ASSIGNMENT 2: COMBINING MODAL LOGIC AND FIRST-ORDER PREDICATE LOGIC

This is a project exploring the logical and metaphysical issues raised by attempting to combine *modal* logic and *first-order* predicate logic. It is designed to test your understanding of both modal logic and first-order predicate logic, and some of the distinctive issues which arise when we attempt to combine them. ¶ Read through all of these questions before attempting to answer them. Write your answers as clearly and explicitly as you can and explain all your working. ¶ Submit your answers as a PDF file on MMS by **November 13, 2023**, and provide a *hardcopy* to Greg either at his office, Edgecliffe 105 (there will be an envelope on the door if Greg is away from the office) or in a lecture class, by **3pm November 14**, to assist with marking. ¶ Each submission must be *anonymous*, and must start with the project coversheet, available on Moodle.

WARNING: This project is not straightforward. It will stretch you, and test your abilities. Do not be surprised or discouraged if you find it challenging.

We'll start by defining *models* for first-order modal logic. Models for first-order predicate logic have the form $M = \langle D, I \rangle$ where D is a domain of objects, and I interprets the predicates and names. The simplest way to add the modal operators of $\Box$ and $\Diamond$ to these models is to add a set W of worlds, and to allow the predicates to vary in interpretation from world to world. Following this scheme, a model is a *triple* $M = \langle D, W, I \rangle$ where D is a non-empty set of objects (the domain), W is a non-empty set of objects (the worlds), and I is an interpretation function, assigning values to the names and predicates of the language. I interprets *names* as usual:

- For each name a, $I(a)$ is an element of D.

So, a name names an *object*, and this does not vary from world to world. Predicates, however, *can* vary in interpretation from world to world. An n-place predicate is interpreted by assigning a truth value for every n-tuple of objects *and* a choice of a world. That is:

- For each n-place predicate F and for each $d_1, \ldots, d_n$ from D and for each w in W, $I(F)(d_1, \ldots, d_n, w)$ is a truth value, either 0 or 1.

As with models of first-order logic, a function v is an assignment of values to the variables when for each variable x it assigns a value $v(x)$ from

D. And as before, we can interpret any term, whether a variable or a name, by assigning

- $I(t, v) = I(t)$, if t is a name.
- $I(t, v) = v(t)$, if t is a variable.

Then we can extend our interpretation I to assign truth values to each formula, given a world w and a valuation v, like this:

- $I(Ft_1 \cdots t_n, w, v) = 1$ iff $I(F)(I(t_1, v), \ldots, I(t_n, v), w) = 1$
- $I(A \wedge B, w, v) = 1$ iff $I(A, w, v) = 1$ and $I(B, w, v) = 1$.
- $I(A \vee B, w, v) = 1$ iff $I(A, w, v) = 1$ or $I(B, w, v) = 1$.
- $I(A \to B, w, v) = 1$ iff $I(A, w, v) = 0$ or $I(B, w, v) = 1$.
- $I(\neg A, w, v) = 1$ iff $I(A, w, v) = 0$.
- $I(\bot, w, v) = 1$ never.
- $I(\Box A, w, v) = 1$ iff $I(A, w', v) = 1$ for *every* world $w' \in W$.
- $I(\Diamond A, w, v) = 1$ iff $I(A, w', v) = 1$ for *some* world $w' \in W$.
- $I(\forall x A, w, v) = 1$ iff $I(A, w, v') = 1$ for *every* x-variant v' of v.
- $I(\exists x A, w, v) = 1$ iff $I(A, w, v') = 1$ for *some* x-variant v' of v.

As with models for modal logic and first-order logic, we use models and to define counterexamples to arguments. The triple of a model M, an assignment v of values to the variables, and a world w serves as a counterexample to the argument $X \succ A$ iff $I(B, w, v) = 1$ for each $B \in X$, while $I(A, w, v) = 0$.

An argument $X \succ A$ is valid iff it has no counterexample. When the argument $X \succ A$ is valid, we write $X \vDash_{\text{CDQS5}} A$. "CDQS5" is for "Constant Domain Quantified S5".

Constant domain quantified S5 is one of the *simplest* ways to combine possible worlds and a domain for quantification. It is said to be "constant domain" because the domain of objects does not vary from world to world.

