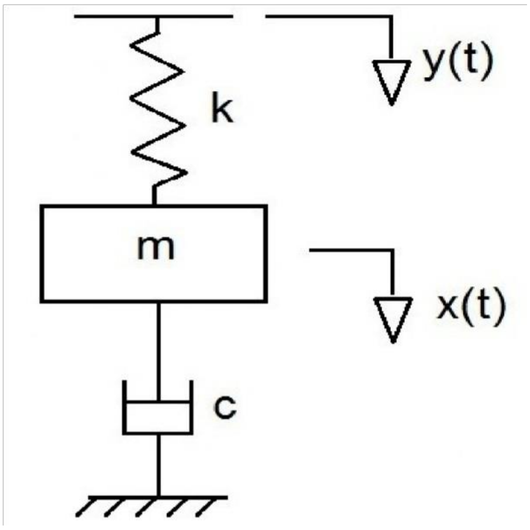
SCHOOL OF ENGINEERING
Year 2 Laboratories



VI- Vibration

MECH215: Dynamic Systems

Lecturer:

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Notation

Symbol	Meaning	Units
m	Suspended mass	kg
x(t)	Deflection of the mass	m
y(t)	End deflection of the spring	m
k	Spring stiffness	N/m
c	Damping coefficient	Ns/m
ω	Frequency	rad/s
ω_d	Frequency of free-damped oscillation	rad/s
ω_n	Natural frequency	rad/s
ξ	Damping ratio	-

1 Introduction

The dynamical response of systems to various disturbances forms a major field of study in engineering. For example, the behaviour of suspension units, loudspeaker cones and more generally, structures, hydraulic systems and many others can be predicted using a system modelling approach. This approach will be used in the experiment to study the behaviour of a second-order mechanical system, and essentially the general procedure for all such problems is to:

- Build a mathematical model of the system
- Predict the behaviour of the system using this model
- Compare the predicted with the real behaviour and draw any relevant conclusions

This exercise is related to the MECH215/ENGG301 notes on **Free Vibrations** and **Forced Harmonic Vibrations**.

1.1 Aims and Objectives

The aims of this lab is to illustrate the concepts or free and forced vibrations through a real dynamic system and the objectives of this virtual lab are:

- To understand the difference between free response and harmonic forced response for an oscillating system.
- To understand the effects of different levels of damping on an oscillating.
- To understand how to use experimental results to identify the parameters of a dynamical system.

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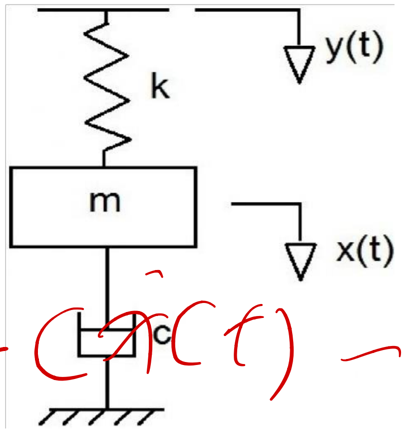
2 Background

2.1 The System Model



$m\ddot{x}(t) = -c\dot{x}(t) - kx(t) + ky(t)$

Figure 1 Model of the oscillator

The system consists of a mass-spring damper arrangement as shown is Fig. 1:

- m is the suspended mass (kg)
- k is the spring stiffness (N/m)
- c is the damping coefficient (Ns/m)
- y(t) is the end deflection of the spring (m)
- x(t) is the deflection of the mass (m)

To obtain the equation of motion of the system, the forces that are associated with the mass are sum together. Then applying the Newton's Second Law of motion yields eq. 1.

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = ky(t)$$

(eq.1)

Equation 1 can then be compared to eq.2 below, which is the standard form of a second order dynamical system and which can be found in the lectures' slides. By comparing the coefficients from eq.1 and eq.2, the values for the damping ratio ξ, natural frequency ω_n and the input u(t) (which is equal to y(t) is this particular case) can be determined.

$$\ddot{x}(t) + 2\xi\omega_n\dot{x}(t) + {\omega_n}^2 x(t) = {\omega_n}^2 u(t).$$

(eq.2)

2.2 Predicted Responses

The response x(t) of the system will depend on the input y(t) and the initial conditions. In addition, x(t) will depend on ξ and ω_n. However in the experiment only ξ will vary. We will examine two types of response: the free response (when y(t)=0, i.e. no input) and the sinusoidal forced response (when y(t) is a sinusoid).

