Proceedings of the 2006 IEEE International Conference on Robotics and Automation
Orlando, Florida - May 2006

Gaits and Gait Transitions for Legged Robots

G. Clark Haynes and Alfred A. Rizzi
The Robotics Institute
Carnegie Mellon University
Email: {gch,arizzi}@cs.cmu.edu

***Abstract*— This paper introduces the concept of gait transitions, acyclic feedforward motion patterns that allow a robot to switch from one gait to another. Legged robots often utilize collections of gait patterns to locomote over a variety of surfaces. Each feedforward gait is generally tuned for a specific surface and set of operating conditions. To enable locomotion across a changing surface, a robot must be able to stably change between gaits while continuing to locomote. By understanding the fundamentals of gaits, we present methods to correctly transition between differing gaits. On two separate robotic platforms, we show how the application of gait transitions enhances each robot's behavioral suite. Using the RHex robotic hexapod, gait transitions are used to smoothly switch from a tripod walking gait to a metachronal wave gait used to climb stairs. We also introduce the RiSE platform, a hexapod robot capable of vertical climbing, and discuss how gait transitions play an important role in achieving vertical mobility.**

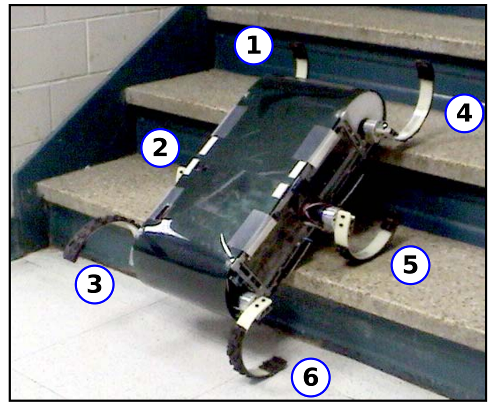

Fig. 1. RHex performing a gait transitioning, switching from normal walking to climbing stairs. Leg numbering conventions are noted in the figure.

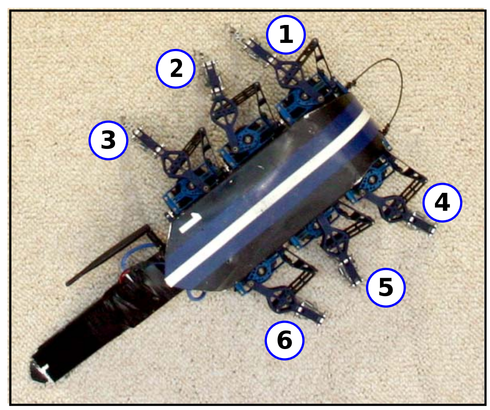

Fig. 2. RiSE on a carpeted wall. The robot uses clawed feet and leg compliance to adhere to the wall. Leg numbering conventions are noted.

I. Introduction

There are at least three obvious approaches to the problem of generating motions for legged robots. One approach is to design a reactive system whose emergent behavior resembles a desired motion. While it is possible to encode stability within such a system, it is difficult to design the emergent behavior, and these systems often lack common intuition. A second, opposite approach is to plan the individual motions for the legs and feet of a robot, while ensuring stability of the resulting overall motion. Legged locomotion, however, necessarily involves complex and often nonholonomic constraints relating to surface contact and conservation of momentum, making the planning approach very difficult to implement.

One popular alternative is to take an abstracted view of legged locomotion by simply specifying the *gaits* a robot may use. A hierarchical approach to robot control can then be taken, locomoting by switching through a sequence of gaits. The control system can focus on selecting appropriate gaits and adjusting parameters of those gaits, such as speed, steering, etc. The question remaining is how to safely transition between gaits, the primary focus of this paper.