QUESTION 1 (4 POINTS)

For the first question in this assignment, you will work with a single model. Consider the model with worlds $W = \{w_1, w_2\}$ and domain $D = \{a, b, c\}$. Interpret the predicates F and G like this:

$I(F)$	a	b	c
w_1	1	0	1
w_2	0	1	1

$I(G)$	a	b	c
w_1	1	0	0
w_2	0	1	0

We can use this model to evaluate statements involving both quantifiers and modal operators.

For example, in world w_1, relative to the assignment $(x : a)$ (assigning x the value a) Gx is true, while in world w_2, relative to that valuation, Gx is false. So, relative to that valuation, $\Box Gx$ is false, while $\Diamond Gx$ is true at both world w_1 and w_2. So, the statement $\exists x \Diamond Gx$ is true in our model (no matter which world, no matter which assignment of values to the variables), since there is a value to assign x (namely, a) that makes $\Diamond Gx$ true. In other words, there is something that is *possibly* G is true in our model, at each of the two worlds.

Task 1: Clearly explain why, in this model, $\Box\exists x Gx$ is *true* at worlds w_1 and w_2, and why $\exists x \Box Gx$ is *false* at worlds w_1 and and w_2. Make your explanations as clear and explicit as possible, showing how the status $\Box\exists x Gx$ at each world depends on the value of $\exists x Gx$ at each world, and how this depends on the value of Gx for each possible assignment of values to the variable x, and similarly, how $\exists x \Box Gx$ at each world depends on the value of $\Box Gx$ at each world for each assignment of values to the variable x, and how this depends on the values of Gx at each world for each assignment of values.

Task 2: When you have completed the explanations for $\Box\exists x Gx$ and $\exists x \Box Gx$, provide the same explanations for the status of the two formulas $\Diamond\forall x Fx$ and $\forall x \Diamond Fx$ at each world in our model, too.

QUESTION 2 (4 POINTS)

One use of models is to evaluate arguments. You have already verified that in this model, $\Box\exists x Gx$ is *true* (at both worlds) while $\exists x \Box Gx$ is false (at both worlds). So, this model, with either world, counts as a counterexample to the argument

$$\Box\exists x Gx \succ \exists x \Box Gx$$

So, this argument is invalid in CDQS5. It is one thing for $\Box\exists x Gx$ to be true (for it to be *necessary* that something has property G). It is another for $\exists x \Box Gx$ is true (there needn't be something that *necessarily* has property G).

The converse of this argument *is* valid. It has no counterexamples.

$$\exists x \Box Gx \succ \Box\exists x Gx$$

Here is why: suppose we have a model $M = \langle D, W, I\rangle$ with domain D, worlds W and interpretation I, that makes $I(\exists x \Box Gx, w, v) = 1$ for some choice of world w and assignment v of values. This means that there is some value $a \in D$ to assign to the variable x in the x-variant assignment v' of v, such that $I(\Box Gx, w, v') = 1$. And this (by the truth conditions for $\Box$) means that $I(Gx, w', v') = 1$ for each world $w' \in W$.

Working backwards to $\Box\exists x Gx$ from this fact, we can see that since $I(Gx, w', v') = 1$ (since v' is an x-variant of the assignment v), we have

$I(\exists xGx, w', v) = 1$ too, for any world w'. Now, since this holds for *any* world w', we have $I(\Box\exists xGx, w, v) = 1$ too.

So, I have shown that in any model, in any world (and assignment of values), if $\exists x\Box Gx$ is true, so is $\Box\exists xGx$. In any of these models, if there is something that *necessarily* has property G, then it is necessary that something has property G too.

Task 1: Explain in your own words, as clearly and precisely as possible, why the arguments

$$\forall x\Box Fx \succ \Box\forall xFx \qquad \Diamond\exists xFx \succ \exists x\Diamond Fx$$

are both *valid* in CDQS5.

These arguments (or rather, the *formulas* corresponding to the arguments: $\Diamond\exists xFx \rightarrow \exists x\Diamond Fx$ and $\forall x\Box Fx \rightarrow \Box\forall xFx$), are famous in modal logic. They were formulated by the American philosopher, Ruth Barcan Marcus (1921–2012), in her pioneering work on quantified modal logic. In the rest of this project, you will explore the status of these *Barcan Formulas/Arguments*.