2.2.1 Predicted Free Response

The initial conditions and the input of the second order free response system are given as (there is only an initial displacement, the initial speed is zero):

$y(t) = 0; \dot{x}(0) = 0; x(0) = x_0 \neq 0$ set 0.05

Applying these conditions to eq.2, gives eq.3 in the classical form.

$$\ddot{x}(t) + 2\xi\omega_n\dot{x}(t) + {\omega_n}^2 x(t) = 0$$

(eq.3)

If ξ < 1 (underdamped case), then the standard solution can be written in the form of eq. 4.

$$x(t) = A(t)cos(\omega_d t - \varphi_\xi)$$

(eq.4)

Where the damped frequency ω_d can be calculated as

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

(eq.5)

where A(t) is the amplitude and φ_ξ is the phase shift, defined as

$$A(t) = \frac{x_0}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t} \qquad \varphi_\xi = arctan\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)$$

Note: If ξ > 0, then the frequency of the free response is **not** the natural frequency ω_n but the damped frequency ω_d. A sketch of x(t), which is modelled by eq.4, can be seen in fig.2.

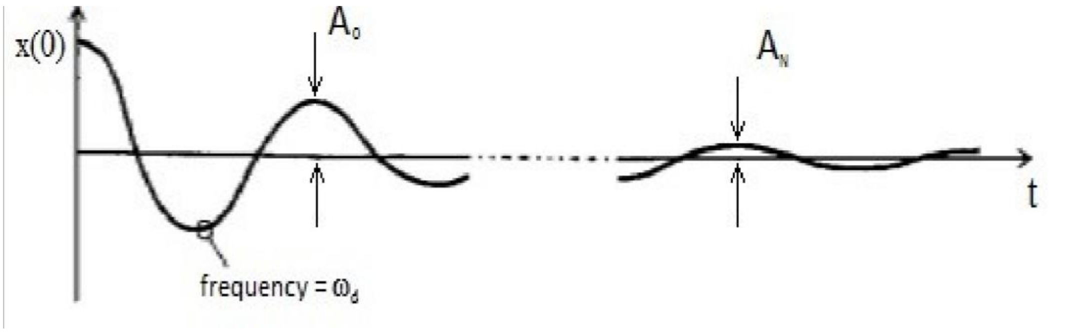


Figure 2: Time domain plot of response indicating the damped frequency and the amplitude decay

The decay of the oscillatory response is often characterised by its **logarithmic decrement** defined in eq. 6:

$$\delta = ln\left(\frac{A_i}{A_{i+1}}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

(eq.6)

where A_i is the amplitude of the i^{th} peak at any time axis and A_{i+1} is the amplitude for the $(i+1)^{th}$ peak. Notice that the logarithm decrement is independent of the initial condition x_0.

2.2.2 Predicted Sinusoidal Response

The initial conditions and input of the second-order forced response system are now given as:

$x(0) = \dot{x}(0) = 0; y(t) = Ysin(\omega_0 t)$

Here ω_0 is the input frequency and Y is the input amplitude of the imposed displacement. In this case the steady-state response can be written as shown by eq.7.

$$x(t) = Y\beta(\omega_0)sin(\omega_0 t + \varphi_H(\omega_0))$$

(eq.7)

The forced (steady-state) response is another sinusoid of the same frequency as y(t), but of **amplitude** $Y\beta(\omega_0)$ and a **phase shift** of $\varphi_H(\omega_0)$ rad/s. A sketch of the response can be seen in fig.3.

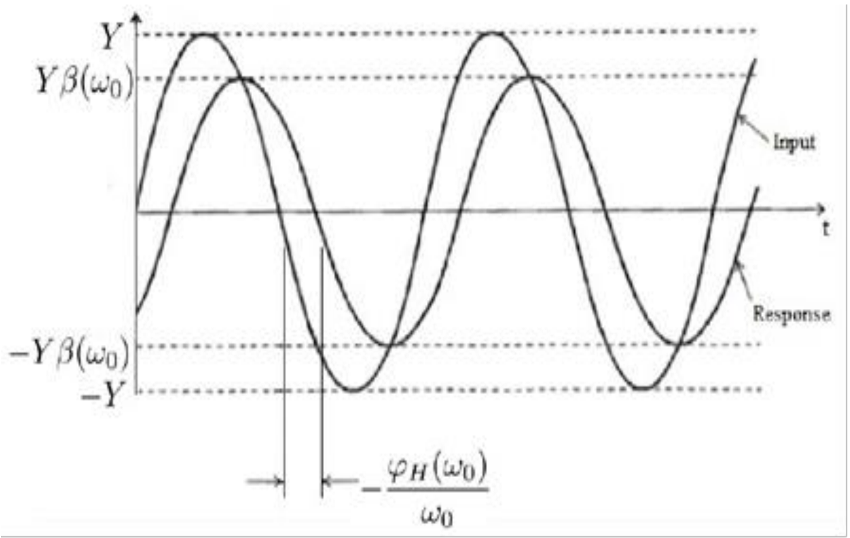


Figure 3: Phase and amplitude for a forced vibration system

From eq.7, the amplification factor β and the phase shift φ_H can be calculated using eq. 8 and eq.9.

