```
? Dot
```

a.*b*.*c* or Dot[*a*, *b*, *c*] gives products of vectors, matrices, and tensors. ≫

```
In[1]:= Commute[mat1_, mat2_] := Dot[mat1, mat2] - Dot[mat2, mat1]

Commute[{{1, 2}, {2, 1}}, {{3, 4}, {1, 2}}]

{{-6, -2}, {2, 6}}
```

Pauli Matrices

```
In[2]:= σ1 = {{0, 1}, {1, 0}};
σ2 = {{0, ⅈ}, {-ⅈ, 0}};
σ3 = {{1, 0}, {0, -1}}

Out[4]= {{1, 0}, {0, -1}}
```

Gellmann Matrices

```
In[5]:= λ1 = {{0, 1, 0}, {1, 0, 0}, {0, 0, 0}};
λ2 = {{0, -ⅈ, 0}, {ⅈ, 0, 0}, {0, 0, 0}};
λ3 = {{1, 0, 0}, {0, -1, 0}, {0, 0, 0}};
λ4 = {{0, 0, 1}, {0, 0, 0}, {1, 0, 0}};
λ5 = {{0, 0, -ⅈ}, {0, 0, 0}, {ⅈ, 0, 0}};
λ6 = {{0, 0, 0}, {0, 0, 1}, {0, 1, 0}};
λ7 = {{0, 0, 0}, {0, 0, -ⅈ}, {0, ⅈ, 0}};
λ8 = 1 / √3 {{1, 0, 0}, {0, 1, 0}, {0, 0, -2}};

In[13]:= GellMann = {λ1, λ2, λ3, λ4, λ5, λ6, λ7, λ8}

Out[13]= {{{0, 1, 0}, {1, 0, 0}, {0, 0, 0}}, {{0, -ⅈ, 0}, {ⅈ, 0, 0}, {0, 0, 0}},
  {{1, 0, 0}, {0, -1, 0}, {0, 0, 0}}, {{0, 0, 1}, {0, 0, 0}, {1, 0, 0}},
  {{0, 0, -ⅈ}, {0, 0, 0}, {ⅈ, 0, 0}}, {{0, 0, 0}, {0, 0, 1}, {0, 1, 0}},
  {{0, 0, 0}, {0, 0, -ⅈ}, {0, ⅈ, 0}}, {{1/√3, 0, 0}, {0, 1/√3, 0}, {0, 0, -2/√3}}}

GellMann[[1]]

{{0, 1, 0}, {1, 0, 0}, {0, 0, 0}}
```

Lie Algebra of Pauli Matrices (SU(2))

```
Commute[σ1, σ2] / ⅈ

{{-2, 0}, {0, 2}}

PauliMatrix[{3}]

{{{1, 0}, {0, -1}}}

Table[PauliMatrix[k], {k, 1, 3}]

{{{0, 1}, {1, 0}}, {{0, -ⅈ}, {ⅈ, 0}}, {{1, 0}, {0, -1}}}

In[15]:= Perm2 = Permutations[{1, 2, 3}, {2}]

Out[15]= {{1, 2}, {1, 3}, {2, 1}, {2, 3}, {3, 1}, {3, 2}}
```

```
In[16]:= PauliPerm = Permutations[Table[PauliMatrix[k], {k, 1, 3}], {2}]

Out[16]= {{{{0, 1}, {1, 0}}, {{0, -i}, {i, 0}}}, {{{0, 1}, {1, 0}}, {{1, 0}, {0, -1}}},
  {{{0, -i}, {i, 0}}, {{0, 1}, {1, 0}}}, {{{0, -i}, {i, 0}}, {{1, 0}, {0, -1}}},
  {{{1, 0}, {0, -1}}, {{0, 1}, {1, 0}}}, {{{1, 0}, {0, -1}}, {{0, -i}, {i, 0}}}}
```

```
PauliCom = Table[Commute[PauliPerm[[k]][[1]], PauliPerm[[k]][[2]]], {k, 1, 6}]

{{{2 i, 0}, {0, -2 i}}, {{0, -2}, {2, 0}}, {{-2 i, 0}, {0, 2 i}},
 {{0, 2 i}, {2 i, 0}}, {{0, 2}, {-2, 0}}, {{0, -2 i}, {-2 i, 0}}}
```

```
Table[{f^ToString[Flatten[Join[Perm2[[k]], Complement[{1, 2, 3}, Perm2[[k]]]]]],
   -i * Dot[PauliCom[[k]], Inverse[Flatten[
        PauliMatrix[Complement[{1, 2, 3}, Perm2[[k]]]], 1]]][[1]][[1]]}, {k, 1, 6}]
```

$\{\{f^{\{1,2,3\}}, 2\}, \{f^{\{1,3,2\}}, -2\}, \{f^{\{2,1,3\}}, -2\}, \{f^{\{2,3,1\}}, 2\}, \{f^{\{3,1,2\}}, 2\}, \{f^{\{3,2,1\}}, -2\}\}$

Note that structure constant of SU(2) is just Levi-Civita symbol (multiplied by 2)

Now, SU(3)

```
In[17]:= GellPerm = Permutations[GellMann, {2}]

In[19]:= GellCom = Table[Commute[GellPerm[[k]][[1]], GellPerm[[k]][[2]]] / i, {k, 1, 56}]

In[21]:= Perm3 = Permutations[{1, 2, 3, 4, 5, 6, 7, 8}, {2}]

Length[Perm3]

56

Commute[GellMann[[1]], GellMann[[2]]] // MatrixForm
```

$$\begin{pmatrix} 2i & 0 & 0 \\ 0 & -2i & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
In[22]:= Manipulate[{GellCom[[k]] // MatrixForm, Perm3[[k]]}, {k, 1, 42, 1}]
```

Out[22]=

k

$$\left\{\begin{pmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \{1, 2\}\right\}$$

