# Cambridge AI+

## Lecture 6: Perceptron

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University of Cambridge, Department of Computer Science and Technology

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# Outline

# Landscape of Machine Learning Algorithms

Training Set provided initially

**Supervised Learning**

Nearest Neighbour, Regression, Logistic Regression, Decision Trees and Random Forests, Perceptron, Naive Bayes, Boosting, Support Vector Machines, Neural Networks

Predict unseen data

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Density Estimation (Maximum Likelihood), Feature Extraction: Dimensionality Reduction, Principal Component Analysis, Singular Value Decomposition, Clustering

Predict unseen data

Extract Knowledge

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Feedback after Decisions

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Nearest Neighbour, Regression, Logistic Regression, Decision Trees and Random Forests, Perceptron, Naive Bayes, Boosting, Support Vector Machines, Neural Networks

**Online/Reinforcement Learning**

Online Perceptron, Weighted-Majority, Markov Chains, Hidden-Markov Models, Markov Decision Processes: Temporal Difference, Q-Learning, SARSA

**Unsupervised Learning**

Density Estimation (Maximum Likelihood), Feature Extraction: Dimensionality Reduction, Principal Component Analysis, Singular Value Decomposition, Clustering

Predict unseen data

Maximise Reward

Extract Knowledge

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Introduction

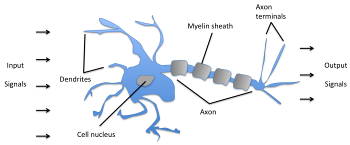
Perceptron

Conclusion, Problems and Solutions

Additional Material: Why Perceptron Works

# Introduction to Perceptron

- The Perceptron algorithm was invented 1958 by Frank Rosenblatt
- inspired by a biological neuron: output 1 only if input is above a certain threshold




Schematic of a biological neuron.

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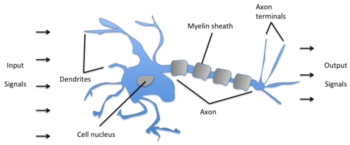


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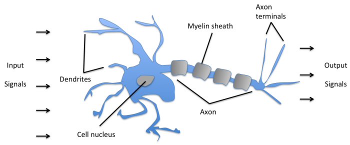


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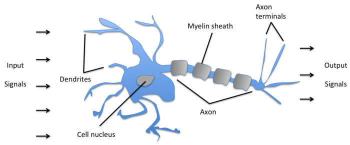


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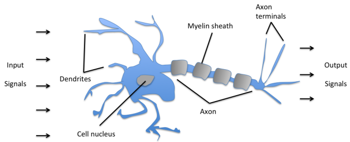


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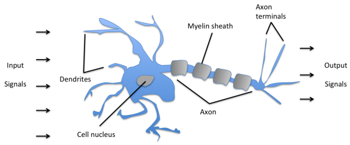


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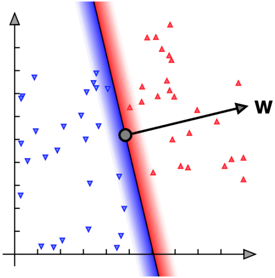

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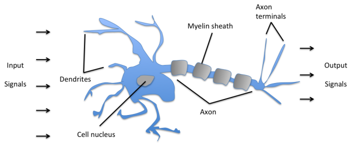


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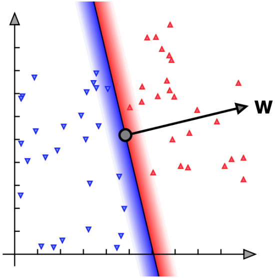

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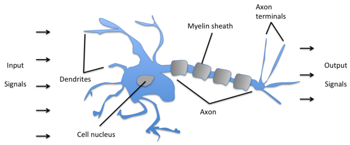


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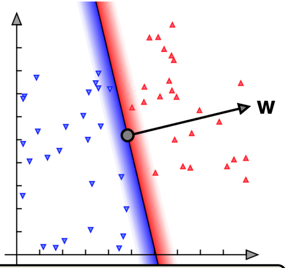


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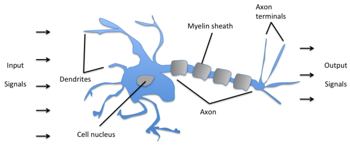


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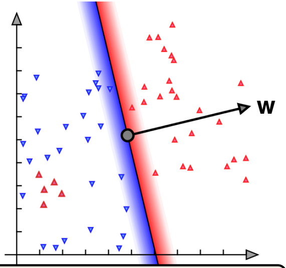


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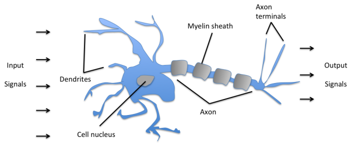


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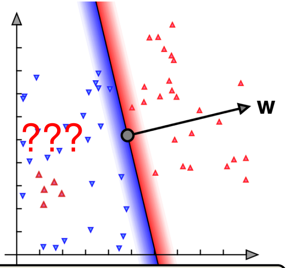


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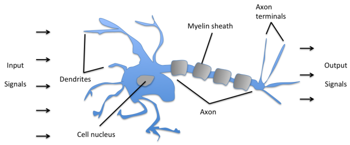


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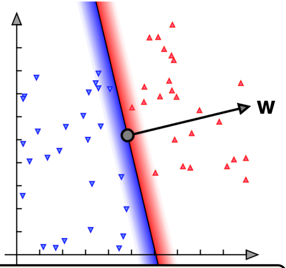


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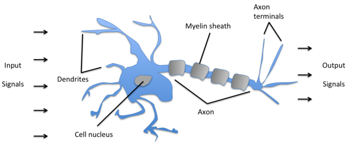


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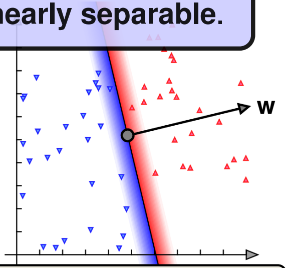


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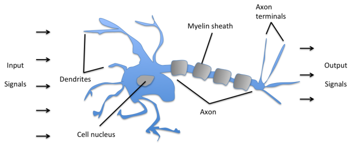

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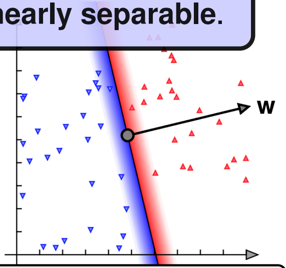


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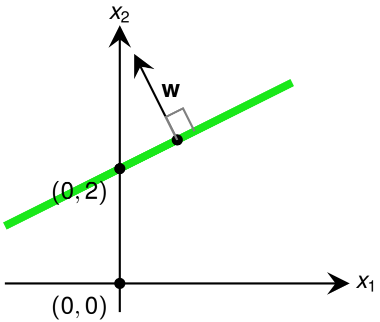

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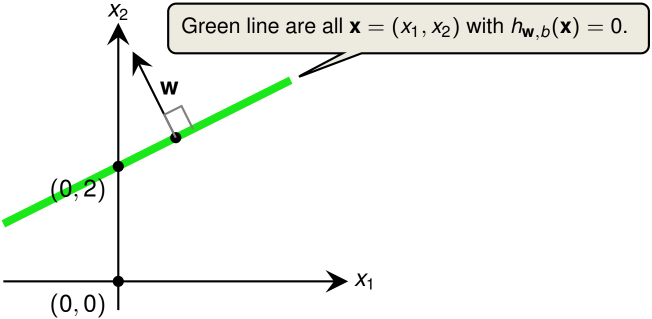

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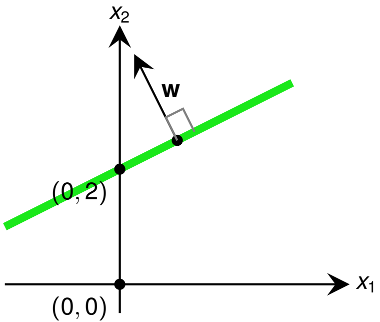

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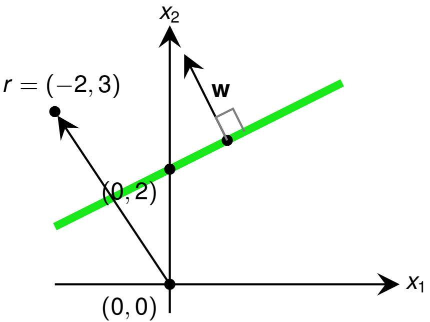

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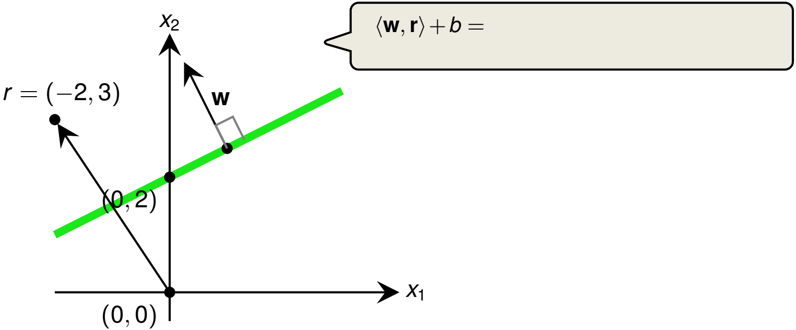

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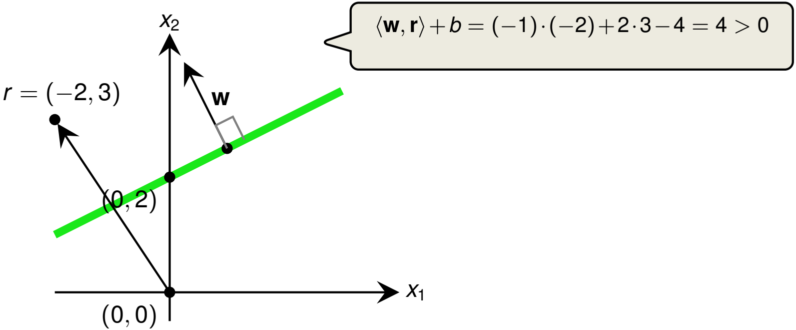

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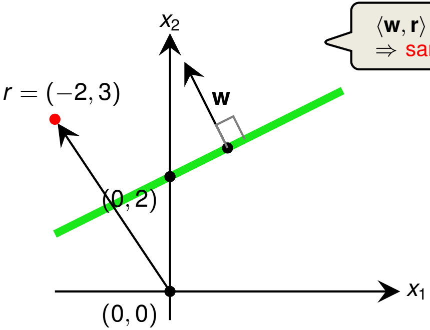


$\langle \mathbf{w}, \mathbf{r} \rangle + b = (-1) \cdot (-2) + 2 \cdot 3 - 4 = 4 > 0$
$\Rightarrow$ same side as $\mathbf{w}$

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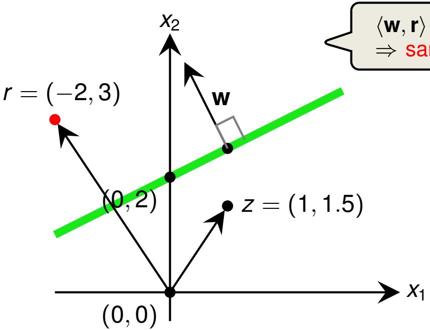


$\langle \mathbf{w}, \mathbf{r} \rangle + b = (-1) \cdot (-2) + 2 \cdot 3 - 4 = 4 > 0$
$\Rightarrow$ same side as $\mathbf{w}$

# Linear Functions, Inner Products and Classifications

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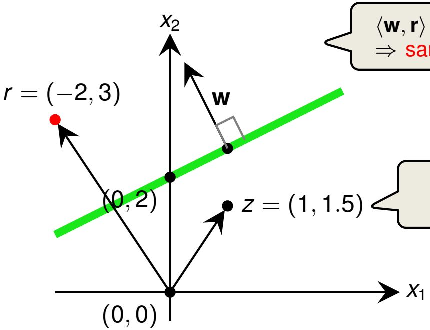


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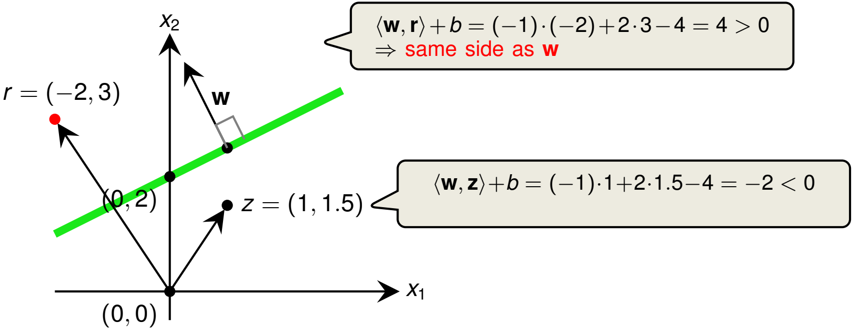

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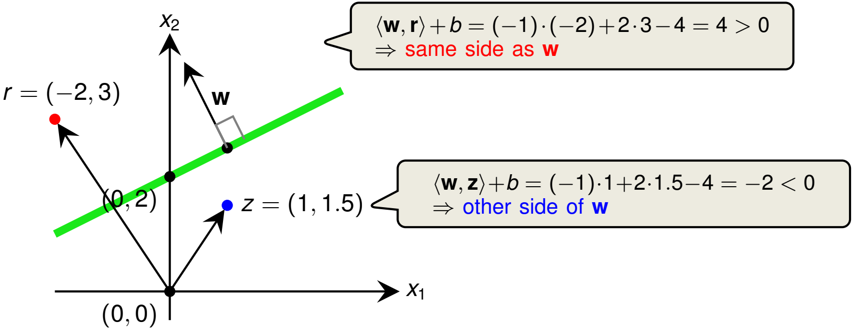

# Linear Regression versus Perceptron

Linear Regression

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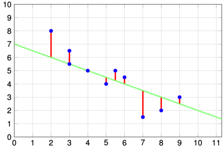

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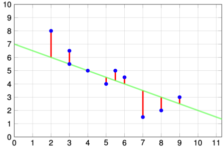

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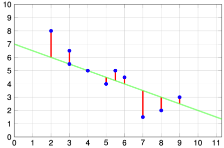


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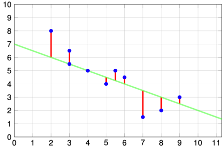


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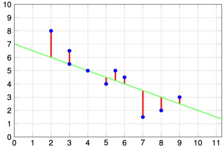


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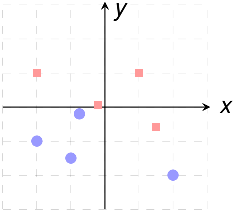

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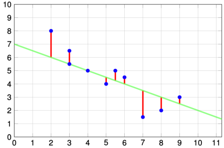


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- Input: Data set with $d$ features and one **class**
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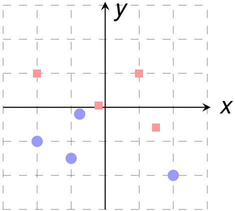

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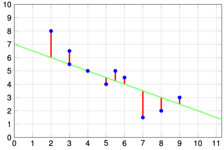


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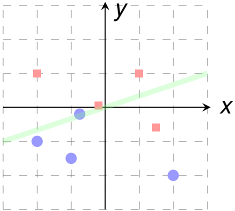

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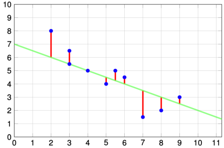


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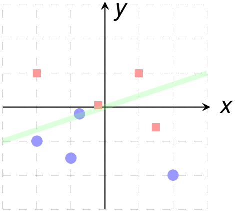

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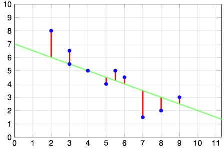


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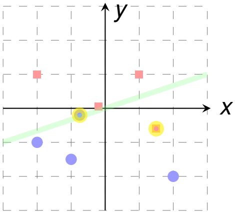

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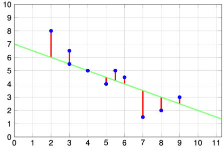


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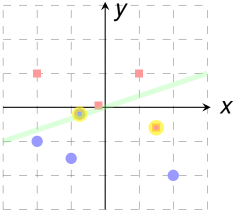

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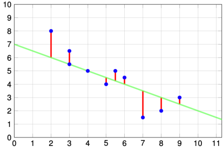


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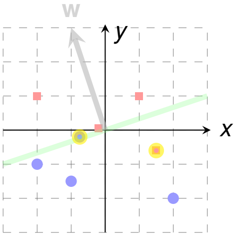
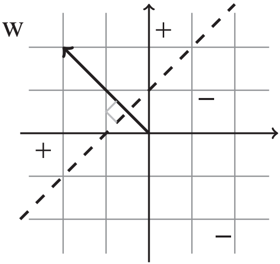


# From Affine Linear Functions to homogenous linear Functions



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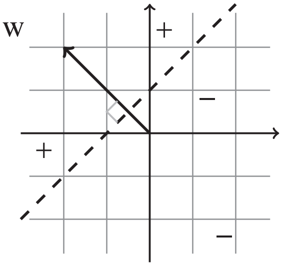
- Let $\mathbf{w}' = (b, w_1, w_2, \ldots, w_d) \in \mathbb{R}^{d+1}$



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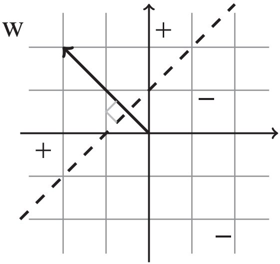

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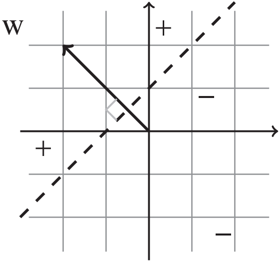

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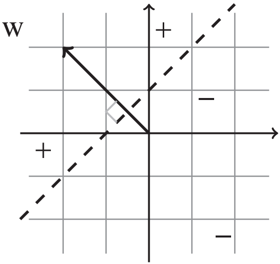

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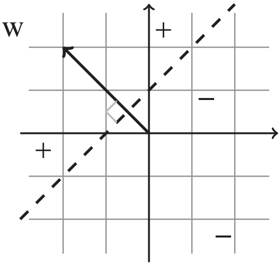

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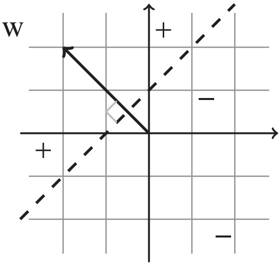

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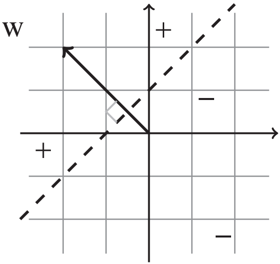

## The Perceptron Algorithm

**Batch Perceptron**

**input:** A training set $(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_m, y_m)$
**initialize:** $\mathbf{w}^{(1)} = (0, \ldots, 0)$
  **for** $t = 1, 2, \ldots$
    **if** $(\exists\, i \;\text{ s.t. }\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
      $\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$
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- It is possible to find a hyperplane directly using Linear Programming
- We will now see an illustration of this algorithm on a small data set

# Illustration of Perceptron (1/2)

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Original Input

- dimension $d = 2$, $m = 8$ points

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- dimension $d = 2$, $m = 8$ points
- $\mathbf{x}_1 = (1, 1)$, $y_1 = 1$ pos.
- $\mathbf{x}_2 = (-2, -1)$, $y_2 = -1$ neg.
- $\mathbf{x}_3 = (-0.75, -0.2)$, $y_3 = -1$ neg.
- $\mathbf{x}_4 = (-2, 1)$, $y_4 = 1$ pos.
- $\mathbf{x}_5 = (-1, -1.5)$, $y_5 = -1$ neg.
- $\mathbf{x}_6 = (2, -2)$, $y_6 = -1$ neg.
- $\mathbf{x}_7 = (1.5, -0.6)$, $y_7 = 1$ pos.
- $\mathbf{x}_8 = (-0.2, 0.05)$, $y_8 = 1$ pos.