More specifically, we address how gait transitions can be used to combine behaviors to produce safe and capable locomotion over terrain. One example we will focus on is the RHex robot (see Fig. 1). RHex utilizes separate gaits for tasks such as walking, jogging, running, pronking, and stair-climbing [1]–[3]. We will also explore the RiSE robot (see Fig. 2), where we use a variety of gaits for climbing on various vertical surfaces. In both cases, we explore strategies for transitioning between gaits. For RHex, we describe an automated behavioral sequence to enable the smooth transition from level ground walking to stair climbing. In the case of RiSE, automatically generated transitions enable operator-"steered" locomotion on a uniform vertical surface, where the transitions switch between parametrically different (up, down, turn) behaviors as well as structurally different (tripod, tetrapod, pentapod) behaviors.

A. Related Research

Ever since the first walking mechanisms were developed, roboticists have been trying to create robots that exhibit complex, animal-like motion. Biologists reverse-engineered the neuronal bases of locomotion [4], [5] while their applied counterparts created robots that used networks of simple reflexes and coordination schemes to locomote [5]–[7]. These policies result in networks of simple computational elements from which gait-like behaviors emerge. There is no concept of “changing between gaits”, as all motions are produced by the reactive policies.

The opposite approach has been deliberate and careful planning of every footfall a robot makes [8], [9]. These methods require very accurate sensor information, accurate modeling of the constraints related to locomotion, and computational power to perform the planning, all of which are difficult to achieve on a mobile platform. While high-level behaviors have been exhibited, these methods have generally been largely unsuccessful on small, fast, and possibly dynamic mobile robots.

An alternative approach is to explicitly store individual gaits, each designed for a specific purpose. In the absence of sensor information, intuitive feedforward motion patterns can be rapidly developed and are often quite successful at various tasks [1]. To improve robustness and performance, learning techniques have been applied in this domain [10], [11]. With a large set of possible gaits, the challenging task becomes understanding how to transition between them, while still adhering to the basic principals of legged locomotion.

Our focus in this paper is on using this class of open-loop behavioral strategies. The reader should note, however, that by using simple feedforward leg trajectories, we do not preclude the use of feedback in a gait. In [12], a method is introduced which continuously mixes both feedforward and feedback information to improve a gait's locomotion over a challenging surface.

II. Gaits for Legged Robots

A. Understanding Gaits

A gait is a cyclic motion pattern that produces locomotion through a sequence of foot contacts with the ground. The legs provide support for the body of the robot while the forces resulting from ground contact propel the robot. Gaits can differ in a variety of ways, and different gaits produce different styles of locomotion.

A motion pattern, the basic building block to define gaits, is a mapping from *phase*, a scaled version of time, to a desired robot configuration. For a gait, this function maps from phase space, $\mathbb{P}$, to the configuration space of the whole robot, $\mathbb{Q}$. Gaits are typically cyclic, so the domain, $\mathbb{P}$, is topologically equivalent to the unit circle, $\mathbb{S}^1$. If $\mathbb{G}$ is the space of all possible gaits, then a gait $g \in \mathbb{G}$ is a periodic, continuous, and injective function from phase angle to desired robot configuration.

$$g : \mathbb{P} \rightarrow \mathbb{Q} \tag{1}$$

On a robotic hexapod, the configuration space, $\mathbb{Q}$, is naturally thought of as the Cartesian product of the individual configuration spaces for each leg, $\mathbb{Q}_i$. Thus, (1) can be rewritten as a collection of functions, one for each leg.

$$g : \quad \mathbb{P} \rightarrow \mathbb{Q}_1 \times \mathbb{Q}_2 \times \mathbb{Q}_3 \times \mathbb{Q}_4 \times \mathbb{Q}_5 \times \mathbb{Q}_6 \tag{2}$$
$$g_i : \quad \mathbb{P} \rightarrow \mathbb{Q}_i \tag{3}$$

