Theory and Practice of Humanoid Walking Control

Jaeheung Park, SNU

Inverse Kinematics for Leg

- Use the material in Homework assignment for Leg IK – HW#1

Walking, Balancing, and Falling

- Walking is bipedal locomotion that moves COM from one position to the other. In general, it involves foot steps.
- Balancing usually means that it can maintain or recover its certain (balanced?) states from disturbed state without falling.
- Falling is a state that the robot cannot recover to a balanced state.
- Maintaining a certain contact state is related to ZMP being within a supporting polygon.
- Determining that a robot will fall or not is not simple especially for a high DOF system.

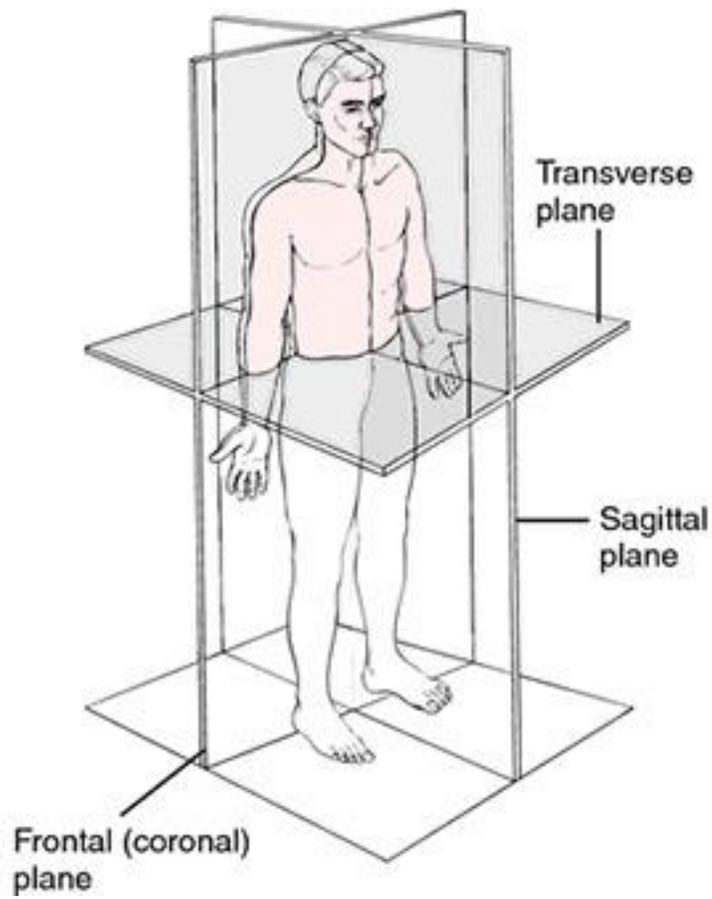
Main Assumption in This Class

- Robot is walking on a flat ground.
- Foot hold is plane. That is, there are plane-contacts between the foot hold and the flat ground.

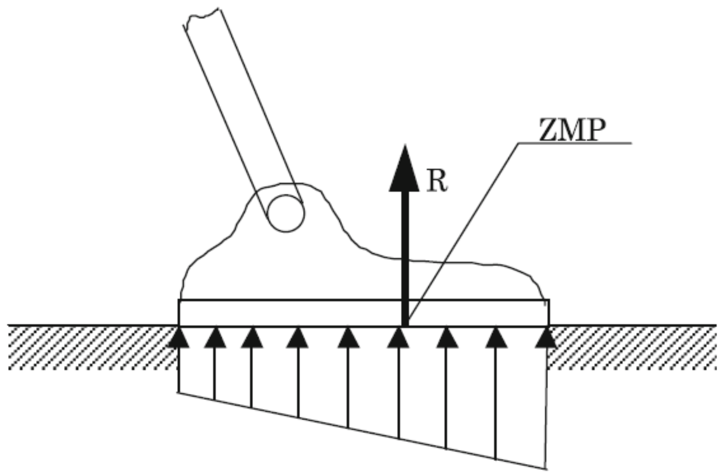
The material in today's lecture is mainly from the following reference book.

