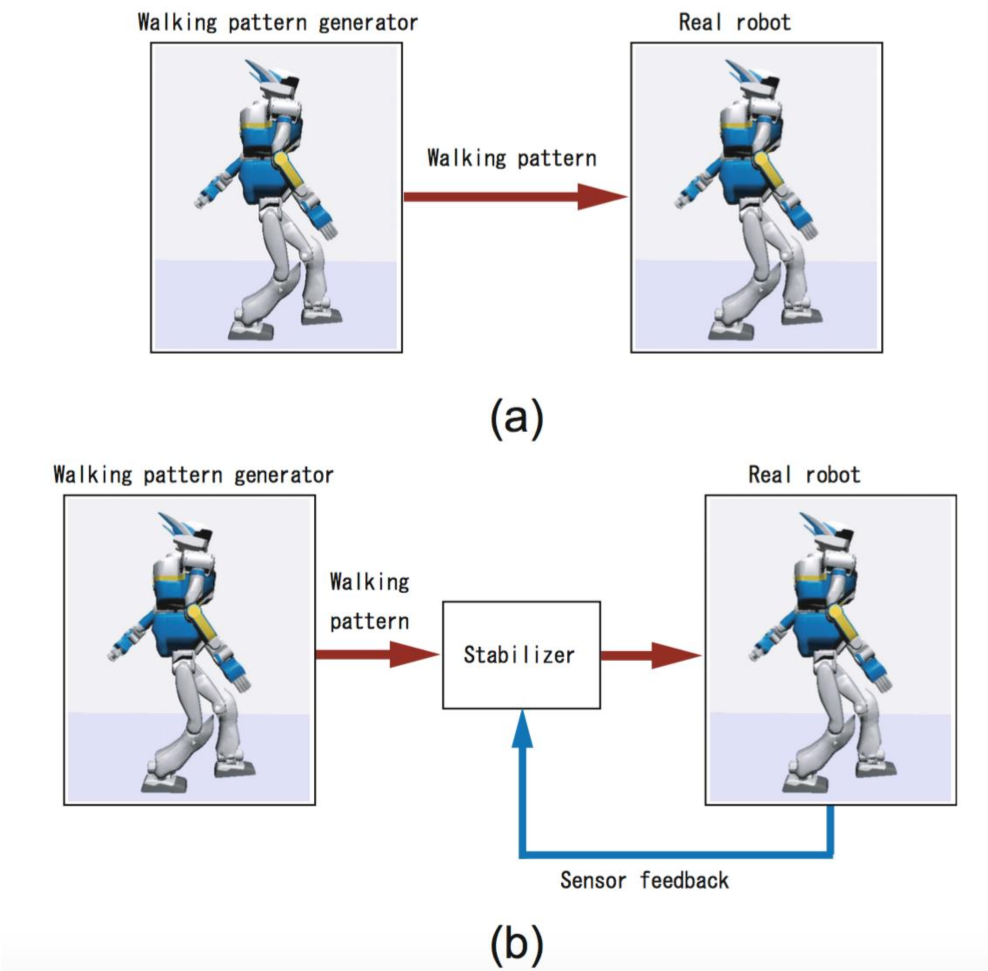
Theory and Practice of Humanoid Walking Control

Jaeheung Park, SNU

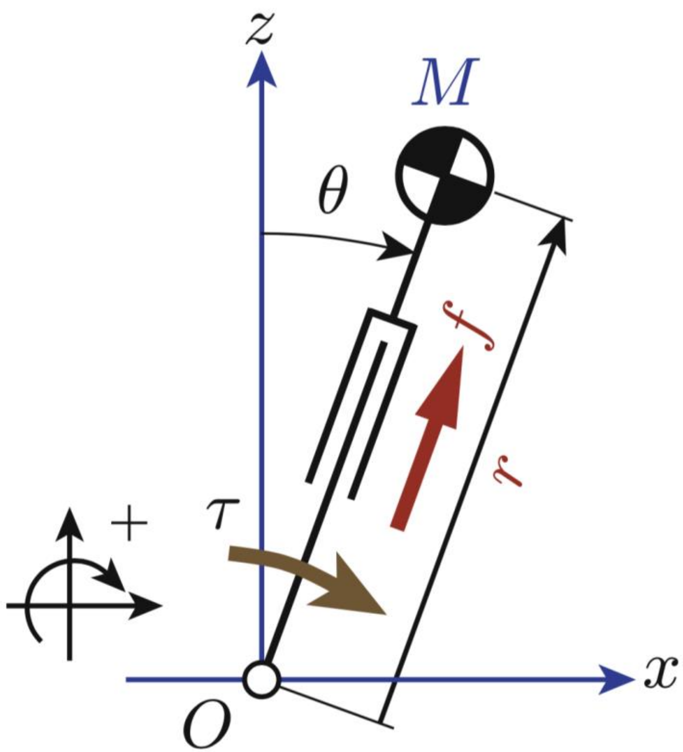
Walking Pattern Generation



Walking pattern is a motion trajectory that would not fall down during walking. That is, ZMP is within supporting polygons all the time.

Only feedforward (or open-loop) control may not work very well.

2D Inverted Pendulum Model

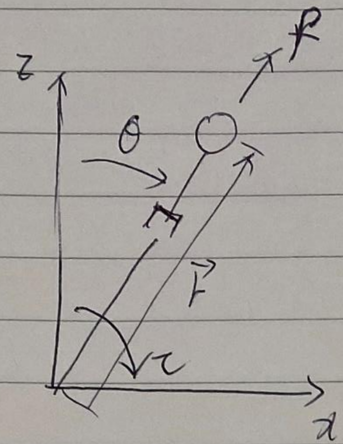


- Point mass
- Massless leg with rotational joint on the ground
- Only sagittal motion

$$r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - gr\sin\theta = \tau/M$$

$$\ddot{r} - r\dot{\theta}^2 + g\cos\theta = f/M$$

Derivation!!!



* $\vec{p} = r\vec{e_r}$

* $\vec{v} = \dot{r}\vec{e_r} + r\dot{\vec{e_r}}$

$\left(\dot{\vec{e_r}} = \vec{\omega} \times \vec{e_r} = \dot{\theta}\,\vec{e_y} \times \vec{e_r} = \dot{\theta}\,\vec{e_\theta}\right)$

$\vec{v} = \dot{r}\vec{e_r} + r\dot{\theta}\vec{e_\theta}$

* $\vec{a} = \ddot{r}\vec{e_r} + \dot{r}\dot{\vec{e_r}} + (\dot{r}\dot{\theta} + r\ddot{\theta})\vec{e_\theta}$
$\qquad + r\dot{\theta}\,\dot{\vec{e_\theta}}$

$\left(\dot{\vec{e_\theta}} = \vec{\omega} \times \vec{e_\theta} = \dot{\theta}\,\vec{e_y} \times \vec{e_\theta} = -\dot{\theta}\,\vec{e_r}\right)$

$\Rightarrow$ * $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e_r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{e_\theta}$

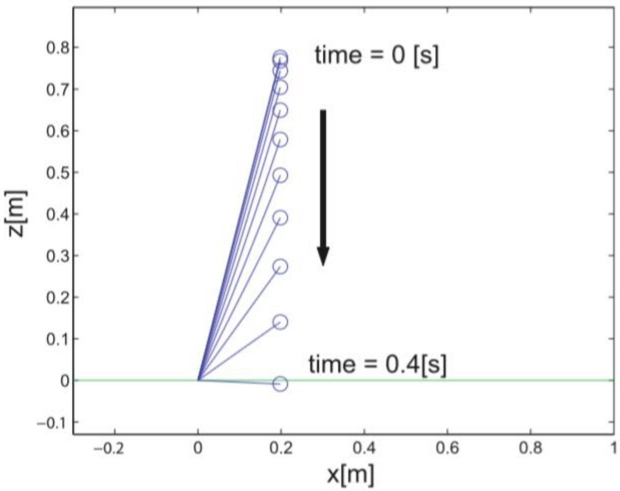
(r-direction) $\quad M(\ddot{r} - r\dot{\theta}^2) = f - Mg\cos\theta$

