PY4669 Modal Logic & Metaphysics

Take-Home Assignment 2
Due Tuesday 19 March at midnight

This assignment will be marked out of 20. It will be worth 20% of your final grade. Each question will carry the points indicated. Some questions have sub-questions (a), (b), ... You must answer each sub-question for the answer to be complete.

By submitting this assignment, you agree that your submission is all your own work. Please do not write your name anywhere in this document.

You are to edit this file, writing your answers in the space provided below each question. Then submit the Word file of your answers on MMS. NB when a maximum number of words is specified for discursive answers, the number is indicative: try to stick to the word limit, but you won't be penalized if you exceed it.

Q1 (a) Provide an exposition of Kripke's argument for the Necessity of Identity:

(NI) $x = y \supset \Box(x = y)$

(b) Explain how (NI) vindicates the Kripkean claim that there are necessary *a posteriori* truths.

[*Max. 500 words overall*]
[**5 points, 2 for sub-question (a) and 3 for (b)**]

Q2 Explain the main differences between two (families of) accounts of the metaphysical status of possible worlds: (1) (extreme-genuine) modal realism (worlds are *concreta*: causally and spatiotemporally isolated maximal mereological sums); (2) modal actualism or ersatzism (worlds are *abstracta*: maximal states of affairs, or maximal properties, or sets of propositions, or of sentences, etc.).

[*Max. 600 words*]
[**4 points]**

Q3 The schematic formula

$\langle \lambda x.\Box\varphi(x)\rangle(c) \equiv \Box\langle \lambda x.\varphi(x)\rangle(c)$

is logically valid when the singular term *c* is rigid. Explain why this is so, given the workings of λ-abstraction.

[**5 points**]

Q4 In the partial semantics of F&M Ch. 11 (General Non-Rigid Interpretation), allowing for non-denoting terms, three of these are valid and one is invalid:

1. $\langle \lambda x.\neg\varphi\rangle(t) \supset \neg\langle \lambda x.\varphi\rangle(t)$
2. $\neg\langle \lambda x.\varphi\rangle(t) \supset \langle \lambda x.\neg\varphi\rangle(t)$
3. $\langle \lambda x.\varphi \supset \psi\rangle(t) \supset [\langle \lambda x.\varphi\rangle(t) \supset \langle \lambda x.\psi\rangle(t)]$
4. $[\langle \lambda x.\varphi\rangle(t) \supset \langle \lambda x.\psi\rangle(t)] \supset \langle \lambda x.\varphi \supset \psi\rangle(t)$

(a) Tell which ones are valid/invalid and (b) explain why. An informal explanation will be enough: no need for a formal proof of validity, or a countermodel.

[**6 points: 4 for (a) (1 for each schematic formula) and 2 for (b)**]

*****END OF ASSIGNMENT*****

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20th March 2024

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QUESTION 1

0/2 for 1(a) 2/3 for 1(b)

There are contingently true identity statement, but this does not entail the contingency of the notion of identity; on the contrary, identity is necessary. Kripke introduced the notion of rigid designator that designates the same object in all possible worlds in which that object exists. Kripke argues if $a = b$ is true and a and b are rigid designators, then $a = b$ is a necessary truth because the rigid designation implies that a and b both designates the same object in all possible worlds. If a or b are non-rigid designators, then $a = b$ might be a contingently true identity statement. For example, "Prime Minister of the UK" is a definite description not applies to a unique object so it is not a rigid designator, then "Prime Minister of the UK = Rishi Sunak" is contingently true because the Prime Minister might be succeeded by someone else in the future. This means that such contingency arises from the semantics of the term (the rigidity of designation), rather than from identity. By stipulating that the participating terms are rigid designations, thus removing the semantic layer of contingency, we find that the identity of objects must be necessary.

