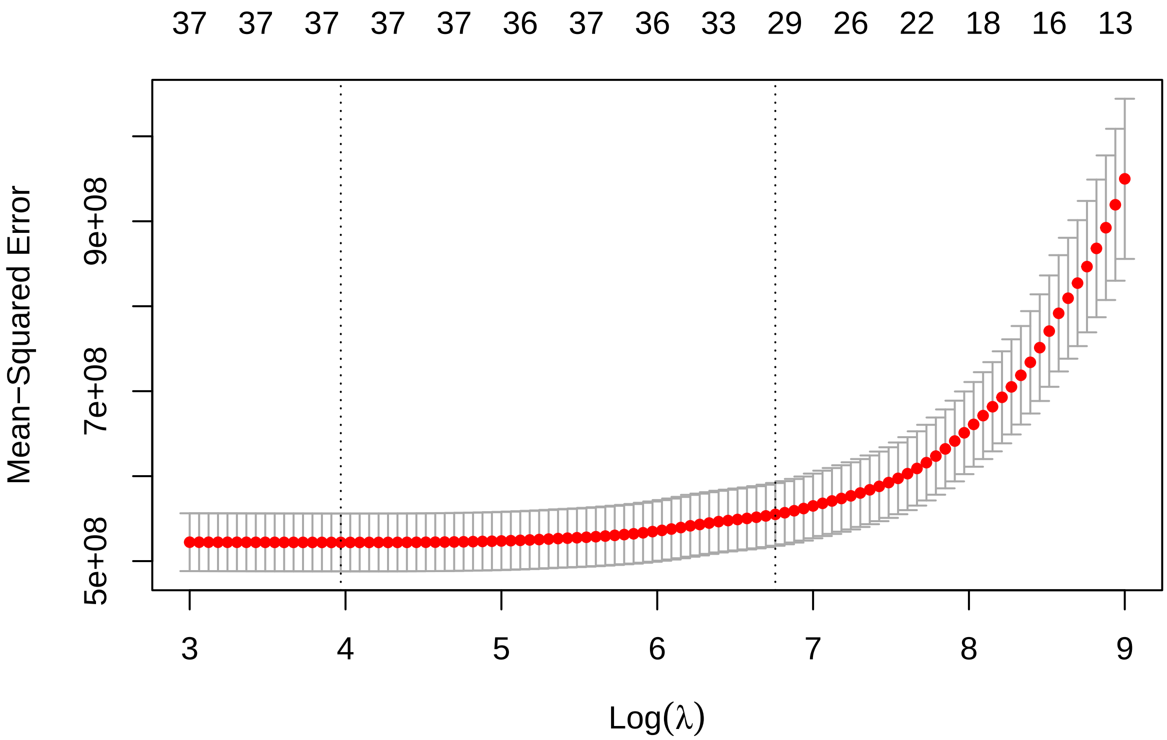
```r
error_table <- data.frame(
  Model = c("LASSO", "Elastic Net", "PLS"),
  Test_RMSE = c(test_rmse_min, test_rmse_enet_min, test_rmse_pls)
)

print(error_table)
```

```
##         Model Test_RMSE
## 1       LASSO  21005.70
## 2 Elastic Net  20929.44
## 3         PLS  21243.27
```