$\Rightarrow \ddot{r} - r\dot{\theta}^2 + g\cos\theta = f/M$

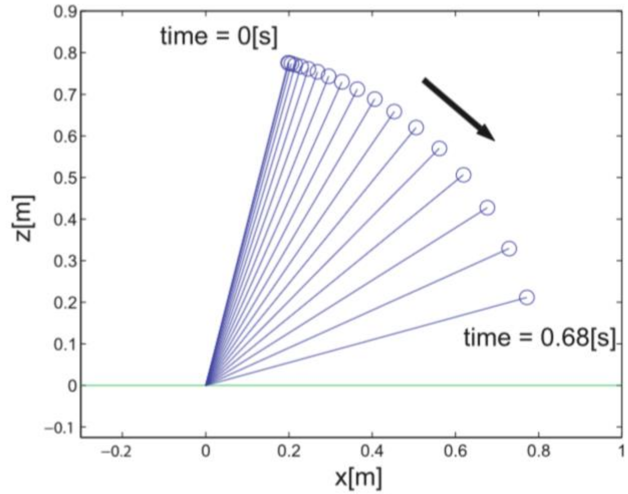
(θ-direction) $\quad M(2\dot{r}\dot{\theta} + r\ddot{\theta}) = \tau/r + Mg\sin\theta$

$\Rightarrow r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - rg\sin\theta = \tau/M$

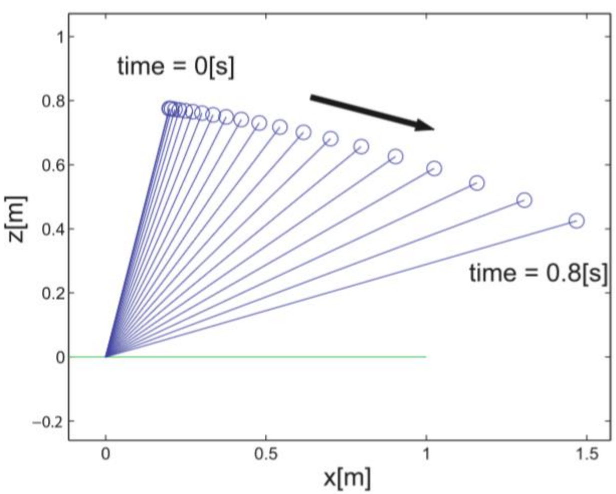
Behavior of 2D Inverted Pendulum



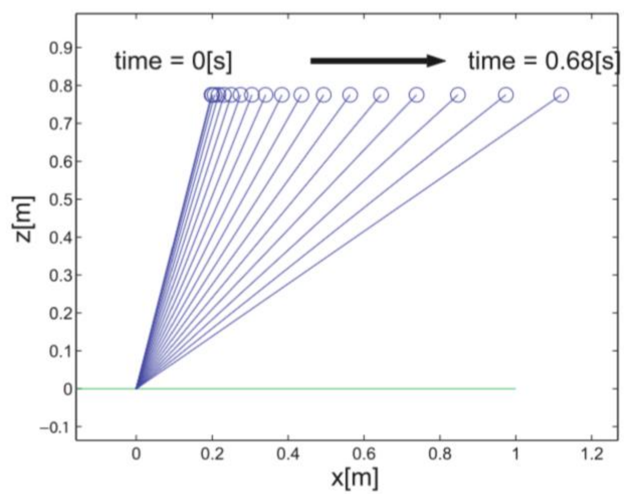
(a) $f = 0$: Free fall of CoM



(b) $f = Mg\cos\theta - Mr\dot{\theta}^2$: Fall down with constant leg length



(c) $f = Mg$: Fall down and acceleration



(d) $f = Mg/\cos\theta$: CoM accelerates while keeping the initial height

When tau=0, the motion of CoM is illustrated in the left figure.

Linear inverted pendulum

- CoM moves horizontally if $$f = \frac{Mg}{\cos\theta}$$
- Horizontal dynamics : $$M\ddot{x} = f\sin\theta$$

$$M\ddot{x} = \frac{Mg}{\cos\theta}\sin\theta = Mg\tan\theta = Mg\frac{x}{z}$$

$$\Rightarrow \ddot{x} = \frac{g}{z}x$$

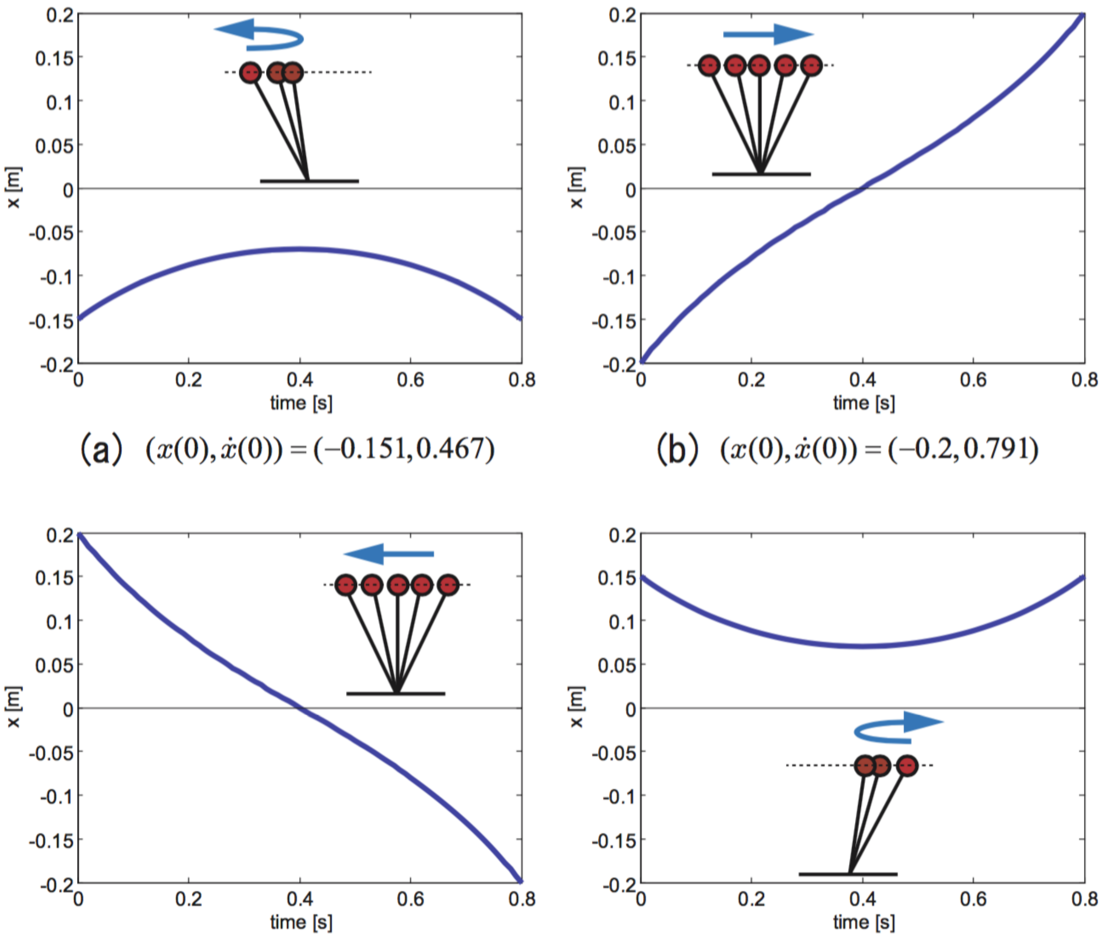
- LIPM 다른 방법으로도 유도

Linear inverted pendulum

- Solution when z is constant $$\ddot{x} = \frac{g}{z}x$$

$$x(t) = x(0)\cosh(t/T_c) + T_c\dot{x}(0)\sinh(t/T_c)$$
$$\dot{x}(t) = x(0)/T_c\sinh(t/T_c) + \dot{x}(0)\cosh(t/T_c)$$
$$T_c \equiv \sqrt{z/g}$$