- Introduction to Humanoid Robotics
  - Shuuji Kajita, Hirohisa Hirukawa, Kensuke Harada, Kazuhito Yokoi

Sagittal, Frontal, Transverse Planes

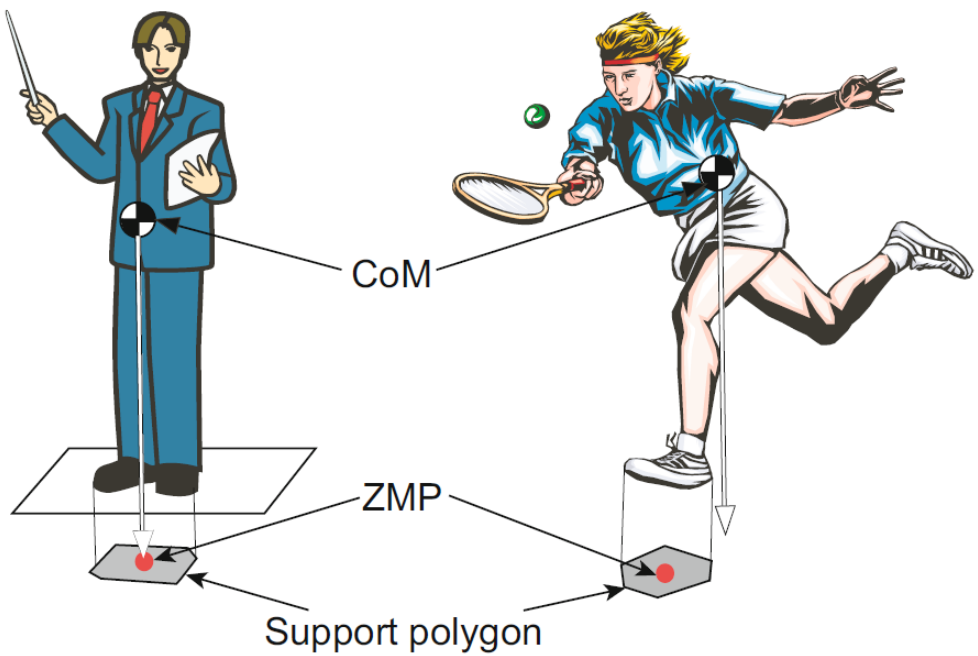


Zero Moment Point (Sagittal Plane)



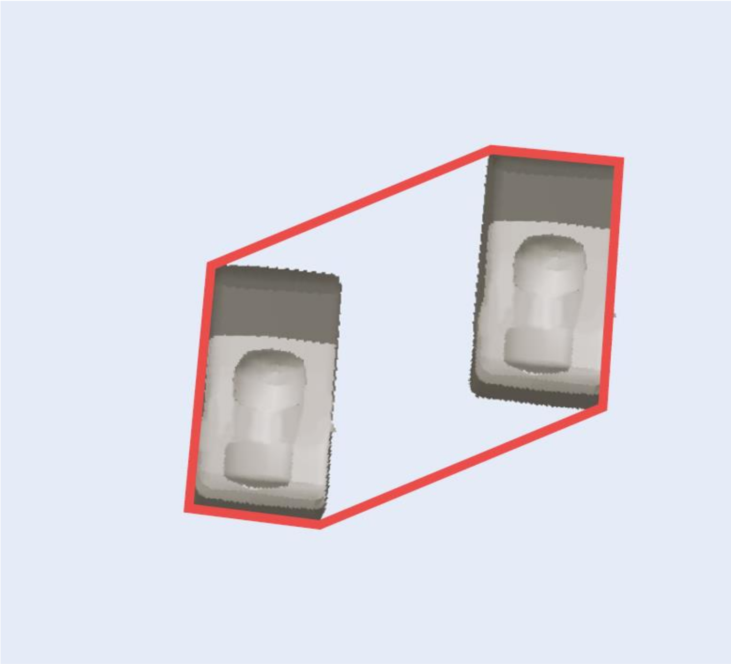
- The reaction forces on the foot can be represented by one resultant force while resultant moment is zero. This point is defined to be zero moment point or ZMP.
- Only vertical forces are considered. The tangential force (friction forces) does not contribute to the resultant moment.

ZMP in supporting polygon



- In a static case, CoM = ZMP which is in the supporting polygon if the system is stable.
- In a dynamic case, they are not the same. However, ZMP is still within the supporting polygon.

