Kalman Filter

Grundlagen

P. Schön (5121059), C. Thein (5121017)

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Technische Hochschule Würzburg-Schweinfurt

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What is the Kalman Filter?

The Kalman filter is a mathematical method for iteratively estimating parameters to describe system states.
In this process, a prediction about a parameter value is repeatedly made, combined with the error-prone measurement, and then used again to make a new prediction.

Process of Kalman Filters

Prediction

1. State prediction: $\hat{x}_k = A\hat{x}_{k-1} + Bu_{k-1}$
2. Covariance prediction: $P_k = AP_{k-1}A^T + Q$

Correction

3. Kalman Gain Prediction: $K_k = P_kH^T(HP_kH^T + R)^{-1}$
4. State Update: $\hat{x}_k = \hat{x}_k + K_k(z_k - H\hat{x}_k)$
5. Covariance Update: $P_k = (I - K_kH)P_k$

Kalman explained

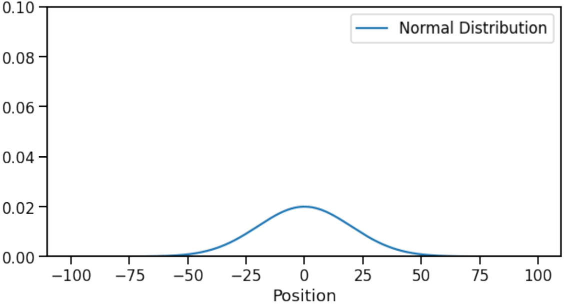

Fig.: Start Postion at t_0

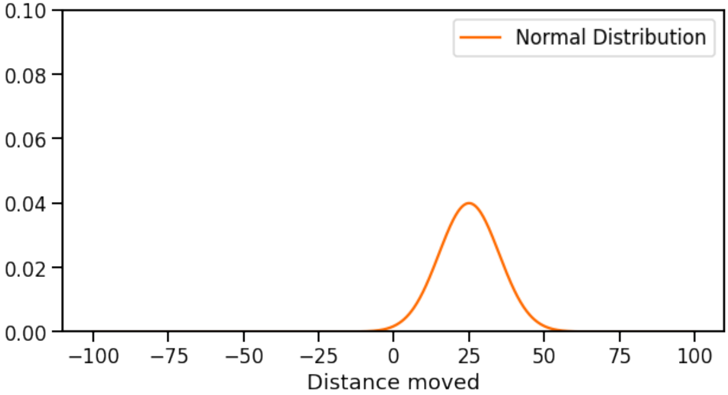

Fig.: Calculated Postion at t_1

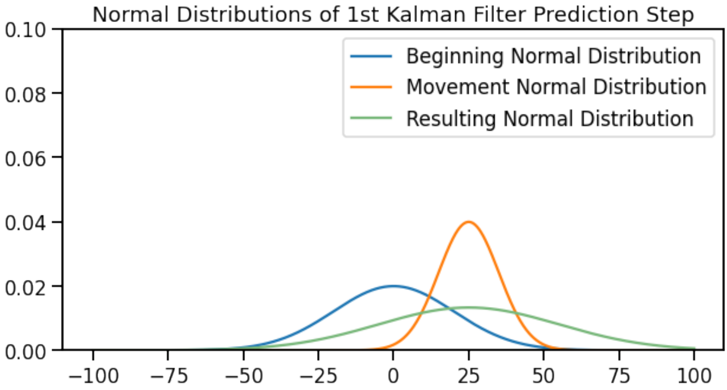

Fig.: Predicted Postion at t_1

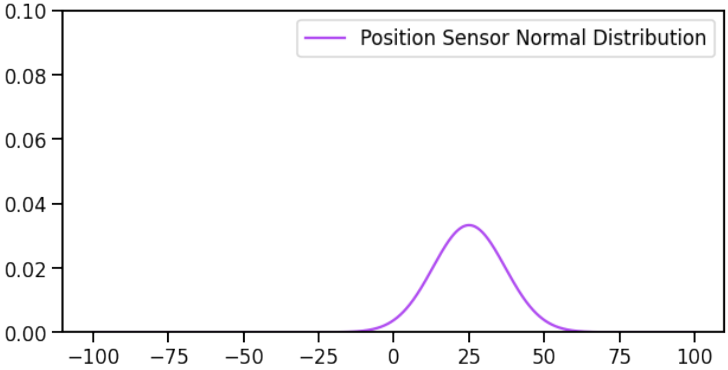

Fig.: Messured Sensor Data of Postion at t_1

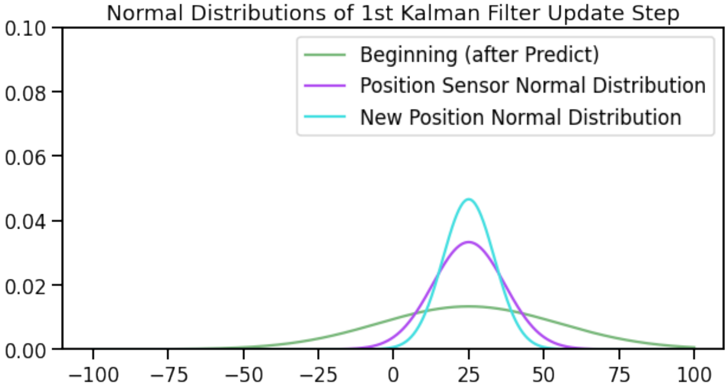

Fig.: Correction of the Kalman Gain after measurement at t_1

Takeaways

- The precision decreases with the prediction
- The precision increases with the correction

State Representation

We define the state vector x to include the position and velocity of the mouse in both x and y directions

$$x = \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}$$

where:

- x is the position on the x-axis
- y is the postion on the y-axis
- v_x is the velocity in the x-direction
- v_y is the velocity in the y-direction

Formulate the Transtition Model