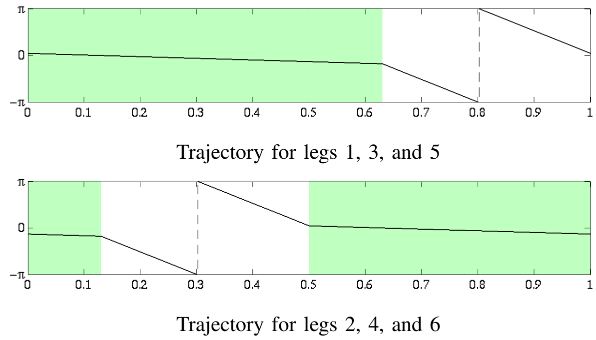

Fig. 3. A tripod gait on RHex, used to walk. The horizontal axis corresponds to clock phase, vertical to leg position, in radians. Wrap-around in leg position is noted by a dashed line. The shaded regions correspond to stance.

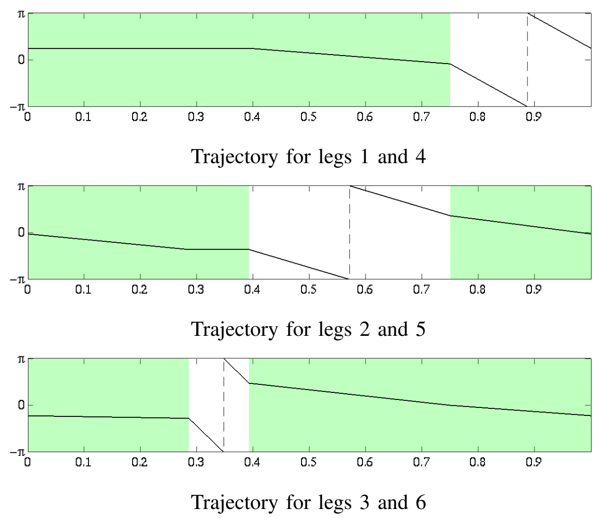

Fig. 4. Gait patterns for the RHex stair-climber gait, phase vs. leg position, stance sections shaded. This is a metachronal wave gait that recirculates legs from back to front.

While this representation of a gait gives us the desired motion patterns for each leg, an important distinction must be made between a leg that is in contact with the ground, termed in *stance*, and a leg that is recirculating through the air, in *flight*. During stance, a leg pushes against the ground, generating forces that move the robot forward. A leg in flight returns to the configuration where stance begins again, completing a cycle of the gait.

The sequence in which legs begin stance, as well as a count of the number of legs in stance, reveals much about the structure of a gait. We will restrict ourselves to discussing types of gaits for six-legged robots, hexapods, but related gait types exist for both bipeds and quadrupeds.

Figure 3 shows an example of a *tripod* gait, one in which a minimum of three legs are in stance at any given time. Each function maps phase angle to the desired leg position. In this gait, legs are separated into groups of three, each set consisting of a front and back leg from one side of the body, as well as the middle leg from the other. Each set of three legs recirculates only when the other set is in stance.

Figure 4 shows a very different gait, one that allows RHex to climb stairs, as detailed in [3]. Rather than moving legs in sets of three, this gait pairs contralateral legs together. The legs are recirculated in these sets of two, in order from back to front, making this a metachronal wave gait.

B. *Useful Gait Parameterizations*

Ignoring the exact spatial trajectory each leg follows, useful parameters for describing gaits exist in the timing of events in these trajectories. These parameters provide a form of semantic information about a gait, indicating the type of gait, when certain legs undergo stance, as well as whether or not the gait will be *valid*, providing proper support for the body of the robot as it locomotes.

Since legs can potentially begin stance at different phases, this suggests our first set of parameters. The *stance phase offset*, $\rho_i \in \mathbb{P} = [0, 1]$, is the phase angle at which a given leg begins stance. If multiple legs have the same stance phase offset, they will make contact with the ground at the same time, and if values differ, they dictate the order in which legs make contact.