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- $\mathbf{x}_8 = (-0.2, 0.05)$, $y_8 = 1$ pos.

Output

Find normal vector $(w_1, w_2)$ and bias $b$ correctly classifying all points.

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- $\mathbf{x}_1 = (1, 1)$, $y_1 = 1$ pos.
- $\mathbf{x}_2 = (-2, -1)$, $y_2 = -1$ neg.
- $\mathbf{x}_3 = (-0.75, -0.2)$, $y_3 = -1$ neg.
- $\mathbf{x}_4 = (-2, 1)$, $y_4 = 1$ pos.
- $\mathbf{x}_5 = (-1, -1.5)$, $y_5 = -1$ neg.
- $\mathbf{x}_6 = (2, -2)$, $y_6 = -1$ neg.
- $\mathbf{x}_7 = (1.5, -0.6)$, $y_7 = 1$ pos.
- $\mathbf{x}_8 = (-0.2, 0.05)$, $y_8 = 1$ pos.

New Input

- dimension $d = 3$, $m = 8$ points

Output

Find normal vector $(w_1, w_2)$ and bias $b$ correctly classifying all points.

# Illustration of Perceptron (1/2)

**Original Input**

- dimension $d = 2$, $m = 8$ points
- $\mathbf{x}_1 = (1, 1)$, $y_1 = 1$ pos.
- $\mathbf{x}_2 = (-2, -1)$, $y_2 = -1$ neg.
- $\mathbf{x}_3 = (-0.75, -0.2)$, $y_3 = -1$ neg.
- $\mathbf{x}_4 = (-2, 1)$, $y_4 = 1$ pos.
- $\mathbf{x}_5 = (-1, -1.5)$, $y_5 = -1$ neg.
- $\mathbf{x}_6 = (2, -2)$, $y_6 = -1$ neg.
- $\mathbf{x}_7 = (1.5, -0.6)$, $y_7 = 1$ pos.
- $\mathbf{x}_8 = (-0.2, 0.05)$, $y_8 = 1$ pos.

**New Input**

- dimension $d = 3$, $m = 8$ points
- $\mathbf{x}_1 = (1, 1, 1)$, $y_1 = 1$ pos.
- $\mathbf{x}_2 = (1, -2, -1)$, $y_2 = -1$ neg.
- $\mathbf{x}_3 = (1, -0.75, -0.2)$, $y_3 = -1$ neg.
- $\mathbf{x}_4 = (1, -2, 1)$, $y_4 = 1$ pos.
- $\mathbf{x}_5 = (1, -1, -1.5)$, $y_5 = -1$ neg.
- $\mathbf{x}_6 = (1, 2, -2)$, $y_6 = -1$ neg.
- $\mathbf{x}_7 = (1, 1.5, -0.6)$, $y_7 = 1$ pos.
- $\mathbf{x}_8 = (1, -0.2, 0.05)$, $y_8 = 1$ pos.

**Output**

Find normal vector $(w_1, w_2)$ and bias $b$ correctly classifying all points.

# Illustration of Perceptron (1/2)

**Original Input**

- dimension $d = 2$, $m = 8$ points
- $\mathbf{x}_1 = (1, 1)$, $y_1 = 1$ pos.
- $\mathbf{x}_2 = (-2, -1)$, $y_2 = -1$ neg.
- $\mathbf{x}_3 = (-0.75, -0.2)$, $y_3 = -1$ neg.
- $\mathbf{x}_4 = (-2, 1)$, $y_4 = 1$ pos.
- $\mathbf{x}_5 = (-1, -1.5)$, $y_5 = -1$ neg.
- $\mathbf{x}_6 = (2, -2)$, $y_6 = -1$ neg.
- $\mathbf{x}_7 = (1.5, -0.6)$, $y_7 = 1$ pos.
- $\mathbf{x}_8 = (-0.2, 0.05)$, $y_8 = 1$ pos.

**New Input**

- dimension $d = 3$, $m = 8$ points
- $\mathbf{x}_1 = (1, 1, 1)$, $y_1 = 1$ pos.
- $\mathbf{x}_2 = (1, -2, -1)$, $y_2 = -1$ neg.
- $\mathbf{x}_3 = (1, -0.75, -0.2)$, $y_3 = -1$ neg.
- $\mathbf{x}_4 = (1, -2, 1)$, $y_4 = 1$ pos.
- $\mathbf{x}_5 = (1, -1, -1.5)$, $y_5 = -1$ neg.
- $\mathbf{x}_6 = (1, 2, -2)$, $y_6 = -1$ neg.
- $\mathbf{x}_7 = (1, 1.5, -0.6)$, $y_7 = 1$ pos.
- $\mathbf{x}_8 = (1, -0.2, 0.05)$, $y_8 = 1$ pos.

**Output**

Find normal vector $(w_1, w_2)$ and bias $b$ correctly classifying all points.

**Output**

Find normal vector $(w'_1, w'_2, w'_3)$ correctly classifying all points.

# Illustration of Perceptron (1/2)

**Original Input**

- dimension $d = 2$, $m = 8$ points
- $\mathbf{x}_1 = (1, 1)$, $y_1 = 1$ pos.
- $\mathbf{x}_2 = (-2, -1)$, $y_2 = -1$ neg.
- $\mathbf{x}_3 = (-0.75, -0.2)$, $y_3 = -1$ neg.
- $\mathbf{x}_4 = (-2, 1)$, $y_4 = 1$ pos.
- $\mathbf{x}_5 = (-1, -1.5)$, $y_5 = -1$ neg.
- $\mathbf{x}_6 = (2, -2)$, $y_6 = -1$ neg.
- $\mathbf{x}_7 = (1.5, -0.6)$, $y_7 = 1$ pos.
- $\mathbf{x}_8 = (-0.2, 0.05)$, $y_8 = 1$ pos.

**New Input**

- dimension $d = 3$, $m = 8$ points
- $\mathbf{x}_1 = (1, 1, 1)$, $y_1 = 1$ pos.
- $\mathbf{x}_2 = (1, -2, -1)$, $y_2 = -1$ neg.
- $\mathbf{x}_3 = (1, -0.75, -0.2)$, $y_3 = -1$ neg.
- $\mathbf{x}_4 = (1, -2, 1)$, $y_4 = 1$ pos.
- $\mathbf{x}_5 = (1, -1, -1.5)$, $y_5 = -1$ neg.
- $\mathbf{x}_6 = (1, 2, -2)$, $y_6 = -1$ neg.
- $\mathbf{x}_7 = (1, 1.5, -0.6)$, $y_7 = 1$ pos.
- $\mathbf{x}_8 = (1, -0.2, 0.05)$, $y_8 = 1$ pos.

**Output**

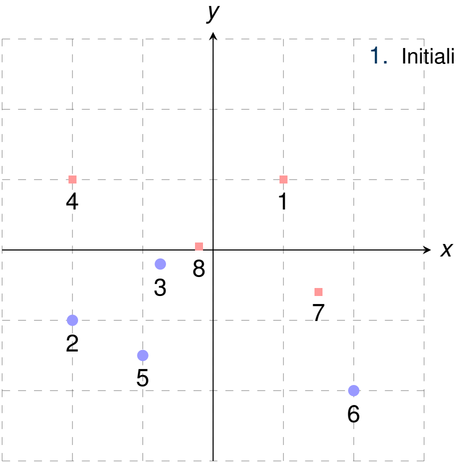
Find normal vector $(w_1, w_2)$ and bias $b$ correctly classifying all points.

**Output**

Find normal vector $(w_1', w_2', w_3')$ correctly classifying all points.

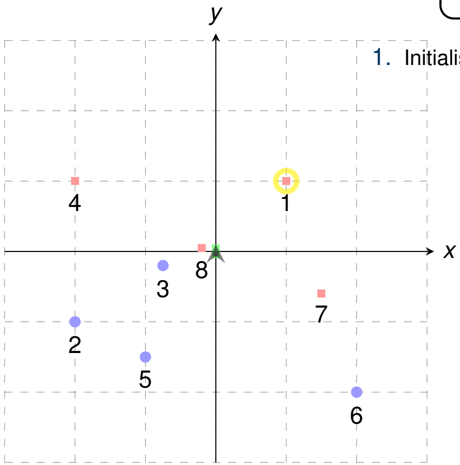
$b = w_1'$, $w_1 = w_2'$ and $w_2 = w_3'$!

## Illustration of Perceptron (2/2)




1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$.

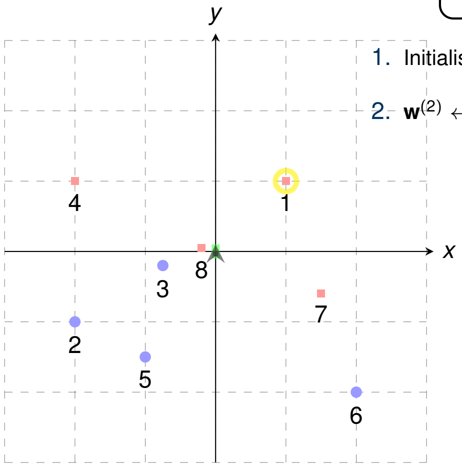
# Illustration of Perceptron (2/2)




$$\textbf{if } (\exists\, i \;\text{ s.t. }\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0) \text{ then}$$
$$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$$

1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$. Pick $\mathbf{x}_1 = (1, 1, 1)$

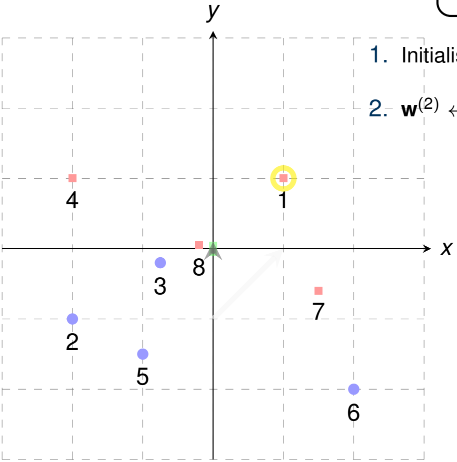
# Illustration of Perceptron (2/2)




$$\textbf{if } (\exists\, i \ \text{ s.t. } \ y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0) \text{ then}$$
$$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$$

1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$. Pick $\mathbf{x}_1 = (1, 1, 1)$
2. $\mathbf{w}^{(2)} \leftarrow (1, 1, 1)$.

# Illustration of Perceptron (2/2)

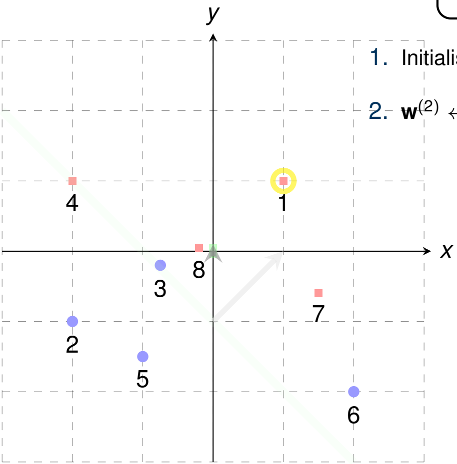



> **if** $(\exists\, i$ s.t. $y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
> $\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$

1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$. Pick $\mathbf{x}_1 = (1, 1, 1)$

2. $\mathbf{w}^{(2)} \leftarrow (1, 1, 1)$.

# Illustration of Perceptron (2/2)

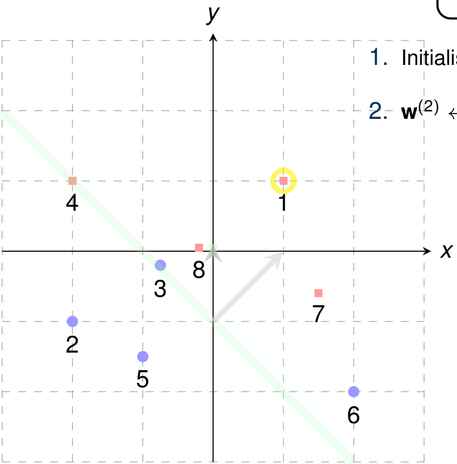



$$\begin{aligned}&\textbf{if } (\exists\, i \ \text{ s.t. } \ y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0) \text{ then}\\ &\quad \mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i\end{aligned}$$

1. Initialise $\mathbf{w}^{(1)} = (0,0,0)$. Pick $\mathbf{x}_1 = (1,1,1)$
2. $\mathbf{w}^{(2)} \leftarrow (1,1,1)$.

# Illustration of Perceptron (2/2)

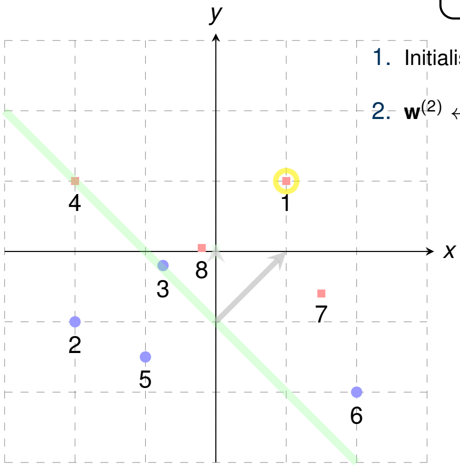
$$\textbf{if } (\exists\, i \;\; \text{s.t.} \;\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0) \text{ then}$$
$$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$$




1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$. Pick $\mathbf{x}_1 = (1, 1, 1)$
2. $\mathbf{w}^{(2)} \leftarrow (1, 1, 1)$.

# Illustration of Perceptron (2/2)

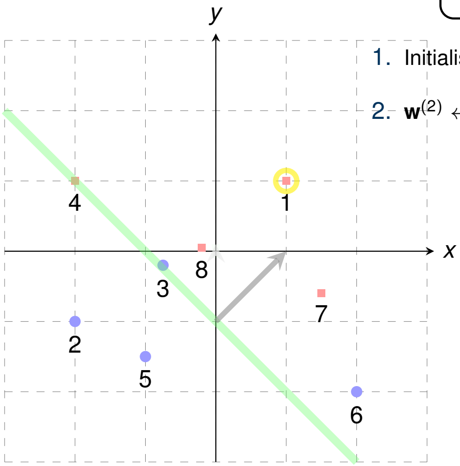
**if** $(\exists\, i \;\; \text{s.t.} \;\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$




1. Initialise $\mathbf{w}^{(1)} = (0,0,0)$. Pick $\mathbf{x}_1 = (1,1,1)$
2. $\mathbf{w}^{(2)} \leftarrow (1,1,1)$.

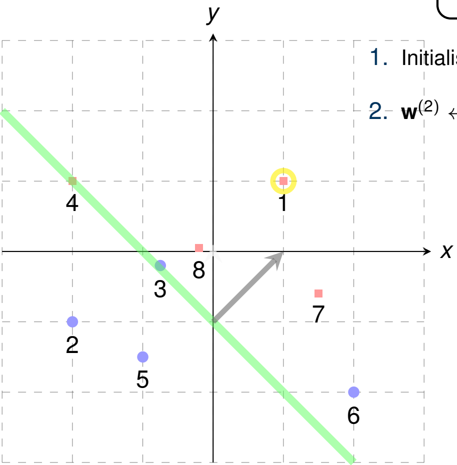
# Illustration of Perceptron (2/2)

**if** $(\exists\, i \;\text{ s.t. }\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$




1. Initialise $\mathbf{w}^{(1)} = (0,0,0)$. Pick $\mathbf{x}_1 = (1,1,1)$
2. $\mathbf{w}^{(2)} \leftarrow (1,1,1)$.

# Illustration of Perceptron (2/2)

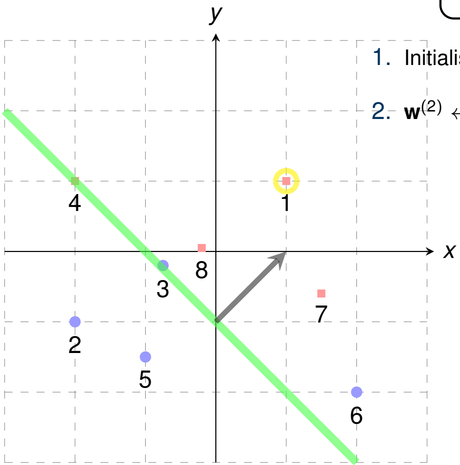



**if** $(\exists\, i \;\text{ s.t. }\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$

1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$. Pick $\mathbf{x}_1 = (1, 1, 1)$
2. $\mathbf{w}^{(2)} \leftarrow (1, 1, 1)$.

