```
In[1]:= λ1 = {{0, 1, 0}, {1, 0, 0}, {0, 0, 0}};
        λ2 = {{0, -ⅈ, 0}, {ⅈ, 0, 0}, {0, 0, 0}};
        λ3 = {{1, 0, 0}, {0, -1, 0}, {0, 0, 0}};
        λ4 = {{0, 0, 1}, {0, 0, 0}, {1, 0, 0}};
        λ5 = {{0, 0, -ⅈ}, {0, 0, 0}, {ⅈ, 0, 0}};
        λ6 = {{0, 0, 0}, {0, 0, 1}, {0, 1, 0}};
        λ7 = {{0, 0, 0}, {0, 0, -ⅈ}, {0, ⅈ, 0}};
        λ8 = 1 / Sqrt[3] {{1, 0, 0}, {0, 1, 0}, {0, 0, -2}};
        Gellmann = {λ1, λ2, λ3, λ4, λ5, λ6, λ7, λ8}
```

Out[9]= $\{\{\{0,1,0\},\{1,0,0\},\{0,0,0\}\},\{\{0,-i,0\},\{i,0,0\},\{0,0,0\}\},\{\{1,0,0\},\{0,-1,0\},\{0,0,0\}\},\{\{0,0,1\},\{0,0,0\},\{1,0,0\}\},\{\{0,0,-i\},\{0,0,0\},\{i,0,0\}\},\{\{0,0,0\},\{0,0,1\},\{0,1,0\}\},\{\{0,0,0\},\{0,0,-i\},\{0,i,0\}\},\{\{\frac{1}{\sqrt{3}},0,0\},\{0,\frac{1}{\sqrt{3}},0\},\{0,0,-\frac{2}{\sqrt{3}}\}\}\}$

```
In[10]:= Gellmann[[1]]
Out[10]= {{0, 1, 0}, {1, 0, 0}, {0, 0, 0}}

In[11]:= CartanH = {λ3, λ8} / 2
```

Out[11]= $\{\{\{\frac{1}{2},0,0\},\{0,-\frac{1}{2},0\},\{0,0,0\}\},\{\{\frac{1}{2\sqrt{3}},0,0\},\{0,\frac{1}{2\sqrt{3}},0\},\{0,0,-\frac{1}{\sqrt{3}}\}\}\}$

```
In[143]:= ladderE = {λ1 + ⅈ λ2, λ4 + ⅈ λ5, λ6 + ⅈ λ7} / 2;
          ladderF = Table[ConjugateTranspose[ladderE[[i]]], {i, 3}]

Out[144]= {{{0, 0, 0}, {1, 0, 0}, {0, 0, 0}},
           {{0, 0, 0}, {0, 0, 0}, {1, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 1, 0}}}

In[17]:= Commute[x_, y_] := Dot[x, y] - Dot[y, x]

In[18]:= Commute[CartanH[[1]], ladderE[[1]]]

Out[18]= {{0, 1, 0}, {0, 0, 0}, {0, 0, 0}}

In[137]:= {Table[ladderE[[i]], {i, 3}]} // MatrixForm
```

Out[137]//MatrixForm=

$$\left(\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix}\ \begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}\ \begin{pmatrix}0&0&0\\0&0&1\\0&0&0\end{pmatrix}\right)$$

```
In[145]:= {Table[ladderF[[j]], {j, 3}]} // MatrixForm
```

Out[145]//MatrixForm=

$$\left(\begin{pmatrix}0&0&0\\1&0&0\\0&0&0\end{pmatrix}\ \begin{pmatrix}0&0&0\\0&0&0\\1&0&0\end{pmatrix}\ \begin{pmatrix}0&0&0\\0&0&0\\0&1&0\end{pmatrix}\right)$$

In[146]:=
```
CommutationRelationE := Table[{{"H"_i, "E"_j},
    "=" MatrixForm[Commute[CartanH[[i]], ladderE[[j]]]]}, {i, 2}, {j, 3}];
TextGrid[CommutationRelationE, Frame → All]
```

