# Online Appendix

**to**

**Frenemies in the Retail Market: A Partnership Between a Physical Retailer and an E-tailer for Consumer Returns**

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***A. Proof of Lemma 1***

We first derive their shopping choices by comparing , , and . We find that  when . For no-cross-return case, we further derive . We consider that , such that showrooming will not dominate e-Direct. For cross-return case, we get . We find  when  for both cross- and no-cross-return cases.

Then we separate our analysis into two cases: (i)  and (ii) . For the case with , we get , which indicates and . Then, we find that (i)  for  , and (ii)  for . If we further have , i.e., , we will have  for . To summarize, when , the consumers with  will choose e-Direct, the consumers with  will choose showrooming, and the consumers with  will choose buy-offline. If , i.e., , none of the consumers will choose buy-offline. The consumers with  will choose e-Direct, and the consumers with  will choose showrooming. We assume that  in order to have , otherwise buy-offline and showrooming would not co-exist at any given  for no-cross-return case.

For the case with , which indicates , there does not exist a region for  as it requires . Hence, there is no showrooming consumer in this case. Instead, we find that  when , which indicates  and . To make sure , we need , more specifically, and . It’s trivial to show. We can further verify that  based on the assumption . In addition, we find that  when . Hence, when , the consumers with  will choose e-Direct, and the consumers with  will choose buy-offline. When , we have . In such a case, the consumers with  will choose buy-offline.

***B. Proof of Lemma 2***

We first set up the consumer demand , based on consumer segmentation from Lemma 1. For simplicity, we introduce the following notation: we use case A to denote Seg F (segment F) from Lemma 1, case B for Seg E-F, case C for Seg E-S-F, and case D for Seg E-S.

* Case A: When , , , ;
* Case B: When , , , ;
* Case C: When , , , ;
* Case D: When , , , .

Now let’s derive offline retailer’s best response functions under each case.

* Case A: When , we get, the total profit function is . We derive positive derivative , so the best response price for physical retailer is . Thus, the total profit for offline retailer in this case is ;
* Case B: When , we get, the total profit function is . We solve the derivative and get  and  Then we evaluate at the upper limit of , . To make , we get . Then we evaluate at the lower limit of , . To make , we get . Note here,  is positive. When , solve the Lagrangian , we get the boundary solution and . When , solve the Lagrangian , we get the boundary solution and ;
* Case C: When , we get, the total profit function is . We derive negative second order derivative , so we get  such that . The total profit in this case is . To reach this optimal price and profit, we need to have . For the upper limit,  when . For the lower limit  when . Notice that , so we have . Next we derive the boundary solution when . We solve the Lagrangian , and get the boundary solution  and . Then when , we solve the Lagrangian , and get the boundary solution  and ;
* Case D: When , we get , the total profit function is . Hence, we have no best response function for this case.

Next, we summarize the offline retailer’s overall best response function by consolidating their best response from above.

* Case A:  and the corresponding total profit is ;
* Case B: When , the boundary solution is and the corresponding total profit is .

When , the interior solution is  and the corresponding total profit is .

When , the boundary solution is and the corresponding total profit is ;

* Case C: When , the boundary solution is  and the corresponding total profit is .

When , the interior solution is  and the corresponding total profit is .

When , the boundary solution is  and the corresponding total profit is .

From the summary, we find , so  is dominated. We also notice that . Hence, we compare the two boundaries  and , and we get . We derive when . Therefore, we have:

* Case F1: 

When ,  and the total profit is .

When ,  and the total profit is .

When ,  and the total profit is .

When ,  and the total profit is .

When ,  and the total profit is ;

When , i.e., , we need to compare  and . Hence, we get  We derive positive second order derivative . Then we evaluate  when , and we get . We evaluate  when , and we get . After solving , we get two roots and . Then to compare  and , we take the difference . When , we have . Since  and , we have . Therefore, the smaller root  is inside the range and we get . Since ,  decrease as  decreases. Next, we will compare  with  and . First, we get  and . Given , have a chance to intersect with  and . Second, let , so we have . Let , so we have . Then, we compare  and , we get . Note that  when . Hence, when  decreases,  will reach  first. Therefore, to summarize, we have:

* Case F2: 

When ,  and the total profit is .

When ,  and the total profit is .

When ,  and the total profit is .

When ,  and the total profit is ;

When , we have , so we need to compare  and . We derive  and the second order derivative  is positive. We first evaluate  when , and get . Then we get . When , we have , assuming . Let , we have . Hence when , we have . Then we evaluate  when , and get . Next, we derive the upper boundary of  by solving . We get two roots and . Then we compare  and , and get . So we pick up the smaller root and have . To evaluate , we first have  and . Then we solve  and get . Hence, we have . To summarize the case, we have:

* Case F3: 

When ,  and the total profit is .

When ,  and the total profit is .

When ,  and the total profit is ;

When , we have , so we need to compare  and . We derive  and after solving , we have . To summarize, we have:

* Case F4: 

When ,  and the total profit is .

When ,  and the total profit is .

Now let’s derive e-retailer’s best response functions  to the offline retailer’s choice of offline price under each case.

* Case A: When , we get, the total profit function is . Hence, there is no best response function in this case.
* Case B: When , we get, the total profit function is  and we derive the second order derivative . Then we solve  and get  and . Note that we have the condition , so we first evaluate the lower boundary . When , we get . We derive negative derivative  and get  when . Hence, we need to have . Then we evaluate the upper boundary . When , we get . We derive positive derivative  and get  when . Hence, we need to have . Then, we check the compatibility and have . So we need to satisfy the condition  in this case. When , solve the Lagrangian , we get the boundary solution  and . When , we get the boundary solution  and .
* Case C: When , we get, the total profit function is  and we derive the second order derivative . Then we solve  and get  and . Note that we have the condition , so we first evaluate the lower boundary . When , we get . We derive negative derivative  and get  when . Hence we need to have . Then we evaluate the upper boundary . When , we get . We derive positive derivative  and get  when . Hence, we need to have . Then, we check the compatibility and have  based on our assumption that . So we need to satisfy the condition  in this case. When , solve the Lagrangian , we get the boundary solution  and . When , solve the Lagrangian , we get the boundary solution  and .
* Case D: When , we get , the total profit function is . We derive positive derivative . Hence we get the boundary solution  and .

Next, we summarize the e-retailer’s overall best response function by consolidating their best response from above. First, we notice that , so case D is dominated. Therefore, we have the following:

* Case B: When , the boundary solution is  and the corresponding total profit is .

When , the interior solution is  and the corresponding total profit is .

When , the boundary solution is and the corresponding total profit is ;

* Case C+D: When , the boundary solution is  and the corresponding total profit is .

When , the interior solution is  and the corresponding total profit is .

When , the boundary solution is and the corresponding total profit is .

First, we notice that . Then we compare the two boundaries  and , and we have  given . So we get . Then we need to discuss the position of the other two boundaries  and . Since , there are two possible positions for , i.e.,  and . Therefore, we look at the two cases separately.

When , i.e. , we have . Since , we get . Then we compare  with , and we get . We derive the second order derivative . Then when , we get . When , we get . Therefore, we derive two roots and  by solving  and we keep the larger root. We have , so we keep . Therefore, to summarize, we have:

* Case B+C+D: 

When , the boundary solution is  and the corresponding total profit is .

When , the interior solution is  and the corresponding total profit is .

When , the interior solution is  and the corresponding total profit is .

When , the boundary solution is and the corresponding total profit is .