# Illustration of Perceptron (2/2)

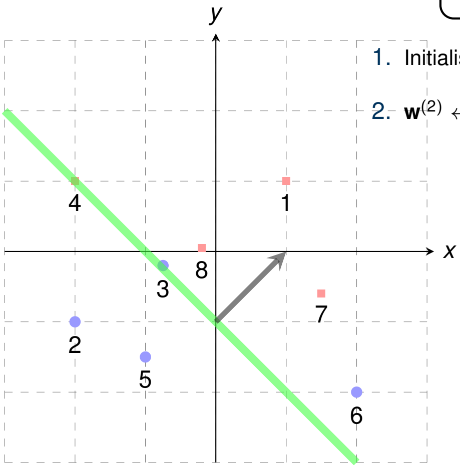
**if** $(\exists\, i \;\; \text{s.t.} \;\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$




1. Initialise $\mathbf{w}^{(1)} = (0,0,0)$. Pick $\mathbf{x}_1 = (1,1,1)$
2. $\mathbf{w}^{(2)} \leftarrow (1,1,1)$.

# Illustration of Perceptron (2/2)

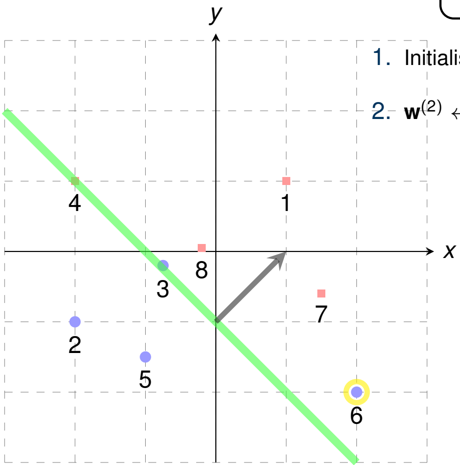
**if** $(\exists\, i \ \text{ s.t. } \ y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$




1. Initialise $\mathbf{w}^{(1)} = (0,0,0)$. Pick $\mathbf{x}_1 = (1,1,1)$
2. $\mathbf{w}^{(2)} \leftarrow (1,1,1)$.

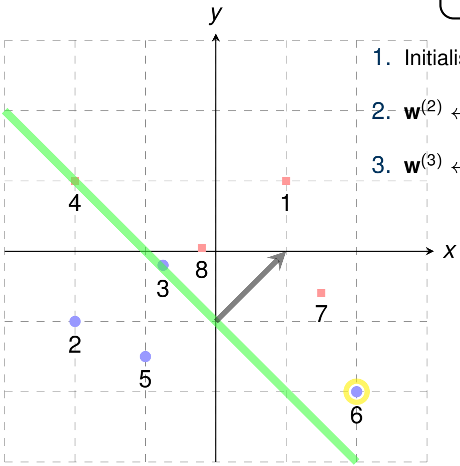
# Illustration of Perceptron (2/2)

$$\textbf{if } (\exists\, i \ \text{ s.t. } \ y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0) \text{ then}$$
$$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$$




1. Initialise $\mathbf{w}^{(1)} = (0,0,0)$. Pick $\mathbf{x}_1 = (1,1,1)$
2. $\mathbf{w}^{(2)} \leftarrow (1,1,1)$. Pick $\mathbf{x}_6 = (1,2,-2)$.

# Illustration of Perceptron (2/2)

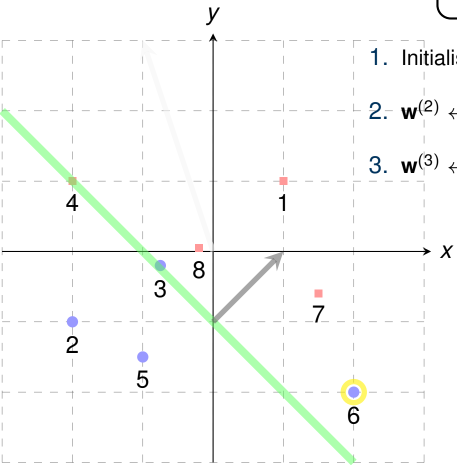



**if** $(\exists\, i \ \text{ s.t. } \ y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$

1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$. Pick $\mathbf{x}_1 = (1, 1, 1)$
2. $\mathbf{w}^{(2)} \leftarrow (1, 1, 1)$. Pick $\mathbf{x}_6 = (1, 2, -2)$.
3. $\mathbf{w}^{(3)} \leftarrow (0, -1, 3)$.

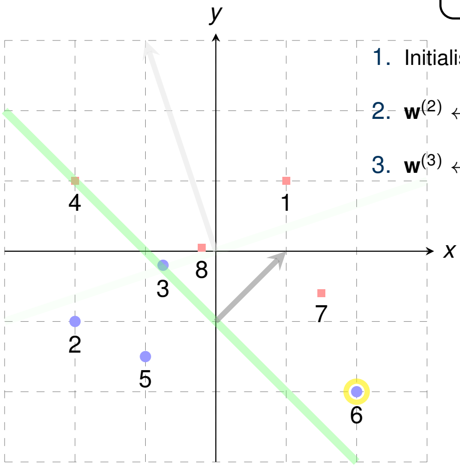
# Illustration of Perceptron (2/2)

**if** $(\exists\, i \;\text{ s.t. }\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$




1. Initialise $\mathbf{w}^{(1)} = (0,0,0)$. Pick $\mathbf{x}_1 = (1,1,1)$
2. $\mathbf{w}^{(2)} \leftarrow (1,1,1)$. Pick $\mathbf{x}_6 = (1,2,-2)$.
3. $\mathbf{w}^{(3)} \leftarrow (0,-1,3)$.

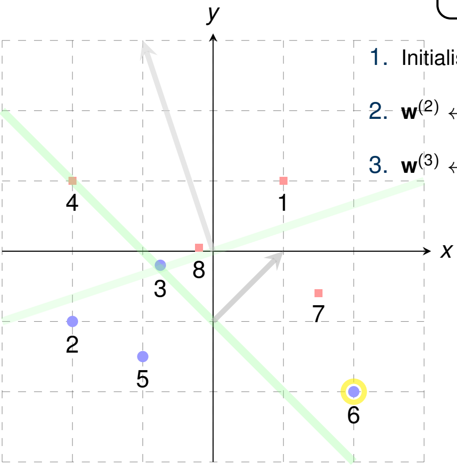
# Illustration of Perceptron (2/2)




**if** $(\exists\, i \;\; \text{s.t.} \;\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$

1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$. Pick $\mathbf{x}_1 = (1, 1, 1)$
2. $\mathbf{w}^{(2)} \leftarrow (1, 1, 1)$. Pick $\mathbf{x}_6 = (1, 2, -2)$.
3. $\mathbf{w}^{(3)} \leftarrow (0, -1, 3)$.

# Illustration of Perceptron (2/2)

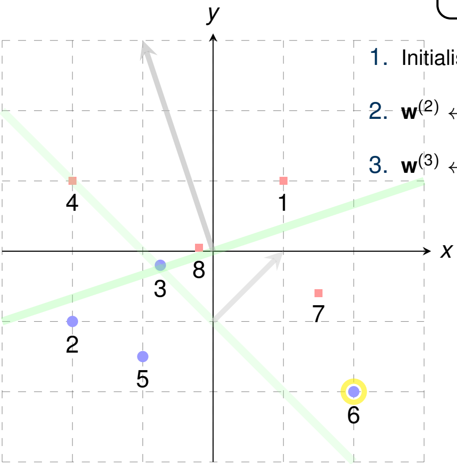



$$\textbf{if } (\exists\, i \;\text{ s.t. }\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0) \text{ then}$$
$$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$$

1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$. Pick $\mathbf{x}_1 = (1, 1, 1)$
2. $\mathbf{w}^{(2)} \leftarrow (1, 1, 1)$. Pick $\mathbf{x}_6 = (1, 2, -2)$.
3. $\mathbf{w}^{(3)} \leftarrow (0, -1, 3)$.

# Illustration of Perceptron (2/2)

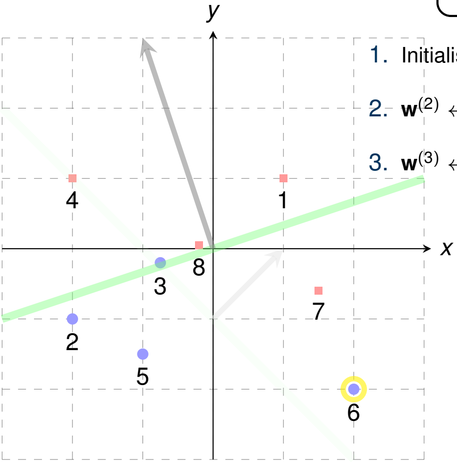
**if** $(\exists\, i \text{ s.t. } y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$




1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$. Pick $\mathbf{x}_1 = (1, 1, 1)$
2. $\mathbf{w}^{(2)} \leftarrow (1, 1, 1)$. Pick $\mathbf{x}_6 = (1, 2, -2)$.
3. $\mathbf{w}^{(3)} \leftarrow (0, -1, 3)$.

# Illustration of Perceptron (2/2)

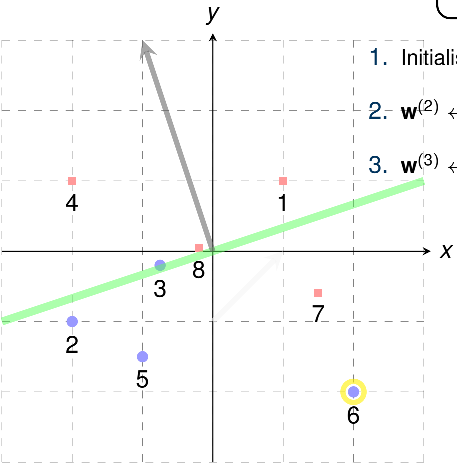
**if** $(\exists\, i \;\text{ s.t. }\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$




1. Initialise $\mathbf{w}^{(1)} = (0,0,0)$. Pick $\mathbf{x}_1 = (1,1,1)$
2. $\mathbf{w}^{(2)} \leftarrow (1,1,1)$. Pick $\mathbf{x}_6 = (1,2,-2)$.
3. $\mathbf{w}^{(3)} \leftarrow (0,-1,3)$.

# Illustration of Perceptron (2/2)

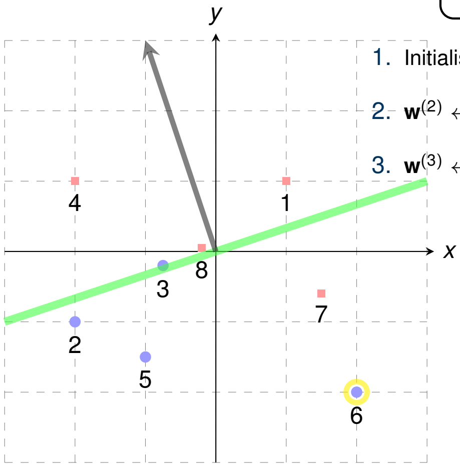
**if** $(\exists\, i \text{ s.t. } y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$




1. Initialise $\mathbf{w}^{(1)} = (0,0,0)$. Pick $\mathbf{x}_1 = (1,1,1)$
2. $\mathbf{w}^{(2)} \leftarrow (1,1,1)$. Pick $\mathbf{x}_6 = (1,2,-2)$.
3. $\mathbf{w}^{(3)} \leftarrow (0,-1,3)$.

# Illustration of Perceptron (2/2)

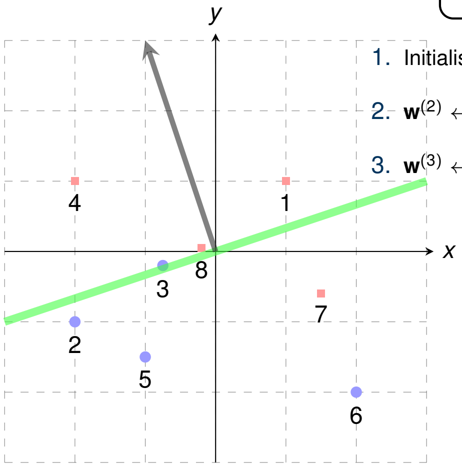
$$\textbf{if } (\exists\, i \ \text{ s.t. } \ y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0) \text{ then}$$
$$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$$




1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$. Pick $\mathbf{x}_1 = (1, 1, 1)$
2. $\mathbf{w}^{(2)} \leftarrow (1, 1, 1)$. Pick $\mathbf{x}_6 = (1, 2, -2)$.
3. $\mathbf{w}^{(3)} \leftarrow (0, -1, 3)$.

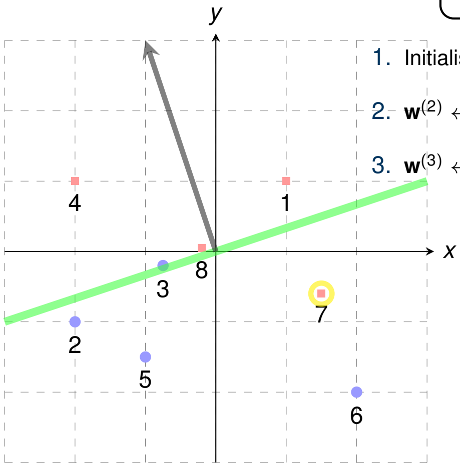
# Illustration of Perceptron (2/2)

if $(\exists\, i \;\text{ s.t. }\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$




1. Initialise $\mathbf{w}^{(1)} = (0,0,0)$. Pick $\mathbf{x}_1 = (1,1,1)$
2. $\mathbf{w}^{(2)} \leftarrow (1,1,1)$. Pick $\mathbf{x}_6 = (1,2,-2)$.
3. $\mathbf{w}^{(3)} \leftarrow (0,-1,3)$.

# Illustration of Perceptron (2/2)

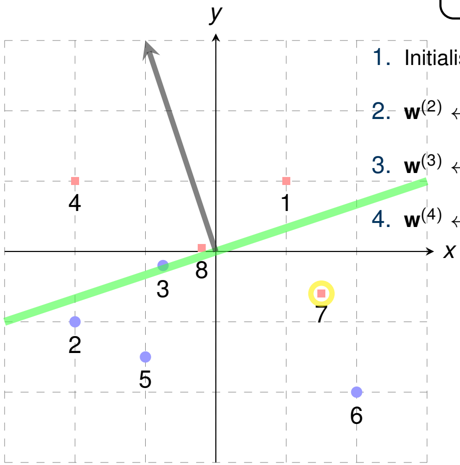



**if** $(\exists\, i \;\text{ s.t. }\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$

1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$. Pick $\mathbf{x}_1 = (1, 1, 1)$
2. $\mathbf{w}^{(2)} \leftarrow (1, 1, 1)$. Pick $\mathbf{x}_6 = (1, 2, -2)$.
3. $\mathbf{w}^{(3)} \leftarrow (0, -1, 3)$. Pick $\mathbf{x}_7 = (1, 1.5, -0.6)$.

**Quiz 1:** What is $w^{(4)}$?

# Illustration of Perceptron (2/2)

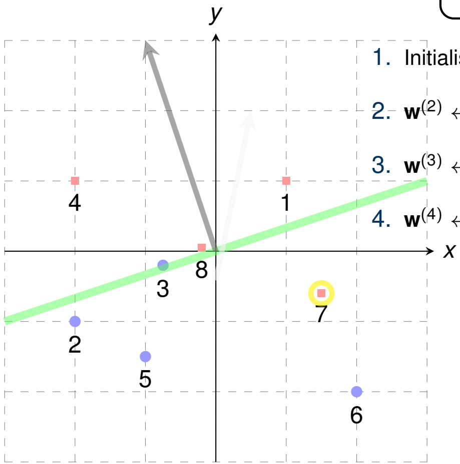



$$\textbf{if } (\exists\, i \ \text{ s.t. } \ y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0) \text{ then}$$
$$\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$$

1. Initialise $\mathbf{w}^{(1)} = (0, 0, 0)$. Pick $\mathbf{x}_1 = (1, 1, 1)$
2. $\mathbf{w}^{(2)} \leftarrow (1, 1, 1)$. Pick $\mathbf{x}_6 = (1, 2, -2)$.
3. $\mathbf{w}^{(3)} \leftarrow (0, -1, 3)$. Pick $\mathbf{x}_7 = (1, 1.5, -0.6)$.
4. $\mathbf{w}^{(4)} \leftarrow (1, 0.5, 2.4)$.