Similarly, the percentage of phase during which stance occurs for a given leg is called the *stance duty factor*, $\delta_i \in [0, 1]$. If one were to increase the duty factor for all legs in a gait, there would be more overlap of stance between legs.

A total of 12 parameters, two for each leg, can be used to describe a gait. Studying the example RHex gaits above, the parameters for the walking gait in Figure 3 are

$$\rho = \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ 0.5 \\ 0 \\ 0.5 \end{bmatrix}, \; \delta = \begin{bmatrix} 0.63 \\ 0.63 \\ 0.63 \\ 0.63 \\ 0.63 \\ 0.63 \end{bmatrix}. \tag{4}$$

Likewise, for the stair-climber gait in Figure 4, the values are

$$\rho = \begin{bmatrix} 0 \\ 0.75 \\ 0.39 \\ 0 \\ 0.75 \\ 0.39 \end{bmatrix}, \; \delta = \begin{bmatrix} 0.75 \\ 0.64 \\ 0.89 \\ 0.75 \\ 0.64 \\ 0.89 \end{bmatrix}. \tag{5}$$

It is straightforward to note, just from these parameters, which legs move together, and when stance occurs for each leg.

A simple method to count the number of legs in stance is given by

$$l \; : \; \mathbb{P} \rightarrow \mathbb{I} \tag{6}$$

$$l(\phi) = \sum_{i=1}^{6} c_i(\phi) \tag{7}$$

$$c_i(\phi) = \begin{cases} 1 & \text{if } \phi \in [\rho_i, \rho_i + \delta_i), \text{ in } \textit{stance} \\ 0 & \text{if } \phi \in [\rho_i + \delta_i, \rho_i), \text{ in } \textit{flight} \end{cases} \tag{8}$$

A hexapod robot with three or more legs on the ground is generally in a *valid* configuration, providing stable support for the body. An *invalid* configuration, likewise, is one with less than three legs on the ground, or a possibly degenerate configuration of more legs (three legs but all on one side, for instance).

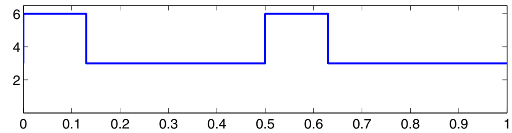

Fig. 5. Count of legs in stance for the RHex tripod walking gait. The horizontal axis corresponds to phase angle, with number of legs on the vertical axis.

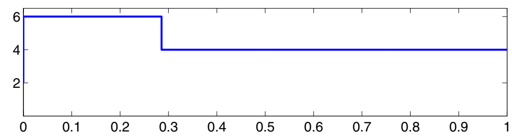

Fig. 6. Stance count for RHex stair-climber gait, phase angle vs. leg count.

Gaits which always keep a minimum of five legs on the ground are generally called *pentapods*. The gait in Fig. 4 is a form a *tetrapod*, as it always has at least four legs on the ground. As mentioned before, the gait in Fig. 3 is a *tripod*, keeping three legs in stance at all times. This can be seen in Figures 5 and 6, the count of legs in stance for each gait, as computed by (7).

The computed *ground speed* of a gait is an important intrinsic value, derived from known parameters. If a gait recirculates at frequency f and if the physical distance a leg travels in the forward direction during stance is given by $\tau_i \in \mathbb{R}$, the average ground speed at which the leg moves the body is

$$\gamma_i = \frac{\tau_i f}{\delta_i}. \tag{9}$$

In most gaits, where all legs move at the same speed, this is the same as the ground speed for the robot as a whole.

III. Gait Transitions

A gait transition is a motion pattern that is inherently acyclic, beginning at a phase angle and robot configuration found in one gait and ending at a phase angle and configuration from another gait. Whereas gaits are meant to be run indefinitely, transitions are finite behaviors that switch between gaits. In order to generate useful transitions, it is important to understand the fundamental properties of gaits, such as their parameterizations, as well as issues like gait speed and gait validity.

