# 华东师范大学软件工程学院实验报告


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|---|---|---|---|
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| 实验编号: | Lab 01 | 实验名称: | Manipulating bits |


## 1 实验目的

1) 熟悉整数和浮点数的位级表示

2) 练习 C 中位运算的使用

## 2 实验内容与实验步骤

### 2.1 实验内容

编写完善 bits.c 中关于整数和浮点数位级表示的 15 个函数，并进行评测。

**1) bitAnd** 要求仅使用 ~ 和 | 完成 & 操作。

考虑到 x & y = ~~(x & y) = ~(~x | ~y)，代码如下：

```c
int bitAnd(int x, int y) {
    return ~(~x | ~y);
}
```

**2) getByte** 获取 x 第 n 个字节的内容

将 x 右移 $n \times 8$ 位（一字节）后与 0xff 相与，代码如下：

```c
int getByte(int x, int n) {
    return (x >> (n << 3)) & 0xff;
}
```

**3) logicalShift** 完成逻辑右移功能

先进行算数右移，然后再与一个前 n 位全为零而其他位全为 1 的数相与，从而对先 n 位进行清零，代码如下：

```c
int logicalShift(int x, int n) {
    return (x >> n) & ~(1 << 31 >> n << 1);
}
```

**4) bitCount** 统计一个数的二进制表示中 1 的个数

首先定义 5 个掩码（mask1 到 mask5）用于按位操作。这些掩码的作用是将整数 x 中的每个不同位按组分组，并将相邻的位相加，以便计算每组中包含的 1 的数量。然后将掩码 mask1-5 应用于 x，以按 2 位、4 位、8 位、16 位和 32 位分组计算 1 的数量。最后，将所有组中 1 的数量相加，得到结果，代码如下：

```c
int bitCount(int x) {
    int mask1 = 0x55 | 0x55 << 8;
    int mask2 = 0x33 | 0x33 << 8;
    int mask3 = 0x0f | 0x0f << 8;
    int mask4 = 0xff | 0xff << 16;          // 0x00ff00ff
    int mask5 = 0xff | 0xff << 8;           // 0x0000ffff
    mask1 = mask1 | mask1 << 16;            // 0x55555555
    mask2 = mask2 | mask2 << 16;            // 0x33333333
    mask3 = mask3 | mask3 << 16;            // 0x0f0f0f0f
    x = (x & mask1) + ((x >> 1) & mask1);   // add count of 1 every
        2 bits
    x = (x & mask2) + ((x >> 2) & mask2);   // every 4 bits
    x = (x & mask3) + ((x >> 4) & mask3);   // every 8 bits
    x = (x & mask4) + ((x >> 8) & mask4);   // every 16 bits
    x = (x & mask5) + ((x >> 16) & mask5); // every 32 bits
    return x;
}
```

**5) bang** 在不使用！的情况下得到！x

首先将 x 与其相反数按位或，在 x 是 0 的情况下得到 0，其他情况下得到最高位为 1 的二进制数。右移 31 位并按位取反后，0 得到全 1 而其他情况得到全 0，与 1 相与得到答案。代码如下：

```c
int bang(int x) {
    return ~((x | (~x + 1)) >> 31) & 1;
    // the highest bit of x | (~x + 1) is 0 only if x = 0
}
```

**6) tmin**　得到 int 类型整数的最小值

即 0x80000000，代码如下：

```
int tmin(void) {
    return 1 << 31;
}
```

**7) fitsBits**　判断 x 是否能被表示成 n 位二进制补码

如果 x 左移 $32-n$ 位再右移 $32-n$ 后结果不变，则 x 可以被表示成 n 位二进制补码，代码如下：

```
int fitsBits(int x, int n) {
    int c = 33 + ~n; // 32 - n
    return !(((x << c) >> c) ^ x);
    // x ^ y equals 0 only if x == y
}
```

