Homework 3: 直接构造法

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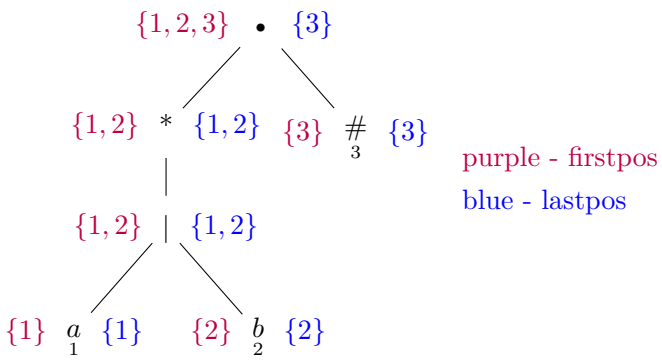
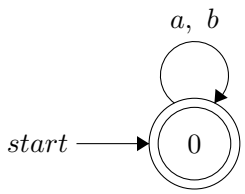
1. 使用直接构造法构造这四个正则表达式的 DFA，并且最小化 DFA。

a) $(a|b)^*$

$(a|b)^* \rightarrow (a_1|b_2)^*\#_3$



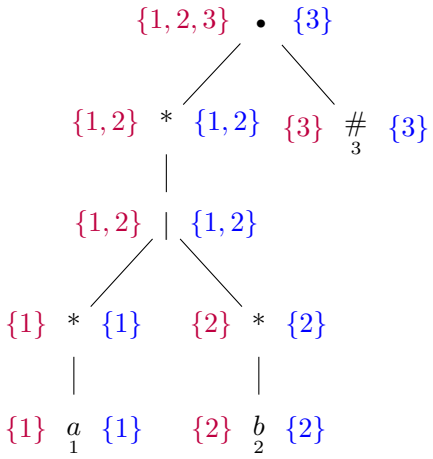
followpos(1) = {1, 2, 3}
followpos(2) = {1, 2, 3}
followpos(3) = {}
S_0 = firstpos(*root*) = {1, 2, 3}
a : followpos(1) = {1, 2, 3} = S_0 move(S_0, a) = S_0
b : followpos(2) = {1, 2, 3} = S_0 move(S_0, b) = S_0



显然，是最简的 DFA。

b) $(a^*|b^*)^*$

$(a^*|b^*)^* \rightarrow (\underset{1}{a}^*|\underset{2}{b}^*)^*\underset{3}{\#}$



purple - firstpos

blue - lastpos

followpos(1) $= \{1, 2, 3\}$

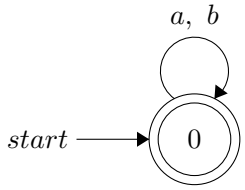
followpos(2) $= \{1, 2, 3\}$

followpos(3) $= \{\}$

$S_0 = \text{firstpos}(root) = \{1, 2, 3\}$

$a : \text{followpos}(1) = \{1, 2, 3\} = S_0 \qquad \text{move}(S_0, a) = S_0$

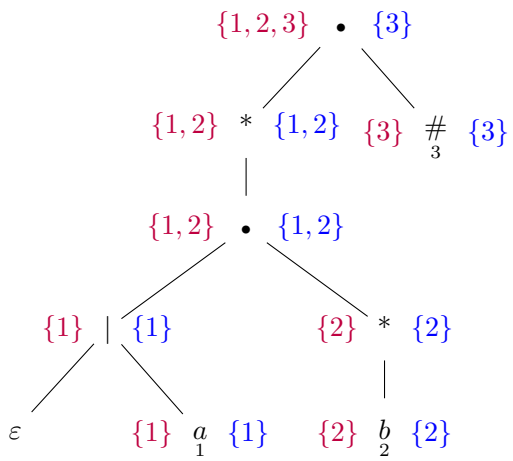
$b : \text{followpos}(2) = \{1, 2, 3\} = S_0 \qquad \text{move}(S_0, b) = S_0$