Gait transitions are motion patterns, like gaits, but without the requirement that they be periodic. Rather than being defined on $\mathbb{S}^1$, transitions are defined for some interval of phase, $[\phi_1, \phi_2] \subset \mathbb{P}$.

$$g_t \; : \; [\phi_1, \phi_2] \rightarrow \mathbb{Q} \tag{10}$$

The property that transitions begin and end in various gaits results in endpoint constraints. If g_t is a gait transition from

g_1 to g_2, where $g_1, g_2 \in \mathbb{G}$, the following must hold,

$$g_t(\phi_1) = \quad g_1(\phi_1) \tag{11}$$

$$g_t(\phi_2) = \quad g_2(\phi_2). \tag{12}$$

While it is useful to describe gaits in terms of temporal parameters such as phase offsets and duty factors, these values are inherently tied to the periodicity of a gait, and do not exist for transitions.

A. Simple Transitions

The simplest case of a gait transition is for two gaits that happen to cross paths in configuration space, $\exists\, \phi_1, \phi_2 \in \mathbb{P}$ such that $g_1(\phi_1) = g_2(\phi_2)$. At that intersection point, a robot can instantaneously switch from one gait to the other.

For gaits that do not intersect, however, this approach is not guaranteed to produce valid robot configurations, motivating the use of more complex gait transitions. A complex transition would be one that continues locomotion, changing a gait's parameters from one gait to another over a finite period of time, while keeping the robot in valid configurations throughout the transition.

B. Complex Transitions

It is possible to produce valid transitions simply by playing leg trajectories from different gaits at the same time, progressively switching each leg from one gait to the other.

While it is important for all legs on a surface to move at the same rate as one another, a leg in flight can potentially move faster or slower than normal, as long as the robot remains within valid configurations. If a leg in flight were to move faster than normal, it would touch down earlier. This reduces the stance phase offset for that leg, effectively putting the robot in an entirely different gait than it was previously. Alternatively, if a leg is slowed or paused, the phase offset for that leg increases.

In the methods developed in section IV, we describe transitions as a series of phase offset modifications, each changing the parameters of a gait slightly, until finally reaching the goal gait. Legs switch, one by one, from one gait to another, resulting in midway configurations where some legs are playing one gait, and the rest the other.

While these techniques are relatively straightforward, care must be taken to ensure that the robot remains in valid configurations at all times. In our applications, these constraints involve assuring statically-stable positions of the body, but could also involve energetics for dynamics gaits.

IV. Experimental Results

Through analysis of gaits, we have applied gait transitions to produce novel behaviors on two different robotic hexapods.

A. RHex Stair-Climber

The RHex robot is capable of locomoting through and over most body-height obstacles with a tripod gait, but it must use a distinctly different gait to climb stairs. Making use of gait transitions, the robot can continue making forward progress as