**8) divpwr2**　计算 $\frac{x}{2^n}$，向 0 舍入。

考虑通过右移代替除法，但其对数字的直接截断在 $x<0$ 时不满足向 0 舍入的要求。考虑设置一个偏移量 bias，当 $x<0$ 时，将其设置为 $2^n-1$，以对 x 进行修正，完成向 0 舍入。代码如下：

```
int divpwr2(int x, int n) {
    int bias = (x >> 31) & ((1 << n) + ~0);
    // equals 0 when x >= 0 and 2 ^ n - 1 when x < 0
    return (x + bias) >> n;
}
```

**9) negate**　得到 $-x$

即 $\sim x+1$，代码如下：

```
int negate(int x) {
    return ~x + 1;
}
```

**10) isPositive** 判断 $x > 0$ 是否成立

考虑将 x 右移 31 位以提取符号位，但要注意特判 $x = 0$，此时应返回 0，代码如下：

```c
int isPositive(int x) {
    return (!(x >> 31)) & !!x;
}
```

**11) isLessOrEqual** 判断 $x \leq y$ 是否成立

考虑通过判断 $y - x \geq 0$ 是否成立来进行判断。但在 $x < 0$ 且 $y > 0$, 或 $x > 0$ 且 $y < 0$ 时, 可能发生溢出，不过在前一种情况下，$x \leq y$ 必然成立，后一种情况下必然不成立，直接特判即可。代码如下：

```c
int isLessOrEqual(int x, int y) {
    int ispositivex = !(x >> 31);            // x >= 0
    int ispositivey = !(y >> 31);            // y >= 0
    return (!ispositivex | ispositivey)      // !(x > 0 && y < 0)
            & (((!ispositivex) & ispositivey) // x < 0 && y > 0
            | !(((y + ~x + 1) >> 31) & 1));   // y - x >= 0
}
```

**12) ilog2** 计算 $\log_2 x$

$\log_2 x$ 的值即为 x 的二进制表示中最高位的位置。可以采用类似分治的思路，从 16 位到 8 位，4 位，2 位，1 位，逐步确定 x 的最高位的位置，代码如下：

```c
int ilog2(int x) {
    int ans = ((!!(x >> 16)) << 4);
    ans = ans | ((!!(x >> (ans + 8))) << 3);
    ans = ans | ((!!(x >> (ans + 4))) << 2);
    ans = ans | ((!!(x >> (ans + 2))) << 1);
    ans = ans | (!!(x >> (ans + 1)));
    return ans;
}
```

**13) float_neg** 计算浮点数 f 的相反数，但当 f 是 NaN 时，返回 f

将 f 的最高位取反即可，但要注意特判 NaN。代码如下：

```
unsigned float_neg(unsigned uf) {
    if (!(((uf << 1) ^ 0xff000000) >> 24) & !!(uf & 0x007fffff))
      return uf;
    else
      return uf + 0x80000000;
}
```

**14) float_i2f** 将整数转换为浮点数

可以通过将 ux 循环右移得到最高位的位置，进而加上偏移量得到阶码值 $e$。注意将 int 类型整数转换为单精度浮点数时，需要舍去最后 9 位，因此需要注意浮点数表示规则中的向偶数舍入。当最后 9 位大于 0x100 时，或最后十位为 0x300 时，满足要求，将结果加一。在代码中，$e$ 的初始值被设置为 $bias-1$，这是因为在二进制表示中，最后一位的权值是 $2^0$。代码如下：

```
unsigned float_i2f(int x) {
    unsigned ux = x < 0 ? -x : x;
    unsigned _ux = ux;
    unsigned e = 127 - 1; // bias - 1
    unsigned ans = 0;
    unsigned frac;
    int shift = 0; // bits to shift left
    int flag;      // round-to-even
    while (ux) {
      e++;
      ux >>= 1;
    }
    if (x == 0) {
      return 0;
    } else if (x < 0) {
      ans |= 0x80000000;
    }
    shift = 159 - e; // 32 - (e - 127)
    frac = _ux << shift;
    flag = (frac & 0x1ff) > 0x100 || (frac & 0x3ff) == 0x300;
    frac = frac >> 9; // 32 - 23
    return (ans | (e << 23) | frac) + flag;
}
```

