PY4669 Modal Logic & Metaphysics

Take-Home Assignment 1
Due Tuesday 13 February at midnight

This assignment will be marked out of 20. It will be worth 20% of your final grade. Each question will carry the points indicated. Some questions have sub-questions (a), (b), ... You must answer each sub-question for the answer to be complete.

By submitting this assignment, you agree that your submission is all your own work. Please do not write your name anywhere in this document.

You are to edit this file, writing your answers in the space provided below each question. Then submit the Word file of your answers on MMS. NB when a maximum number of words is specified for discursive answers, the number is indicative: try to stick to the word limit, but you won't be penalized if you exceed it.

Q1 (a) Explain the connection between modal *axioms* and *conditions on the accessibility relation* $\mathcal{R}$ between worlds in propositional modal frames $<\mathcal{G}, \mathcal{R}>$. (b) Which conditions correspond, respectively, to (T) $\Box P \supset P$, (B) $P \supset \Box\Diamond P$, (S4) $\Box P \supset \Box\Box P$, and (S5) $\Diamond P \supset \Box\Diamond P$?

[**4 points, 3 for sub-question (a) and 1 for (b)**]

Q2 What is the *smallest* countermodel to the (D)-principle $\Box P \supset \Diamond P$, i.e., the one with fewest worlds? Describe such a countermodel and explain why it works.

[**5 points**]

Q3 (a) Explain what each of Quine's three grades of modal involvement consists in. (b) Explain Quine's argument for the conclusion that modalities cannot be features of objects, but rather of how we describe them. (c) Give a *brief* critical assessment of Quine's argument.

[*Max. 500 words overall*]
[**5 points: 1 for sub-question (a), 2 each for (b) and (c)**]

Q4 (a) Why is the Barcan (schematic) Formula

(BF) $\Diamond(\exists x)X \supset (\exists x)\Diamond X$

valid in constant domain semantics? (I.e., all instances of BF are true at all worlds of all constant domain models.) (b) Explain why one may question the validity of BF. (c) Explain how the BF is invalidated in variable domain semantics.

[*Max. 500 words overall*]
[**6 points, 2 per sub-question**]

*****END OF ASSIGNMENT*****

Mark 15.5 / 20

PY4669 Modal Logic & Metaphysics

Take-Home Assignment 1

13th February 2024

I hereby declare that the attached piece of written work is my own work and that I have not reproduced, without acknowledgement, the work of another.

QUESTION 1

2/3

a) Modal axioms are formulas that are added to a modal logic system to capture certain intuitions about necessity, possibility, and other modal concepts. Possible worlds semantics provides a way to validate specific modal axioms: a modal axiom is valid if modal frames $\langle \mathcal{G}, \mathcal{R} \rangle$ whose accessibility relation $\mathcal{R}$ between worlds satisfies certain algebraic condition required by the axiom. Different modal axioms impose specific conditions on the accessibility relation. This means that, under possible worlds semantics, we can define modal logics by specifying sets of frames that meet various algebraic conditions so as to validate intended modal axioms and invalidate unintended modal axioms.

1/1

b) Each axiom scheme is valid in frames that meet the corresponding algebraic condition.

Axiom Scheme	Algebraic Condition
(T) $\Box P \supset P$	reflexive
(B) $P \supset \Box\Diamond P$	symmetric
(S4) $\Box P \supset \Box\Box P$	transitive
(S5) $\Diamond P \supset \Box\Diamond P$	Euclidean

QUESTION 2

$$\boxed{\Gamma} \Vdash$$

5/5

Γ is the only possible world in this model that cannot access itself. It follows that $\Gamma \Vdash \Box P$ since $(\forall \Delta \in \mathcal{G})\, \Gamma\mathcal{R}\Delta \supset \Delta \Vdash P$ (When $\Delta \leftarrow \Gamma$, the antecedent $\Gamma\mathcal{R}\Gamma$ is false so the conditional is true. Because there is only one world Γ, the universal quantification is true.). $\Gamma \nVdash \Diamond P$ since $(\exists \Delta \in \mathcal{G})\, \Gamma\mathcal{R}\Delta \wedge \Delta \Vdash P$ is false (When $\Delta \leftarrow \Gamma$, the left conjunct $\Gamma\mathcal{R}\Gamma$ is false so the conjunction is false. Because there is only one world Γ, the existential quantification is false.). Consequently, $\Gamma \nVdash \Box P \supset \Diamond P$.

