WEEK 1 EXERCISES 1

1. [Chapter 1, Basic Question 3.]

   Which of these are formulas in our formal language Form, and which are not?

   $$p \lor q \qquad p \lor q \to r \qquad \neg\neg p \qquad q \neg p \qquad p \land (q \lor r) \to \bot$$

   $$(p \to q) \to ((p \to (q \to r)) \to (p \to r)) \qquad p \land q \land r$$

   For those that aren't formulas, are they ambiguous? (Could they be made into correct formulas in different ways by adding parentheses?) If they are, disambiguate them, by listing all of the different ways they can be made formulas, and consider for yourself the different things they could *mean*.

2. [Chapter 2, Basic Question 1 (i).]

   Read this proof from top to bottom, and at every inference step, list which assumptions each formula depends on.

   $$\dfrac{\dfrac{\dfrac{p \to q \qquad \dfrac{[p \land r]^1}{p}\,{\scriptstyle \land E}}{q}\,{\scriptstyle \to E} \qquad \dfrac{[p \land r]^1}{r}\,{\scriptstyle \land E}}{q \land r}\,{\scriptstyle \land I}}{(p \land r) \to (q \land r)}\,{\scriptstyle \to I^1}$$

3. [Chapter 2, Basic Question 2, (iii,iv,v).]

   Construct proofs for the following arguments:

   - From the assumption $p \to q$ to $(r \to p) \to (r \to q)$.
   - From p to $q \to (p \land q)$.
   - From $p \land (q \to r)$ to $q \to (p \land r)$.

4. [Chapter 2, Challenge Question 2, (three parts).]

   Show that the following rules are derived rules of the proof system defined in Chapter 2.

   (a) $\dfrac{A \to B \qquad A \to C}{A \to (B \land C)}\ (\to\land)$

   (b) $\dfrac{A \to (B \to C)}{(A \land B) \to C}\ (\mathit{Import})$

   (c) $\dfrac{(A \land B) \to C}{A \to (B \to C)}\ (\mathit{Export})$

KEY CONCEPTS AND SKILLS

- ☐ You can identify premises and conclusions in a course of reasoning presented in a natural language argument.
- ☐ You understand the definitions of the concepts *partial order* and *tree*. You know how to check if a partial order is also a tree, and you can construct examples of partial orders that aren't trees. You can represent finite trees in tree diagrams.
- ☐ You can represent the structure of reasoning of simple arguments in the form of a tree, distinguishing premises and conclusions, individual inference steps and recognising the ultimate conclusion of a proof.
- ☐ You can construct formulas in the formal propositional language Form. You know how to read formulas, recognising conjunction ($\wedge$), disjunction ($\vee$), the conditional ($\to$) and negation ($\neg$), and you are able to detect whether something is actually a formula or if it is not formed using the formation rules of the formal language Form.
- ☐ You can identify the main connective of a complex formula, and the subformulas of a formula.
- ☐ You should be able to *read* tree proofs using the rules $\wedge$E, $\wedge$I, $\to$E and $\to$I. You should be able to check that a proof follows the rules, and you should be able to keep track of which assumptions are active at each stage of the proof.
- ☐ You should be able to *construct* simple tree proofs using the rules $\wedge$E, $\wedge$I, $\to$E and $\to$I.
- ☐ You can perform *reductions* on tree proofs which involve detours, using the reduction steps.