**15) float_twice** 计算一个浮点数的两倍，但当 f 是 NaN 时，返回 f

分情况讨论。当 f 是规格化的浮点数时，直接将阶码加一，当 x 是非规格化的浮点数时，直接将尾数乘以 2，当 f 是 NaN 时，不变。代码如下：

```
unsigned float_twice(unsigned uf) {
    if ((uf & 0x7f800000) == 0) {
        // is denormalized
        uf = (uf & 0x007fffff) << 1 | (uf & 0x80000000);
        // frac *= 2
    } else if ((uf & 0x7f800000) != 0x7f800000) {
        // is not NaN
        uf += 0x00800000; // exp += 1
    }
    return uf;
}
```

## 2.2 实验步骤

1) 安装必要的软件和库，如 make, gcc-multilib 等

```
linux> sudo apt install make gcc gcc-multilib
```

2) 将 datalab-handout 在 Linux 下进行解打包

```
linux> tar -xvf datalab-handout
```

3) 编辑 bits.c，完成其中的 15 个函数

4) 编译

```
linux> make
```

5) 运行 driver.pl 进行评测

```
linux> ./driver.pl
```

## 3 实验过程与分析

在实验过程中，我遇到了一些困难和问题。

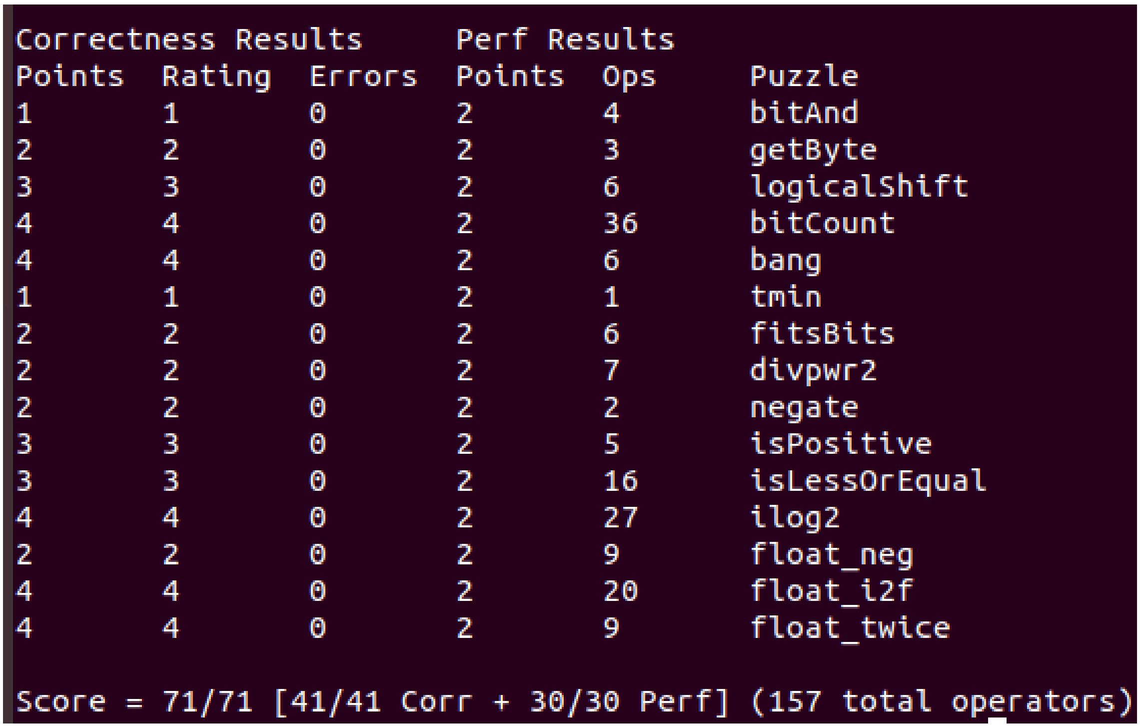
首先便是题目的难度较高。bitCount 和 ilog2 两题在独立思考较长时间后，仍没有较好的思路，在网上查阅资料后，最终才完成这两题。在某些题目的舍入上也遇到了一些问题，讲过多次修改最终才通过测试。