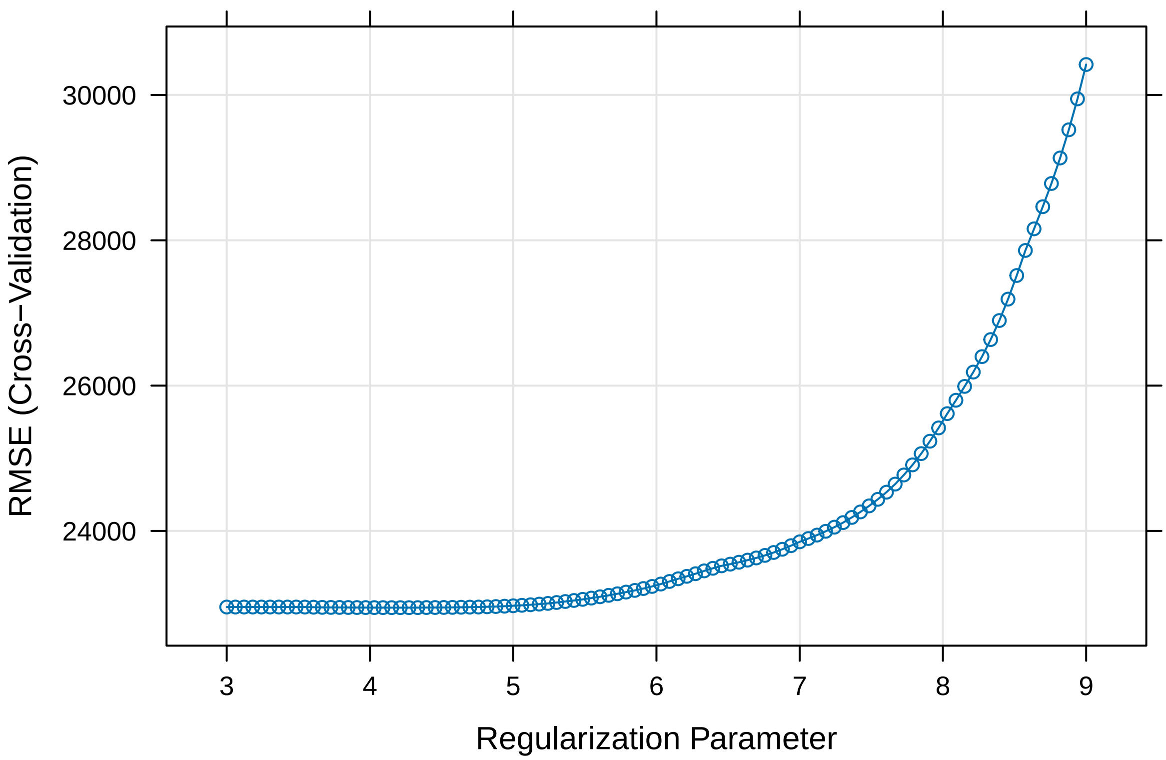
(e)

```r
set.seed(2025)
lasso.fit_caret <- train(Sale_Price ~ ., data = house_train,
                         method = "glmnet",
                         tuneGrid = expand.grid(alpha = 1,
                                                lambda = exp(seq(9, 3, length = 100))),
                         trControl = ctrl1)


lambda_glm <- cv.lasso$lambda.min
```

```r
lambda_caret <- lasso.fit_caret$bestTune$lambda
plot(cv.lasso)
```




```r
plot(lasso.fit_caret, xTrans = log)
```



By `glmnet`, the tuning parameter $\lambda = 52.9684774$; by `caret`, $\lambda = 67.499569$. Even though we specify using glmnet method in caret model fitting, there is still difference in selection of "best" λ. However, we consider this as an acceptable difference especially when considering within the exponential grid($log(\lambda_{glm}) = 3.969697$; $log(\lambda_{caret}) = 4.2121212$). The source of such difference might be from how each function sets up cross-validation or potentially different default error metrics or tie-breaking decisions.