$$\beta(\omega_0) = \frac{1}{\sqrt{\left(1-\left(\frac{\omega_0}{\omega_n}\right)^2\right)^2 + 4\xi^2\left(\frac{\omega_0}{\omega_n}\right)^2}}$$

(eq.8)

$$\varphi_H(\omega_0) = -arctan\left(\frac{2\xi\frac{\omega_0}{\omega_n}}{1-\left(\frac{\omega_0}{\omega_n}\right)^2}\right)$$

(eq.9)

The phase shift is simply the lag of the response behind the input, multiplied by the input frequency. Sketching the values of the amplification factor and the phase against ω_0 for different values of ξ yields a family of curves, fig.4.

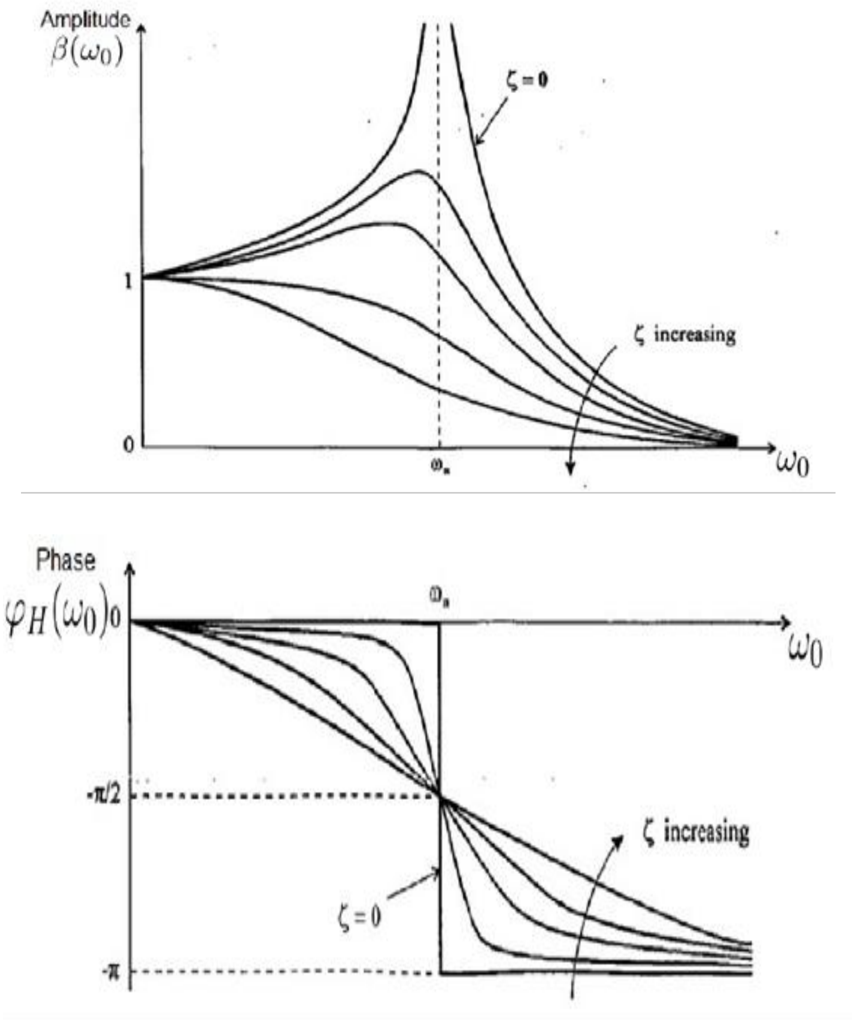


Figure 4: Amplification factor and phase shift example plots for a forced vibration system

The negative angle indicates phase lag.