在实验中，我还遇到了一些 BUG。在题目 fitsBits 的评测中，似乎出现了一些问题。起初，我是用的平台为 WSL2 中的 Ubuntu 20.04.6 LTS，内核版本为 5.15.90.1-microsoft-standard-WSL2，gcc 版本为 9.4.0。在评测 fitsBits 时，我遇到了错误

```
ERROR: Test fitsBits(-2147483648[0x80000000],32[0x20]) failed...
...Gives 1[0x1]. Should be 0[0x0]
```

然而，0x80000000 明显时可以用 32 位补码表示的。后来，我用相同的代码在 Vmware 中的 Ubuntu 20.04.6 LTS 上重新进行了评测，其内核版本为 5.0.0-23-generic，gcc 版本为 7.5.0。在此版本的 Linux 上，没有遇到这个问题。经过我在网上的搜索和线下的实践，我怀疑此 BUG 可能与 gcc 版本有关。

实验最后的评测截图如下：



| Correctness Results | | | Perf Results | | |
|---|---|---|---|---|---|
| Points | Rating | Errors | Points | Ops | Puzzle |
| 1 | 1 | 0 | 2 | 4 | bitAnd |
| 2 | 2 | 0 | 2 | 3 | getByte |
| 3 | 3 | 0 | 2 | 6 | logicalShift |
| 4 | 4 | 0 | 2 | 36 | bitCount |
| 4 | 4 | 0 | 2 | 6 | bang |
| 1 | 1 | 0 | 2 | 1 | tmin |
| 2 | 2 | 0 | 2 | 6 | fitsBits |
| 2 | 2 | 0 | 2 | 7 | divpwr2 |
| 2 | 2 | 0 | 2 | 2 | negate |
| 3 | 3 | 0 | 2 | 5 | isPositive |
| 3 | 3 | 0 | 2 | 16 | isLessOrEqual |
| 4 | 4 | 0 | 2 | 27 | ilog2 |
| 2 | 2 | 0 | 2 | 9 | float_neg |
| 4 | 4 | 0 | 2 | 20 | float_i2f |
| 4 | 4 | 0 | 2 | 9 | float_twice |

Score = 71/71 [41/41 Corr + 30/30 Perf] (157 total operators)