Kripke's argument for the Necessity of Identity underpins his claim regarding the existence of necessary a posteriori truths. Traditionally, necessary truths were associated with a priori knowledge, and contingent truths with a posteriori knowledge. Kripke challenges this dichotomy by showing how some truths are both necessary and a posteriori. The key to understanding this lies in the nature of rigid designation. For example, we once believed that Hesperus (the evening star) and Phosphorus (the morning star) rigidly designated two different objects, but in fact, they are the same planet. This implies that there are necessary a posteriori truths because, while identity sentences involving two rigid designators are necessary if true, the realisation of their identity is a posteriori, resulting from empirical means and not from logical entailment or semantic analysis alone. In other word, if the morning star is the evening star, then the morning star is necessarily the evening star, but we may not know that the morning star is the evening star, i.e., they designate the same planet, since this is a posteriori knowledge.

QUESTION 2 2/4

The difference lies in their ontological commitments. Genuine modal realism commits to a plurality of concrete worlds, each as real as our own, but inaccessible to us. Modal actualism, on the other hand, maintains a more conservative ontology, positing that only the actual world is concrete, and possible worlds are merely abstract ways of talking about how certain things could have been different.

Under the view of genuine modal realism, possible worlds are concrete entities, complete with their own space, time, and causal relations. They are real in the same sense that our world is real, albeit completely separate from our own. For Lewis, this means that when we talk about how things could have been, we are referring to the way things are in some other concrete world. This approach provides a straightforward way to analyse modal claims by referring to what is true in these concrete worlds. However, this ontological generosity postulates an infinite number of fully-fledged, concrete universes, which is seen as a costly commitment.

In contrast, modal actualism avoids such ontological extravagance by claiming that all possible worlds are abstract rather than concrete. These abstract entities can take various forms, such as maximal consistent sets of propositions, sentences, states of affairs, or properties. They are not real worlds but representations of how the world could be. On this view, when we talk about possibilities, we are not referring to other concrete worlds but rather to abstract representations of different ways the world might have been. This approach allows for a discussion of modal claims without committing to the existence of an infinite number of fully-fledged, concrete universes. However, modal actualism struggles to define the notion of possibility without committing to primitive modality. Modal actualism also faces the issue of cross-world identity. While genuine modal realism can straightforwardly claim that there are distinct counterparts in different concrete worlds, modal actualism must navigate the more abstract terrain of identifying entities across different possible representations. Moreover, the abstract nature of possible worlds in modal actualism may not provide the same richness and depth of explanatory power as the concrete worlds posited by genuine modal realism.

QUESTION 3 5/5

Proof. Let $\mathcal{M} = \langle \mathcal{G}, \mathcal{R}, \mathcal{D}, \mathcal{I} \rangle$ be a model.

We want to show that, for any arbitrary valuation v and possible world $w \in \mathcal{G}$, we have

$$\mathcal{M}, w \Vdash_v \Box\langle \lambda x.\Phi(x)\rangle(c) \iff \mathcal{M}, w \Vdash_v \langle \lambda x.\Box\Phi(x)\rangle(c)$$

To prove this, we proceed as follows:

$$\begin{aligned}\mathcal{M}, w \Vdash_v \Box\langle\lambda x.\Phi(x)\rangle(c) &\iff \mathcal{M}, w' \Vdash_v \langle\lambda x.\Phi(x)\rangle(c), \forall w' \colon w\mathcal{R}w' \\ &\iff \mathcal{M}, w' \Vdash_{v'} \Phi(x), \forall w' \colon w\mathcal{R}w', \text{where } v' \sim_x v \text{ such that } v'(x) = \mathcal{I}(c, w') \\ &\iff \mathcal{M}, w \Vdash_{v'} \Box\Phi(x), \text{where } v' \sim_x v \text{ such that } v'(x) = \mathcal{I}(c, w')\end{aligned}$$

The singular term c is rigid, which means for all possible world $w \in \mathcal{G}$, $\mathcal{I}(c, w)$ is the same if it is defined; accordingly, $\mathcal{I}(c, w') = \mathcal{I}(c, w)$, then

$$\begin{aligned}&\iff \mathcal{M}, w \Vdash_{v'} \Box\Phi(x), \text{where } v' \sim_x v \text{ such that } v'(x) = \mathcal{I}(c, w) \\ &\iff \mathcal{M}, w \Vdash_v \langle\lambda x.\Box\Phi(x)\rangle(c)\end{aligned}$$