**Quiz 1:** What is $w^{(4)}$?

## Illustration of Perceptron (2/2)




> **if** $(\exists\, i \;\text{ s.t. }\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
> $\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$

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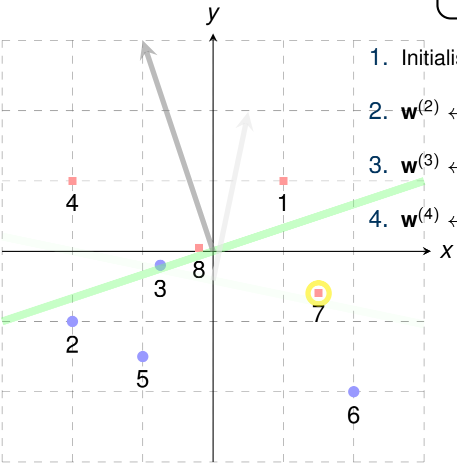
# Illustration of Perceptron (2/2)

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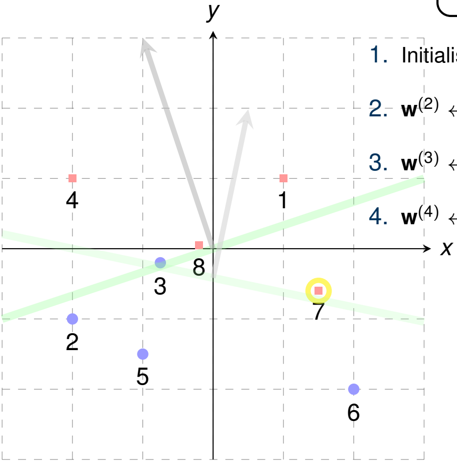


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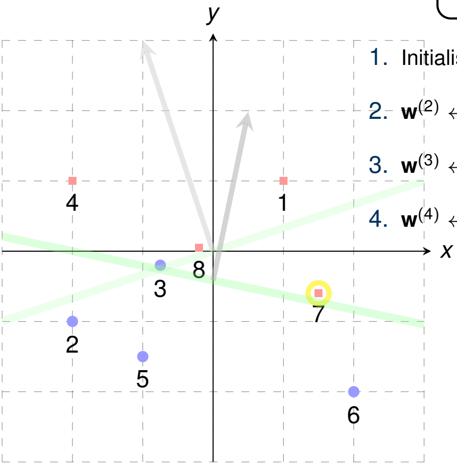


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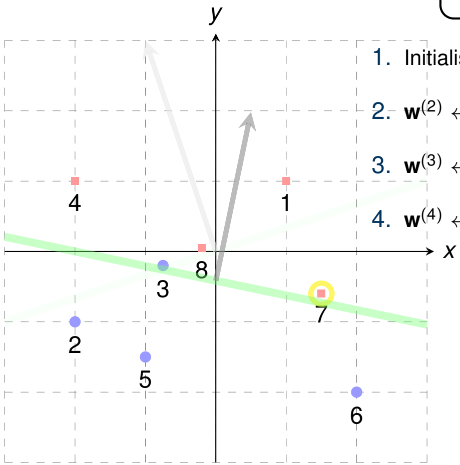


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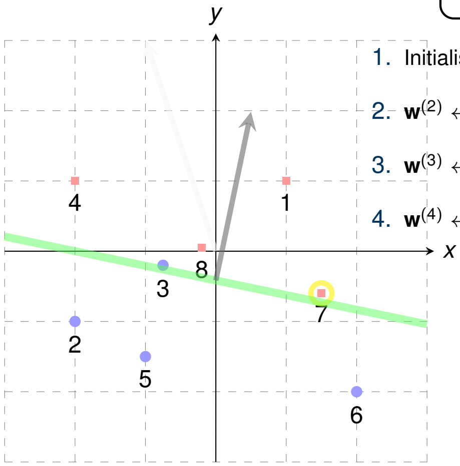


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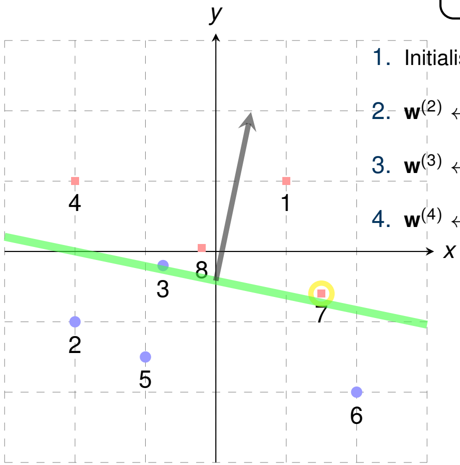


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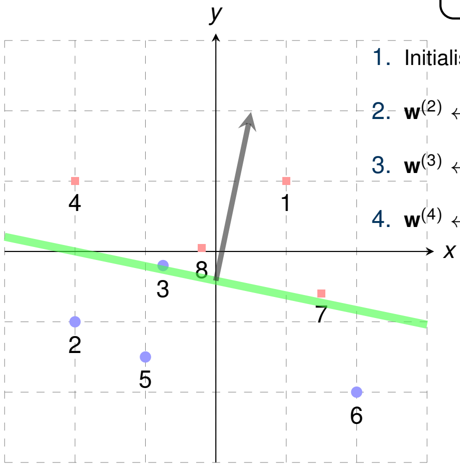


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## Illustration of Perceptron (2/2)

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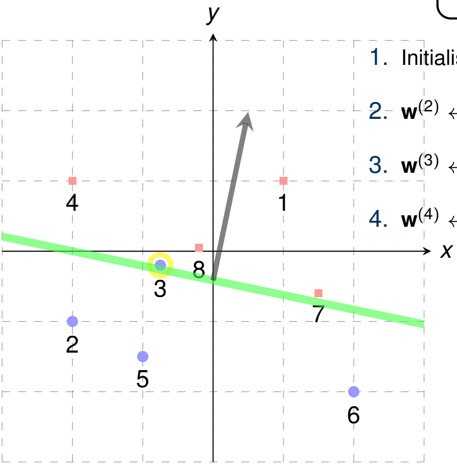


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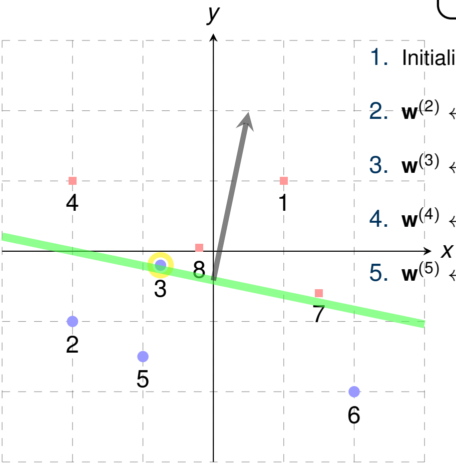
# Illustration of Perceptron (2/2)

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# Illustration of Perceptron (2/2)

**if** $(\exists\, i \;\; \text{s.t.} \;\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
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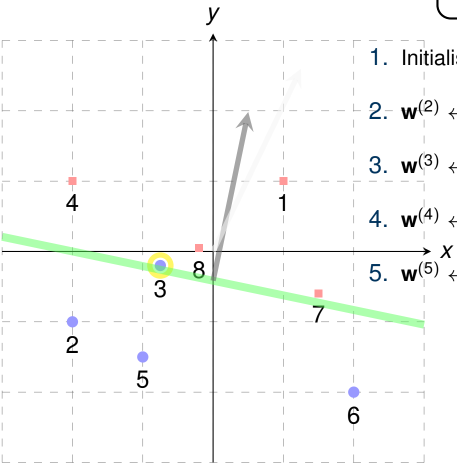


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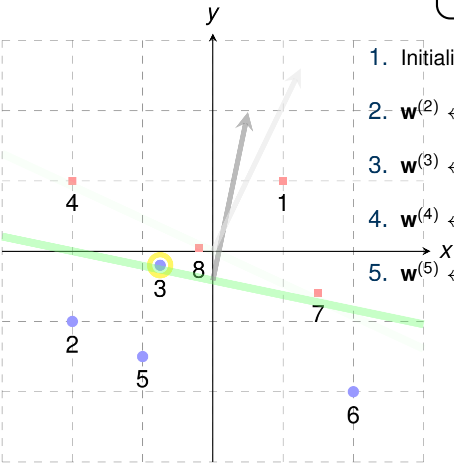


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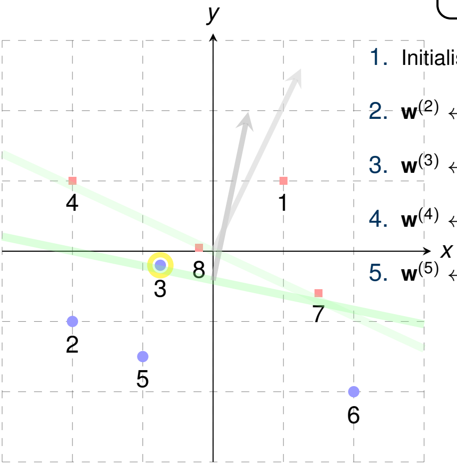


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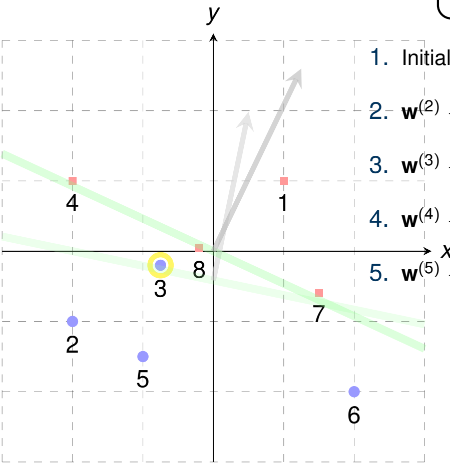


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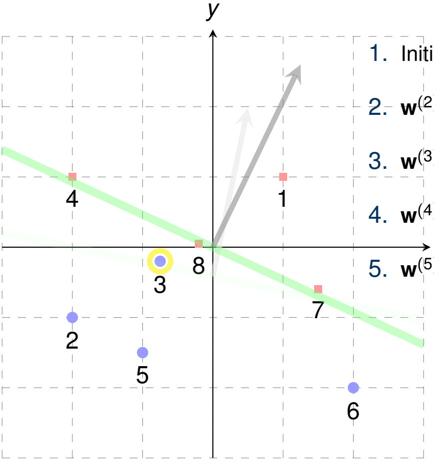


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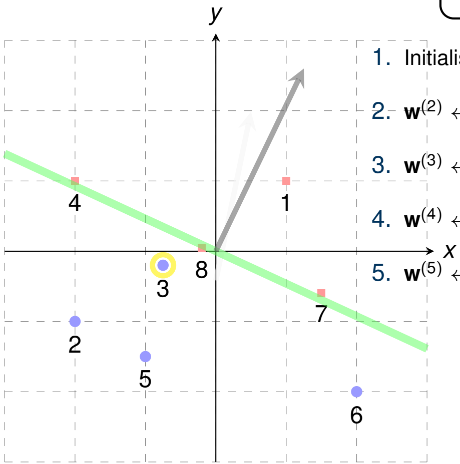


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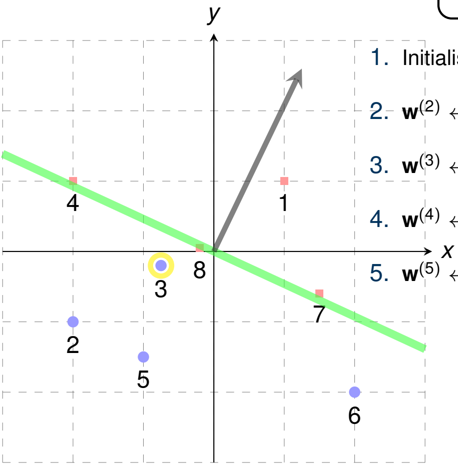


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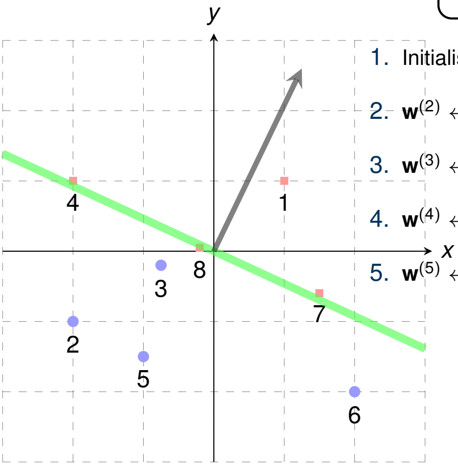


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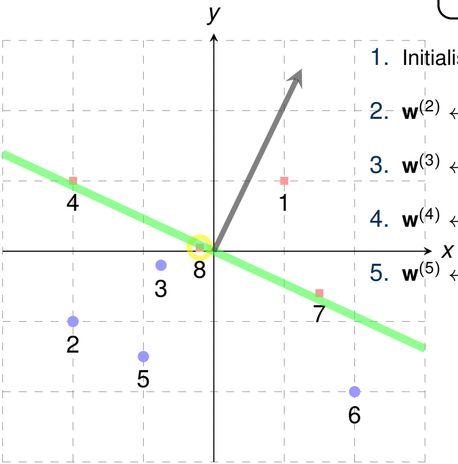


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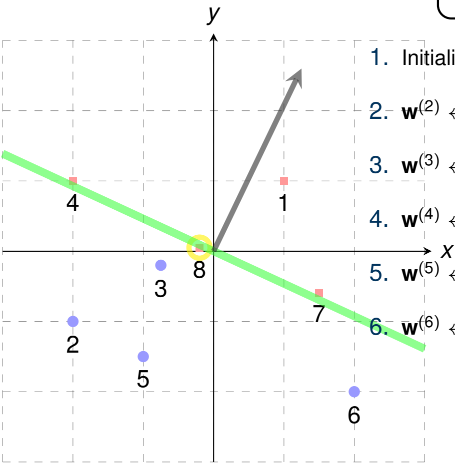
# Illustration of Perceptron (2/2)

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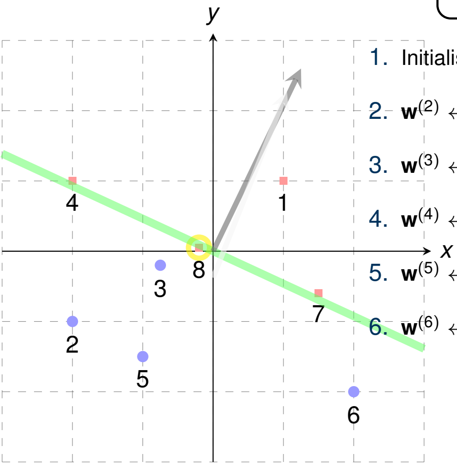


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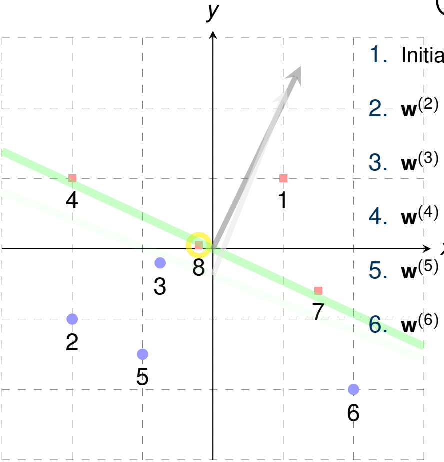
# Illustration of Perceptron (2/2)

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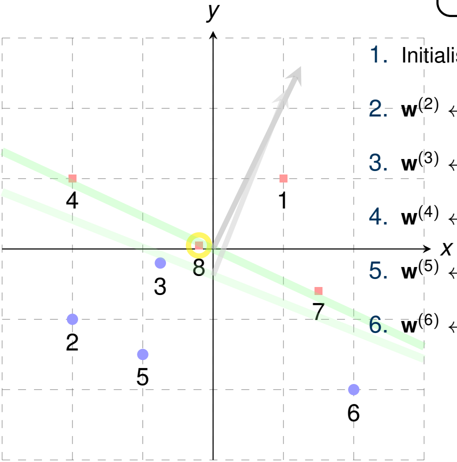


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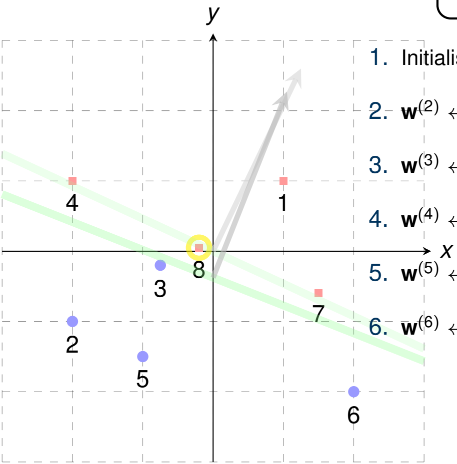


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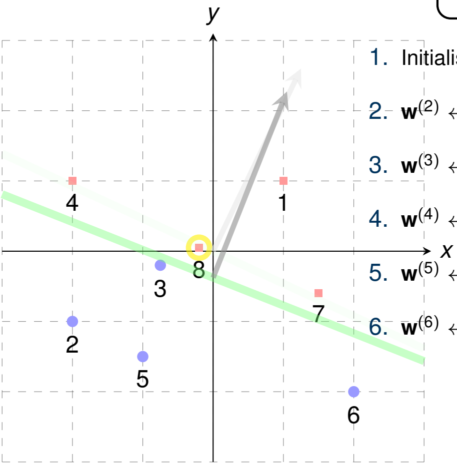


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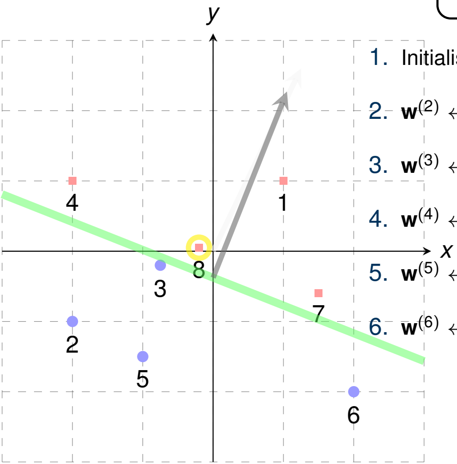


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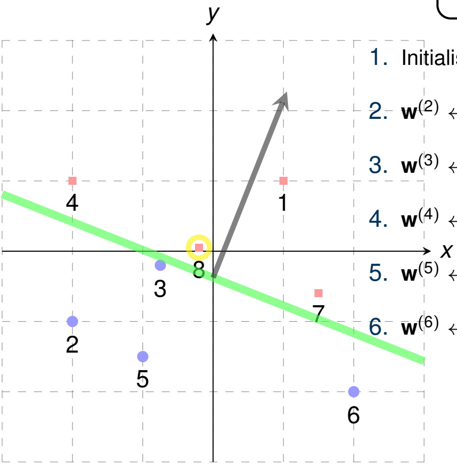


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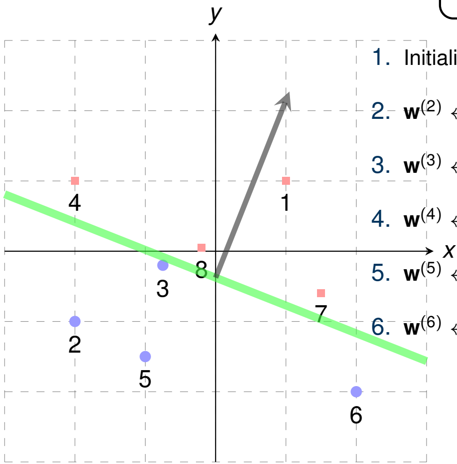


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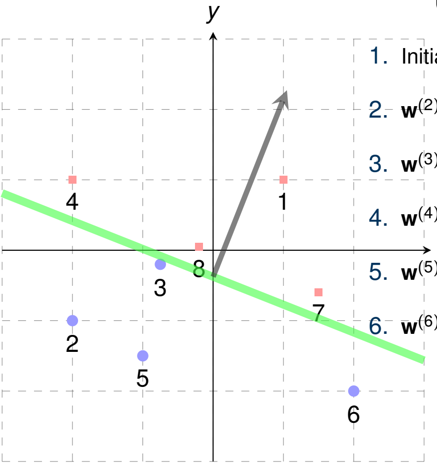


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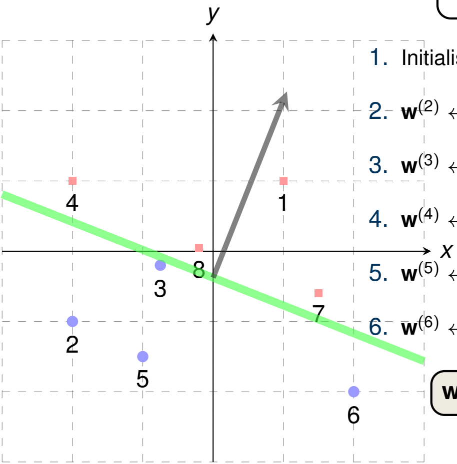


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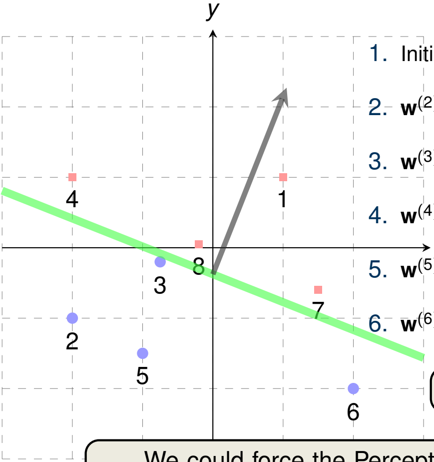


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We could force the Perceptron to run forever by adding more and more points near the green line!