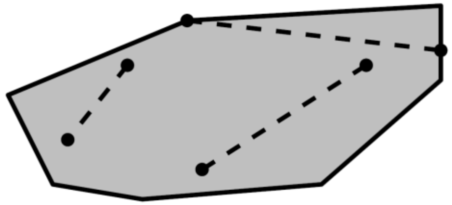
Supporting Polygon on flat surface



- The support polygon is defined as a convex hull, which is the smallest convex set including all contact points.

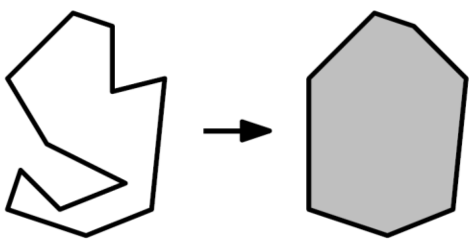
Convexity

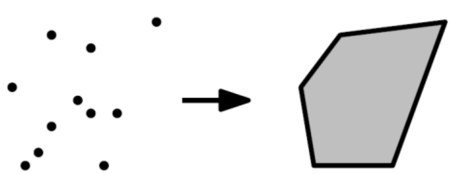
- A shape or set is convex if for any two points that are part of the shape, the whole connecting line segment is also part of the shape.



Convex hull

- For any subset of the plane, its convex hull is the smallest convex set that contains that subset.

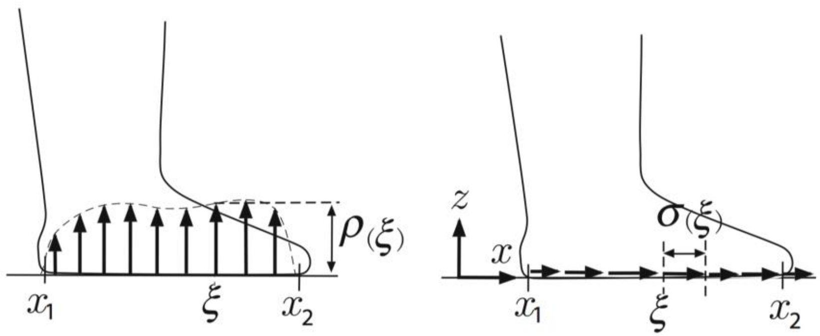




ZMP in supporting polygon

- The condition that ZMP is in supporting polygon is a necessary condition.
- The ZMP of each foot also has to be within each foothold not to lose the plane contact.

2D analysis of ZMP

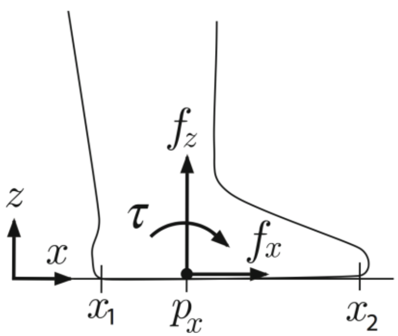


(a) Vertical force (b) Horizontal force

$$f_x = \int_{x_1}^{x_2} \sigma(\xi) d\xi$$

$$f_z = \int_{x_1}^{x_2} \rho(\xi) d\xi$$

$$\tau(p_x) = -\int_{x_1}^{x_2} (\xi - p_x)\rho(\xi) d\xi$$



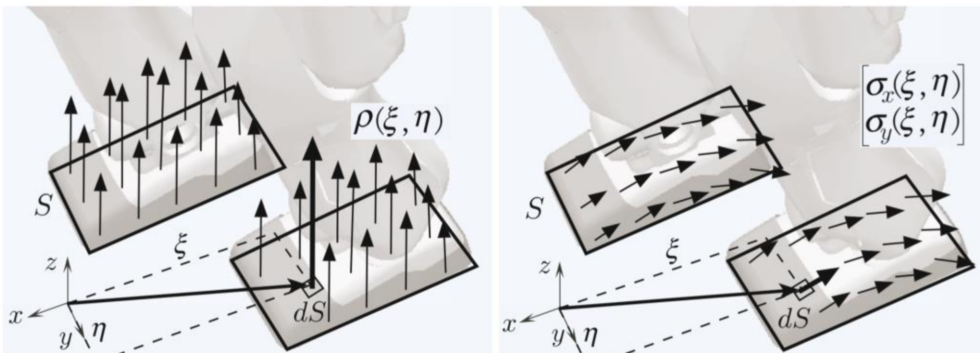
$$p_x = \frac{\int_{x_1}^{x_2} \xi\rho(\xi) d\xi}{\int_{x_1}^{x_2} \rho(\xi) d\xi}$$

- Px is the center of pressure where the moment is zero. This is the definition of ZMP.