图 1: 评测截图

# 4 实验结果总结

通过本次实验，我进一步深入理解了整数和浮点数的位级表示，同时，也学习到了在 C 语言中进行有关位运算的一些技巧，还了解到了一些 Linux 上的基本操作。

# 5 附录（源代码）

bits.c 的源代码如下（此代码已同时以文件的形式提交）：

```
 /*
 * CS:APP Data Lab
 *
 * <Please put your name and userid here>
 *
 * bits.c - Source file with your solutions to the Lab.
 *          This is the file you will hand in to your instructor.
 *
 * WARNING: Do not include the <stdio.h> header; it confuses the dlc
 * compiler. You can still use printf for debugging without including
 * <stdio.h>, although you might get a compiler warning. In general,
 * it's not good practice to ignore compiler warnings, but in this
 * case it's OK.
 */

#if 0
/*
 * Instructions to Students:
 *
 * STEP 1: Read the following instructions carefully.
 */

You will provide your solution to the Data Lab by
editing the collection of functions in this source file.

INTEGER CODING RULES:
```

```

Replace the "return" statement in each function with one
or more lines of C code that implements the function. Your code
must conform to the following style:

int Funct(arg1, arg2, ...) {
    /* brief description of how your implementation works */
    int var1 = Expr1;
    ...
    int varM = ExprM;

    varJ = ExprJ;
    ...
    varN = ExprN;
    return ExprR;
}

Each "Expr" is an expression using ONLY the following:
1. Integer constants 0 through 255 (0xFF), inclusive. You are
    not allowed to use big constants such as 0xffffffff.
2. Function arguments and local variables (no global variables).
3. Unary integer operations ! ~
4. Binary integer operations & ^ | + << >>

Some of the problems restrict the set of allowed operators even further.
Each "Expr" may consist of multiple operators. You are not restricted to
one operator per line.

You are expressly forbidden to:
1. Use any control constructs such as if, do, while, for, switch, etc.
2. Define or use any macros.
3. Define any additional functions in this file.
4. Call any functions.
5. Use any other operations, such as &&, ||, -, or ?:
6. Use any form of casting.
```

```
  7. Use any data type other than int.  This implies that you
     cannot use arrays, structs, or unions.


  You may assume that your machine:
  1. Uses 2s complement, 32-bit representations of integers.
  2. Performs right shifts arithmetically.
  3. Has unpredictable behavior when shifting an integer by more
     than the word size.

EXAMPLES OF ACCEPTABLE CODING STYLE:
  /*
   * pow2plus1 - returns 2^x + 1, where 0 <= x <= 31
   */
  int pow2plus1(int x) {
     /* exploit ability of shifts to compute powers of 2 */
     return (1 << x) + 1;
  }

  /*
   * pow2plus4 - returns 2^x + 4, where 0 <= x <= 31
   */
  int pow2plus4(int x) {
     /* exploit ability of shifts to compute powers of 2 */
     int result = (1 << x);
     result += 4;
     return result;
  }

FLOATING POINT CODING RULES

For the problems that require you to implent floating-point operations,
the coding rules are less strict.  You are allowed to use looping and
conditional control.  You are allowed to use both ints and unsigneds.
You can use arbitrary integer and unsigned constants.
```

```

You are expressly forbidden to:
  1. Define or use any macros.
  2. Define any additional functions in this file.
  3. Call any functions.
  4. Use any form of casting.
  5. Use any data type other than int or unsigned.  This means that you
     cannot use arrays, structs, or unions.
  6. Use any floating point data types, operations, or constants.


NOTES:
  1. Use the dlc (data lab checker) compiler (described in the handout) to
     check the legality of your solutions.