The state transition model describes how the state evolves from one time step to the next. If we assume a constant velocity model, the state transition can be expressed as:

$$x_{k+1} = Ax_k + w_k$$

where w_k represents the proces noise, assumed to be Gaussian with zero mean and covariance Q.

Create the Transition Matrix

The transition matrix A for a constant velocity Model is:

$$A = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Δt is the time intervall between mesurements.

Observation Model

The observation model realtes the state to the measurements:

$$z_k = Hx_k + v_k$$

where v_k represents the measurement noise, assumed to be Gaussian with zero mean and covariance R.
The observation matrix H for direct measurement of position is:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Prediction Step

The prediction step estimates the next state and its uncertainty:

- Predict the state:

$$x_{k+1} = Ax_k \tag{1}$$

- Predict the error coavriance:

$$P_{k+1} = AP_kA^T + Q \tag{2}$$

Correction Step

The correction step updates the state estimate with the new measurement:

- Compute the Kalman gain:

$$K_k = P_k H^T (H P_k H^T + R)^{-1} \tag{3}$$

- Update the state estimate:

$$x_k = x_k + K_k (z_k - H x_k) \tag{4}$$

- Update the error covariance:

$$P_k = (I - K_k H) P_k \tag{5}$$

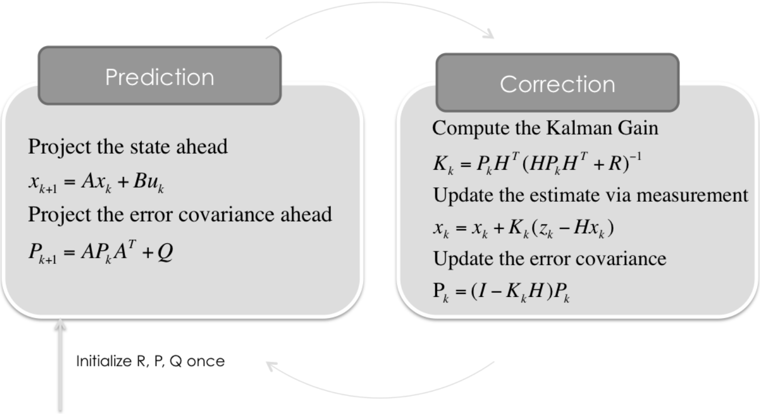

Fig.: Iterative Nature of the Kalman filter

Cheat Sheet - Math Symbols

Symbol	Meaning
k	Interval / iteration of the Kalman filter
z_k	Measurement from the sensor
x_k	Value of the current estimation
x_{k-1}	Value of the previous estimation
P_k	Value of the current error covariance
P_{k-1}	Value of the previous error covariance
R	Variance of the measurements
Q	Process noise covariance
A	Transition Matrix
H	Observation Matrix
Bu_{k-1}	Control signal

Summary

KALMA KALMA !