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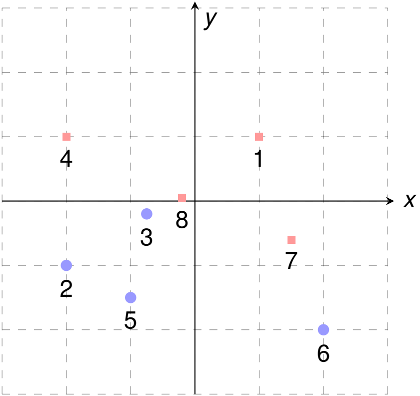
Why does the Perceptron Algorithm make progress?



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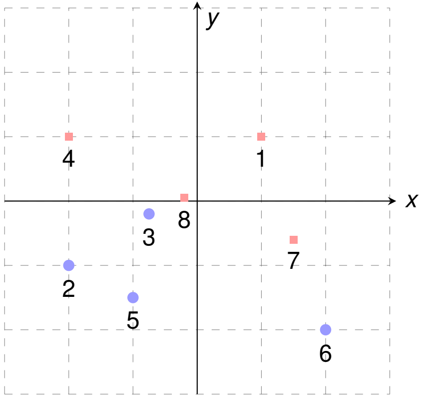

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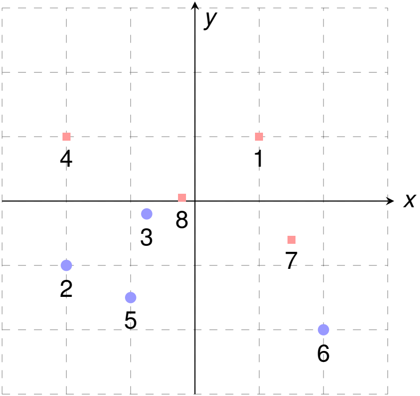

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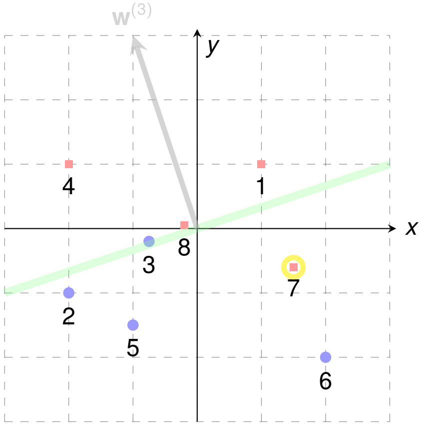

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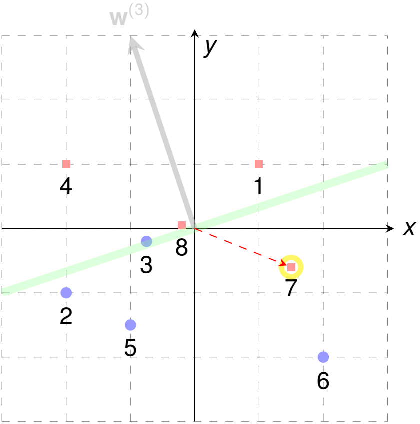

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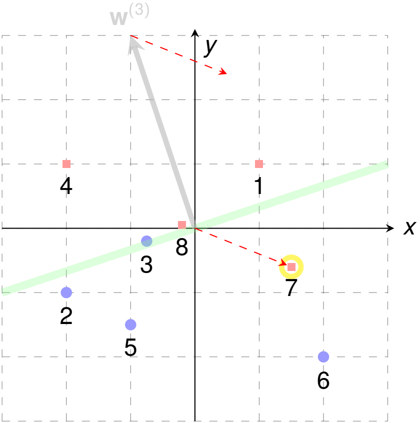

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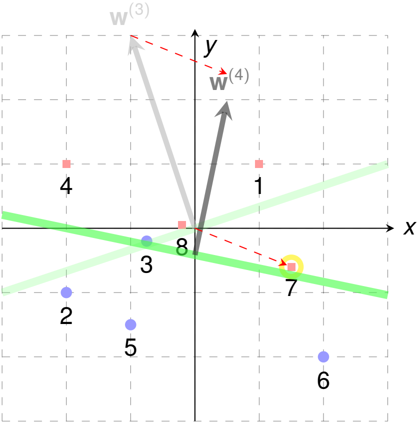

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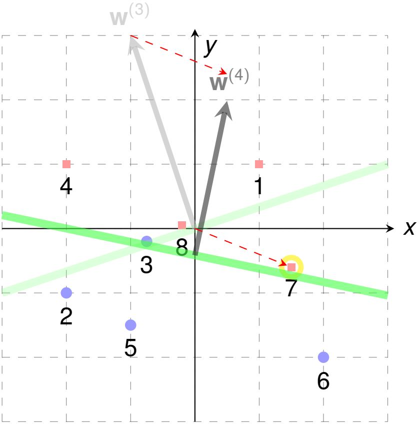

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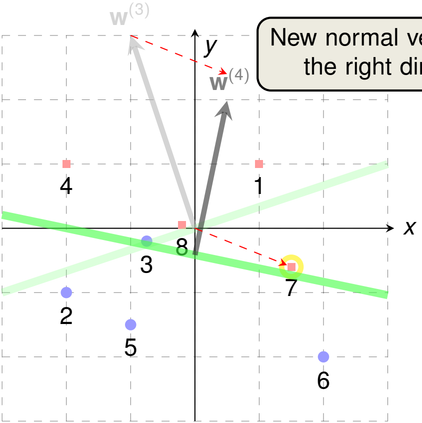


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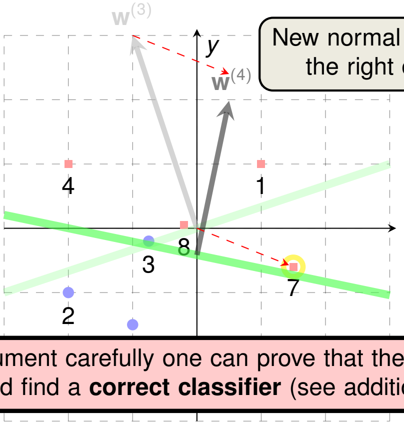


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Using this argument carefully one can prove that the Perceptron will **terminate** and find a **correct classifier** (see additional material)!

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**Quiz 2:** Classify email that contains words "buy", "offer", "the" and "of"

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# Outline

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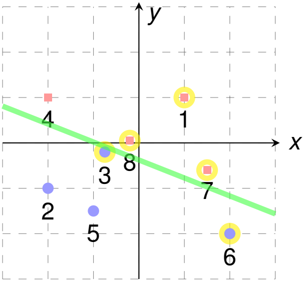


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  - small margin $\rightsquigarrow$ potentially slow convergence

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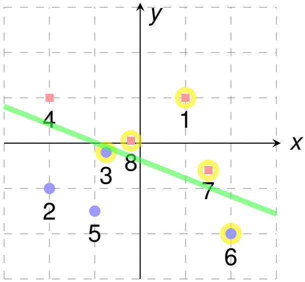


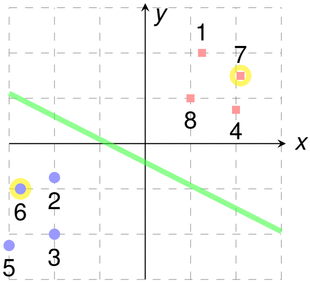


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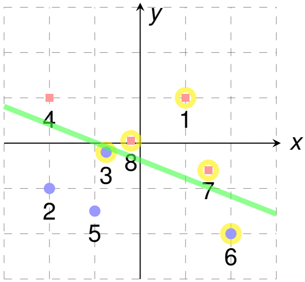


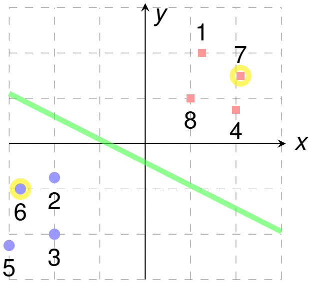


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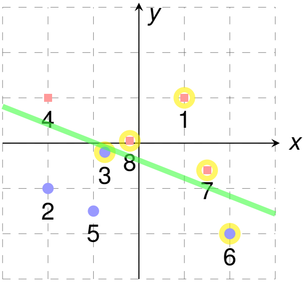


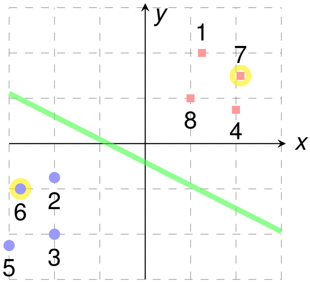


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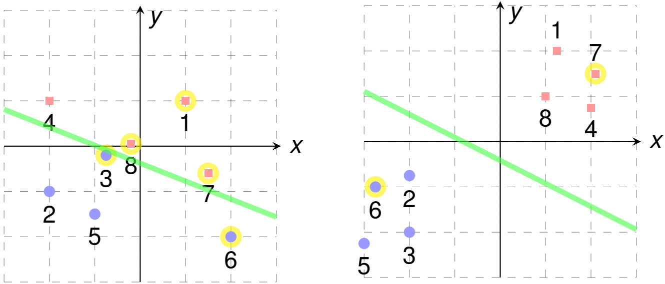


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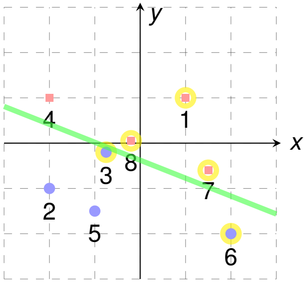


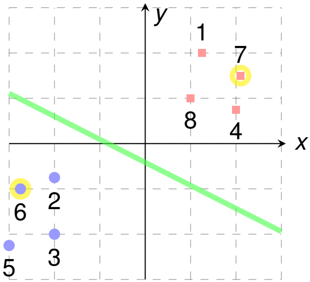


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- **Big Problem:** Perceptron cannot handle data that is not linearly separable
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  - if points are far from linearly separable, better use other methods like SVM, SVM with Kernels or Neutral Networks

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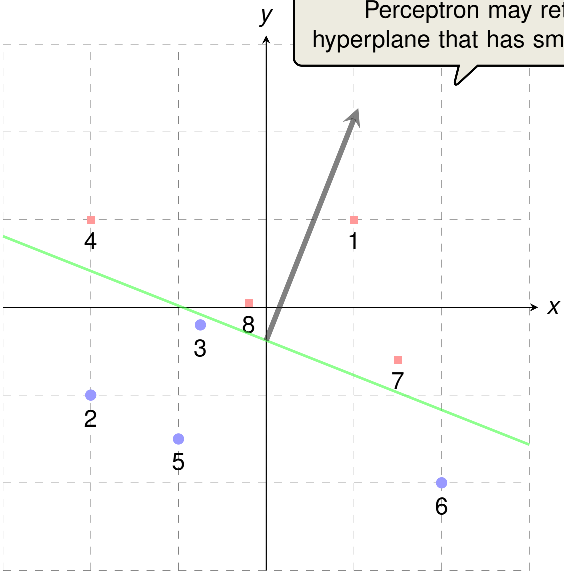


Perceptron may return a hyperplane that has small margin.

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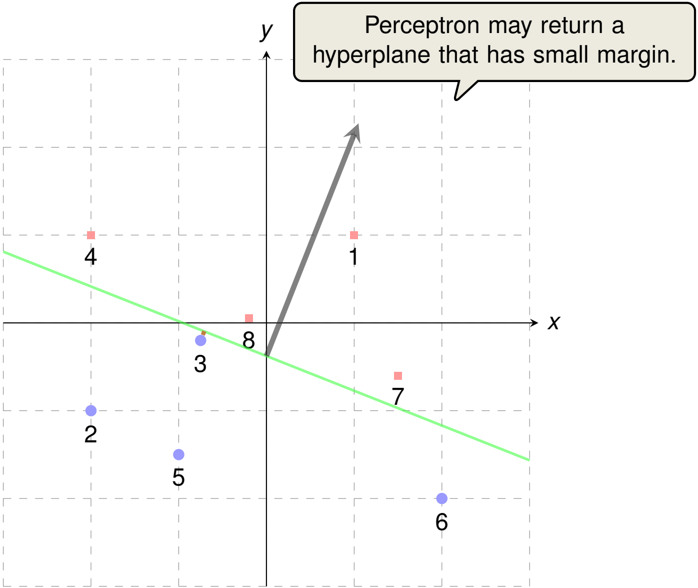

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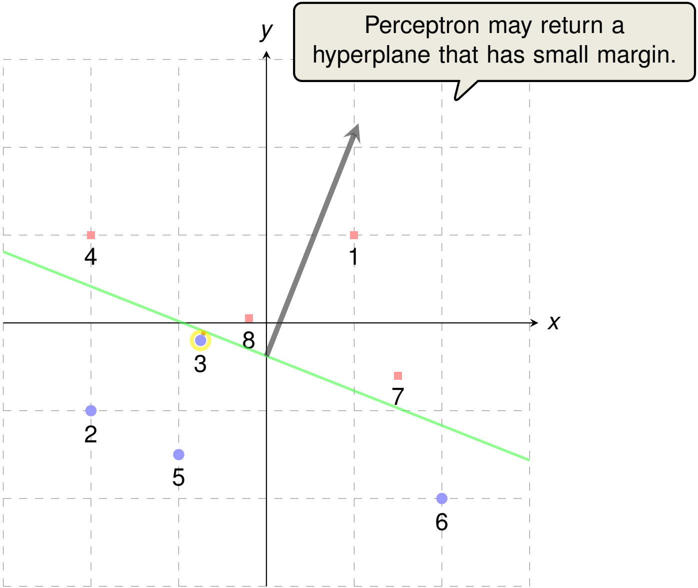

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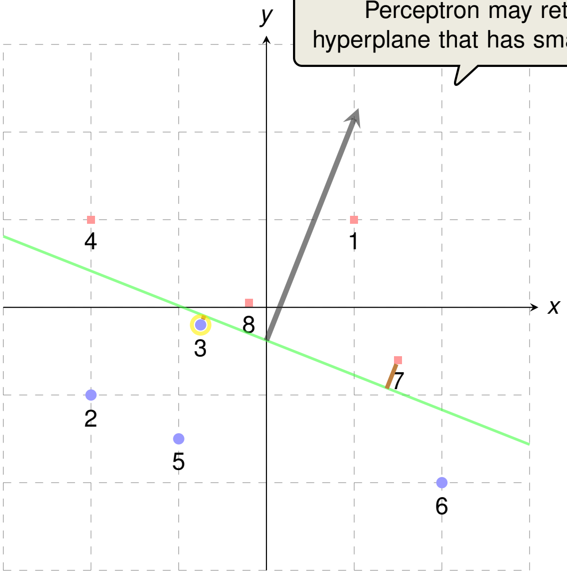


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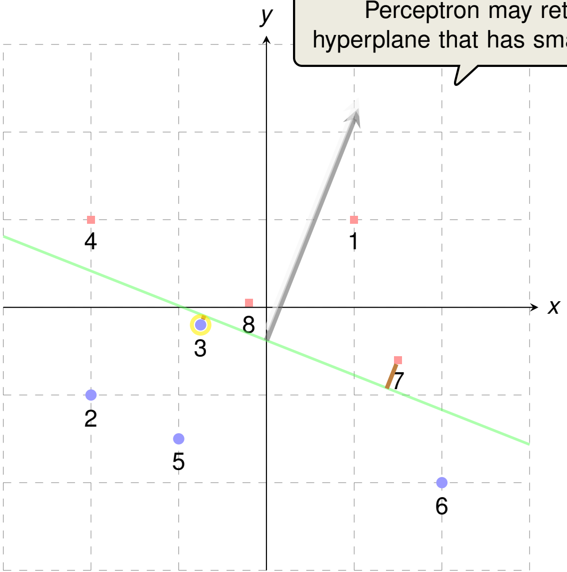