Task 2: Explain in your own words, as clearly and precisely as possible, these two arguments (the converses of the previous two)

$$\Box\forall xFx \succ \forall x\Box Fx \qquad \exists x\Diamond Fx \succ \Diamond\exists xFx$$

are also *valid* in CDQS5.

QUESTION 3 (4 POINTS)

The arguments $\forall x\Box Fx \succ \Box\forall xFx$ and $\Diamond\exists xFx \succ \exists x\Diamond Fx$ and their converses have no counterexamples in CDQS5 models. In this question, your job is to find *proofs* for each of these arguments, using the proof rules $\Box$I, $\Box$E, $\Diamond$I and $\Diamond$E.

Task 1: Construct proofs, using the natural deduction proof rules for predicate logic, and for S5, for these two arguments. (You will need to use the double negation elimination rule DNE for the second argument, but not the first.)

$$\Box\forall xFx \succ \forall x\Box Fx \qquad \exists x\Diamond Fx \succ \Diamond\exists xFx$$

Be careful to explain why the side conditions for the $\Box$I, $\Diamond$E, $\forall$I and $\exists$E rules are satisfied whenever they are used in your proofs.

Task 2: You will find that given the original side conditions on the rules $\Box$I and $\Diamond$E, as they are given in Chapter 9, you *cannot* find proofs for the Barcan arguments that satisfy those side conditions. You will have to expand the definition of *modal formula* so that your proofs satisfy the modal side conditions. However, it is possible to extend the notion of a modal formula so as to allow for proofs for these two arguments:

$$\forall x\Box Fx \succ \Box\forall xFx \qquad \Diamond\exists xFx \succ \exists x\Diamond Fx$$

So, Task 2 involves three components: (1) clearly specify a wider definition of what can count as *constant domain modal formula* in the language of first-order modal logic for CDQS5. (2) Explain why this choice of modal formulas results in $\Box$I and $\Diamond$E rules that are *sound* for validity defined in CDQS5 models, by showing that if a constant domain modal formula (as you have defined it) is true at some world in a CDQS5 model, then it is true at *all* worlds of that model. (3) Using these rules, write out full proofs for the arguments $\forall x \Box Fx \succ \Box \forall x Fx$ and $\Diamond \exists x Fx \succ \exists x \Diamond Fx$, explaining, in your own words, why it is that the side conditions for each $\Box$I, $\Diamond$E, $\forall$I and $\exists$E inference are satisfied in the proofs you write.

QUESTION 4 (8 POINTS)

The Barcan formulas/arguments are controversial. Not everyone thinks that they are, as a matter of fact, *valid*. Some say that (for some properties F) there *could have been* something that has property F, even there *is* no thing where *it* could have been F. (I could have had a younger sister—had things gone differently in my family—but I find it very hard believe that there *is* something that could have been my younger sister.) For this last question, you will explore this issue.

First, read the following articles that give an account of the debate.

- James Garson "Modal Logic" (2018) *Stanford Encyclopedia of Philosophy*, concentrating on Section 15 on Quantifiers in Modal Logic.[1]
- Chris Menzel "Actualism" (2014), also from the *Stanford Encyclopedia of Philosophy*,[2]
- Bernard Linsky and Ed Zalta (1994) "In Defense of the Simplest Quantified Modal Logic," *Philosophical Perspectives*, (Logic and Language), 8: 431–458.[3]

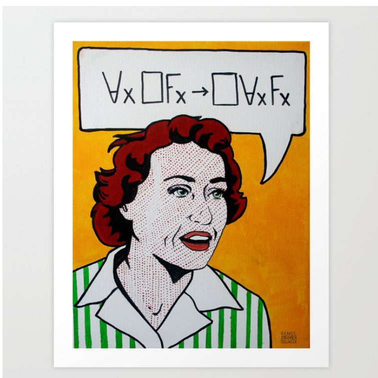
Once you have read these papers (and perhaps, followed up on other references contained in them), you have a choice between two tasks.

OPTION 1: Write a short essay (no more than two pages) either defending constant domain quantified S5 from its critics, explaining why the Barcan arguments are, in fact, *valid*, or explaining why as a matter of fact these argument should be taken to be *invalid*. In either case, defend the choices you make, being as clear as you can in your reasoning, and paying attention to the metaphysical commitments incurred by the models you choose.