Location of ZMP

- If $\rho(\xi) \geq 0$, then $x_1 \leq p_x \leq x_2$

3D analysis of ZMP



(a) Vertical reaction forces (b) Horizontal reaction forces

- Effect of vertical reaction forces

$$f_z = \int_S \rho(\xi,\eta)dS$$

- The moment $\tau_n(p)$ is the moment due to the normal force f_z.

3D analysis of ZMP due to vertical forces

$$\tau_n(p) \equiv [\tau_{nx}\, \tau_{ny}\, \tau_{nz}]^T$$

$$\tau_{nx} = \int_S (\eta - p_y)\rho(\xi, \eta) dS$$

$$\tau_{ny} = -\int_S (\xi - p_x)\rho(\xi, \eta) dS$$

$$\tau_{nz} = 0.$$

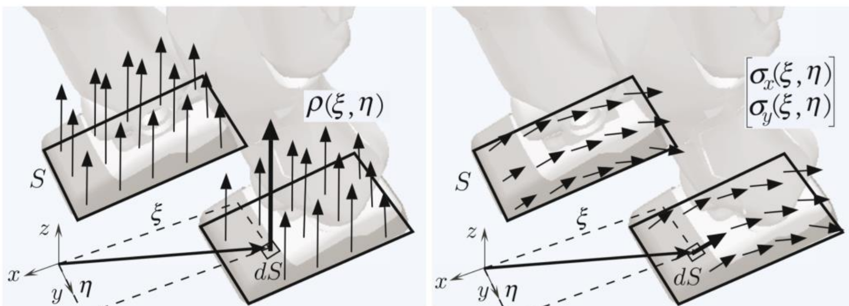
- To make $\tau_{nx},\ \tau_{ny}$ be zero,

$$p_x = \frac{\int_S \xi \rho(\xi, \eta) dS}{\int_S \rho(\xi, \eta) dS}$$

$$p_y = \frac{\int_S \eta \rho(\xi, \eta) dS}{\int_S \rho(\xi, \eta) dS}$$

- Because rho is the pressure, p is the center of pressure.

Effect of the horizontal component



(a) Vertical reaction forces (b) Horizontal reaction forces

$$f_x = \int_S \sigma_x(\xi,\eta)dS$$

$$f_y = \int_S \sigma_y(\xi,\eta)dS$$

$$\tau_t(p) \equiv [\tau_{tx}\,\tau_{ty}\,\tau_{tz}]^T$$

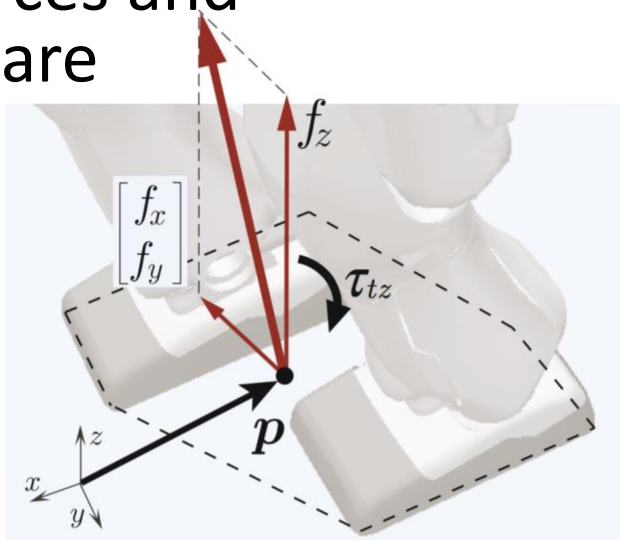
$$\tau_{tx} = 0,\ \tau_{ty} = 0,$$

$$\tau_{tz} = \int_S \{(\xi - p_x)\sigma_y(\xi,\eta) - (\eta - p_y)\sigma_x(\xi,\eta)\}dS$$

- The moment tau_t is the moment due to the horizontal forces.