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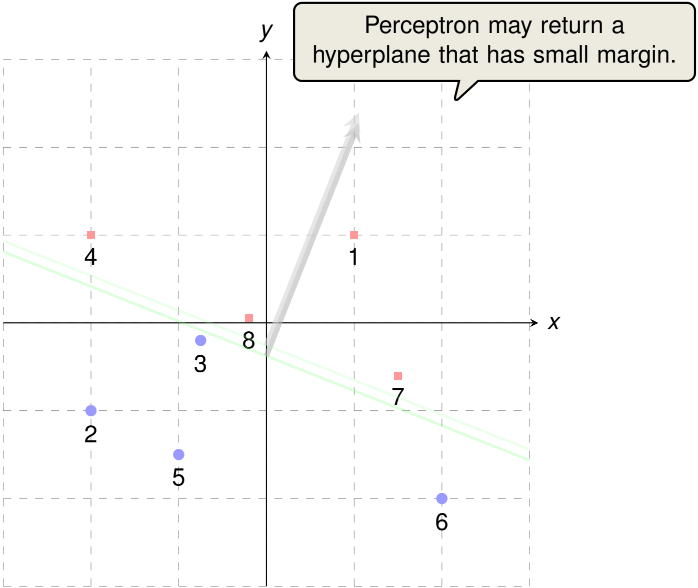

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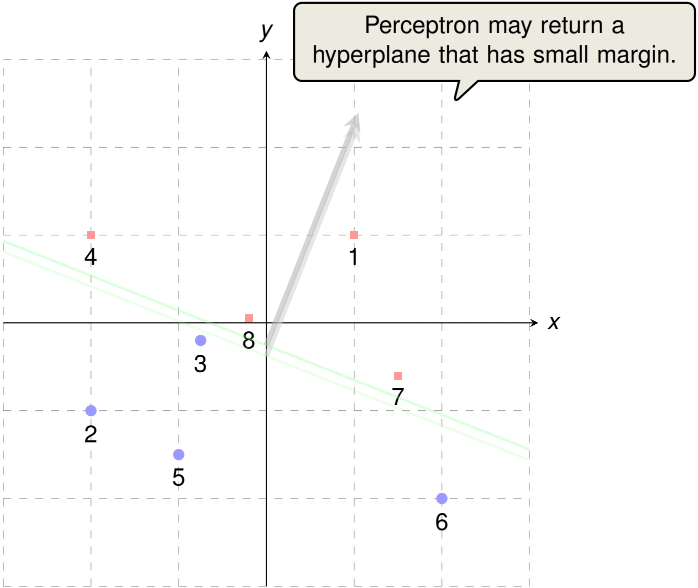

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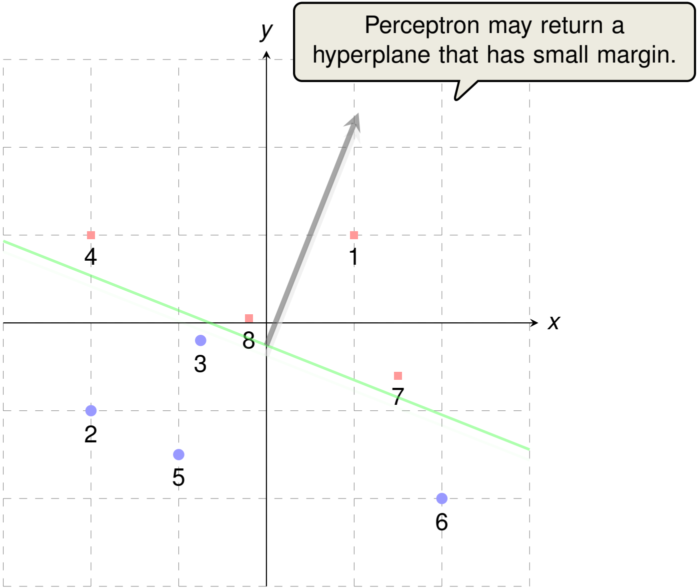

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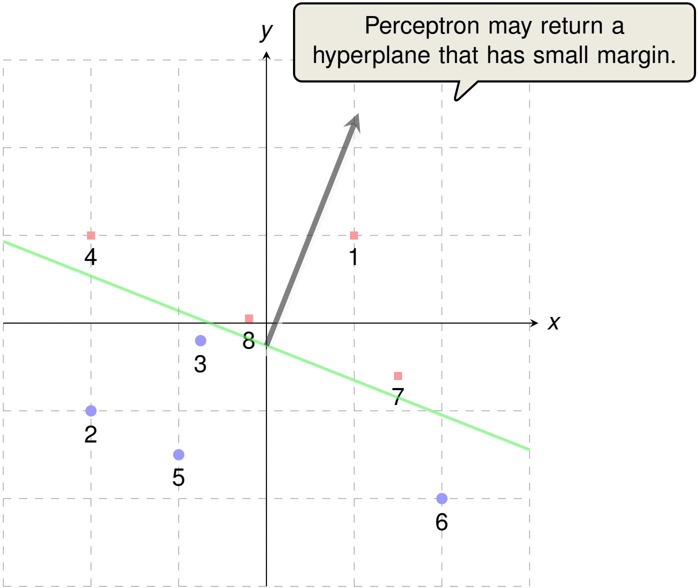

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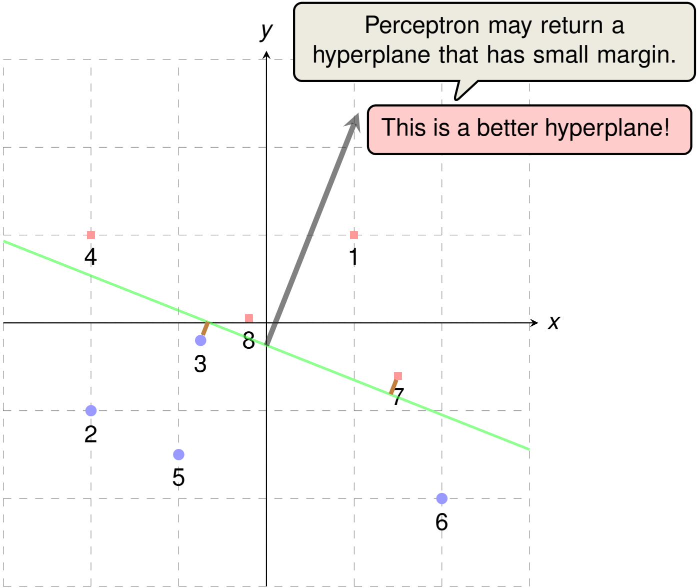

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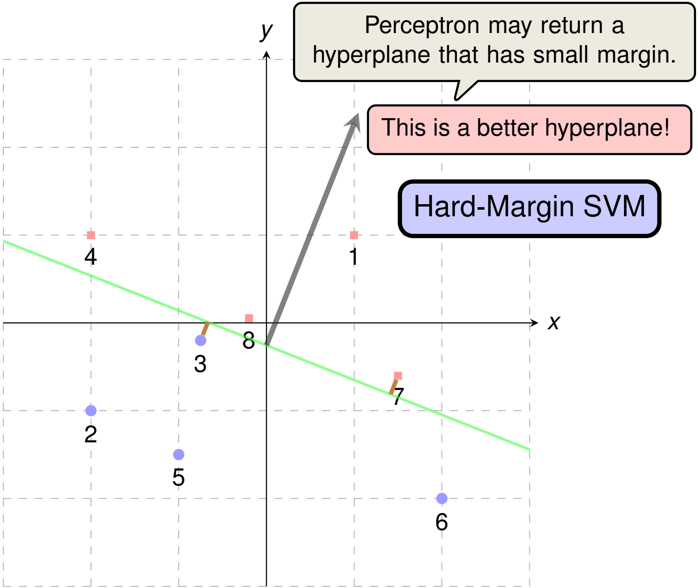

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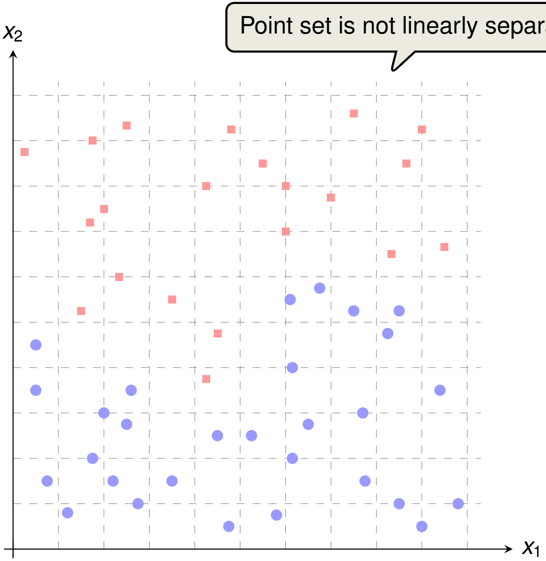

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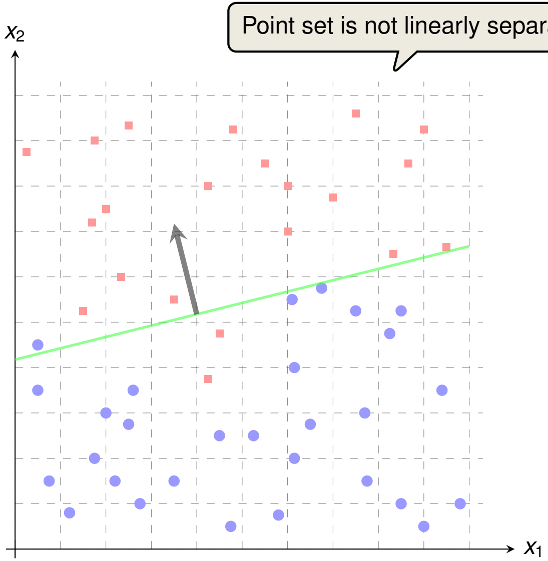

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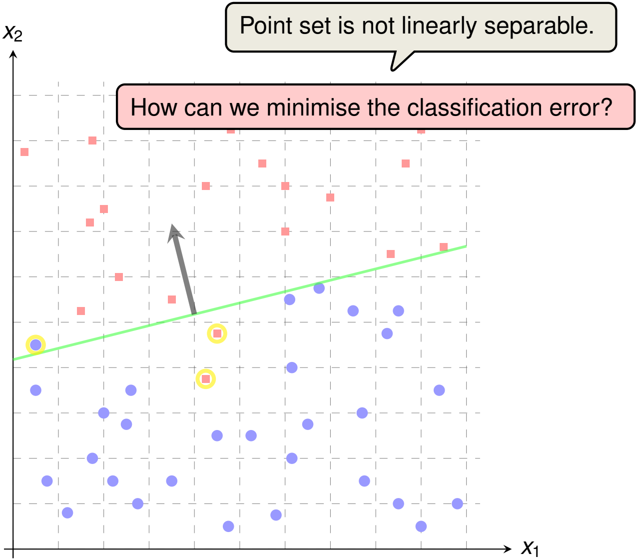

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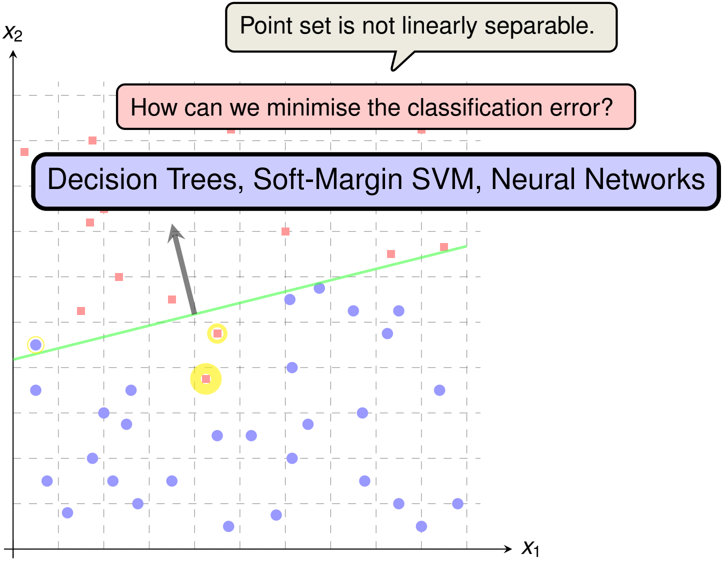

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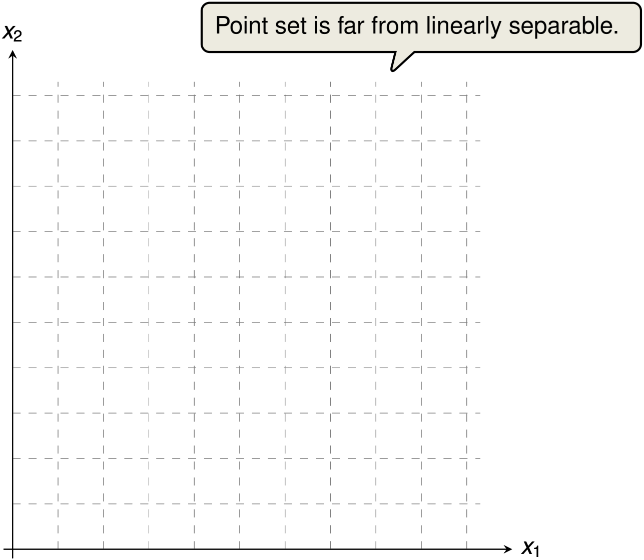

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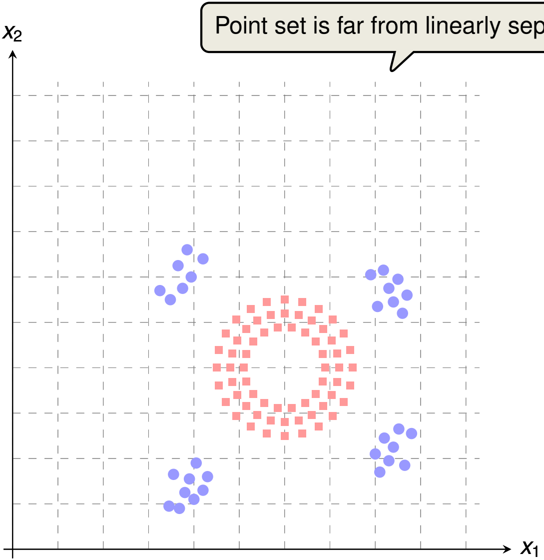

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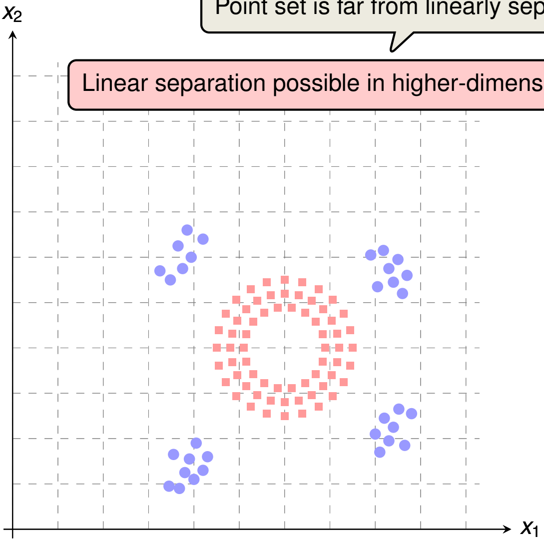

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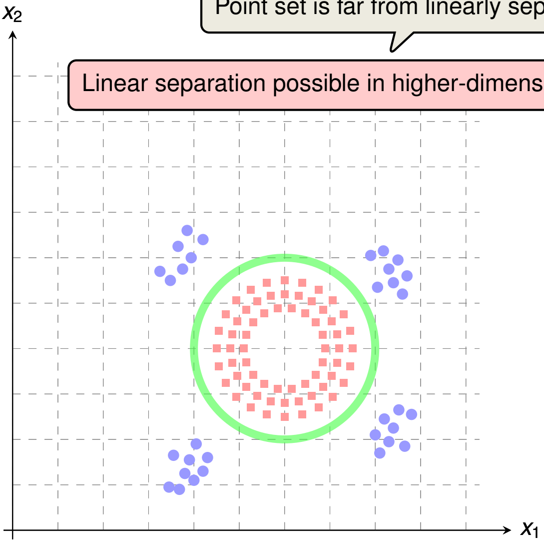

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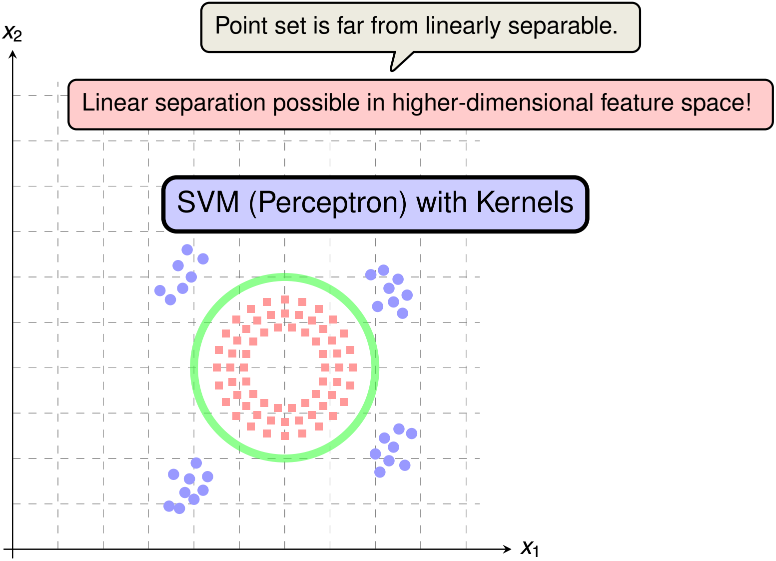
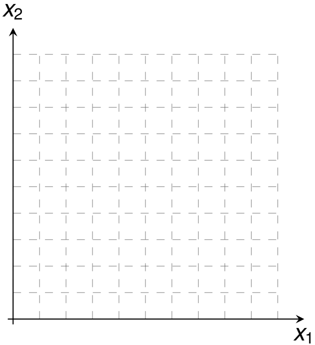