OPTION 2: Define some *other* class of models for first-order modal logic that *invalidate* the Barcan arguments. Give truth conditions for $\Box$ and $\Diamond$ formulas and for quantified formulas in your models (as well as for atomic formulas) and then use these models to give counterexamples to

[1] https://plato.stanford.edu/entries/logic-modal/
[2] https://plato.stanford.edu/entries/actualism/
[3] https://mally.stanford.edu/Papers/simple-qml.pdf

the Barcan arguments. Be as clear and precise as you can, and defend the choice of models that you give. A good defence will involve explaining the significance of any way your models diverge from CDQS5 models, saying something more informative than “these models give us counterexamples to the Barcan arguments.” Explain how your models can represent ways things could be.



Ruth Barcan Marcus, portrait by Renee Bolinger,
https://www.reneebolinger.com/portraits.html

University of St Andrews

Department of Philosophy

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QUESTION 1

Task 1

Claim. $\Box\exists x Gx$ *is true at worlds* w_1 *and* w_2.

Proof. $I(Gx, w_1, v^x_a) = 1$, so $I(\exists x Gx, w_1, v) = 1$.

Similarly, $I(Gx, w_2, v^x_b) = 1$, so $I(\exists x Gx, w_2, v) = 1$.

$\forall w' \in W, I(\exists x Gx, w', v) = 1. \therefore I(\Box\exists x Gx, w, v) = 1.$ $\blacksquare$

Claim. $\exists x\Box Gx$ *is false at worlds* w_1 *and* w_2.

Proof. $I(Gx, w_1, v^x_a) = 1$ and $I(Gx, w_2, v^x_a) = 0$, so $I(\Box Gx, w, v^x_a) = 0$.

Similarly, $I(Gx, w_1, v^x_b) = 0$ and $I(Gx, w_2, v^x_b) = 1$, so $I(\Box Gx, w, v^x_b) = 0$.

Similarly, $I(Gx, w_1, v^x_c) = 0$ and $I(Gx, w_2, v^x_c) = 0$, so $I(\Box Gx, w, v^x_c) = 0$.

$\forall v' : v' \sim_x v, I(\Box Gx, w, v') = 0. \therefore I(\exists x\Box Gx, w, v) = 0.$ $\blacksquare$

Task 2

Claim. $\Diamond\forall x Fx$ *is false at worlds* w_1 *and* w_2.

Proof. $I(\forall x Fx, w_1, v) = 0$ since there is a v^x_b such that $I(Fx, w_1, v^x_b) = 0$.

Similarly, $I(\forall x Fx, w_2, v) = 0$ since there is a v^x_a such that $I(Fx, w_2, v^x_a) = 0$.

$\forall w' \in W, I(\forall x Fx, w', v) = 0. \therefore I(\Diamond\forall x Fx, w, v) = 0.$ $\blacksquare$

Claim. $\forall x\Diamond Fx$ *is true at worlds* w_1 *and* w_2.

Proof. $I(\Diamond Fx, w, v^x_a) = 1$ since $I(Fx, w_1, v^x_a) = 1$.

Similarly, $I(\Diamond Fx, w, v^x_b) = 1$ since $I(Fx, w_2, v^x_b) = 1$.

Similarly, $I(\Diamond Fx, w, v^x_c) = 1$ since $I(Fx, w_1, v^x_c) = 1$.

No matter which assignment of values to variable x, $\Diamond Fx$ is true. $\therefore I(\forall x\Diamond Fx, w, v) = 1.$ $\blacksquare$

QUESTION 2

The example suggests a proof pattern that can be applied to all these four arguments.

Task 1

Claim. $\forall x \Box Fx \vDash_{CDQS5} \Box \forall x Fx$

Proof. Suppose we have a model $M = \langle D, W, I \rangle$ such that $I(\forall x \Box Fx, w, v) = 1$.

$$
\begin{aligned}
I(\forall x \Box Fx, w, v) = 1 \ &\textit{iff}\ I(\Box Fx, w, v') = 1, \forall v' : v' \sim_x v \\
&\textit{iff}\ I(Fx, w', v') = 1, \forall v' : v' \sim_x v, \forall w' \in W \\
&\textit{iff}\ I(\forall x Fx, w', v) = 1, \forall w' \in W \\
&\textit{iff}\ I(\Box \forall x Fx, w, v) = 1
\end{aligned}
$$

It is better to symbolically distinguish object- and metalevel quantifiers.