3D analysis

- As a summary, the ground reaction forces and moments over the surface of the sole are



$$f = [f_x\ f_y\ f_z]^T,$$

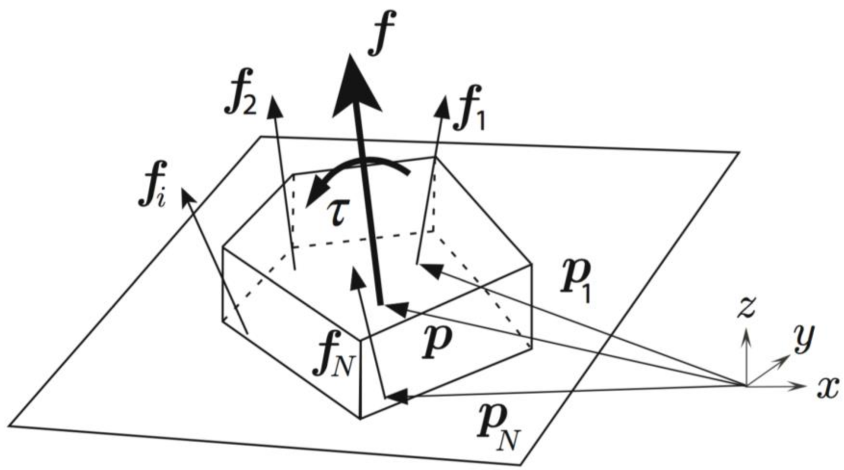
$$\tau_p = \tau_n(p) + \tau_t(p)$$

$$= [0\ 0\ \tau_{tz}]^T$$

- In general cases, the moment about z direction is not zero. Therefore, the ZMP (p) is the point where the horizontal components of the ground reaction moments are zero.

This can be a problem…

Region of ZMP in 3D



$$f = \sum_{i=1}^{N} f_i$$

$$\tau(p) = \sum_{i=1}^{N} (p_i - p) \times f_i$$

- The x and y components of tau(p) is zero at ZMP. So,

$$p = \frac{\sum_{i=1}^{N} p_i f_{iz}}{\sum_{i=1}^{N} f_{iz}}$$

Region of ZMP in 3D

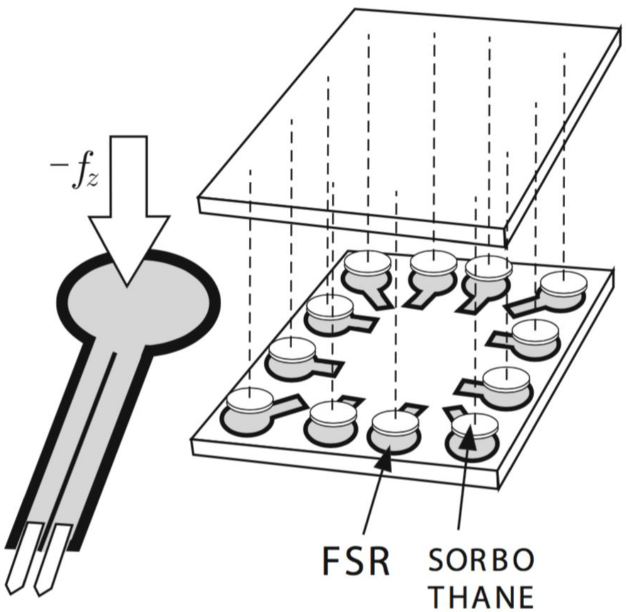
$$p = \frac{\sum_{i=1}^{N} p_i f_{iz}}{\sum_{i=1}^{N} f_{iz}}$$

- Normally f_{iz} >= 0, then

$$p = \sum_{i=1}^{N} \alpha_i p_i \qquad \alpha_i \geq 0 \text{ and } \sum_{i=1}^{N} \alpha_i = 1$$

- Therefore, ZMP is within the convex hull of supporting polygon.

Measurement of ZMP?



- Measuring f_z at multiple points could be enough.

$$p = \frac{\sum_{i=1}^{N} p_i f_{iz}}{\sum_{i=1}^{N} f_{iz}}$$

where

$$p = [p_x,\ p_y,\ 0]^T$$

ZMP for double support

- The forces on both feet should be accounted for.
- Global stability(?) is assessed by looking at the ZMP and supporting polygon.
- Individual contact states of each foot needs to be handled by ZMP and supporting polygon of each foot.