Out[147]=

$\{\{H_1, E_1\}, = \begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{H_1, E_2\}, = \begin{pmatrix}0&0&\frac{1}{2}\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{H_1, E_3\}, = \begin{pmatrix}0&0&0\\0&0&-\frac{1}{2}\\0&0&0\end{pmatrix}\}$
$\{\{H_2, E_1\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{H_2, E_2\}, = \begin{pmatrix}0&0&\frac{\sqrt{3}}{2}\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{H_2, E_3\}, = \begin{pmatrix}0&0&0\\0&0&\frac{\sqrt{3}}{2}\\0&0&0\end{pmatrix}\}$

In[148]:=
```
CommutationRelationF := Table[{{"H"_i, "F"_j},
    "=" MatrixForm[Commute[CartanH[[i]], ladderF[[j]]]]}, {i, 2}, {j, 3}];
TextGrid[CommutationRelationF, Frame → All]
```

Out[149]=

$\{\{H_1, F_1\}, = \begin{pmatrix}0&0&0\\-1&0&0\\0&0&0\end{pmatrix}\}$	$\{\{H_1, F_2\}, = \begin{pmatrix}0&0&0\\0&0&0\\-\frac{1}{2}&0&0\end{pmatrix}\}$	$\{\{H_1, F_3\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&\frac{1}{2}&0\end{pmatrix}\}$
$\{\{H_2, F_1\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{H_2, F_2\}, = \begin{pmatrix}0&0&0\\0&0&0\\-\frac{\sqrt{3}}{2}&0&0\end{pmatrix}\}$	$\{\{H_2, F_3\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&-\frac{\sqrt{3}}{2}&0\end{pmatrix}\}$

In[150]:=
```
CommutationRelationEE := Table[{{"E"_i, "E"_j},
    "=" MatrixForm[Commute[ladderE[[i]], ladderE[[j]]]]}, {i, 3}, {j, 3}];
TextGrid[CommutationRelationEE, Frame → All]
```

Out[151]=

$\{\{E_1, E_1\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{E_1, E_2\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{E_1, E_3\}, = \begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}\}$
$\{\{E_2, E_1\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{E_2, E_2\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{E_2, E_3\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$
$\{\{E_3, E_1\}, = \begin{pmatrix}0&0&-1\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{E_3, E_2\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{E_3, E_3\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$

In[152]:=
```
CommutationRelationFF := Table[{{"F"_i, "F"_j},
    "=" MatrixForm[Commute[ladderF[[i]], ladderF[[j]]]]}, {i, 3}, {j, 3}];
TextGrid[CommutationRelationFF, Frame → All]
```

Out[153]=

$\{\{F_1, F_1\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{F_1, F_2\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{F_1, F_3\}, = \begin{pmatrix}0&0&0\\0&0&0\\-1&0&0\end{pmatrix}\}$
$\{\{F_2, F_1\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{F_2, F_2\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{F_2, F_3\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$
$\{\{F_3, F_1\}, = \begin{pmatrix}0&0&0\\0&0&0\\1&0&0\end{pmatrix}\}$	$\{\{F_3, F_2\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$	$\{\{F_3, F_3\}, = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\}$

In[154]:=
```
CommutationRelationEF := Table[{{"E"i, "F"j},
    "=" MatrixForm[Commute[ladderE[[i]], ladderF[[j]]]]}, {i, 3}, {j, 3}];
TextGrid[CommutationRelationEF, Frame → All]
```

Out[155]=

$\left\{\{E_1, F_1\}, = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right\}$	$\left\{\{E_1, F_2\}, = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}\right\}$	$\left\{\{E_1, F_3\}, = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right\}$
$\left\{\{E_2, F_1\}, = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}\right\}$	$\left\{\{E_2, F_2\}, = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}\right\}$	$\left\{\{E_2, F_3\}, = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right\}$
$\left\{\{E_3, F_1\}, = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right\}$	$\left\{\{E_3, F_2\}, = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right\}$	$\left\{\{E_3, F_3\}, = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\right\}$