Linear inverted pendulum



(a) $(x(0), \dot{x}(0)) = (-0.151, 0.467)$

(b) $(x(0), \dot{x}(0)) = (-0.2, 0.791)$

(c) $(x(0), \dot{x}(0)) = (0.2, -0.791)$

(d) $(x(0), \dot{x}(0)) = (0.151, -0.467)$

- Under various initial conditions

Linear inverted pendulum

$$x_1 = x_0 \cosh(\tau/T_c) + T_c \dot{x}_0 \sinh(\tau/T_c)$$
$$\dot{x}_1 = x_0/T_c \sinh(\tau/T_c) + \dot{x}_0 \cosh(\tau/T_c)$$

- Then using the definition of sinh and cosh,

$$x_1 = \frac{x_0 + T_c\dot{x}_0}{2} e^{\tau/T_c} + \frac{x_0 - T_c\dot{x}_0}{2} e^{-\tau/T_c}$$
$$\dot{x}_1 = \frac{x_0 + T_c\dot{x}_0}{2T_c} e^{\tau/T_c} - \frac{x_0 - T_c\dot{x}_0}{2T_c} e^{-\tau/T_c}$$

Linear inverted pendulum

- If initial and final positions are given as x_0 and x_1, the time duration tau can be computed.

$$\tau = T_c \ln \frac{x_1 + T_c \dot{x}_1}{x_0 + T_c \dot{x}_0}$$

Or

$$\tau = T_c \ln \frac{x_0 - T_c \dot{x}_0}{x_1 - T_c \dot{x}_1}$$

Except singularity

Orbital Energy

$$\dot{x}(\ddot{x} - \frac{g}{z}) = 0$$

Typo x 들어가야 함

$$\int \{\ddot{x}\dot{x} - \frac{g}{z}x\dot{x}\}dt = constant$$

- Then

$$\frac{1}{2}\dot{x}^2 - \frac{g}{2z}x^2 = constant \equiv E$$

for unit mass

Orbital Energy

- E can be used to predict its behavior if the values of a certain state is given.

- If it cannot reach the top,

$$E = -\frac{g}{2z} x_{rev}^2 \Rightarrow |x_{rev}| = \sqrt{-\frac{2zE}{g}}$$

E is negative

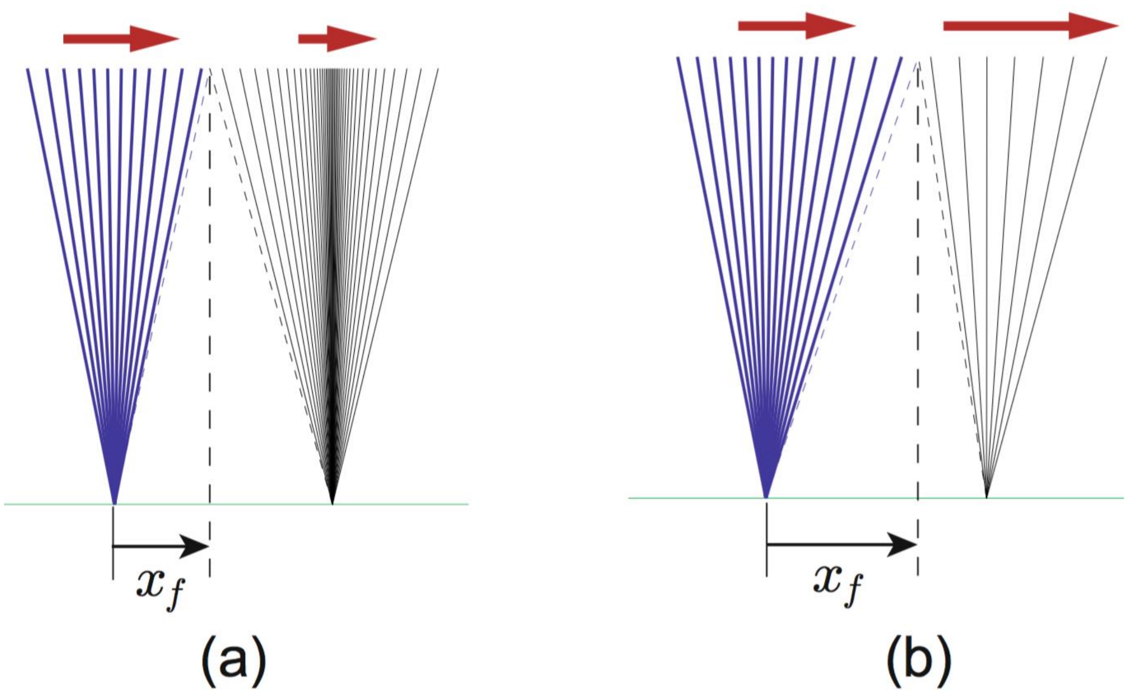
- If it can pass the top,

$$E = \frac{1}{2} \dot{x}_{top}^2 \Rightarrow |\dot{x}_{top}| = \sqrt{2E}$$

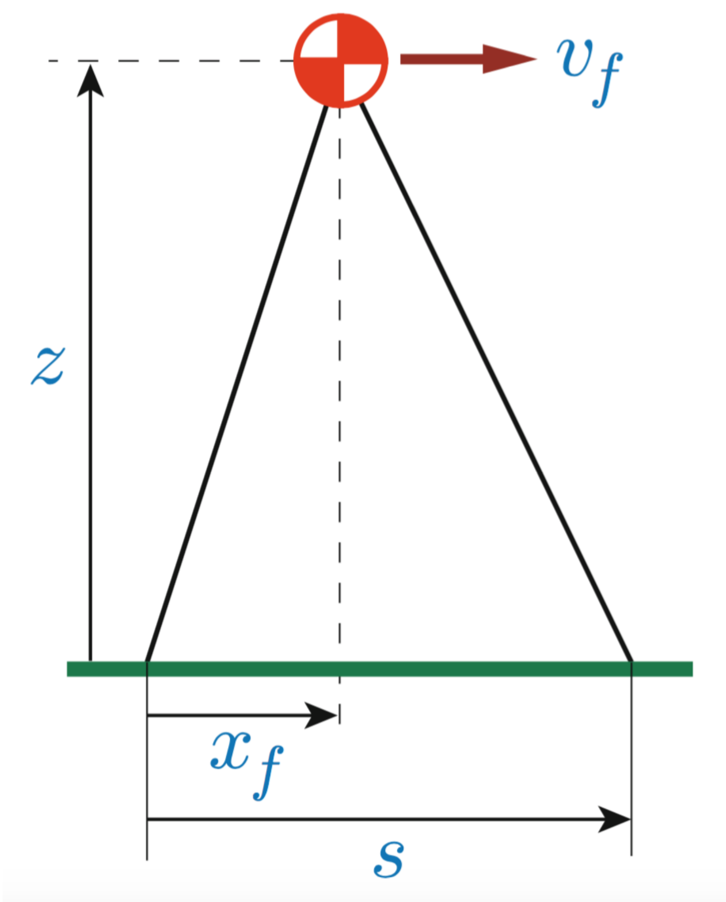
E is positive

Supporting Leg Exchange using orbital energy

- Given a fixed step length, early touch down slows down walking. Later touch down will speed it up.