# A Glimpse at the The Kernel Method



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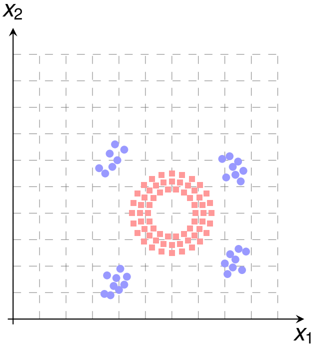

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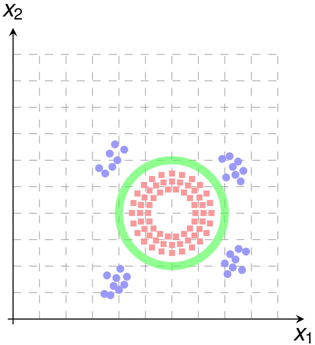

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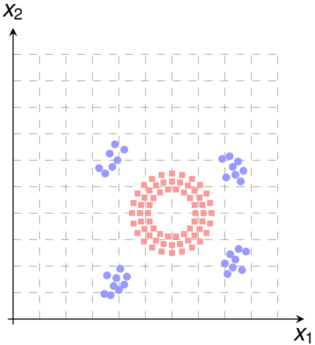

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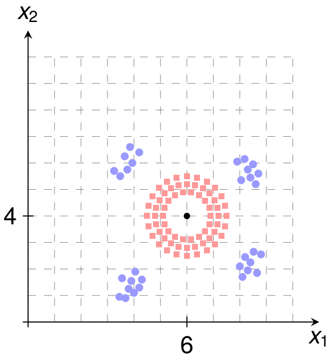

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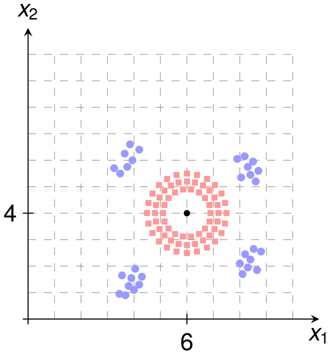

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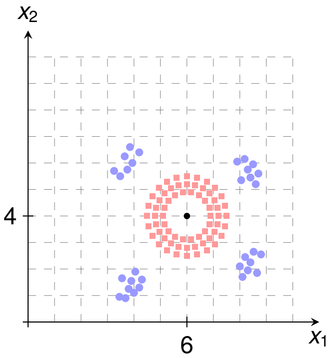


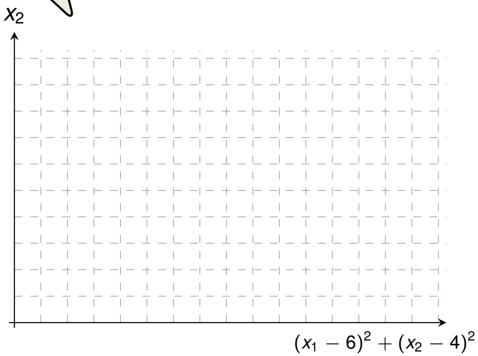

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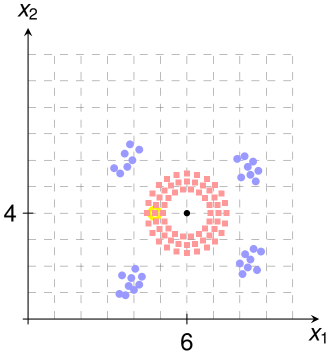


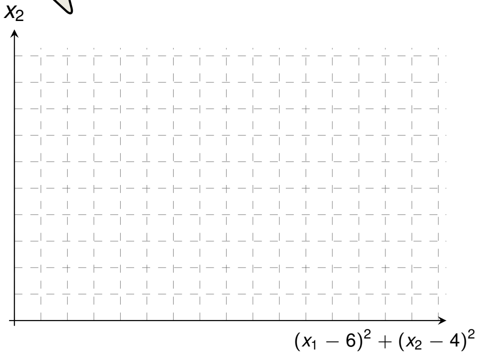

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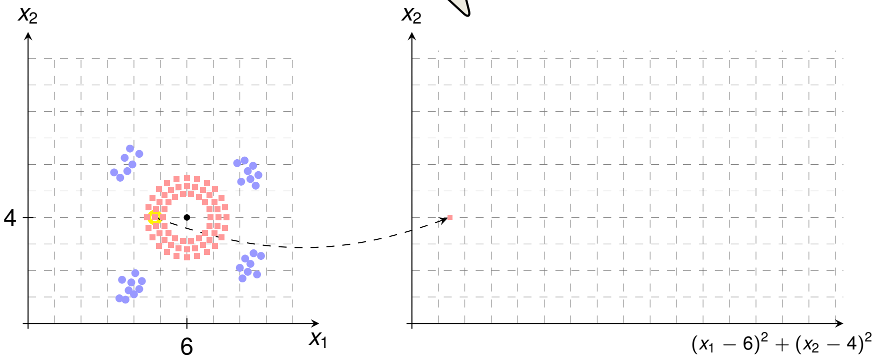

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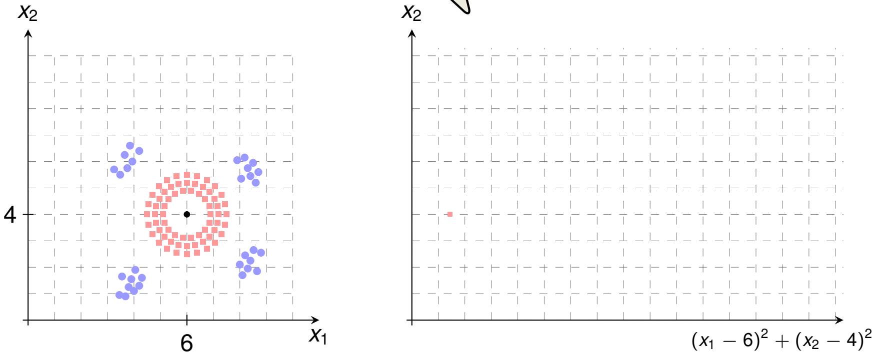

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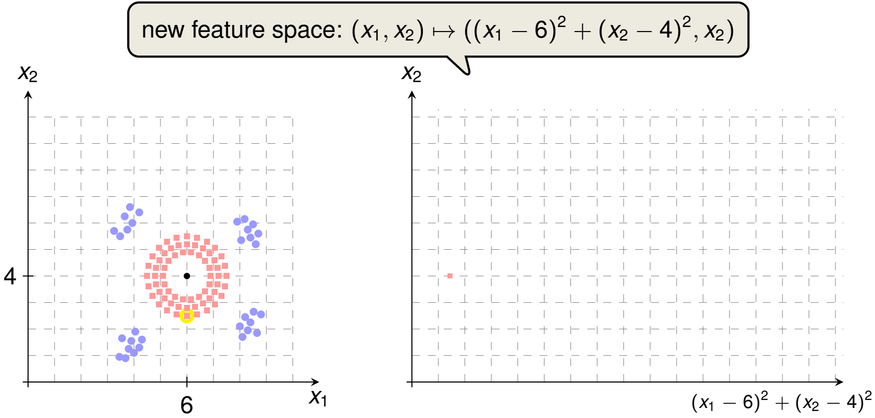

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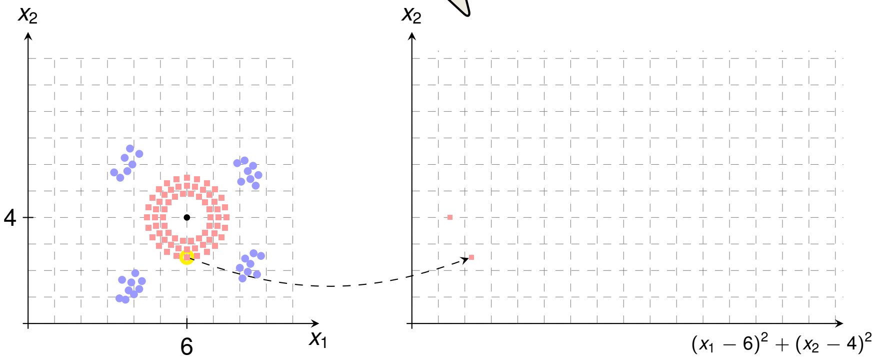

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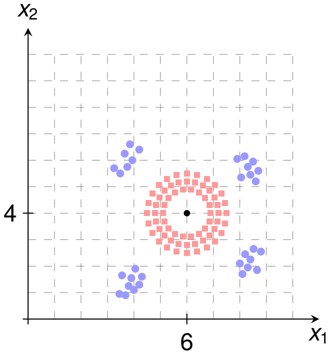


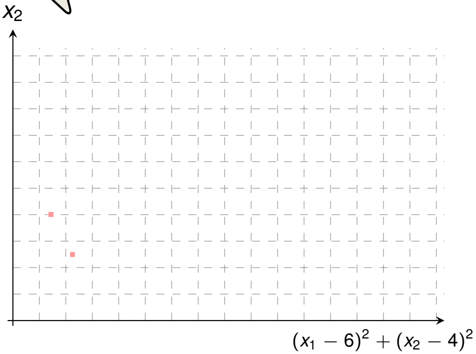

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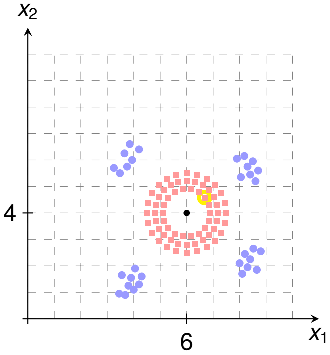


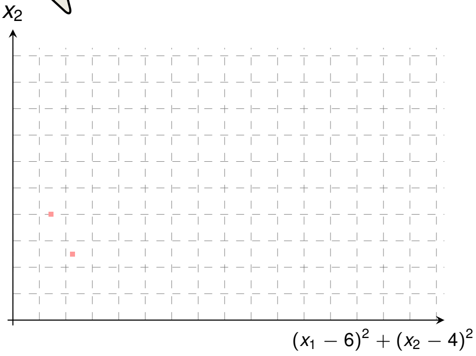

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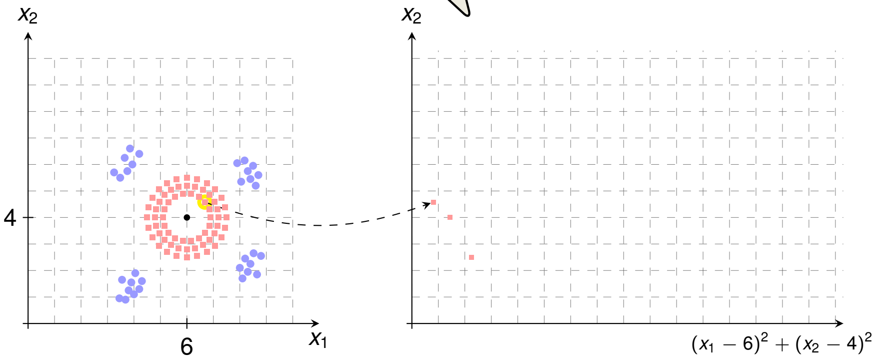

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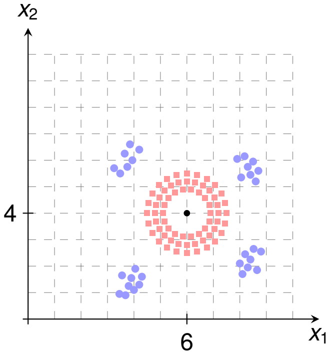


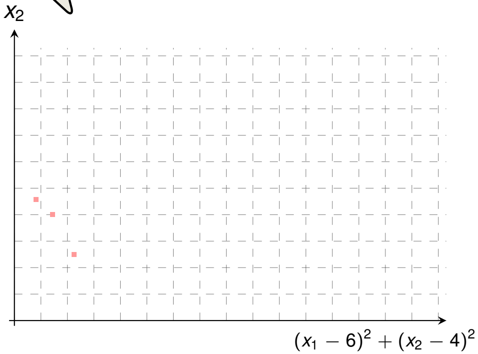

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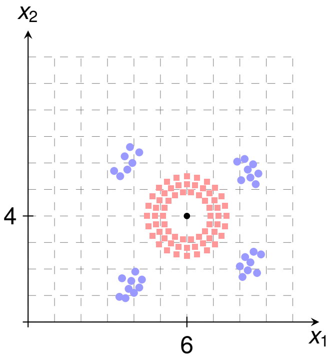


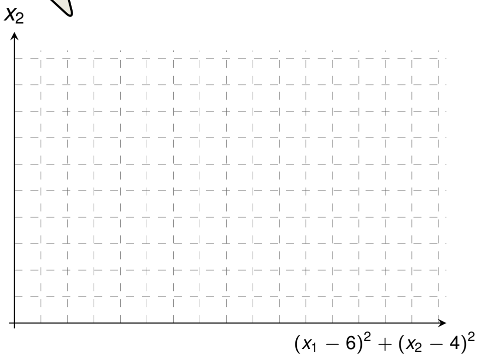

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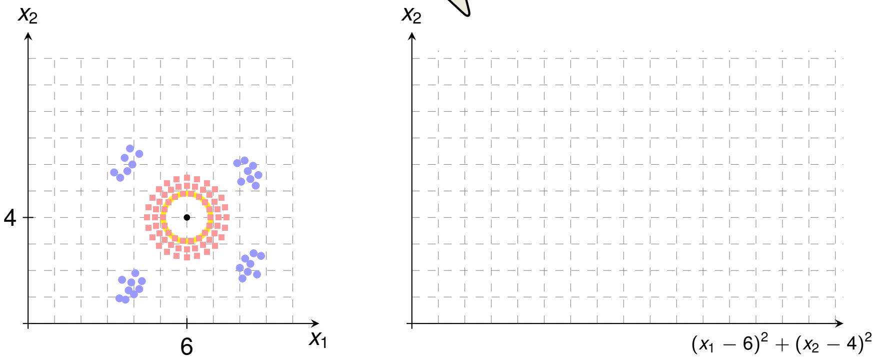

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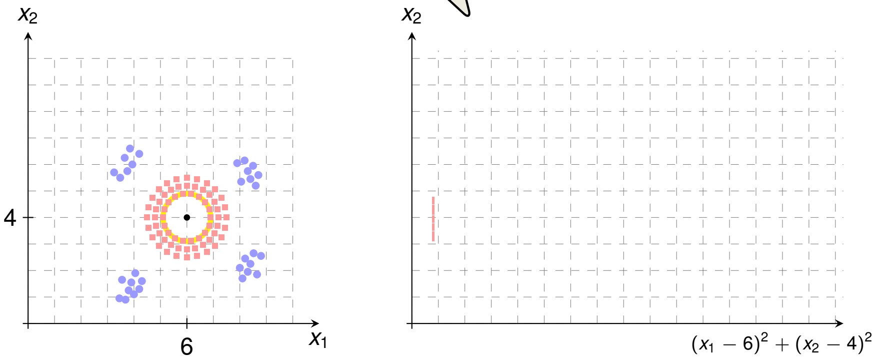

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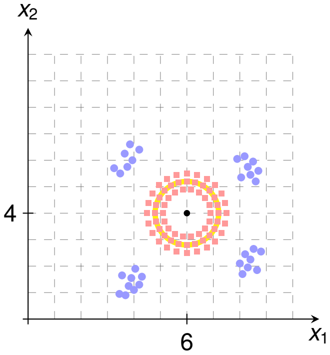


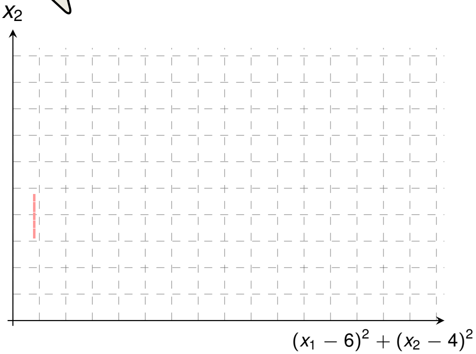

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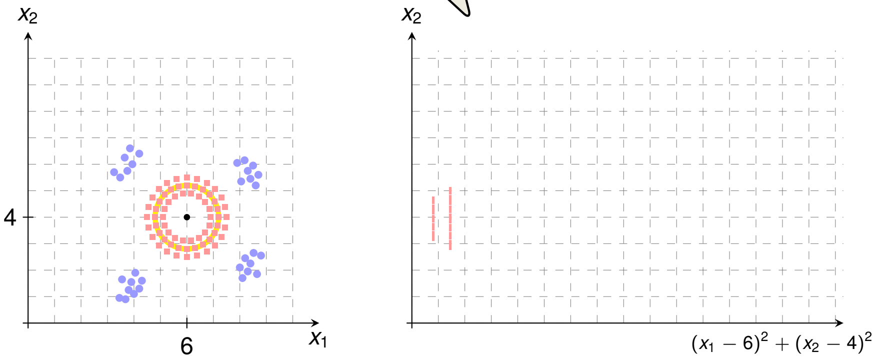

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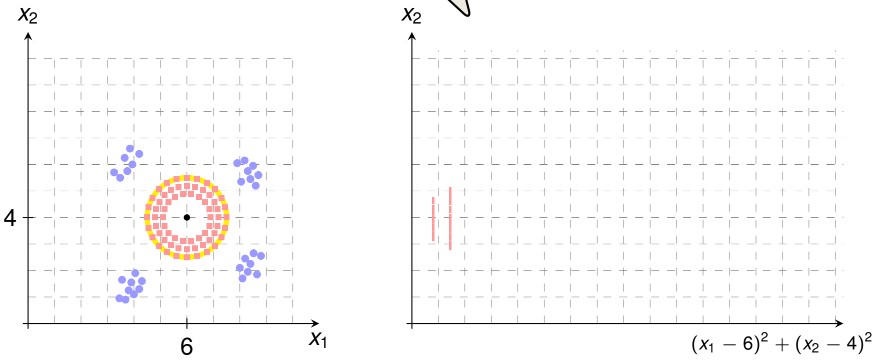