∎

QUESTION 4 6/6

Formula 2 is invalid. Suppose t does not designate at the world of evaluation. By Definition 11.1.4.10 in the Fitting-Mendelsohn textbook 1st ed., $\langle\lambda x.\Phi\rangle(t)$ is false at that world, so $\neg\langle\lambda x.\Phi\rangle(t)$ is true at that world. On the other hand, by the same definition, $\langle\lambda x.\neg\Phi\rangle(t)$ is false at that world. This world at which t does not designate provides a counterexample of $\neg\langle\lambda x.\Phi\rangle(t) \supset \langle\lambda x.\neg\Phi\rangle(t)$.

Formula 1, 3, and 4 are valid. Let $\mathcal{M} = \langle\mathcal{G}, \mathcal{R}, \mathcal{D}, \mathcal{I}\rangle$ be a model, v be a valuation, and $w \in \mathcal{G}$.

Proof.

$$\begin{aligned}\mathcal{M}, w \Vdash_v \langle\lambda x.\neg\Phi\rangle(t) &\iff \mathcal{M}, w \Vdash_{v'} \neg\Phi, \text{where } v' \sim_x v \text{ such that } v'(x) = \mathcal{I}(t, w) \\ &\iff \mathcal{M}, w \nVdash_{v'} \Phi, \text{where } v' \sim_x v \text{ such that } v'(x) = \mathcal{I}(t, w) \\ &\iff \mathcal{M}, w \nVdash_v \langle\lambda x.\Phi\rangle(t) \\ &\iff \mathcal{M}, w \Vdash_v \neg\langle\lambda x.\Phi\rangle(t)\end{aligned}$$

$$\begin{aligned}\mathcal{M}, w \Vdash_v \langle\lambda x.(\Phi \supset \Psi)\rangle(t) &\iff \mathcal{M}, w \Vdash_{v'} \Phi \supset \Psi, \text{where } v' \sim_x v \text{ such that } v'(x) = \mathcal{I}(t, w) \\ &\iff \mathcal{M}, w \Vdash_{v'} \Phi \implies \mathcal{M}, w \Vdash_{v'} \Psi, \text{where } v' \sim_x v \text{ such that } v'(x) = \mathcal{I}(t, w) \\ &\iff \mathcal{M}, w \Vdash_v \langle\lambda x.\Phi\rangle(t) \implies \mathcal{M}, w \Vdash_v \langle\lambda x.\Psi\rangle(t) \\ &\iff \mathcal{M}, w \Vdash_v \langle\lambda x.\Phi\rangle(t) \supset \langle\lambda x.\Psi\rangle(t)\end{aligned}$$

$$
\begin{aligned}
\mathcal{M}, w \Vdash_v \langle \lambda x.\Phi \rangle(t) \supset \langle \lambda x.\Psi \rangle(t) &\iff \\
\mathcal{M}, w \Vdash_v \langle \lambda x.\Phi \rangle(t) \implies \mathcal{M}, w \Vdash_v \langle \lambda x.\Psi \rangle(t) &\iff \\
\mathcal{M}, w \Vdash_{v'} \Phi \implies \mathcal{M}, w \Vdash_{v'} \Psi, \text{where } v' \sim_x v \text{ such that } v'(x) = \mathcal{I}(t, w) &\iff \\
\mathcal{M}, w \Vdash_{v'} \Phi \supset \Psi, \text{where } v' \sim_x v \text{ such that } v'(x) = \mathcal{I}(t, w) &\iff \\
\mathcal{M}, w \Vdash_v \langle \lambda x.(\Phi \supset \Psi) \rangle(t) &
\end{aligned}
$$

In summary, these formulae are valid because the truth of the antecedent guarantees that t is designated at the world being evaluated. ∎