Supporting leg exchange

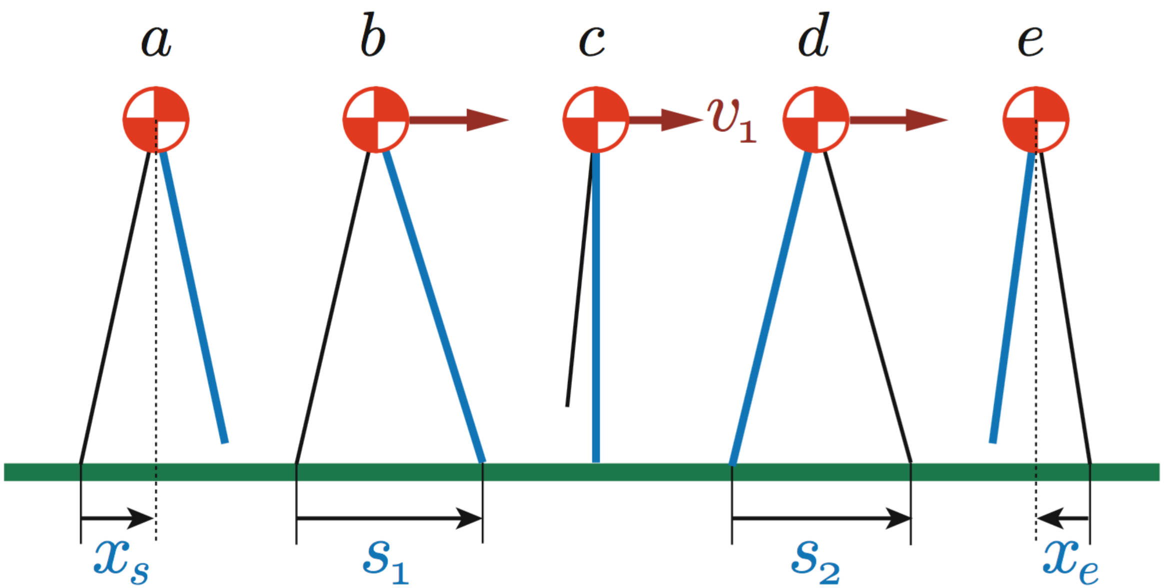


$$E_1 = -\frac{g}{2z}x_f^2 + \frac{1}{2}v_f^2$$

$$E_2 = -\frac{g}{2z}(x_f - s)^2 + \frac{1}{2}v_f^2$$

- V_f should be the same. S is the step length. So, either E1 and E2 can be specified or x_f can be specified.

Example of simple gait planning



- Start (a->b)

$$E_0 = -\frac{g}{2z}x_s^2$$

- Step (b->c->d)

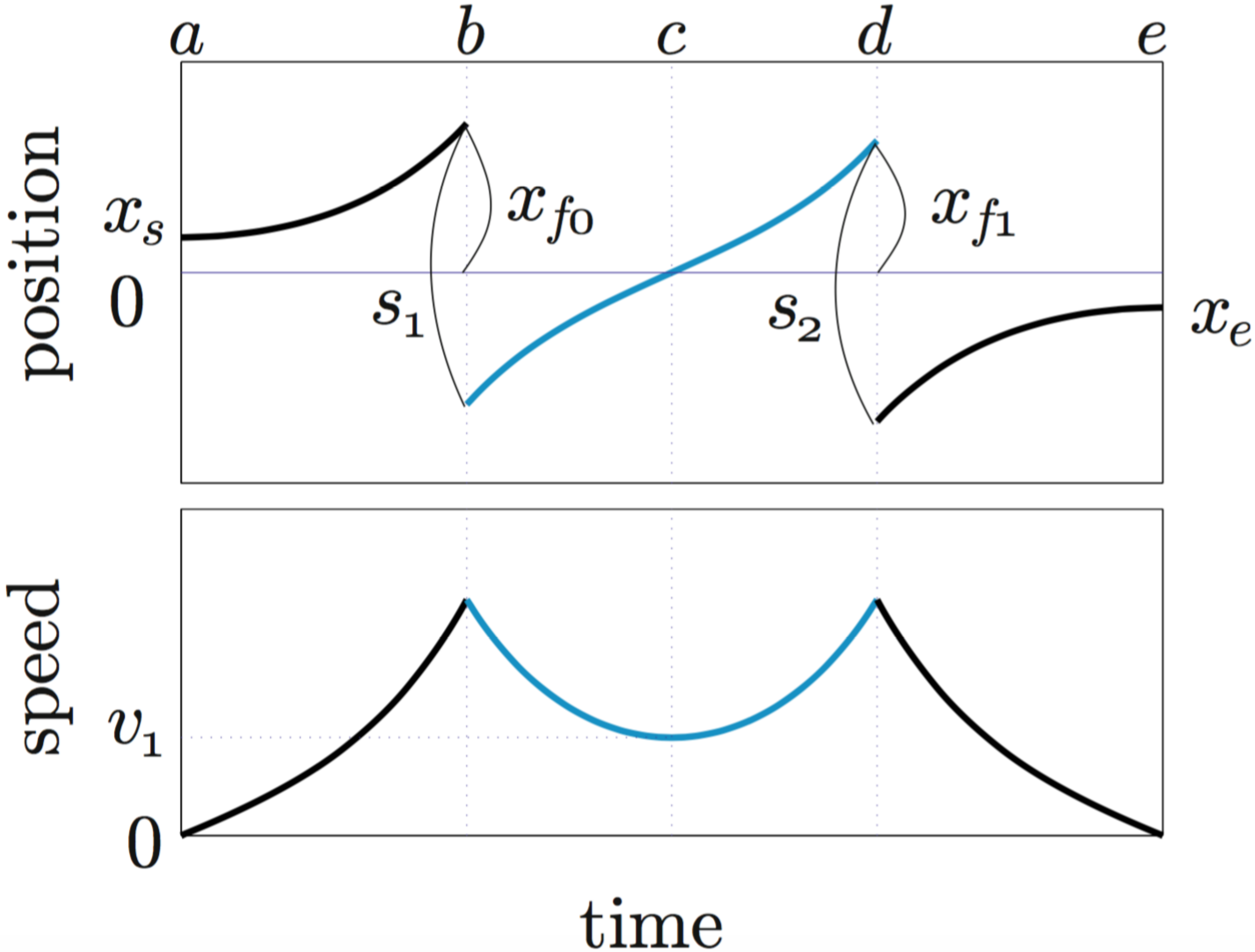
$$E_1 = \frac{1}{2}v_1^2$$

- Finish (d->e)

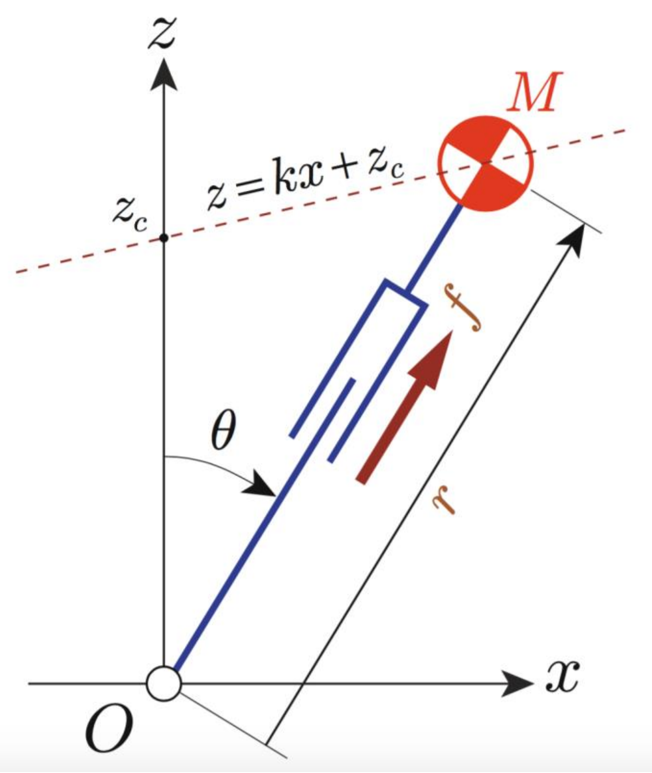
$$E_2 = -\frac{g}{2z}x_e^2$$

- Given E0, E1, E2, and s1, s2, we can compute the switching positions x_f0 and x_f1.

