

Kalman Filter

Grundlagen

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..... What is the Kalman Filter?

The Kalman filter is a mathematical method for iteratively estimating parameters to describe system states.

In this process, a prediction about a parameter value is repeatedly made, combined with the error-prone measurement, and then used again to make a new prediction.

Kalman Filter

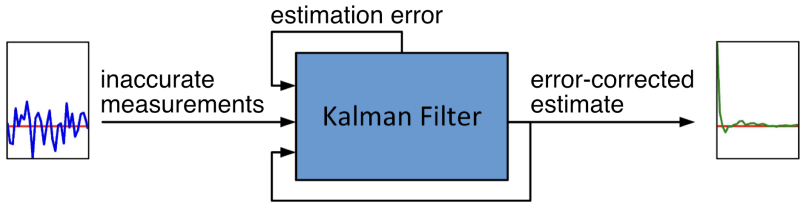


Fig.: Procedure of the Kalman Filter¹

Sensors susceptible to failure

- Light sensor (photoresistor)
- Ultrasonic sensor
- Infrared sensor
- Temperature sensor (thermoelectric)
- Magnetic field sensor
- Gas and air quality sensor
- Humidity sensor
- Motion sensor (accelerometer)
- Pressure sensor
- Sound sensor

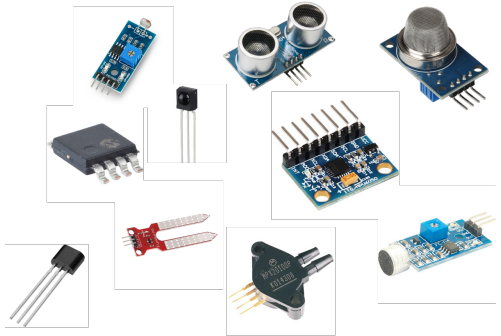


Fig.: sensors

The Mean



$$\mu = \frac{1}{N} \sum_{n=1}^N V_n = \frac{1}{5} (5 + 5 + 10 + 10 + 10) = 8 \text{cent}$$

..... The standarddeviation

The variance

The covariance

Prediction

1. State prediction: $\hat{x}_k = A\hat{x}_{k-1} + Bu_{k-1}$
2. Covariance prediction: $P_k = AP_{k-1}A^T + Q$

Correction

3. Kalman Gain Prediction: $K_k = P_k H^T (H P_k H^T + R)^{-1}$
4. State Update: $\hat{x}_k = \hat{x}_k + K_k(z_k - H\hat{x}_k)$
5. Covariance Update: $P_k = (I - K_k H)P_k$

