HW2

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2.12 Solution:

(e) converse example:

when $S = \{-1, 1\}$, $T = \{0\}$,

then $\{x \mid \mathrm{dis}(x, S) \le \mathrm{dist}(x, T)\}$

$= \{x \in R \mid x \le -\frac{1}{2} \text{ or } x \ge \frac{1}{2}\}$

Clearly, this is not a convex set.

(f) $x + S_2 \subseteq S_1$ if $x + y \in S_1$ for all $y \in S_2$.

Then $\{x \mid x + S_2 \subseteq S_1\} = \bigcap_{y \in S_2} \{x \mid x + y \in S_1\} = \bigcap_{y \in S_2} (S_1 - y)$

It's the intersection of convex set $S_1 - y$.

(g) $\{x \mid \|x-a\|_2 \le \theta \|x-b\|_2\}$

$= \{x \mid \|x-a\|_2^2 \le \theta^2 \|x-b\|_2^2\}$

$= \{x \mid (1-\theta^2) x^T x - 2(a - \theta^2 b)^T x + (a^T a - \theta^2 b^T b) \le 0\}$

If $\theta = 1$, $\{x \mid -2(a-b)^T x + (a^T a - b^T b) \le 0\}$, this is halfspace.

If $0 \le \theta < 1$, $\{x \mid x^T x - 2\left(\frac{a - \theta^2 b}{1-\theta^2}\right)^T x + \frac{a^T a - \theta^2 b^T b}{1-\theta^2} \le 0\}$

$= \{x \mid (x - x_0)^T (x - x_0) \le R^2\}$

$x_0 = \frac{a - \theta^2 b}{1 - \theta^2}$, $R = \left(\frac{\theta^2 \|b\|_2^2 - \|a\|_2^2}{1-\theta^2} - \|x_0\|_2^2\right)^{\frac{1}{2}}$

3.20 (c) Solution: $f(x) = \mathrm{tr}(A_0 + x_1 A_1 + \cdots + x_n A_n)^{-1}$

from 3.18, we know that $f(x) = \mathrm{tr}(X^{-1})$ is convex on $\mathrm{dom}\, f = S_{++}^n$

then $x \to A_0 + x_1 A_1 + \cdots + x_n A_n \in S_{++}^m$

$f(x)$ is a convex function.

3.21 (a) Solution: First, $f(x) = \max_{i=1,\dots,k} \|A^{(i)} x - b^{(i)}\|$, where $A^{(i)} \in R^{m \times n}$

$b \in R^m$. $\|\cdot\|$ is a norm on R^{m}.

f is a pointwise maximum of k functions $\|A^{(i)} x - b^{(i)}\|$.

Each of these function is convex, because it's the composition of an affine transformation and a norm.

Then this function is convex.