Example of simple gait planning



Walking pattern on uneven terrain



- If we constrain the motion of CoM as

$$z = kx + z_c$$

$$f_x = f\sin\theta = (x/r)f$$

$$f_z = f\cos\theta = (z/r)f$$

The force must be parallel with the constraint line.

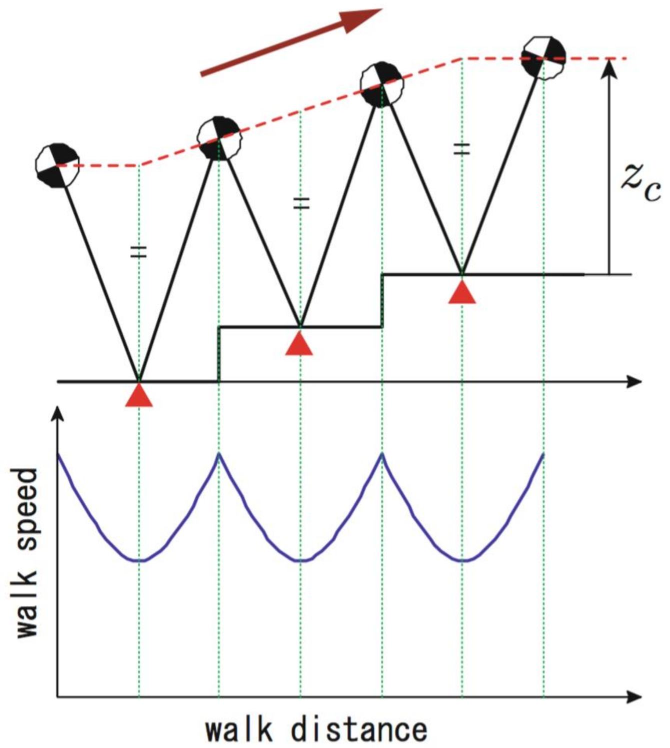
$$f_x : f_z - Mg = 1 : k$$

$$\Rightarrow f = \frac{Mgr}{z - kx} = \frac{Mgr}{z_c}$$

- Then, again

$$\ddot{x} = \frac{g}{z_c}x$$

Walking pattern on uneven terrain



- This part is from 4.2.6. of the reference book.

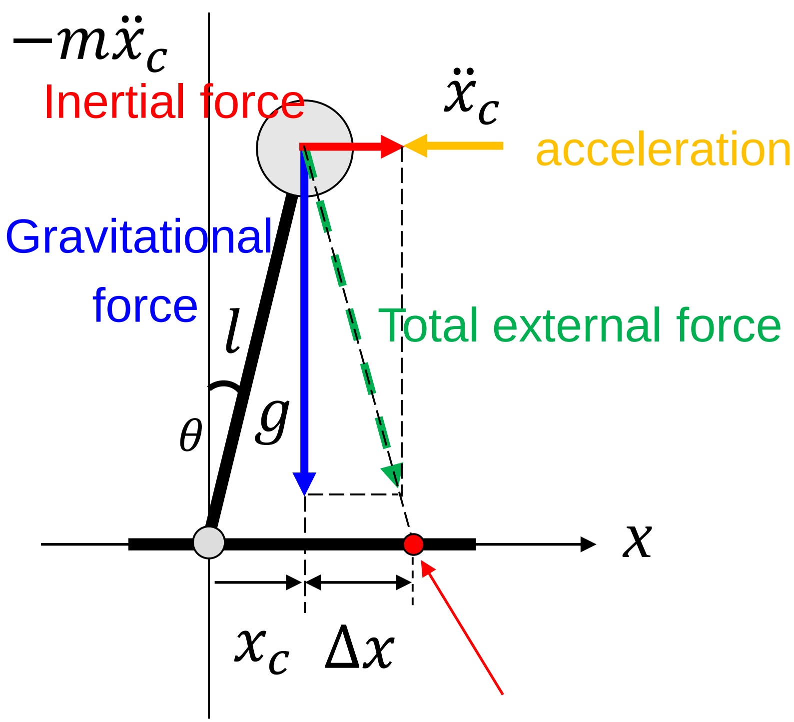
Introduction to Humanoid Robotics
Shuuji Kajita, Hirohisa Hirukawa, Kensuke Harada, Kazuhito Yokoi

Derivation of LIPM: different approaches

Inverted pendulum – Force Equilibrium

✓ Zero Moment Point (ZMP) : The point inside the support area for which it holds that ΣT = 0 (dynamically balanced state).

✓ The point where the total external force passes through the ground.



✓ Assumption

1) $\cos\theta \approx 1$ (linearization)

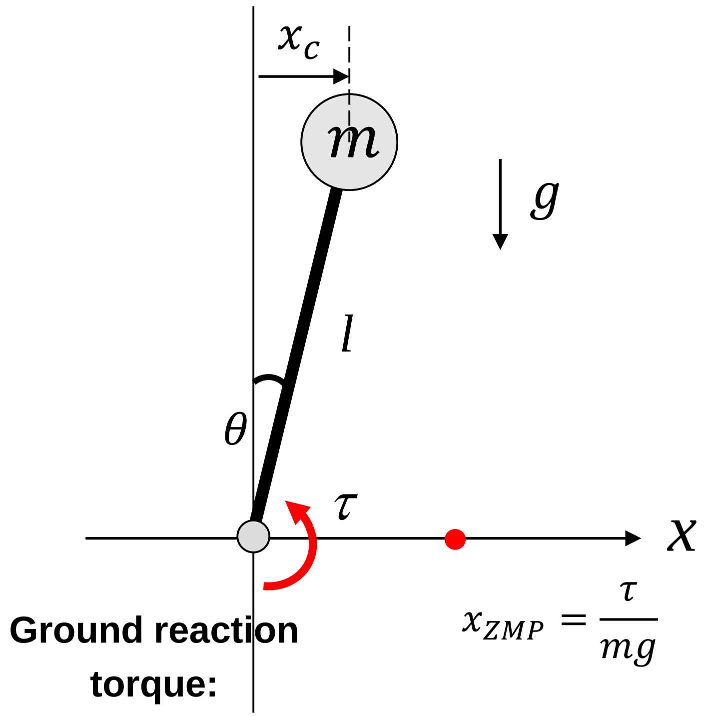
$$g : -\ddot{x}_c = l\cos\theta : \Delta x$$

$$\rightarrow \Delta x = -\frac{l\cos\theta}{g}\ddot{x}_c$$

$$\therefore \Delta x = -\frac{l}{g}\ddot{x}_c$$

$$x_{ZMP} = x_c + \Delta x = x_c - \frac{l}{g}\ddot{x}_c \longrightarrow x_{ZMP} = x_c - \frac{l}{g}\ddot{x}_c$$