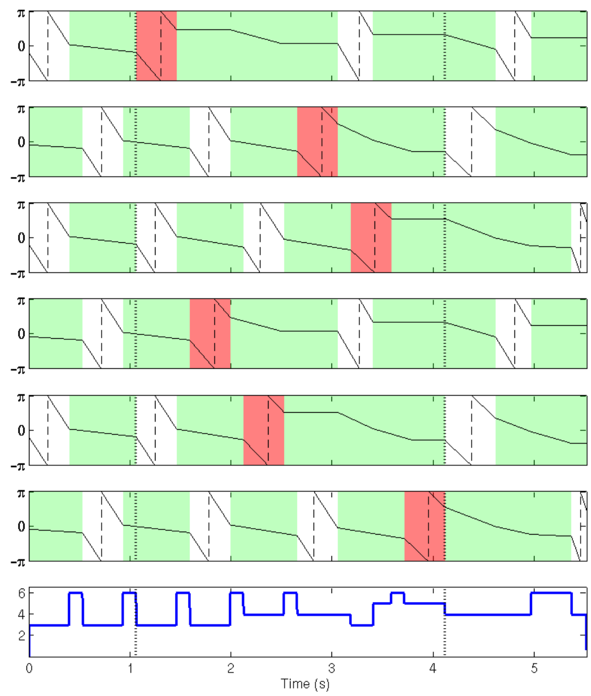

Fig. 7. Leg trajectories for a transitional behavior between walking and climbing stairs, legs 1-6. One complete cycle of the tripod walking gait takes place before launching the gait transition (note correspondence to Fig. 3, marked by the dotted line. The transition finishes at the second dotted line, after which the robot executes one cycle of the stair-climber gait (Fig. 4). A count of the number of legs in stance throughout the transition is shown at the bottom.

it walks onto and up a set of stairs. Prior to our work, the task of initiating a climb required an awkward manual positioning of the robot at the base of the stairs, a pause, and execution of a "first step" set of leg trajectories that propped the front of the robot onto the first stair, but dropped the back onto the floor [3].

1) Gaits Involved: The two gaits involved in a transition from walking to climbing differ in many ways, not only in gait parameters, but also in how they interact with the surface.

The tripod gait, shown in Figure 3, is an extremely simple RHex gait. Three legs recirculate while the other three go through stance, and a minimal overlap exists while the robot is transferring its weight from one tripod to the other. In this gait, legs on opposing sides of the body are always out-of-phase with each other.

The stair-climber gait, shown in Figure 4, is drastically different. Rather than recirculating legs in tripods, the legs recirculate in a metachronal wave gait, progressing from the back to the front. While climbing stairs, the robot uses the unique geometry of its legs to properly nestle them into each stair, preventing the robot from sliding backwards. Contrasted with the tripod gait, the stair-climber gait is symmetric across its body, always keeping legs on opposing sides in phase.

2) Transition Technique: As a transition can be described as switching legs individually from one gait to another, the physical barrier of a stair dictates when to switch each leg. Upon reaching the first stair, a leg interacts with a different

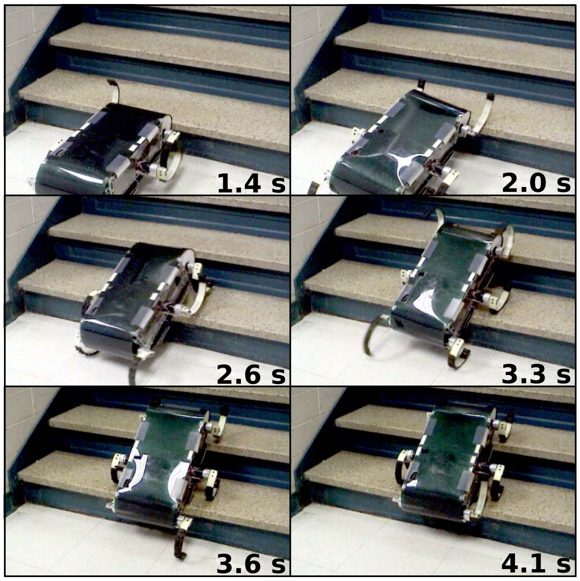

Fig. 8. RHex executing the walk to stair transition. Noted times correspond to those shown in Fig. 7.

surface geometry, requiring the use of a different trajectory to continue locomotion.

The principal difference between these two gaits lies in the phase relationship between opposing legs. As a tripod gait approaches the stairs, a single front leg will hit the stair first, signaling the need to change gaits. This leg, rather than pushing through stance, pauses on the stair momentarily, and waits for the leg on the opposite side of the body to catch-up. This brings the two legs into phase with one another, and these legs begin playing the stair-climber gait. Even though other legs continue to push the body forward, a paused leg will simply prop the body up, sliding forward along the stair.