The maximum amplitude is reached for $\omega_0 = \omega_r$, where $\omega_r = \omega_n\sqrt{1-2\xi^2}$ is the resonance frequency. The maximum amplitude is then given by the formula

$\beta_{max} = \beta(\omega_r) = \frac{1}{2\xi\sqrt{1-\xi^2}}$

(eq.10)

Concerning the phase plots, we have the property

$\varphi_H(\omega_n) = -\frac{\pi}{2}$ rad,

(eq.11)

independently of the value of the damping ratio.

For systems where a peak in the amplitude curve exists, the 'strength' of the peak is measured by the **bandwidth**, defined as the frequency range over which, eq.12:

$\beta(\omega_0) > \frac{\beta_{max}}{\sqrt{2}}$.

(eq.12)

A sketch to illustrate the bandwidth is shown by fig.5:

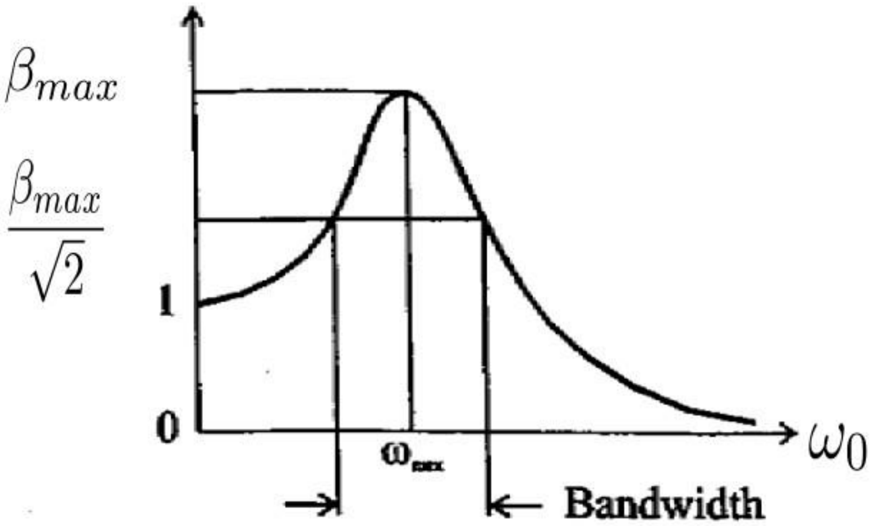


Figure 5: Definition of bandwidth

It can be shown that for **small** values of ξ,

$Bandwidth \simeq 2\xi\omega_n$

$\frac{5}{6.5 \times 2} = \xi$

(eq.12)

3 Health and Safety

There are no significant hazards in this experiment.

Students are reminded that they are required by law to comply with the Department's basic rules of lab safety given to them at the start of the semester.

4 Experimental Procedure

The necessary data for the experiments are: mass of each disc = 1kg; mass of frame + damper piston = 2.1kg. Thus the total mass is 4.1 kg. All tables require a title and all recordings require units

This is a virtual lab. The response of the oscillator is then simulated and animated using a Matlab program. Friction forces and measurements errors are taken into account in this program. They are generated randomly based on the student number. As a consequence, each student will work with unique experimental measurements.

4.1 D1: Free Response

Use the Matlab program "Lab_VI_virtual_simulation.m" to simulate the oscillator.

i. Undamped case: In the Matlab program, choose the option 'no damping' case, choose an initial position of the mass and a simulation duration. Repeat the procedure 5 times with the same configuration (but changing the run number). From the recorded results, find the period T_n and determine the natural frequency ω_n. Assume here that ξ is negligible. Determine also the stiffness of the spring.

ii. Damped case: Now choose the low of damping, repeat 5 runs and for each run, report the amplitude value for the two first peaks.

4.2 D2: Sinusoidal Response

The amplification factor $\beta(\omega_0)$ is equal to the ratio of the response amplitude to the input amplitude.

i. With the damper at its low level of damping, determine the amplification factor and phase for increasing frequency and use this data to construct amplification and phase curves. Use frequency values of 0.4 – 4 Hz and take at least 10 recordings.

ii. Now for the critically damped case, i.e. so that there is **just** no overshoot in the free response, repeat the procedure for measuring amplification and phase and plot the new curves on your original graph.

iii. Now for the intermediate value of damping between, repeat the procedure for measuring amplification and phase, and again plot the new curves on your original graph.