Inverted pendulum - 2



✓ Assumption

1) $I = ml^2$ (Point mass), massless rod
2) $\sin\theta \approx \theta$(linearization)

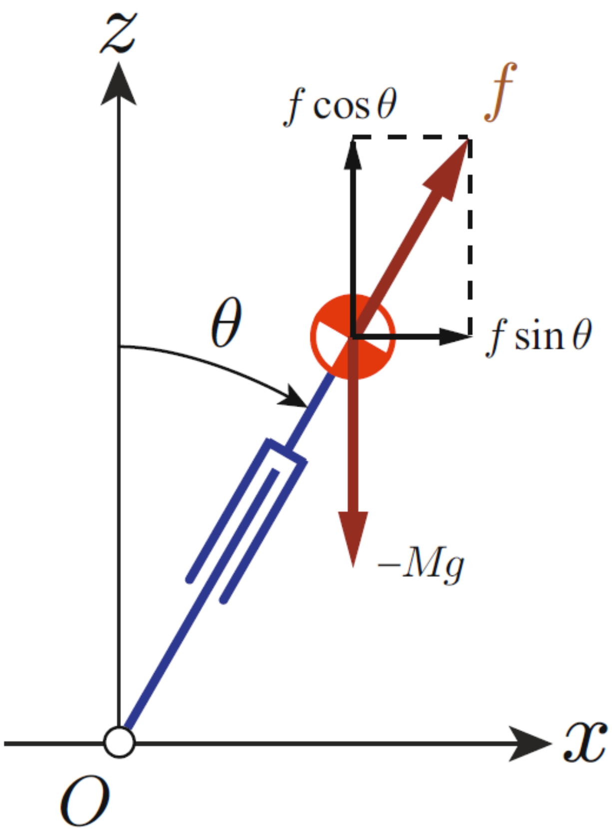
$$I\ddot{\theta} = mg \cdot l sin\theta - \tau$$

$$\rightarrow ml^2\ddot{\theta} = mgl\theta - \tau$$

$$\rightarrow \frac{\tau}{mg} = l\theta - \frac{l}{g}(l\ddot{\theta})$$

$$\therefore x_{ZMP} = x_c - \frac{l}{g}\ddot{x}_c$$

2D–Linear inverted pendulum



- ✓ Assumption

1) Vertical component is canceled by gravity → Horizontal component accelerates the CoM horizontally.
2) z_c is constant

- ✓ Horizontal Dynamics

$$M\ddot{x} = f sin\theta, f = \frac{Mg}{cos\theta}$$

$$M\ddot{x} = Mgtan\theta = Mg\frac{x}{z}$$

If X, Y zmp = 0

$$\ddot{x}_c = g\frac{x_c}{z_c} \qquad \ddot{y}_c = g\frac{y_c}{z_c}$$

✓ 3D–Linear inverted pendulum

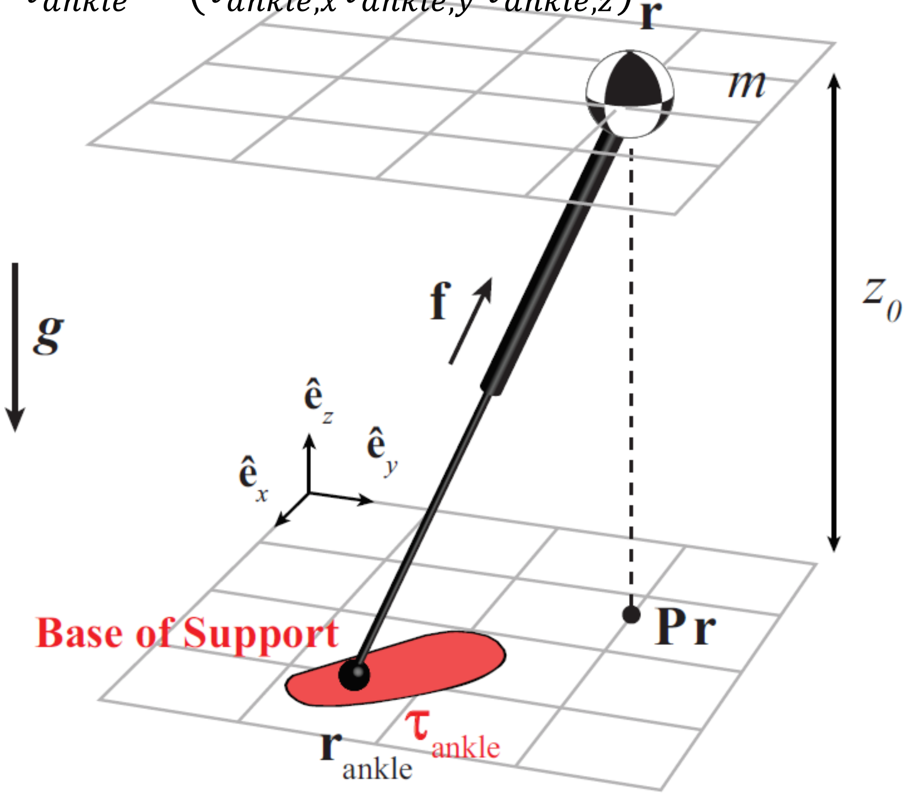
$$r = (x \;\; y \;\; z\,)^T$$

$$r_{ankle} = (x_{ankle} \;\; y_{ankle} \;\; z_{ankle})^T$$

$$f = (f_x \; f_y \; f_z)^T = (m\ddot{x} \; m\ddot{y} \; m\ddot{z} + mg)^T, f_z = mg$$

$$g = (0 \;\; 0 \;\; -g)^T, \omega_0 = \sqrt{\frac{g}{z_0}}$$

$$\tau_{ankle} = \left(\tau_{ankle,x} \tau_{ankle,y} \; \tau_{ankle,z}\right)$$



✓ Model constraint

1) $z = z_0, \dot{z} = \ddot{z} = 0$

✓ Equations of Motion

$$m\ddot{r} = f + mg$$

✓ Moment balance for the massless link

$$-(r - r_{ankle}) \times f + \tau_{ankle} = 0$$

$$-\begin{bmatrix} x - x_{ankle} \\ y - y_{ankle} \\ z_0 \end{bmatrix} \times \begin{pmatrix} f_x \\ f_y \\ mg \end{pmatrix} + \begin{pmatrix} \tau_{ankle,x} \\ \tau_{ankle,y} \\ \tau_{ankle,z} \end{pmatrix} = 0$$