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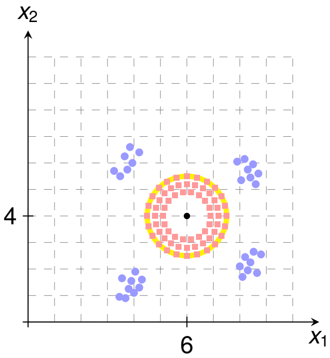


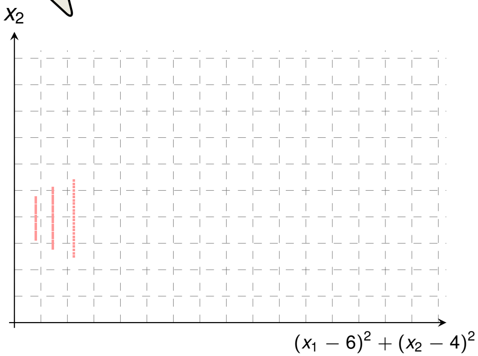

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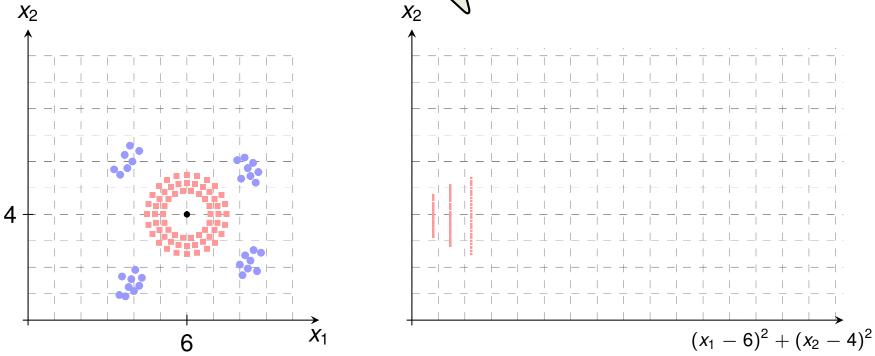

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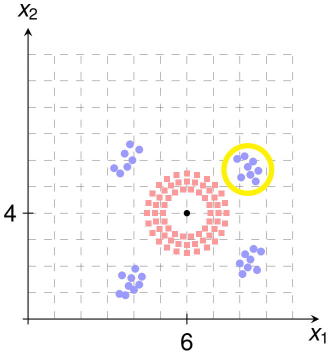


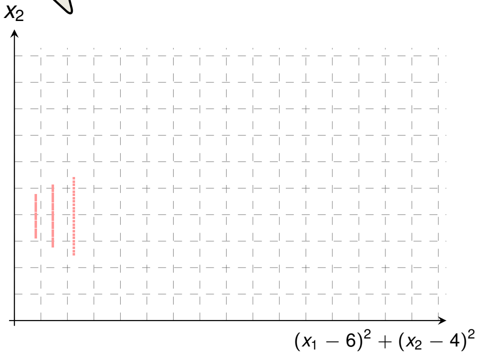

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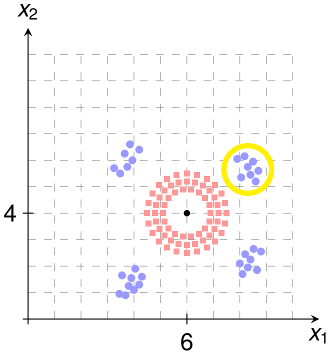


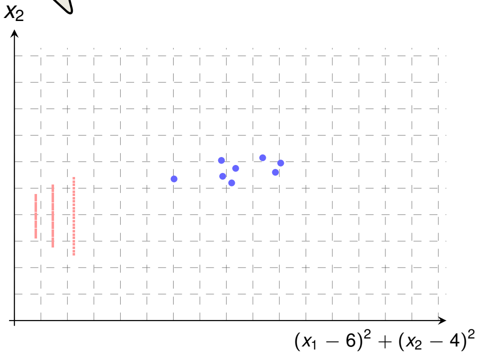

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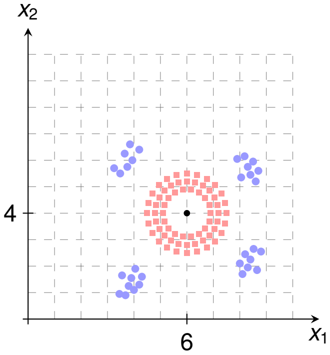


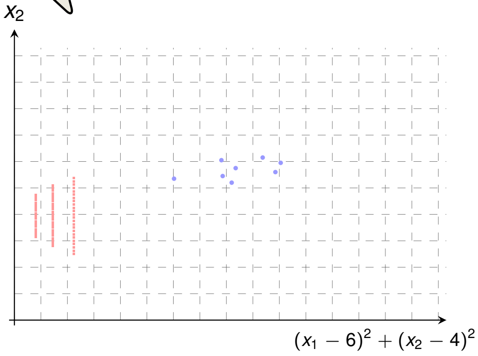

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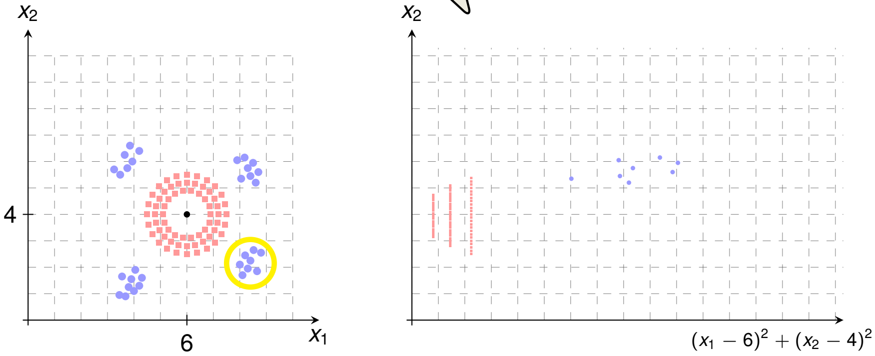

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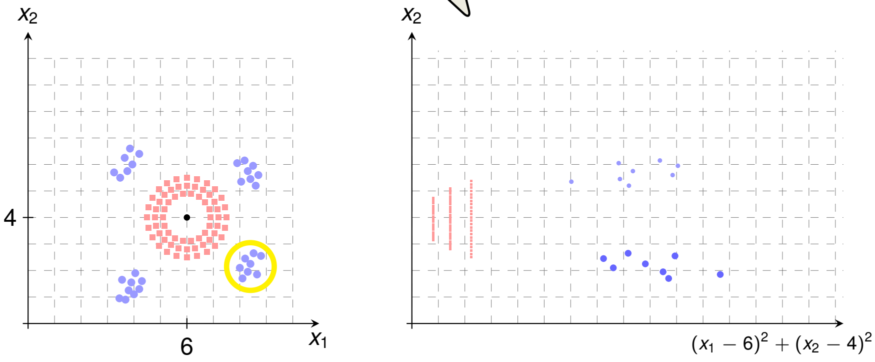

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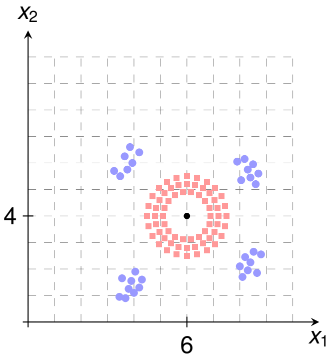


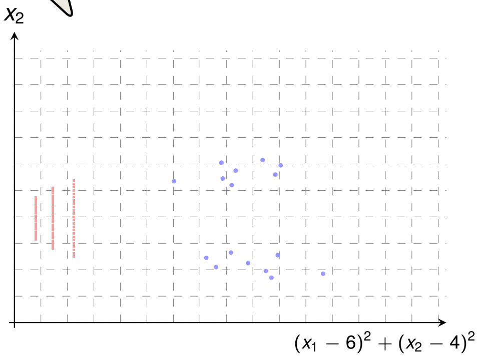

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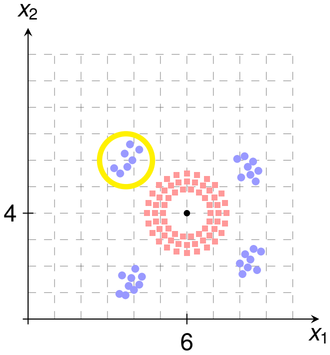


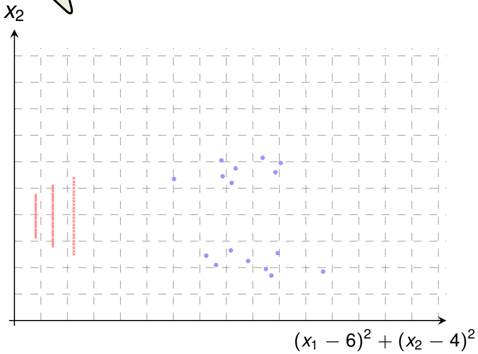

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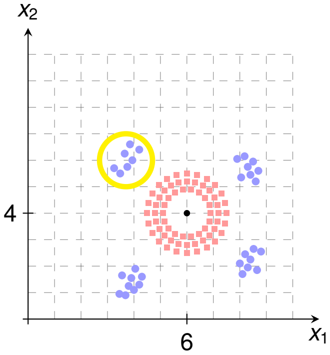


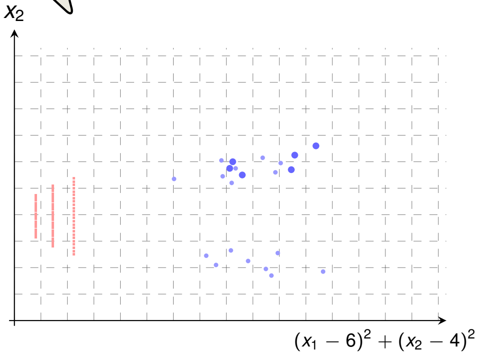

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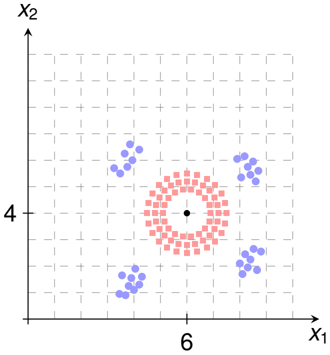


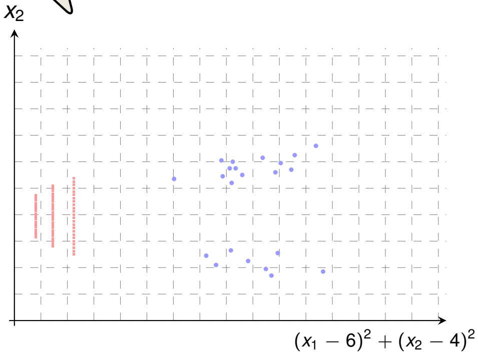

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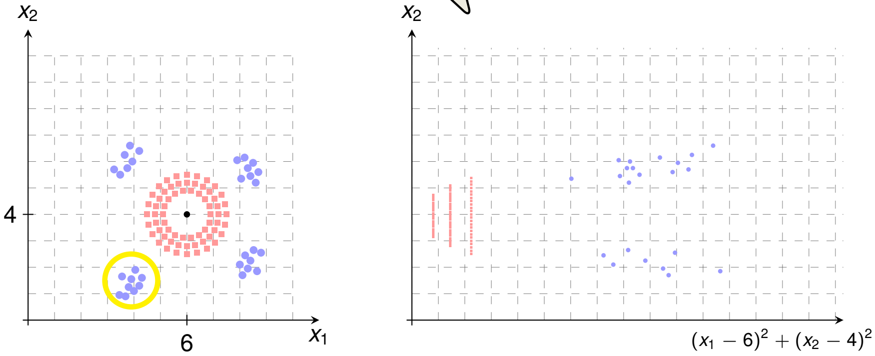

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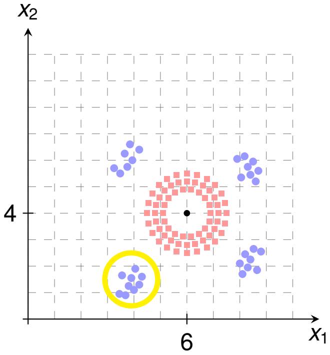


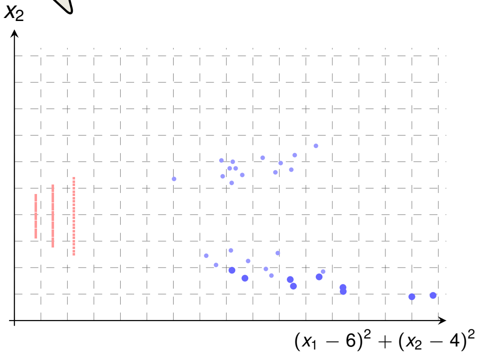

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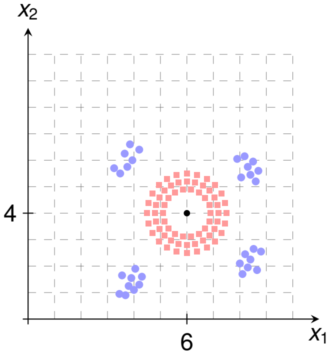


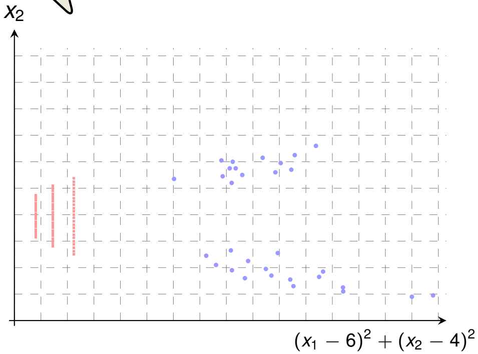

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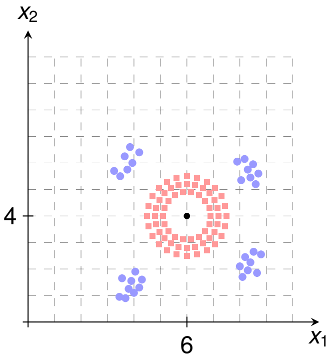


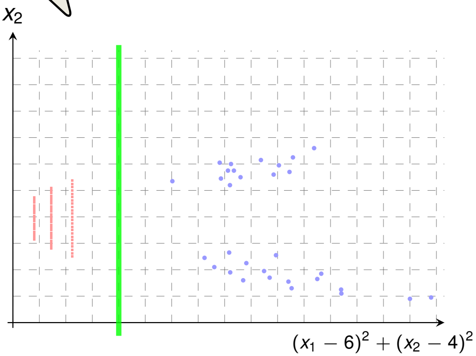

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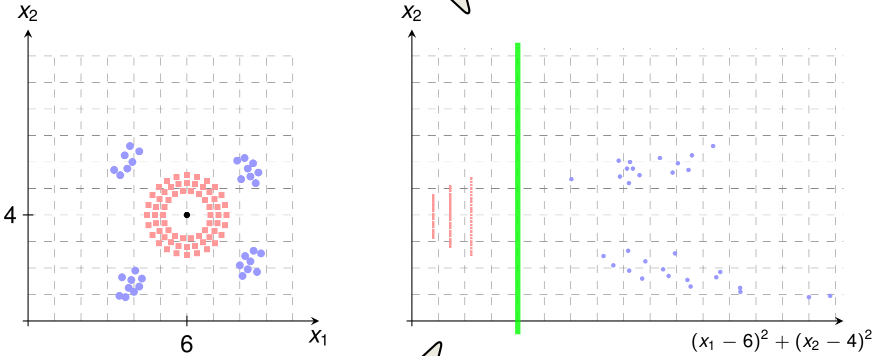


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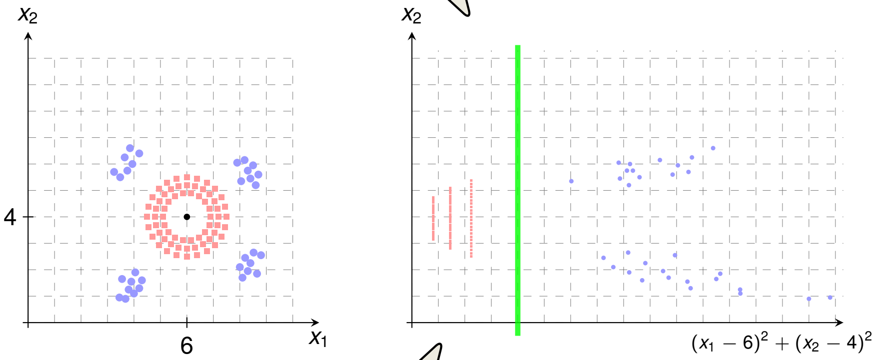


in new feature space, a linear separation is possible!

- How do we choose a proper feature space?
- How do we compute the linear classifier efficiently?
- How do we obtain a non-linear classifier in the original space?

# Outline

Introduction

Perceptron

Conclusion, Problems and Solutions

Additional Material: Why Perceptron Works

# Analysis of the Perceptron Algorithm

**Batch Perceptron**

**input:** A training set $(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_m, y_m)$
**initialize:** $\mathbf{w}^{(1)} = (0, \ldots, 0)$
  **for** $t = 1, 2, \ldots$
    **if** $(\exists\, i \;\text{ s.t. }\; y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$ then
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Proof consists of 3 **Key Steps**:

1. **Step 1**: We can always find a $\mathbf{w}^*$ with $\|\mathbf{w}^*\| = 1$ and $\langle \mathbf{w}^*, \mathbf{x} \rangle \geq \gamma > 0$.
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Runtime bound independent of number of points and dimensionality!