We have shown that in any model, in any world, and any assignments of values, if $\forall x \Box Fx$ is true, so is $\Box \forall x Fx$. ∎ ✓

Claim. $\Diamond \exists x Fx \vDash_{CDQS5} \exists x \Diamond Fx$

Proof. Suppose we have a model $M = \langle D, W, I \rangle$ such that $I(\Diamond \exists x Fx, w, v) = 1$.

$$
\begin{aligned}
I(\Diamond \exists x Fx, w, v) = 1 \ &\textit{iff}\ I(\exists x Fx, w', v) = 1, \exists w' \in W \\
&\textit{iff}\ I(Fx, w', v') = 1, \exists w' \in W, \exists v' : v' \sim_x v \\
&\textit{iff}\ I(\Diamond Fx, w, v') = 1, \exists v' : v' \sim_x v \\
&\textit{iff}\ I(\exists x \Diamond Fx, w, v) = 1
\end{aligned}
$$

We have shown that in any model, in any world, and any assignments of values, if $\Diamond \exists x Fx$ is true, so is $\exists x \Diamond Fx$. ∎ ✓

Task 2

Claim. $\Box \forall x Fx \vDash_{CDQS5} \forall x \Box Fx$

Proof. Suppose we have a model $M = \langle D, W, I\rangle$ such that $I(\Box\forall x Fx, w, v) = 1$.

$$\begin{aligned} I(\Box\forall x Fx, w, v) = 1 \ &\textit{iff}\ I(\forall x Fx, w', v) = 1, \forall w' \in W \\ &\textit{iff}\ I(Fx, w', v') = 1, \forall w' \in W, \forall v' : v' \sim_x v \\ &\textit{iff}\ I(\Box Fx, w, v') = 1, \forall v' : v' \sim_x v \\ &\textit{iff}\ I(\forall x \Box Fx, w, v) = 1 \end{aligned}$$

We have shown that in any model, in any world, and any assignments of values, if $\Box\forall x Fx$ is true, so is $\forall x \Box Fx$. ∎ ✓

Claim. $\exists x \Diamond Fx \vDash_{CDQS5} \Diamond \exists x Fx$

Proof. Suppose we have a model $M = \langle D, W, I\rangle$ such that $I(\exists x \Diamond Fx, w, v) = 1$.

$$\begin{aligned} I(\exists x \Diamond Fx, w, v) = 1 \ &\textit{iff}\ I(\Diamond Fx, w, v') = 1, \exists v' : v' \sim_x v \\ &\textit{iff}\ I(Fx, w', v') = 1, \exists v' : v' \sim_x v, \exists w' \in W \\ &\textit{iff}\ I(\exists x Fx, w', v) = 1, \exists w' \in W \\ &\textit{iff}\ I(\Diamond \exists x Fx, w, v) = 1 \end{aligned}$$

We have shown that in any model, in any world, and any assignments of values, if $\exists x \Diamond Fx$ is true, so is $\Diamond \exists x Fx$. ∎ ✓

QUESTION 3

Task 1

Claim. $\Box\forall xFx \vdash_{CDQS5} \forall x\Box Fx$

Proof.

$$\dfrac{\dfrac{\dfrac{\dfrac{\Box\forall xFx}{\forall xFx}\,\Box E}{Fa}\,\forall E}{\Box Fa}\,\Box I}{\forall x\Box Fx}\,\forall I$$

Here, since $\{\Box\forall xFx\}$ contains only modal formula, the $\Box I$ side condition is satisfied. Additionally, since the name a does not occur in the conclusion $\forall x\Box Fx$ of the $\forall I$ inference step and in any assumption in $\{\Box\forall xFx\}$ upon which the premise of this step depends, the eigenvariable condition of $\forall I$ is satisfied. ∎

Claim. $\exists x\Diamond Fx \vdash_{CDQS5} \Diamond\exists xFx$

Proof.