Kalman explained

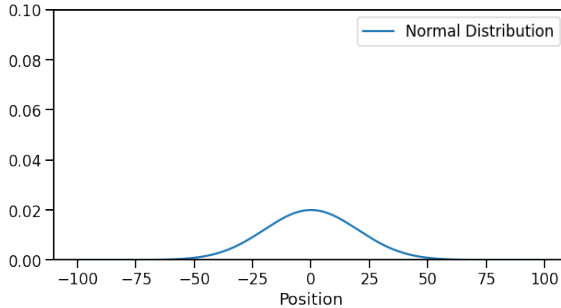


Fig.: Start Postion at t_0

Kalman explained

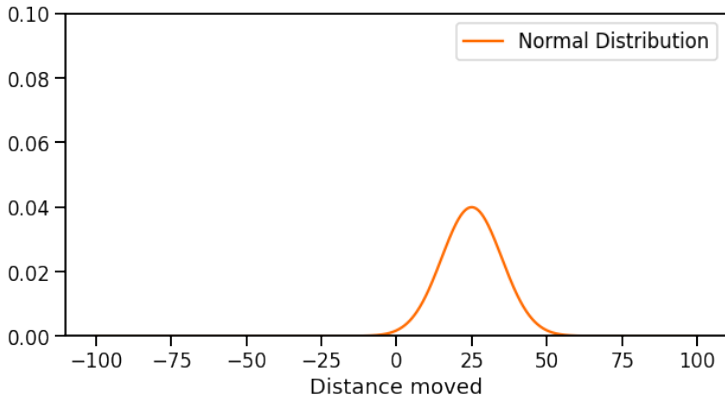


Fig.: Calculated Position at t_1

Kalman explained

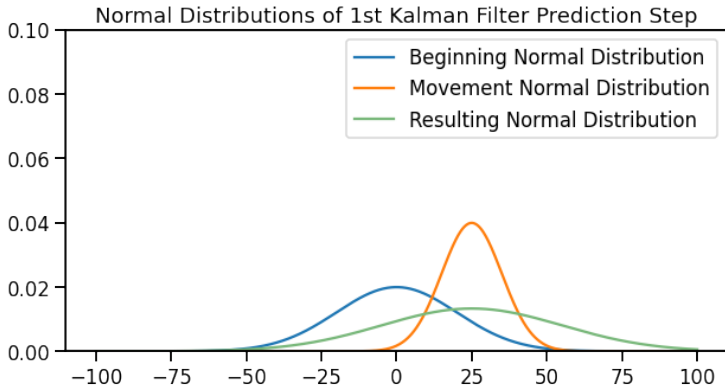


Fig.: Predicted Position at t_1

Kalman explained

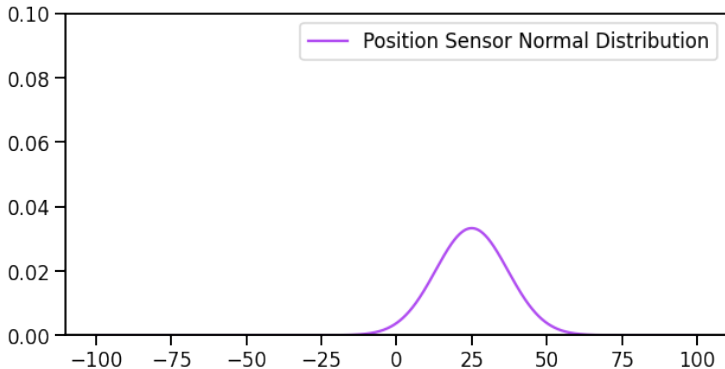


Fig.: Messured Sensor Data of Postion at t_1

Kalman explained

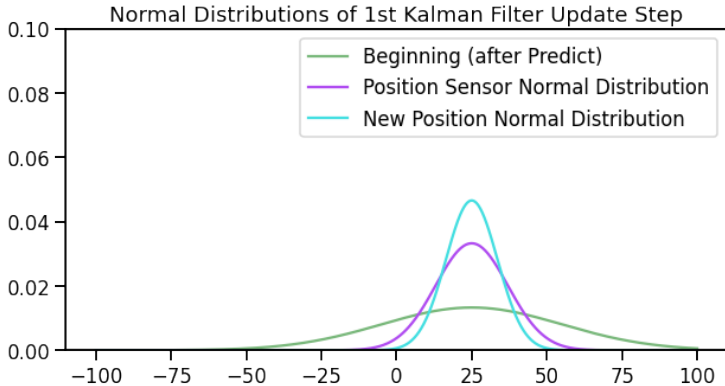


Fig.: Correction of the Kalman Gain after measurement at t_1

Takeaways

- The precision decreases with the prediction
- The precision increases with the correction

State Representation

We define the state vector x to include the position and velocity of the mouse in both x and y directions

$$x = \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}$$

where:

- x is the position on the x-axis
- y is the position on the y-axis
- v_x is the velocity in the x-direction
- v_y is the velocity in the y-direction

Formulate the Transition Model

The state transition model describes how the state evolves from one time step to the next. If we assume a constant velocity model, the state transition can be expressed as:

$$x_{k+1} = Ax_k + w_k$$

where w_k represents the process noise, assumed to be Gaussian with zero mean and covariance Q .

..... Create the Transition Matrix

The transition matrix A for a constant velocity Model is:

$$A = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Δt is the time intervall between measurements.

The observation model relates the state to the measurements:

$$z_k = Hx_k + v_k$$

where v_k represents the measurement noise, assumed to be Gaussian with zero mean and covariance R .

The observation matrix H for direct measurement of position is:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Prediction Step

The prediction step estimates the next state and its uncertainty:

- Predict the state:

$$x_{k+1} = Ax_k \quad (1)$$

- Predict the error covariance:

$$P_{k+1} = AP_kA^T + Q \quad (2)$$

Correction Step

The correction step updates the state estimate with the new measurement:

- Compute the Kalman gain:

$$K_k = P_k H^T (H P_k H^T + R)^{-1} \quad (3)$$

- Update the state estimate:

$$x_k = x_k + K_k (z_k - H x_k) \quad (4)$$

- Update the error covariance:

$$P_k = (I - K_k H) P_k \quad (5)$$

Kalman Filter

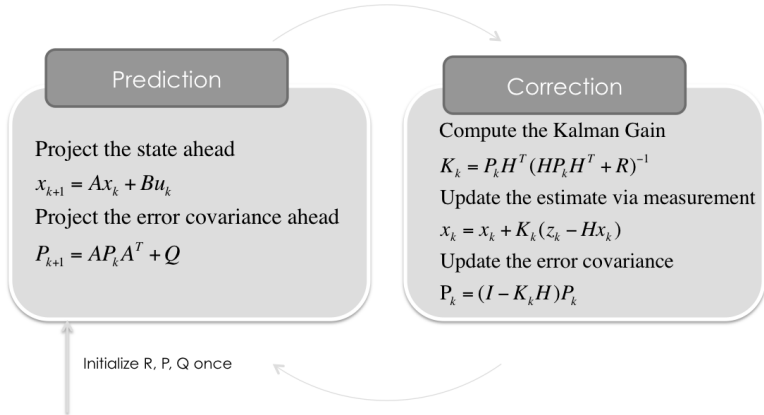


Fig.: Iterative Nature of the Kalman filter

Cheat Sheet - Math Symbols

Symbol	Meaning
k	Interval / iteration of the Kalman filter
z_k	Measurement from the sensor
x_k	Value of the current estimation
x_{k-1}	Value of the previous estimation
P_k	Value of the current error covariance
P_{k-1}	Value of the previous error covariance
R	Variance of the measurements
Q	Process noise covariance
A	Transition Matrix
H	Observation Matrix
Bu_{k-1}	Control signal

KALMA KALMA !

List of figures