The two front legs play through stance in the stair-climber gait, pulling the body forward as the legs in the tripod gait push. Middle legs, upon reaching the first stair, switch next, individually. In order to synchronize legs in the stair gait, small pauses are added to ensure correct stance phase offsets. The back legs, which have been continuously running the tripod gait, reach the stairs, and also switch, at which point the robot is in a configuration that is found in the stair-climber gait. By pausing legs momentarily before they begin stance, the robot manages to keep statically-stable leg configurations at all times.

3) Implementation: The transition used to switch between walking and climbing stairs was mostly designed by hand, but key portions were parameterized, and a reactive system was implemented for autonomous operation.

The result of this transition is shown in Figure 7, showing the transition where the front left leg hits the stair first. A mirror image of this transition exists for times when the right leg hits first.

Hand tuning was performed on several key variables. Relative speeds of the two gaits were adjusted, as the tripod gait progresses at a constant speed, while the stair-climber pauses while climbing. Additionally, since the robot changes body pitch as it mounts the stairs, leg angle offsets were added to the tripod gait, and subtracted from the stair-climber gait, throughout the transition. Upon reaching a stair, the robot launches the correct transition (Fig. 7 or its mirror image) depending upon which leg hits the stair first. A simple reactive system detects a stair by looking at motor currents associated with the front legs.

In tests, this behavior was effective at autonomously mounting and climbing a set of stairs without any human intervention, as seen in the photos included in Figure 8.

B. RiSE Gait Families

The RiSE robotic hexapod, Fig. 2, is a robot capable of climbing a variety of vertical surfaces, in addition to level terrain locomotion. On RiSE, we apply gait transitions to achieve vertical mobility, seamlessly chaining together various gaits. Gait transitions allow a human operator to drive the robot around a carpeted vertical surface, changing direction, turning, and even switching between structurally unrelated gaits.

1) Gaits Involved: To exhibit vertical mobility, a multitude of gaits were hand-designed and tuned on the RiSE robot. These included gaits for ascending and descending a carpeted wall, turning while climbing, as well as traversing horizontally across a wall. The specifics of the geometrically-different leg trajectories are beyond the scope of this paper.

A large set of tripod gaits were generated to use these different leg trajectories, climbing upward, downward, and even turning while climbing by producing yawing motions. As the tripod gaits are all related, their configuration space trajectories all intersect, allowing for simple transitions amongst this set of gaits.

Tripod gaits, however, were unsuccessful at traversing a wall sideways, as the robot would have to hold its weight on a single leg at times, motivating the use of a pentapod or tetrapod gait. A pentapod, for instance, allows the robot to hang off of two or three legs at all times, rather than one. Large sets of pentapods, as well as tetrapods, were generated, and, likewise, simple transitions were automatically created to switch within these sets of gaits.

2) Transition Technique: While switching between related gaits can be done instantly, transitioning between structurally unrelated gaits, such as a tripod and a pentapod, is done by switching legs from one gait to the other on a per leg basis over a finite period of time.

Several constraints were placed on finding a valid transition. Foremost, ground speed was normalized between gaits, using (9), recirculating a nominally slower gait at a proportionally higher rate. Furthermore, whereas the RHex robot allows legs to pause and slip along a surface, slipping on RiSE inevitably means breaking stable contact with the surface, possibly risking a fall. A leg that touches down and attaches must continue through stance, therefore making flight the only portion of a trajectory where the leg can be run faster or slower than normal.