QUESTION 3

1/1

a) The first degree is attaching "□" as a metalinguistic predicate to names of statements (a statement may have different formulation).

$$\Box\text{"}9 > 5\text{"}$$

The second degree is *de dicto* modality; The symbol "□" is attached to the statements themselves. This conflates metalanguage and object language.

$$\Box 9 > 5$$

The third degree is *de re* modality. The symbol "□" is attached to a formula contains free occurrence of variable x:

$$\Box x > 5$$

This means that at the third degree, modalisers can appear within the scope of quantification. This is distinct from the second degree, where modalisers can only appear outside the scope of quantification, and thus can only modify closed formulae.

1/2

b) Quine's argument against modalities as features of objects is also tied to his rejection of essentialism — the idea that objects have some essential properties that define their identity across all possible worlds. For Quine, there are no necessary or essential properties of objects in this sense; rather, all properties are contingent upon the ways in which we choose to describe the world.

0,5/2

c) *de re* modality does not imply any essential properties. It just suggest that discussing in such terms is logical.

QUESTION 4

2/2

a) *Proof.* Consider an arbitrary constant domain first-order model $\mathcal{M} = \langle \mathcal{G}, \mathcal{R}, \mathcal{D}, \mathcal{I} \rangle$. We want to show that, for any arbitrary valuation v and possible world $w \in \mathcal{G}$, we have $\mathcal{M}, w \Vdash_v$

$\forall x \Box Fx \iff \mathcal{M}, w \Vdash_v \Box \forall x Fx.$

$$\begin{aligned}
\mathcal{M}, w \Vdash_v \forall x \Box Fx &\iff \mathcal{M}, w \Vdash_{v'} \Box Fx, \forall v' \colon v' \sim_x v \\
&\iff \mathcal{M}, w' \Vdash_{v'} Fx, \forall v' \colon v' \sim_x v, \forall w' \colon w \mathcal{R} w' \\
&\iff \mathcal{M}, w' \Vdash_v \forall x Fx, \forall w' \colon w \mathcal{R} w' \\
&\iff \mathcal{M}, w \Vdash_v \Box \forall x Fx
\end{aligned}$$

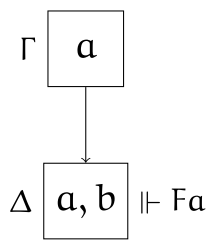
We have shown that in any constant domain first-order model, in any world, and any assignments of values, $\forall x \Box Fx \iff \Box \forall x Fx$ is valid. Barcan Formula is the only if direction of this conclusion. ∎

b) 1/2 Consider the equivalent form of Barcan Formula:

$$\Diamond \exists x Fx \supset \exists x \Diamond Fx$$

It blurs the lines between existence in possible worlds and existence in the actual world. We can interpret this formula as follows: In some possbile worlds, there exists a person who is my sister implies that there exists a person in the actual world who possibly is my sister. This is counterintuitive because it suggests that objects exist in possible world also exist in the actual world that has access to it.

c) *Proof.* We want to show that $\forall x \Box Fx \supset \Box \forall x Fx$ is invalid in some varying domain first-order models. 2/2



This is a varying domain countermodal to Barcan Formula. In this model, we have

$$\mathcal{M}, \Delta \Vdash_v Fa$$

Since Γ can only access Δ where $\Delta \Vdash_v Fa$,

$$\mathcal{M}, \Gamma \Vdash_v \Box Fa$$

$v'(x) = a$ is the only x-variant of v at Γ, so

$$\mathcal{M}, \Gamma \Vdash_v \forall x \Box Fx$$

On the other hand, suppose we had

$$\mathcal{M}, \Gamma \Vdash_v \Box \forall x Fx$$

Subsequently,

$$\mathcal{M}, \Delta \Vdash_v \forall x Fx$$

However, this is not true because $\Delta \nVdash_{v'} Fb$. Thus,

$$\mathcal{M}, \Gamma \nVdash_v \Box \forall x Fx$$

Therefore, Barcan Formula is invalid in varying domain first-order models whose domains grow when moving to accessible worlds. ∎