显然，是最简的 DFA。

c) $((\varepsilon|a)b^*)^*$

$((\varepsilon|a)b^*)^* \rightarrow ((\varepsilon|\underset{1}{a}\underset{2}{b^*})^*\underset{3}{\#}$



purple - firstpos

blue - lastpos

followpos(1) $= \{1, 2, 3\}$

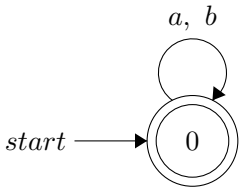
followpos(2) $= \{1, 2, 3\}$

followpos(3) $= \{\}$

$S_0 = \text{firstpos}(root) = \{1, 2, 3\}$

$a : \text{followpos}(1) = \{1, 2, 3\} = S_0 \qquad \text{move}(S_0, a) = S_0$

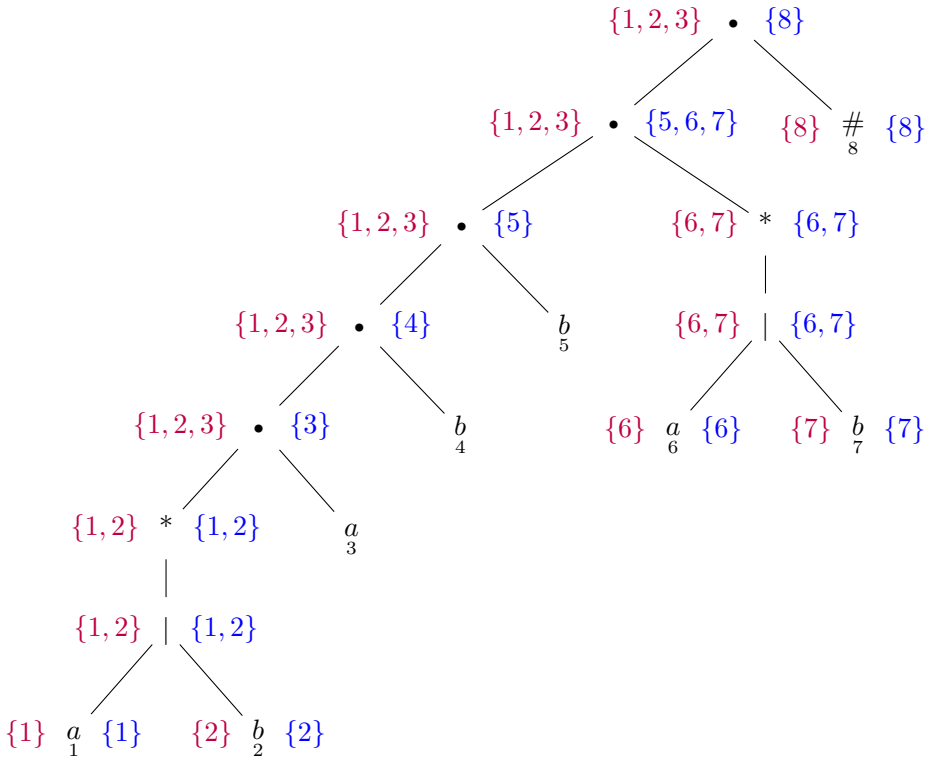
$b : \text{followpos}(2) = \{1, 2, 3\} = S_0 \qquad \text{move}(S_0, b) = S_0$



显然，是最简的 DFA。

d) $(a|b)^*abb(a|b)^*$

$(a|b)^*abb(a|b)^* \rightarrow (\underset{1}{a}|\underset{2}{b})^*\underset{3}{a}\underset{4}{b}\underset{5}{b}(\underset{6}{a}|\underset{7}{b})^*\underset{8}{\#}$



purple - firstpos

blue - lastpos

followpos(1) = {1, 2, 3}
followpos(2) = {1, 2, 3}
followpos(3) = {4}
followpos(4) = {5}
followpos(5) = {6, 7, 8}
followpos(6) = {6, 7, 8}
followpos(7) = {6, 7, 8}
followpos(8) = {}