$$m\ddot{x}(t) + c\cancel{\dot{x}(t)} + kx(t) = \cancel{ky(t)}$$

undamped   free response

$$\ddot{x}(t) + \underbrace{\frac{k}{m}}_{=\,\omega_n^2} x(t) = 0$$

$$k = m\omega_n^2$$

$$\omega_n = \frac{2\pi}{T}$$

5 Tables

Table 1: Free response for natural frequency

Run Number	T1 (s)	T2 (s)	T_n (s)	ω_n (rad/s)	k (N/n)
1					
2					
3					
4					
5					
Average ω_n and k					

Table 2: Free response for lowest damping ratio

Run Number	A0 (mm)	A1 (mm)	A0/A1
1			
2			
3			
4			
5			
Average A0/A1			

*Zero calibration must be done in this step

Table 3: Sinusoidal response of the lowest damping ratio

Run Number	Input Frequency ω_0 (Hz)	Input Frequency ω_0 (rad/s)	Amplification, $\beta(\omega_0)$	Phase (rad), $\varphi_H(\omega_0)$
1				
2				
3				
4				
5				
6				
7				

8				
9				
10				
11				
12				
13				

Table 4: Sinusoidal response of critical damping ratio ξ=1

Run Number	Input Frequency ω_0 (Hz)	Input Frequency ω_0 (rad/s)	Amplification, $\beta(\omega_0)$	Phase (rad), $\varphi_H(\omega_0)$
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				

Table 5: Sinusoidal response of intermediate damping ratio

Run Number	Input Frequency ω_0 (Hz)	Input Frequency ω_0 (rad/s)	Amplification, $\beta(\omega_0)$	Phase (rad), $\varphi_H(\omega_0)$
1				

2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				

6 Technical Note Submission

6.1 Assessment Criteria

- **Technical Writing 5%**
  Correct spelling, grammar, formatting and concise.
- **Introduction 10%**
  Roughly 500 words. This should include a brief context/why this lab is important, what you did, how you did it and a brief summary of the results.
- **Results, calculations and graphs 60 %**
  Part I: Free Response
  1. Show the two tables of results for the free response (i.e. table 1 and 2)
  2. Calculate the average natural frequency (ω_n), the average stiffness (k) and for the low damping case, the average amplitude ratio (A0/A1).
  3. Calculate the low damping ratio (ξ) using the logarithmic decrement formula.

  $$\xi = \frac{1}{\sqrt{\left(\frac{2\pi}{\ln(1.3733)}\right)^2 + 1}}$$

  Part II: Forced Response
  1. Show the three tables of results for the forced response (i.e. table 3, 4 and 5)
  2. Plot graphs for amplification factor vs input frequency (amplification factor plot) and phase shift vs input frequency (phase plot) considering the three levels of damping. (Refer figure 4)
  3. Estimate the natural frequency using the phase plot for the **low** value of damping. (This is where the phase = -π/2 rad/s)
  4. Compare the natural frequency obtained from II-3 and I-2.

  $$\left(\left(\frac{2\pi}{\ln(1.3733)}\right)^2 + 1\right)\xi^2 = 1$$

$$\ln(1.3733) = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \qquad \left(\frac{2\pi\xi}{\ln(1.3733)}\right)^2 = 1-\xi^2$$

$$\beta_{max} = \beta(\omega_r) = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

6.8990

$$\xi = 0.9974$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$17.5929 = 16.5\sqrt{1-2\xi^2}$$

5. Estimate the damping ratio via the amplification factor plot for the **low** value of damping using:
    a. The value of the maximum amplitude.
    b. The value of the resonance frequency.
    c. The bandwidth approximation.
6. Calculate the average of the damping ratio for the **low** value of damping i.e. 5a, 5b, 5c and I-3.
7. Estimate the damping ratio via the amplification plot for the **intermediate** value of damping using:
    a. The value of the maximum amplitude.
    b. The value of the resonance frequency.
    c. The bandwidth approximation.
8. Calculate the average of the damping ratio for the **intermediate** value of damping i.e. 7a, 7b and 7c.

- **Discussion 15%**
  Draw meaning from each plot. Match results to theory. Did what happen match what should have happened?
  Comment upon the shapes of your amplitude and phase curves.
  Give an example of a real mechanical system that can be represented by a similar oscillating system, as has been seen in the lab exercise.
- **Conclusion 5%**
  Draw numerical conclusions from each plot. Summarise findings of numerical experiments

6.2 Submission Instructions

Download the **Technical Note Template** for the Lab VI available on Canvas.

Rename the document to include your name before submitting it – for example “VI Technical Note John Smith.docx”.

Complete the technical note and upload it on Canvas with the link available in the Virtual Lab section.

The deadline is the 15th of May at 23:59.