$$f_x = m\omega_0^2(x - x_{ankle}) + \frac{\tau_{ankle,y}}{z_0}$$

$$f_y = m\omega_0^2(y - y_{ankle}) - \frac{\tau_{ankle,x}}{z_0}$$

✓ 3D–Linear inverted pendulum

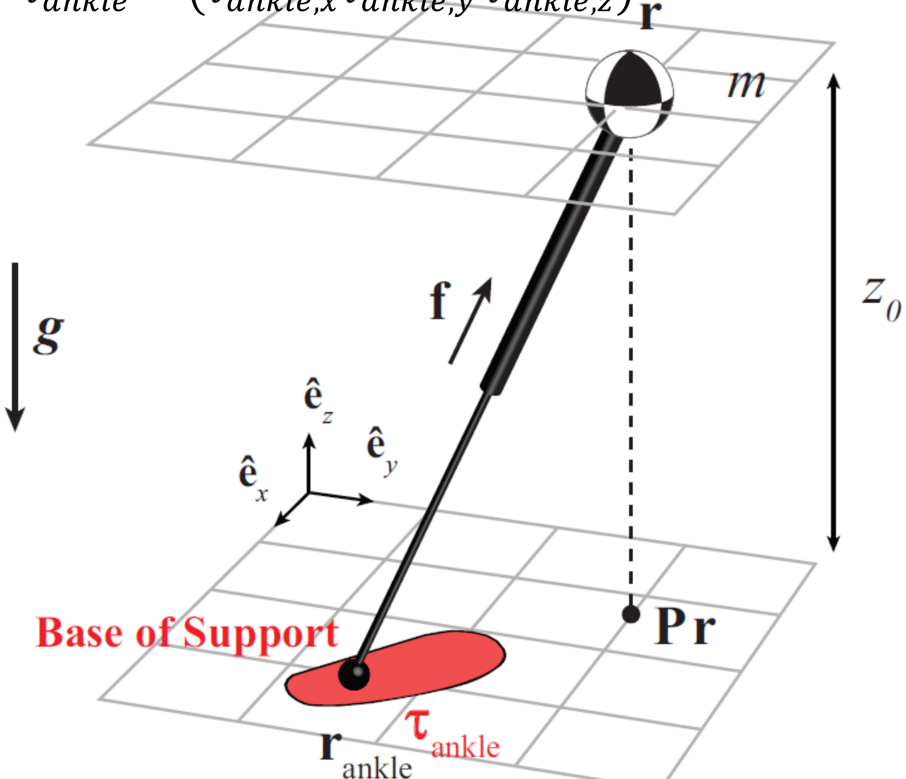
$$r = (x\ \ y\ \ z\,)^T$$

$$r_{ankle} = (x_{ankle}\ \ y_{ankle}\ \ z_{ankle})^T$$

$$f = (f_x\ f_y\ f_z)^T = (m\ddot{x}\ m\ddot{y}\ m\ddot{z} + mg)^T, f_z = mg$$

$$g = (0\ \ 0\ \ -g)^T, \omega_0 = \sqrt{\frac{g}{z_0}}$$

$$\tau_{ankle} = \left(\tau_{ankle,x} \tau_{ankle,y}\ \tau_{ankle,z}\right)$$



✓ Equations of Motion

$$m\ddot{r} = f + mg$$

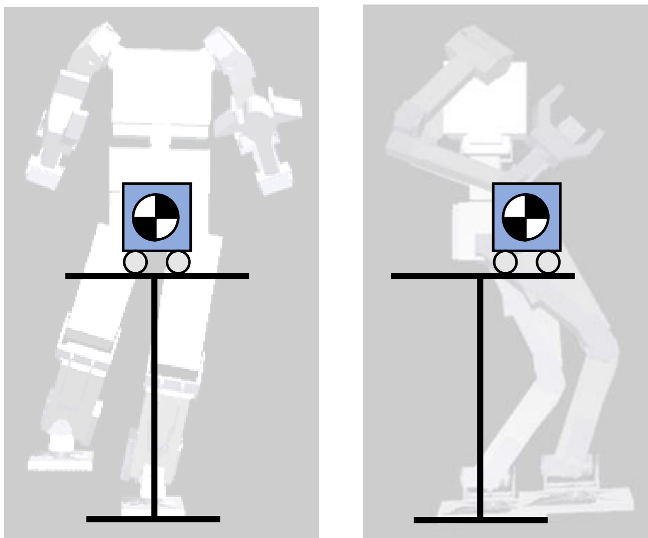
CoM ZMP

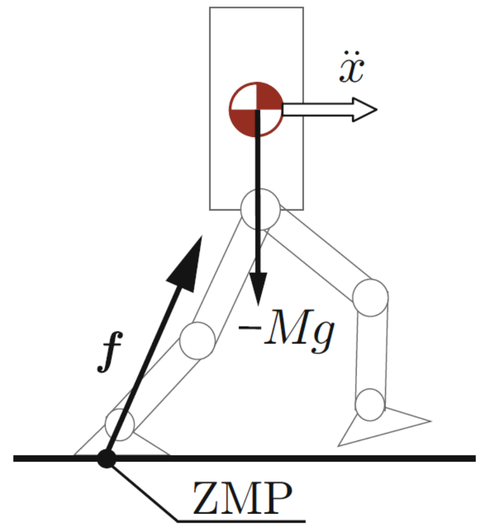
$$\ddot{r}_x = \omega_0^2 \left(x - x_{ankle} + \frac{\tau_{ankle,y}}{mg} \right)$$

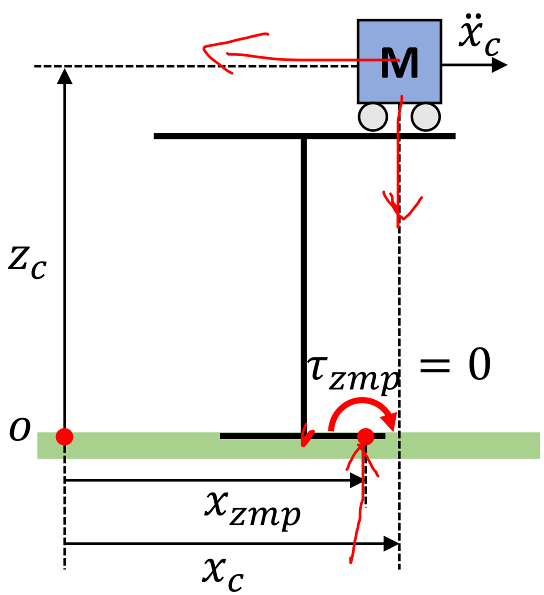
$$\ddot{r}_y = \omega_0^2 \left(y - y_{ankle} - \frac{\tau_{ankle,x}}{mg} \right)$$

$$x_{zmp} = r_x - \frac{1}{\omega_0^2}\ddot{r}_x \qquad y_{zmp} = r_y - \frac{1}{\omega_0^2}\ddot{r}_y$$

✓ Cart Table model







✓ Assumption

1) z_c → constant
2) Massless table

$$M\ddot{x}_c \cdot z_c - Mg \cdot (x_c - x_{zmp}) = 0$$

$$\rightarrow x_{zmp} = x_c - \frac{z_c}{g}\ddot{x}_c$$

Modifiable walking pattern using allowable ZMP region in the foot hold

Applying Newton–Euler equations for the angular momentum taken around the contact point between the pendulum model and ground surface gives

$$\sum \mathbf{T}_{\mathrm{gr}} + \mathbf{r}_{\mathrm{cm}} \times \mathbf{F}_{\mathrm{gr}} = \frac{d}{dt}(\mathbf{r}_{\mathrm{cm}} \times \mathbf{L}) \tag{1}$$

where $\mathbf{T}_{\mathrm{gr}} = [T_x \ T_y \ T_z]^T$ represents the torque created by the ground reaction forces, $\mathbf{F}_{\mathrm{gr}}$ represents gravity, $\mathbf{r}_{\mathrm{cm}} = [x \ y \ z]^T$ is the vector from the contact point to the CM, and $\mathbf{L}$ is the linear momentum of

Modifiable Walking Pattern of a Humanoid Robot by Using Allowable ZMP Variation
Bum-Joo Lee, Daniel Stonier, Yong-Duk Kim, Jeong-Ki Yoo, and Jong-Hwan Kim
TRO Vol 24, No 4, Aug, 2008

the CM. Since the height of the CM is constant, Z_c, the equations of motion are obtained from (1) as follows [3]:

$$\begin{bmatrix} \ddot{y} - \dfrac{g}{Z_c} y \\ \ddot{x} - \dfrac{g}{Z_c} x \end{bmatrix} = \begin{bmatrix} -\dfrac{\sum T_x}{m Z_c} \\ \dfrac{\sum T_y}{m Z_c} \end{bmatrix}. \tag{2}$$

The ZMP can be used to represent the sum of the torques created by the ground reaction forces as follows:

$$\sum \mathbf{T}_{\mathrm{gr}} - \mathbf{r}_{\mathrm{zmp}} \times \mathbf{F}_{\mathrm{gr}} = \begin{bmatrix} 0 & 0 & M_z \end{bmatrix}^T \tag{3}$$

where $\mathbf{r}_{\mathrm{zmp}} = [x_{\mathrm{zmp}} \quad y_{\mathrm{zmp}} \quad 0]^T$ represents the ZMP and M_z is the yawing moment. Following equations can be obtained by substituting (3) into (2):

$$\begin{bmatrix} \ddot{y} - \dfrac{g}{Z_c} y \\ \ddot{x} - \dfrac{g}{Z_c} x \end{bmatrix} = -\frac{g}{Z_c} \begin{bmatrix} y_{\mathrm{zmp}} \\ x_{\mathrm{zmp}} \end{bmatrix}. \tag{4}$$

These equations provide the relationship between the ZMP and the CM motions.