$$\dfrac{\dfrac{\exists x\Diamond Fx \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\Diamond Fa]^1 \qquad \dfrac{\dfrac{[Fa]^2}{\Box Fa}\,\Box I \qquad [\neg\Box Fa]^3}{\bot}\,\neg E}{\bot}\,\Diamond E^2}{\neg\neg\Box Fa}\,\neg I^3}{\Box Fa}\,DNE}{Fa}\,\Box E}{\exists xFx}\,\exists I}{\exists xFx}\,\exists E^1}{\Diamond\exists xFx}\,\Diamond I$$

Satisfaction of side or eigenvalue condition of inference steps will be explained from top to bottom. ¶ The side condition of $\Box I$ is satisfied because there is no assumption upon which $[Fa]^2$ depends. ¶ The side condition of $\Diamond E^2$ is satisfied because $\{Fa, \neg\Box Fa\} \setminus \{Fa\}$ contains only model formula. Notice that the excluded set contains the discharged assumption of this $\Diamond E^2$ step and steps before the minor premise. ¶ The eigenvalue condition of $\exists E^1$ is satisfied because the name a does not occur in $\{\exists x\Diamond Fx, \exists xFx\} \cup \{\Diamond Fa, Fa, \neg\Box Fa\} \setminus \{Fa, \neg\Box Fa, \Diamond Fa\}$. Notice that the excluded set contains the discharged assumption of this $\exists E^1$ step and steps before the minor premise. ∎

[Fa] itself is a non-necessitive assumption

Task 2

Definition 1 (Constant Domain Modal Formula). A formula is a *constant domain modal formula* if and only if it has the form, $\forall x A(x)$, $\neg\exists x A(x)$, $\exists x A(x)$, or $\neg\forall x A(x)$, where $A(x)$ is a modal formula, namely, $\Box B(x)$, $\neg\Diamond B(x)$, $\Diamond B(x)$, or $\neg\Box B(x)$, where $B(x)$ is a formula.

Definition 2 (CDQS5 Side Condition). All assumptions upon which the premise of an application of the rule $\Box I$ and the assumptions upon which the minor premise of an application of $\Diamond E$ depend, excluding the assumption discharged by $\Diamond E$, are all *constant domain modal formulas*.

Theorem 1 (Soundness). For any set X of formulas, and for any formula A in the language of constant domain quantified S5 (CDQS5) logic, if $X \vdash_{CDQS5} A$, then $X \vDash_{CDQS5} A$.

Proof. The soundness of the rules other than $\Box I$ and $\Diamond E$ is unaffected by the new definitions. We will focus on the soundness of these two affected rules.

Suppose a CDQS5 model, $M = \langle D, W, I \rangle$. Note that the accessibility relation R between worlds is an equivalance relation (reflexive, transitive, and symmetric) since CDQS5 is based on S5. Thus, all modal formula true at world $w_1 \in W$ are true at any world $w_2 \in W$ where $w_1 R w_2$.

We will first prove the soundness of $\Box I$ rule.

$$\dfrac{\begin{array}{c} X \\ \Pi \\ A \end{array}}{\Box A}\ \Box I$$

Suppose $X \succ A$ has no counterexample at any world and where the premises of $\Box I$ step satisfies the *CDQS5 side condition*.

Suppose a world $w_1 \in W$ is a counterexample to $X \succ \Box A$, then each member of X is true at world w_1, and a world $w_2 \in W$ would be a counterexample to the argument $X \succ A$. Since the side condition is satisfied, the members of X are *constant domain model formula*: $\forall x B(x)$, $\neg\exists x B(x)$, $\exists x B(x)$, or $\neg\forall x B(x)$, where $B(x)$ is a modal formula. The strategy is to prove that each memebr of X is also true at w_2, then w_2 is a counterexample to $X \succ A$, which contradicts the hypothesis that $X \succ A$ has no counterexample at any world, so the assumption is false.

$\forall x B(x)$ is true at w_1 iff $B(x)[o/x]$ is true for every object $o \in D_{w_1}$, where D_{w_1} denotes the domain of w_1. Since $w_1 R w_2$ and $B(x)[o/x]$ is a modal formula, we must have $B(x)[o/x]$ true at w_2 too. The domain of each world is the same as per the model definition, i.e., $D_{w_1} = D_{w_2}$, so $B(x)[o/x]$ is true for every object $o \in D_{w_2}$, as desired. Therefore, $\forall x B(x)$ is true at w_2.