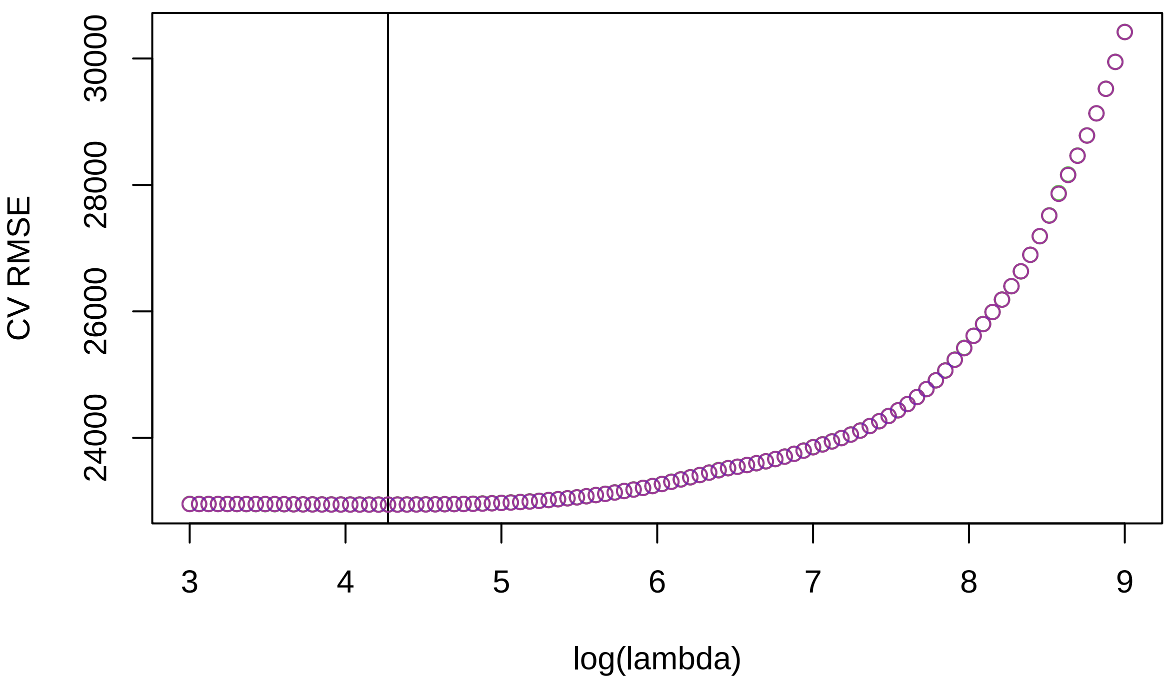
```
set.seed(2025)
resamp <- resamples(list(
  Lasso = lasso.fit_caret,
  ElasticNet = enet.fit_caret_bestPar,
  PartialLeastSq = pls.fit_caret
))

summary(resamp)
```

```
##
## Call:
## summary.resamples(object = resamp)
##
## Models: Lasso, ElasticNet, PartialLeastSq
## Number of resamples: 10
##
## MAE
##                    Min.  1st Qu.   Median     Mean  3rd Qu.     Max. NA's
## Lasso          14741.81 15558.94 16287.66 16679.12 17493.05 19165.85    0
## ElasticNet     14793.66 15493.40 16258.31 16648.24 17495.94 19152.10    0
## PartialLeastSq 14848.76 15656.74 16411.28 16743.46 17517.21 19194.05    0
##
## RMSE
##                    Min.  1st Qu.   Median     Mean  3rd Qu.     Max. NA's
## Lasso          19791.29 21882.13 22646.63 22944.34 23780.51 27025.10    0
## ElasticNet     19835.20 21896.69 22623.14 22939.56 23780.08 27076.72    0
## PartialLeastSq 19837.01 21953.53 22716.43 22945.08 23772.46 26935.47    0
##
## Rsquared
##                     Min.   1st Qu.    Median      Mean   3rd Qu.      Max. NA's
## Lasso          0.8774196 0.8893621 0.9002852 0.9021467 0.9163729 0.9249267    0
## ElasticNet     0.8778045 0.8894517 0.9004570 0.9021908 0.9159685 0.9255766    0
## PartialLeastSq 0.8774820 0.8893300 0.8997448 0.9020588 0.9164506 0.9243377    0
```

In last question, we found the Elastic Net slightly outperform the other two model by measuring RMSE. Since we just fit the lasso model w/ caret, we can directly compare how well three model predicts on test data. And the result aligns w/ our belief that Elastic Net still perform better than other two models overall.

```
set.seed(2025)
train_id_list <- lasso.fit_caret$control$index

dat_dummy <- data.frame(Sale_Price = y, x)
M <- 10
lambda.grid <- exp(seq(9, 3, length = 100))
rmse <- rmse_caret <- matrix(NA, ncol = 100, nrow = M)

for (m in 1:M)
{
  tsdata <- dat_dummy[train_id_list[[m]],]
  vsdata <- dat_dummy[-train_id_list[[m]],]
```

```r
  x1 <- as.matrix(tsdata[,-1])
  y1 <- tsdata[,1]
  x2 <- as.matrix(vsdata[,-1])
  y2 <- vsdata[,1]

  fit <- glmnet(x1, y1, alpha = 1,
                lambda = lambda.grid)

  # caret implementation did not specify lambda
  # the default grid of lambda is different from lambda.grid
  fit_caret <- glmnet(x1, y1, alpha = 1)

  pred <- predict(fit, newx = x2, s = lambda.grid)
  pred_caret <- predict(fit_caret, newx = x2, s = lambda.grid)

  rmse[m,] <- sqrt(colMeans((y2 - pred)^2))
  rmse_caret[m,] <- sqrt(colMeans((y2 - pred_caret)^2))
}

# curve from glmnet (correct)
plot(log(lambda.grid), colMeans(rmse), col = 3, xlab = "log(lambda)", ylab = "CV RMSE")
abline(v = log(lambda.grid[which.min(colMeans(rmse))]))

# caret results
points(log(lasso.fit_caret$results$lambda), lasso.fit_caret$results$RMSE, col = 2)

# try to reproduce caret results from scratch
points(log(lambda.grid), colMeans(rmse_caret), col = rgb(0,0,1,alpha = 0.3))
```




```r
# selected lambda
lambda.grid[which.min(colMeans(rmse))]
```

```
## [1] 71.71696
```

```r
# the corresponding CV RMSE
min(colMeans(rmse))
```

```
## [1] 22945.54
```

As being awared of an small implementation error of caret model, we adjust the algorithm of caret. And the selection of lambda is still in an acceptable range comparing to glmnet.