The general solutions of (4) with ZMP functions $p(t)$ and $q(t)$ for the sagittal and lateral motions, respectively, are obtained by applying Laplace transform as follows:

sagittal motion:

$$\begin{bmatrix} x_f \\ T_c v_f \end{bmatrix} = \begin{bmatrix} C(T) & S(T) \\ S(T) & C(T) \end{bmatrix} \begin{bmatrix} x_i \\ T_c v_i \end{bmatrix} - \frac{1}{T_c} \begin{bmatrix} \int_0^T S(t)\overline{p}(t)dt \\ \int_0^T C(t)\overline{p}(t)dt \end{bmatrix}$$

lateral motion:

$$\begin{bmatrix} y_f \\ T_c w_f \end{bmatrix} = \begin{bmatrix} C(T) & S(T) \\ S(T) & C(T) \end{bmatrix} \begin{bmatrix} y_i \\ T_c w_i \end{bmatrix} - \frac{1}{T_c} \begin{bmatrix} \int_0^T S(t)\overline{q}(t)dt \\ \int_0^T C(t)\overline{q}(t)dt \end{bmatrix} \tag{5}$$

where $(x_i, v_i)/(x_f, v_f)$ and $(y_i, w_i)/(y_f, w_f)$ represent initial/final position and velocity of the CM in the sagittal and the lateral planes, respectively, T is the remaining single support time, $S(t)$ and $C(t)$ are defined as $\cosh(t/T_c)$ and $\sinh(t/T_c)$ with time constant $T_c = \sqrt{Z_c/g}$, and lastly, $\overline{p}(t) = p(T-t)$ and $\overline{q}(t) = q(T-t)$. The first terms on the right-hand side of (5) indicate homogeneous (nonforced) solutions. The latter terms represent additional states (particular or forced solutions) that allow more extensive and unrestricted motions by varying ZMP trajectories with ZMP functions $p(t)$ and $q(t)$.

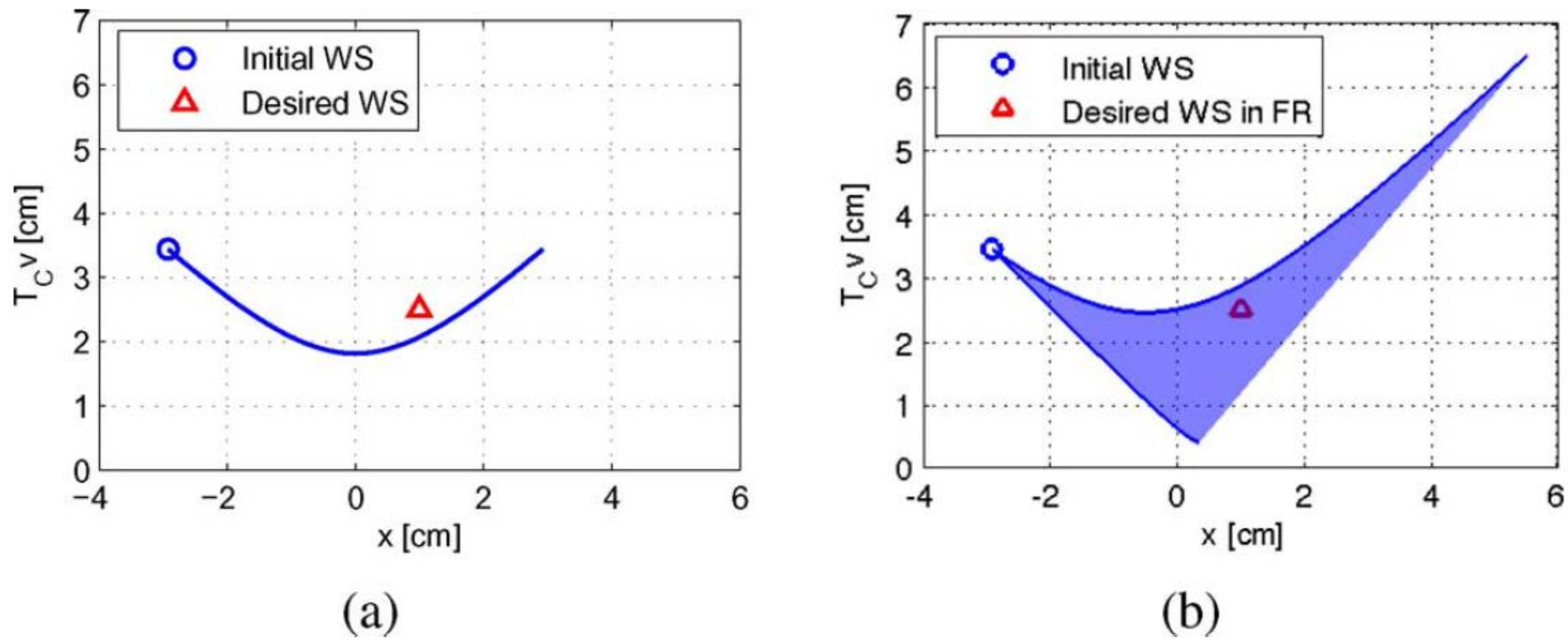


Fig. 4. Comparison of two WS transitions. (a) 3D-LIPM. (b) Modifiable walking pattern (proposed method).

WS: Walking State

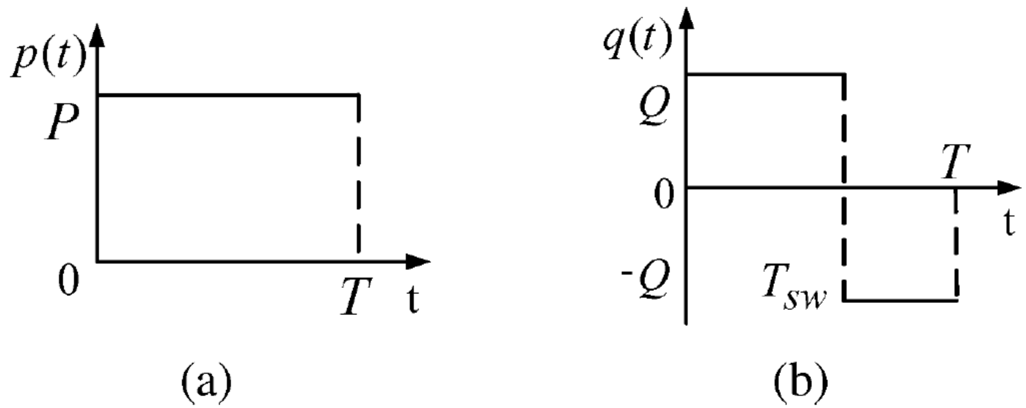


Fig. 5. Proposed two ZMP functions. (a) Constant function for sagittal motion. (b) Step function for lateral motion.