By the De Morgan principles for quantifiers, $\neg\exists x C(x) \vdash \forall x \neg C(x)$. If $C(x)$ is a modal formula, by definition, so is $\neg C(x)$. By semantic substitution and replacing $\forall x B(x)$ in the above reasoning with this De Morgan relation, we prove that $\neg\exists x B(x)$ is true at w_2.

$\exists x B(x)$ is true at w_1 iff there is an object $o \in D_{w_1}$ such that $B(x)[o/x]$ is true. Since $B(x)[o/x]$ is a modal formula, we must have $B(x)[o/x]$ true at w_2 too. The domain of each world is the same, so $B(x)[o/x]$ is indeed true at w_2 since o is also in D_{w_2}. Therefore, $\exists x B(x)$ is true at w_2.

By the similar reasoning for $\neg\exists x C(x)$, we prove that $\neg\forall x B(x)$ is true at w_2.

We have proved that each constant domain modal formula is true at w_2, i.e., each member of X is true at w_2, so A must be false at w_2 (recall that w_2 would be a counterexample to $X \succ A$). This contradicts with the hypothesis that there is no counterexample to $X \succ A$ at any world. Thus, it is not the case that there is a world serves a counterexample to $X \succ \Box A$. Therefore, the $\Box$I rule is *sound* with the CDQS5 side condition.

In addition, we proved that all constant domain modal formula true at a world are true at any world accessible from that world because in the reasoning, the choice of w_1 and w_2 is arbitrary as long as they satisfy $w_1 R w_2$. We will use this lemma to prove the soundness of $\Diamond$E rule.

$$\dfrac{\Diamond A \qquad \begin{matrix} [A]^1 \quad X \\ \Pi \\ \bot \end{matrix}}{\bot}\ \Diamond E^1$$

Suppose $X, A \succ \bot$ has no counterexample at any world and where the minor premise of $\Diamond$E satisfies the CDQS5 side condition.

Suppose a world $w_1 \in W$ is a counterexample to $\Diamond A, X \succ \bot$. Then, $\Diamond A$ and each member of X are true at w_1. In consequence, A must be true at some world w_2 where $w_1 R w_2$. We have known that all constant domain modal formula true at a world are true at any world accessible from that world. Thus, each member of X are also true at w_2. Consequently, w_2 is a counterexample to $\Diamond A, X \succ \bot$, which contradicts the hypothesis that $X, A \succ \bot$ has no counterexample at any world. Thus, it is not the case that there is a world serves a counterexample to $\Diamond A, X \succ \bot$. Therefore, the $\Diamond$E rule is sound with the CDQS5 side condition. ∎

Claim. $\forall x\Box Fx \vdash_{CDQS5} \Box\forall xFx$, *with the CDQS5 side condition.*

Proof.

$$\dfrac{\dfrac{\dfrac{\dfrac{\forall x\Box Fx}{\Box Fa}\ \forall E}{Fa}\ \Box E}{\forall xFx}\ \forall I}{\Box\forall xFx}\ \Box I$$

Here, since the name a does not occur in the conclusion $\forall xFx$ of the $\forall I$ inference step and in any assumption in $\{\forall x\Box Fx\}$ upon which the premise of this step depends, the eigenvariable condition of $\forall I$ is satisfied. Furthermore, since $\{\forall x\Box Fx\}$ contains only *constant domain modal formula*, the CDQS5 side condition of $\Box I$ is satisfied. ∎

Claim. $\Diamond\exists xFx \vdash_{CDQS5} \exists x\Diamond Fx$, *with the CDQS5 side condition.*

Proof.

$$\dfrac{\dfrac{\dfrac{\Diamond\exists xFx \qquad \dfrac{[\exists xFx]^1 \qquad \dfrac{\dfrac{\dfrac{[Fa]^2}{\Diamond Fa}\ \Diamond I}{\exists x\Diamond Fa}\ \exists I \qquad [\neg\exists x\Diamond Fx]^3}{\bot}\ \neg E}{\bot}\ \exists E^2}{\bot}\ \Diamond E^1}{\neg\neg\exists x\Diamond Fx}\ \neg I^3}{\exists x\Diamond Fx}\ DNE$$

The eigenvalue condition of $\exists E^2$ is satisfied because the name a does not occur in $\{\exists xFx, \bot\}\cup\{Fa, \neg\exists x\Diamond Fx\}\setminus\{Fa\}$. Furthermore, the CDQS5 side condition of $\Diamond E^1$ is satisfied because $\{\exists xFx, Fa, \neg\exists x\Diamond Fx\} \setminus \{\exists xFx, Fa\}$ contains only *constant domain model formula*. ∎

QUESTION 4

which project?