  2. Each function has a maximum number of operators (! ~ & ^ | + << >>)
     that you are allowed to use for your implementation of the function.
     The max operator count is checked by dlc. Note that '=' is not
     counted; you may use as many of these as you want without penalty.
  3. Use the btest test harness to check your functions for correctness.
  4. Use the BDD checker to formally verify your functions
  5. The maximum number of ops for each function is given in the
     header comment for each function. If there are any inconsistencies
     between the maximum ops in the writeup and in this file, consider
     this file the authoritative source.

/*
 * STEP 2: Modify the following functions according the coding rules.
 *
 *   IMPORTANT. TO AVOID GRADING SURPRISES:
```

```
 *   1. Use the dlc compiler to check that your solutions conform
 *      to the coding rules.
 *   2. Use the BDD checker to formally verify that your solutions produce
 *      the correct answers.
 */


#endif
/*
 * bitAnd - x&y using only ~ and |
 *   Example: bitAnd(6, 5) = 4
 *   Legal ops: ~ |
 *   Max ops: 8
 *   Rating: 1
 */
int bitAnd(int x, int y) {
  return ~(~x | ~y);
}
/*
 * getByte - Extract byte n from word x
 *   Bytes numbered from 0 (LSB) to 3 (MSB)
 *   Examples: getByte(0x12345678,1) = 0x56
 *   Legal ops: ! ~ & ^ | + << >>
 *   Max ops: 6
 *   Rating: 2
 */
int getByte(int x, int n) {
  return (x >> (n << 3)) & 0xff;
}
/*
 * logicalShift - shift x to the right by n, using a logical shift
 *   Can assume that 0 <= n <= 31
 *   Examples: logicalShift(0x87654321,4) = 0x08765432
 *   Legal ops: ! ~ & ^ | + << >>
 *   Max ops: 20
 *   Rating: 3
 */
```

```c
int logicalShift(int x, int n) {
  return (x >> n) & ~(1 << 31 >> n << 1);
}
/*
 * bitCount - returns count of number of 1's in word
 *   Examples: bitCount(5) = 2, bitCount(7) = 3
 *   Legal ops: ! ~ & ^ | + << >>
 *   Max ops: 40
 *   Rating: 4
 */
int bitCount(int x) {
  int mask1 = 0x55 | 0x55 << 8;
  int mask2 = 0x33 | 0x33 << 8;
  int mask3 = 0x0f | 0x0f << 8;
  int mask4 = 0xff | 0xff << 16;         // 0x00ff00ff
  int mask5 = 0xff | 0xff << 8;          // 0x0000ffff
  mask1 = mask1 | mask1 << 16;           // 0x55555555
  mask2 = mask2 | mask2 << 16;           // 0x33333333
  mask3 = mask3 | mask3 << 16;           // 0x0f0f0f0f
  x = (x & mask1) + ((x >> 1) & mask1);  // add count of 1 every 2
     bits
  x = (x & mask2) + ((x >> 2) & mask2);  // every 4 bits
  x = (x & mask3) + ((x >> 4) & mask3);  // every 8 bits
  x = (x & mask4) + ((x >> 8) & mask4);  // every 16 bits
  x = (x & mask5) + ((x >> 16) & mask5); // every 32 bits
  return x;
}
/*
 * bang - Compute !x without using !
 *   Examples: bang(3) = 0, bang(0) = 1
 *   Legal ops: ~ & ^ | + << >>
 *   Max ops: 12
 *   Rating: 4
 */
int bang(int x) {
  return ~((x | (~x + 1)) >> 31) & 1; // the highest bit of x | (~x
      + 1) is 0 only if x = 0
}
```

```c
/*
 * tmin - return minimum two's complement integer
 *   Legal ops: ! ~ & ^ | + << >>
 *   Max ops: 4
 *   Rating: 1
 */
int tmin(void) {
  return 1 << 31;
}
/*
 * fitsBits - return 1 if x can be represented as an
 *  n-bit, two's complement integer.
 *   1 <= n <= 32
 *   Examples: fitsBits(5,3) = 0, fitsBits(-4,3) = 1
 *   Legal ops: ! ~ & ^ | + << >>
 *   Max ops: 15
 *   Rating: 2
 */
int fitsBits(int x, int n) {
  int c = 33 + ~n; // 32 - n
  return !(((x << c) >> c) ^ x);
}
/*
 * divpwr2 - Compute x/(2^n), for 0 <= n <= 30
 *  Round toward zero
 *   Examples: divpwr2(15,1) = 7, divpwr2(-33,4) = -2
 *   Legal ops: ! ~ & ^ | + << >>
 *   Max ops: 15
 *   Rating: 2
