Homework on Newton's methods

Leave your name and uni here

Problem 1: Univariate optimizations.

In this problem, you will compare three common methods for univariate minimization:Newton's Method, Bisection Method and Golden-Section Search.

Function A:

$$f_1(x) = \ln(1 + x^2) + x^2$$

Function B:

$$f_2(x) = x^4 - 6x^2 + 4x + 8$$

Answer the following questions:

1. Computer $f'(x)$ and $f''(x)$
2. Are $f_1(x)$ and $f_2(x)$ unimodal functions? If not, how many modes it contains?
3. Implementing Bisection, Golden Search and Newtown's methods to find global minimum of $f_1(x)$ and $f_2(x)$
4. Discuss your results – which method is fastest in terms of iteration counts? which method is easist to apply if you only know a broad interval containing the mininum? which method fails or converges poorly if started badly? how the shape of the function influences the performance of these algorithms?
5. Please show all relevant R code for bisection, golden-section, and Newton's methods.

Answer: your answer starts here…

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#R codes:
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Problem 2: Newton's Method in Two Dimensions

$g(x, y)$ is a 2D funciton

$$g(x, y) = x^2 + xy + y^2 - 4x - 3y + 7$$

Answer the folling questions:

1. Derive the gradient $\Delta g(x, y)$ and the Hessian matrix $\Delta^2 g(x, y)$.
2. Implementation a Newton's algorithm to find its minimizer.
3. Choose two different starting values, and compare the resulting solutions, iteration counts and path to convergence
4. Create a coutour plot of g around its minium, and overlay the sequence of iterates from Newton's method to show the path to the minimum.

Answer: your answer starts here...

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#R codes:
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Problem 3

Suppose we have data $(x_i, y_i), i = 1, ..., n$, with y_i follows a conditional expotential distribution

$$Y_i \mid x_i \sim \exp(\lambda_i), \text{ where } \log(\lambda_i) = \alpha + \beta x_i.$$

Please complete the following tasks:

1. Derive the log-likelihood of (x_i, y_i), as well as its Gradient and Hession Matrix
2. Generate a syntheic data with true $\alpha = 0.5$, true $\beta = 1.2$ and sample size $n = 200$.
3. Implement an Newton's algorthm to its MLE
4. Implement a modified Newton's algorithm, where you incoporate both of the step-having and ascent direction check. If a direction is not ascent, you can swith to a simpler gradient descent.
5. For a generalized linear models, one can replace the observed Hessian with the expected Hessian (the Fisher information), which might lead to stable updates akin to a Fisher scoring approach. Implement another modified newton's algorithm with Fisher scoring toestimate MLE.
6. Compare the final parameter estimates, iteration counts and convergences of optimization, and summarize your findings.