The whole project seems to gradually shift the spotlight to the Barcan Formula (BF), implying that BF is the core of the controversy. Nonetheless, from readings one and two, it is understood that BF merely reflects one aspect of the controversy of Actualism. Another inference's validity, the Converse Barcan Formula (CBF), is more controversial than BF. This is because if CBF holds, it would deduce a logically impossible conclusion that there could be something that is distinct from itself. This suggests that the real focus might be on the logical challenges of Actualism. However, through reading three, it is found that the focus is not just on the debate, either metaphysical or logical, between Possibilism and Actualism, but more on Simple Quantified Modal Logic (SQML). The wording in Option 1 also hints that we are expected to defend a type of SQML, CDQS5, but to merely explain the metaphysical validity of BF.

is this logical impossibility? — worth discussing

In reading three, although authors claim that they are Possibilists, they take the position of Actualism in that paper. They propose a new semantic called contingently unconcrete, trying to repair the elegant SQML for Actualism, and hope to stop the continuous development and patching of ugly new logics in response to their logical challenges. This strange act of helping the opponent reveals that the authors' interests are not in which metaphysical stance but in SQML. The title of the article is not about defending Possibilism or Actualism, but SQML. That is to say, their real interest is in promoting SQML.

explain?

Possibilism is primarily concerned with SQML and does not care much about the validity of BF, and might even not be concerned about whether Possibilism is justified in metaphysics. For Possibilists, whether BF is valid or not is merely a choice. They can easily make BF valid by modifying semantics. For instance, Possibilists propose that the existence quantifier could be read as existence unloaded, or, as in reading three, consider some objects as contingently unconcrete. However, so far, the means by which Possibilism protects SQML inevitably need to appeal to possibilia, a concept opposed by Actualism. Possibilists rarely choose to invalidate BF, as it seems challenging to find an SQML that satisfies this condition, contradicting their pursuit of simple and elegant logic.

explain?

Since Actualism insists that the existence quantifier is existence loaded, they must reject BF. Rejecting BF forces them to seek a logic system beyond SQML. The logic proposed by Kripke revealed the feasibility of what they were pursuing. Kripke's Quantified Modal Logic, using varying domains and restricted quantifiers, solved three logical challenges faced by Actualism: BF, CBF, and NE, though it still use the concept of possibilia. Even with such a logical revelation, Actualism has not yet found a consistent logic system for their thesis.

Actualism's stance also seems incompatible with current developments in social metaphysics. Social metaphysics proposes social kinds, which exist within networks of social practices. As the network structure changes, existing social kinds may disappear, and nonexistent social kinds may emerge. For example, some social realities of the medieval period no longer exist today, and modern concepts of gender are being created. In this view, social kinds are a form of possibilia. Actualism's thesis would also deny social metaphysics, so not only in logic, but in metaphysics as well, Actualism faces severe challenges.

The successful interpretation of SQML by Possibilists, the various unsuccessful responses made by Actualism to logical challenges like BF, and the challenges faced by Actualism in contemporary metaphysics development all seem to hint at the triumph of SQML. However, this situation might only be temporary. We do not have sufficient arguments to prove that Actualism's claims have failed, nor enough evidence to justify the metaphysical correctness of Possibilism. As long as Actualism has not completely failed, they still hold the hope of finding a consistent logic for themselves. Logicians without a clear metaphysical stance, like the authors in reading three, can attempt to help Actualism find such logic, even though they appreciate the elegance of SQML more. Perhaps this is the motivation for setting the option 2 for this question.

You have shown some good engagement with the literature, although your exposition would have benefitted from presenting and explaining the arguments in the readings, rather than mere positions. Examples would have aided your exposition. Although you show some independent engagement with the problem, this is brief and under-developed. Arguments could also help here. Unfortunately, you did not state the position you are defending. You need to provide a bibliography and clearer referencing and citations. No further reading is referenced. Try engaging with the BF on a formal level.

- attention to literature ✓✗
- independent reading
- correctness of formal details ✗
- philosophical understanding & creativity ✓