 */
int divpwr2(int x, int n) {
  int bias = (x >> 31) & ((1 << n) + ~0); // equals 0 when x >= 0
      and 2 ^ n - 1 when x < 0
  return (x + bias) >> n;
}
/*
 * negate - return -x
 *   Example: negate(1) = -1.
```

```c
 *   Legal ops: ! ~ & ^ | + << >>
 *   Max ops: 5
 *   Rating: 2
 */
int negate(int x) {
  return ~x + 1;
}
/*
 * isPositive - return 1 if x > 0, return 0 otherwise
 *   Example: isPositive(-1) = 0.
 *   Legal ops: ! ~ & ^ | + << >>
 *   Max ops: 8
 *   Rating: 3
 */
int isPositive(int x) {
  return (!(x >> 31)) & !!x;
}
/*
 * isLessOrEqual - if x <= y  then return 1, else return 0
 *   Example: isLessOrEqual(4,5) = 1.
 *   Legal ops: ! ~ & ^ | + << >>
 *   Max ops: 24
 *   Rating: 3
 */
int isLessOrEqual(int x, int y) {
  int ispositivex = !(x >> 31);
  int ispositivey = !(y >> 31);
  return (!ispositivex | ispositivey) & (((!ispositivex) &
      ispositivey) | !(((y + ~x + 1) >> 31) & 1));
}
/*
 * ilog2 - return floor(log base 2 of x), where x > 0
 *   Example: ilog2(16) = 4
 *   Legal ops: ! ~ & ^ | + << >>
 *   Max ops: 90
 *   Rating: 4
 */
int ilog2(int x) {
```

```
  int ans = ((!!(x >> 16)) << 4);
  ans = ans | ((!!(x >> (ans + 8))) << 3);
  ans = ans | ((!!(x >> (ans + 4))) << 2);
  ans = ans | ((!!(x >> (ans + 2))) << 1);
  ans = ans | (!!(x >> (ans + 1)));
  return ans;
}
/*
 * float_neg - Return bit-level equivalent of expression -f for
 *   floating point argument f.
 *   Both the argument and result are passed as unsigned int's, but
 *   they are to be interpreted as the bit-level representations of
 *   single-precision floating point values.
 *   When argument is NaN, return argument.
 *   Legal ops: Any integer/unsigned operations incl. ||, &&. also
     if, while
 *   Max ops: 10
 *   Rating: 2
 */
unsigned float_neg(unsigned uf) {
  if (!(((uf << 1) ^ 0xff000000) >> 24) & !!(uf & 0x007fffff))
    return uf;
  else
    return uf + 0x80000000;
}
/*
 * float_i2f - Return bit-level equivalent of expression (float) x
 *   Result is returned as unsigned int, but
 *   it is to be interpreted as the bit-level representation of a
 *   single-precision floating point values.
 *   Legal ops: Any integer/unsigned operations incl. ||, &&. also
     if, while
 *   Max ops: 30
 *   Rating: 4
 */
unsigned float_i2f(int x) {
  unsigned ux = x < 0 ? -x : x;
  unsigned _ux = ux;
```

```c
  unsigned e = 127 - 1;
  unsigned ans = 0;
  unsigned frac;
  int shift = 0; // bits to shift left
  int flag;      // round-to-even
  while (ux) {
    e++;
    ux >>= 1;
  }
  if (x == 0) {
    return 0;
  } else if (x < 0) {
    ans |= 0x80000000;
  }
  shift = 159 - e; // 32 - (e - 127)
  frac = _ux << shift;
  flag = (frac & 0x1ff) > 0x100 || (frac & 0x3ff) == 0x300;
  frac = frac >> 9; // 32 - 23
  return (ans | (e << 23) | frac) + flag;
}
/*
 * float_twice - Return bit-level equivalent of expression 2*f for
 *   floating point argument f.
 *   Both the argument and result are passed as unsigned int's, but
 *   they are to be interpreted as the bit-level representation of
 *   single-precision floating point values.
 *   When argument is NaN, return argument
 *   Legal ops: Any integer/unsigned operations incl. ||, &&. also
    if, while
 *   Max ops: 30
 *   Rating: 4
 */
unsigned float_twice(unsigned uf) {
  if ((uf & 0x7f800000) == 0) {                       // is
     denormalized
    uf = (uf & 0x007fffff) << 1 | (uf & 0x80000000); // frac *= 2
  } else if ((uf & 0x7f800000) != 0x7f800000) {       // is not NaN
    uf += 0x00800000;                                 // exp += 1
```

```
    }
    return uf;
}
```