```
In[24]:= Aset = {a1, a2, a3, a4, a5, a6, a7, a8}

Out[24]= {a1, a2, a3, a4, a5, a6, a7, a8}

In[25]:= Aset

Out[25]= {a1, a2, a3, a4, a5, a6, a7, a8}
```

In[29]:=
```
GellSet = Table[Flatten[Aset /.
    Solve[Sum[Aset[[j]] GellMann[[j]], {j, 1, 8}] == GellCom[[k]], Aset], 2], {k, 1, 56}]
```

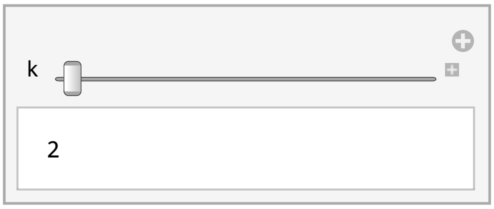
Out[29]=
```
{{0, 0, 2, 0, 0, 0, 0, 0}, {0, -2, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -2, 0, 0, 0, 0, 0},
 {2, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0},
 {0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 2, 0, 0, 0, 0, 0, 0}, {-2, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, -1, 0, 0},
 {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, √3}, {0, 1, 0, 0, 0, 0, 0, 0},
 {1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -√3, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, -√3},
 {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, √3, 0, 0, 0, 0},
 {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0},
 {0, -1, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, √3},
 {0, 0, 0, 0, 0, 0, -√3, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, -√3}, {0, 0, 0, 0, 0, √3, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, √3, 0, 0, 0},
 {0, 0, 0, -√3, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, √3, 0}, {0, 0, 0, 0, 0, -√3, 0, 0}}
```

Now define new function which detect non-zero values and tell where they are.

In[42]:=
```
Detect[list_] := Which[
  Total[list] > 0, {Max[list], Ordering[list, -1]},
  Total[list] == 0, {0, {0}},
  Total[list] < 0, {Min[list], Ordering[list, 1]}
 ]
```

In[45]:=
```
Manipulate[Detect[GellSet[[k]]][[1]], {k, 1, 56, 1}]
```

Out[45]=



In[49]:=
```
Table[{f^ToString[Join[Perm3[[k]], Detect[GellSet[[k]]][[2]]]],
  Detect[GellSet[[k]]][[1]]}, {k, 1, 56, 1}]
```

Out[49]=
$\{\{f^{\{1,2,3\}},2\},\{f^{\{1,3,2\}},-2\},\{f^{\{1,4,7\}},1\},\{f^{\{1,5,6\}},-1\},$
$\{f^{\{1,6,5\}},1\},\{f^{\{1,7,4\}},-1\},\{f^{\{1,8,0\}},0\},\{f^{\{2,1,3\}},-2\},\{f^{\{2,3,1\}},2\},$
$\{f^{\{2,4,6\}},1\},\{f^{\{2,5,7\}},1\},\{f^{\{2,6,4\}},-1\},\{f^{\{2,7,5\}},-1\},\{f^{\{2,8,0\}},0\},$
$\{f^{\{3,1,2\}},2\},\{f^{\{3,2,1\}},-2\},\{f^{\{3,4,5\}},1\},\{f^{\{3,5,4\}},-1\},\{f^{\{3,6,7\}},-1\},$
$\{f^{\{3,7,6\}},1\},\{f^{\{3,8,0\}},0\},\{f^{\{4,1,7\}},-1\},\{f^{\{4,2,6\}},-1\},\{f^{\{4,3,5\}},-1\},$
$\{f^{\{4,5,8\}},\sqrt{3}\},\{f^{\{4,6,2\}},1\},\{f^{\{4,7,1\}},1\},\{f^{\{4,8,1\}},-\sqrt{3}\},\{f^{\{5,1,6\}},1\},$
$\{f^{\{5,2,7\}},-1\},\{f^{\{5,3,4\}},1\},\{f^{\{5,4,3\}},-\sqrt{3}\},\{f^{\{5,6,1\}},-1\},$
$\{f^{\{5,7,2\}},1\},\{f^{\{5,8,4\}},\sqrt{3}\},\{f^{\{6,1,5\}},-1\},\{f^{\{6,2,4\}},1\},\{f^{\{6,3,7\}},1\},$
$\{f^{\{6,4,2\}},-1\},\{f^{\{6,5,1\}},1\},\{f^{\{6,7,8\}},\sqrt{3}\},\{f^{\{6,8,1\}},-\sqrt{3}\},$
$\{f^{\{7,1,4\}},1\},\{f^{\{7,2,5\}},1\},\{f^{\{7,3,6\}},-1\},\{f^{\{7,4,1\}},-1\},\{f^{\{7,5,2\}},-1\},$
$\{f^{\{7,6,1\}},-\sqrt{3}\},\{f^{\{7,8,6\}},\sqrt{3}\},\{f^{\{8,1,0\}},0\},\{f^{\{8,2,0\}},0\},\{f^{\{8,3,0\}},0\},$
$\{f^{\{8,4,5\}},\sqrt{3}\},\{f^{\{8,5,1\}},-\sqrt{3}\},\{f^{\{8,6,7\}},\sqrt{3}\},\{f^{\{8,7,1\}},-\sqrt{3}\}\}$