Calculation of ZMP from Robot's motion

- The ground reaction force tau can be expressed by using ZMP (p), the force, and the moment tau_p at the point.

$$\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{f} + \boldsymbol{\tau}_p$$

- The linear and angular momentums are expressed as the following force and moment equilibrium.

$$\dot{\mathcal{P}} = M\boldsymbol{g} + \boldsymbol{f}$$
$$\dot{\mathcal{L}} = \boldsymbol{c} \times M\boldsymbol{g} + \boldsymbol{\tau}$$

- Moment equilibrium equation becomes as the following by substituting tau and f with the first two equations.

$$\boldsymbol{\tau}_p = \dot{\mathcal{L}} - \boldsymbol{c} \times M\boldsymbol{g} + (\dot{\mathcal{P}} - M\boldsymbol{g}) \times \boldsymbol{p}$$

- The first two rows of this equation can be used to compute ZMP.

$$\tau_{px} = \dot{\mathcal{L}}_x + Mgy + \dot{\mathcal{P}}_y p_z - (\dot{\mathcal{P}}_z + Mg)\boxed{p_y} = 0$$
$$\tau_{py} = \dot{\mathcal{L}}_y - Mgx - \dot{\mathcal{P}}_x p_z + (\dot{\mathcal{P}}_z + Mg)\boxed{p_x} = 0$$

$$\mathcal{P} = [\mathcal{P}_x\ \mathcal{P}_y\ \mathcal{P}_z]^T$$
$$\mathcal{L} = [\mathcal{L}_x\ \mathcal{L}_y\ \mathcal{L}_z]^T$$
$$\boldsymbol{c} = [x\ y\ z]^T$$
$$\boldsymbol{g} = [0\ 0\ -g]^T.$$

Calculation of ZMP from Robot's motion

$$p_x = \frac{Mgx + p_z\dot{\mathcal{P}}_x - \dot{\mathcal{L}}_y}{Mg + \dot{\mathcal{P}}_z}$$

$$p_y = \frac{Mgy + p_z\dot{\mathcal{P}}_y + \dot{\mathcal{L}}_x}{Mg + \dot{\mathcal{P}}_z}$$

- Simplified model by assuming all the links have no rotational inertias.

$$p_x = \frac{\sum_{i=1}^{N} m_i\{(\ddot{z}_i + g)x_i - (z_i - p_z)\ddot{x}_i\}}{\sum_{i=1}^{N} m_i(\ddot{z}_i + g)}$$

$$p_y = \frac{\sum_{i=1}^{N} m_i\{(\ddot{z}_i + g)y_i - (z_i - p_z)\ddot{y}_i\}}{\sum_{i=1}^{N} m_i(\ddot{z}_i + g)}$$

Calculation of ZMP from Robot's motion

$$p_x = \frac{Mgx + p_z\dot{\mathcal{P}}_x - \dot{\mathcal{L}}_y}{Mg + \dot{\mathcal{P}}_z}$$

$$p_y = \frac{Mgy + p_z\dot{\mathcal{P}}_y + \dot{\mathcal{L}}_x}{Mg + \dot{\mathcal{P}}_z}$$

- Simplified model by assuming the whole robot is a point mass.

$$\mathcal{P} = M\dot{\boldsymbol{c}}$$

$$\mathcal{L} = \boldsymbol{c} \times M\dot{\boldsymbol{c}}$$

$$\begin{bmatrix} \dot{\mathcal{P}}_x \\ \dot{\mathcal{P}}_y \\ \dot{\mathcal{P}}_z \end{bmatrix} = \begin{bmatrix} M\ddot{x} \\ M\ddot{y} \\ M\ddot{z} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\mathcal{L}}_x \\ \dot{\mathcal{L}}_y \\ \dot{\mathcal{L}}_z \end{bmatrix} = \begin{bmatrix} M(y\ddot{z} - z\ddot{y}) \\ M(z\ddot{x} - x\ddot{z}) \\ M(x\ddot{y} - y\ddot{x}) \end{bmatrix}$$

$$\Rightarrow$$

$$p_x = x - \frac{(z - p_z)\ddot{x}}{\ddot{z} + g}$$

$$p_y = y - \frac{(z - p_z)\ddot{y}}{\ddot{z} + g}$$

Limitation of ZMP?

- We need to think about what the ZMP is.
- Friction constraints are not considered.

- ZMP paper:
  - Vukobratović, Miomir and Borovac, Branislav. Zero-moment point—Thirty five years of its life. International Journal of Humanoid Robotics, Vol. 1, No. 1, pp. 157–173, 2004.