S_0 = firstpos(*root*) = {1, 2, 3}
a : followpos(1)∪followpos(3) = {1, 2, 3, 4} = S_1 move(S_0, a) = S_1
b : followpos(2) = {1, 2, 3} = S_0 move(S_0, b) = S_0
⇓ mark S_1
a : followpos(1)∪followpos(3) = {1, 2, 3, 4} = S_1 move(S_1, a) = S_1
b : followpos(2) ∪ followpos(4) = {1, 2, 3, 5} = S_2 move(S_1, b) = S_2

$\Downarrow$ mark S_2

a : followpos(1)$\cup$followpos(3) $= \{1, 2, 3, 4\} = S_1$ move$(S_2, a) = S_1$

b : followpos(2)$\cup$followpos(5) $= \{1, 2, 3, 6, 7, 8\} = S_3$ move$(S_2, b) = S_3$

$\Downarrow$ mark S_3

a : followpos(1) $\cup$ followpos(3) $\cup$ followpos(6) $= \{1, 2, 3, 4, 6, 7, 8\} = S_4$ move$(S_3, a) = S_4$

b : followpos(2)$\cup$followpos(7) $= \{1, 2, 3, 6, 7, 8\} = S_3$ move$(S_3, b) = S_3$

$\Downarrow$ mark S_4

a : followpos(1) $\cup$ followpos(3) $\cup$ followpos(6) $= \{1, 2, 3, 4, 6, 7, 8\} = S_4$ move$(S_4, a) = S_4$

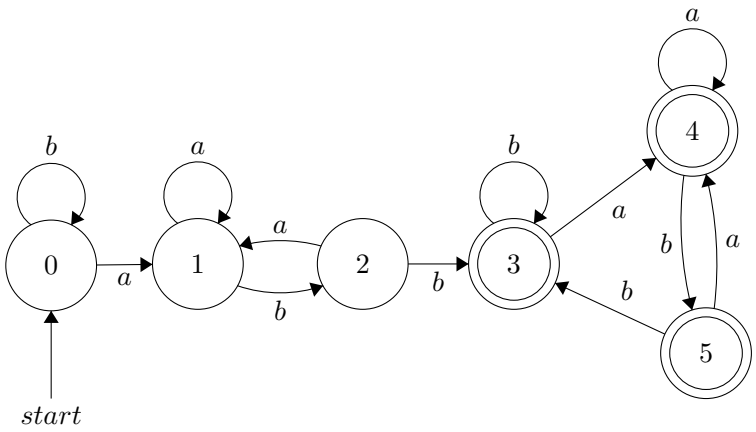
b : followpos(2) $\cup$ followpos(4) $\cup$ followpos(7) $= \{1, 2, 3, 5, 6, 7, 8\} = S_5$ move$(S_4, b) = S_5$

$\Downarrow$ mark S_5

a : followpos(1) $\cup$ followpos(3) $\cup$ followpos(6) $= \{1, 2, 3, 4, 6, 7, 8\} = S_4$ move$(S_5, a) = S_4$

b : followpos(2)$\cup$followpos(5)$\cup$followpos(7) $= \{1, 2, 3, 6, 7, 8\} = S_3$ move$(S_5, b) = S_3$

	a	b
S_0	S_1	S_0
S_1	S_1	S_2
S_2	S_1	S_3
S_3	S_4	S_3
S_4	S_4	S_5
S_5	S_4	S_3



$G_1 = \{3, 4, 5\}$
$G_2 = \{0, 1, 2\}$

a	b
$3 \to 4$	$3 \to 3$
$4 \to 4$	$4 \to 5$
$5 \to 4$	$5 \to 3$

所以 G_1 是不可分的。

a	b
$0 \to 1$	$0 \to 0$
$1 \to 1$	$1 \to 2$
$2 \to 1$	$2 \to 3$

所以 G_2 可以分成 $\{0\}$, $\{1\}$, $\{2\}$。

化简后的 DFA 如下：