University of St Andrews

PY2010 Intermediate Logic

taught by Prof Greg Restall

Exercises Week I

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I hereby declare that the attached piece of written work is my own work and that I have not reproduced, without acknowledgement, the work of another.

1. Forms that are formulas:

   - $p \vee q$
   - $\neg\neg p$
   - $(p \to q) \to ((p \to (q \to r)) \to (p \to q))$

   Ambiguous forms:

   - $p \vee q \to r$ which can be disambiguated to $(p \vee q) \to r$ or $p \vee (q \to r)$
   - $p \wedge (q \vee r) \to \bot$ which can be disambiguated to $(p \wedge (q \vee r)) \to \bot$ or $p \wedge ((q \vee r) \to \bot)$
   - $p \wedge q \wedge r$ which can be disambiguated to $(p \wedge q) \wedge r$ or $p \wedge (q \wedge r)$

   Finally, $q\neg p$ is not a formula because the negation ($\neg$) is a unary logical connective.

2. - $p \leqslant \{p \wedge r\}$
   - $q \leqslant \{p \to q, p \wedge r\}$ and $r \leqslant \{p \wedge r\}$
   - $q \wedge r \leqslant \{p \to q, p \wedge r \text{ left}, p \wedge r \text{ right}\}$
   - $(p \wedge r) \to (q \wedge r) \leqslant \{p \to q\}$

3. -
   $$\dfrac{\dfrac{\dfrac{[r \to p]^2 \quad [r]^1}{p}\to E \qquad p \to q}{q}\to E}{\dfrac{r \to q}{(r \to p) \to (r \to q)}\to I^1}\to I^2$$
   -
   $$\dfrac{\dfrac{p \quad [q]^1}{p \wedge q}\wedge I}{q \to (p \wedge q)}\to I^1$$
   -
   $$\dfrac{\dfrac{p \wedge (q \to r)}{p}\wedge E \qquad \dfrac{\dfrac{p \wedge (q \to r)}{q \to r}\wedge E \qquad [q]^1}{r}\to E}{\dfrac{p \wedge r}{q \to (p \wedge r)}\wedge I}\to I^1$$

4. (a)

$$\dfrac{\dfrac{\dfrac{\overset{\Pi_1}{A \to B} \quad [A]^1}{B}\,{\to}E \quad \dfrac{\overset{\Pi_2}{A \to C} \quad [A]^1}{C}\,{\to}E}{B \wedge C}\,{\wedge}I}{A \to (B \wedge C)}\,{\to}I^1$$

(b)

$$\dfrac{\dfrac{\overset{\Pi}{A \to (B \to C)} \quad \dfrac{[A \wedge B]^1}{A}\,{\wedge}E}{B \to C}\,{\to}E \quad \dfrac{[A \wedge B]^1}{A}\,{\wedge}E}{C}\,{\to}E}{(A \wedge B) \to C}\,{\to}I^1$$

(c)

$$\dfrac{\dfrac{\dfrac{\overset{\Pi}{(A \wedge B) \to C} \quad \dfrac{[A]^2 \quad [B]^1}{A \wedge B}\,{\wedge}I}{C}\,{\to}E}{B \to C}\,{\to}I^1}{A \to (B \to C)}\,{\to}I^2$$