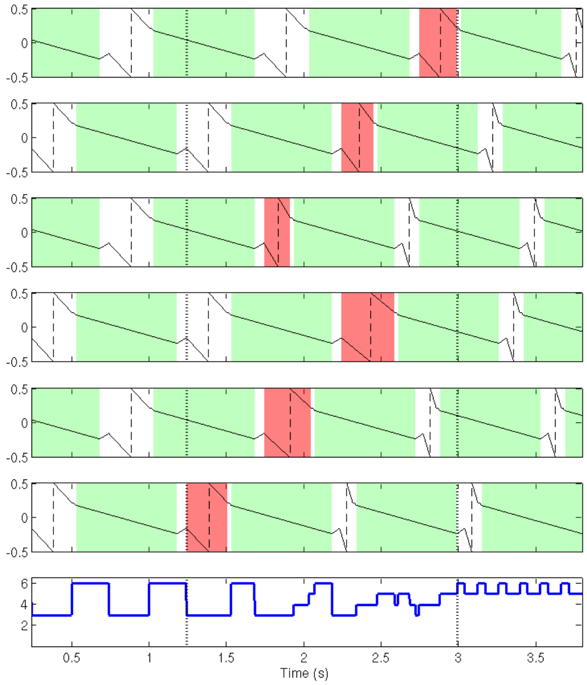

Fig. 9. A gait transition from a RiSE tripod gait to a pentapod gait. The transition occurs between the vertical dotted lines, consisting of speeding or slowing individual legs during flight to match gait parameters. Darker shaded regions correspond to these per leg modifications during flight.

Calculation of actual switching times for each leg is done as follows. Consider two gaits, starting together at some phase angle, and running side by side. If, at a particular phase angle, a leg is in flight in both gaits, that phase angle is a possible switch point between gaits. The leg could lift off in one gait, recirculate, and touch down in the second gait. After normalizing ground speed, gaits recirculate at different rates, so multiple repetitions of each gait are considered while determining the order in which legs are switched. Furthermore, the actual phase angle where the gaits start together is a free parameter, and it is optimized to find the maximum recirculation time for each leg, all the while trying to remain within valid robot configurations.

3) Implementation Results: With a total of approximately 20 gaits, tied together with around 200 transitions, all of which are automatically generated, RiSE can be piloted much like remote-controlled car, while climbing on a vertical wall. The robot manages its leg motions and performs simple transitions automatically when needed, to turn or change direction. Furthermore, a press of a button executes finite transitions between structurally unrelated gaits.

One such transition, from an upward climbing tripod to a high duty factor pentapod, is shown in Figure 9. The darker shaded regions indicate switching times between gaits, during which a leg may speed up or slow down in flight before attaching. Once a leg attaches, it has the phase offset of the pentapod gait.

V. Conclusion and Future Work

Through a greater understanding of gaits, we have shown how gait transitions lead to novel behaviors on two robotic platforms. Switching between gaits instantaneously, or progressively switching legs, one by one, results in motion patterns that continue locomoting, while keeping the robot's configuration valid.

Future directions for this research branch out in a variety of ways. Our newfound understanding of gaits is allowing us to consider a continuum of gaits, rather than isolated gaits. Using this continuous representation, we intend to apply control by evolving a gait over time, performing local feedback by moving throughout a local neighborhood of gaits. We are also interested in techniques of leg coordination, to encode gaits and gait transitions in a continuous framework. Lastly, we intend to study the potential of applying this work to quadrapedal robots, in addition to continuing our work with hexapods.

Acknowledgments

The authors would like to thank Uluc Saranli for his helpful comments motivating and enlightening this work, as well as the RiSE and RHex project team members for developing excellent robotic platforms for research. This work was supported in part by the DSO Biodynotics Program, within the Defense Advanced Research Projects Agency, under contract DARPA/SPAWAR N66001-03-C-8045.

References

[1] R. Altendorfer, N. Moore, H. Komsuoglu, M. Buehler, H. B. Brown, D. McMordie, U. Saranli, R. Full, and D. Koditschek, "Rhex: A biologically inspired hexapod runner," *Autonomous Robots*, vol. 11, p. 207, 2001.

[2] D. McMordie and M. Buehler, "Towards pronking with a hexapod robot," in *4th Int. Conf. on Climbing and Walking Robots*, Karlsruhe, Germany, September 2001.

[3] E. Z. Moore, D. Campbell, F. Grimminger, and M. Buehler, "Reliable stair climbing in the simple hexapod 'rhex'," in *IEEE Int. Conf. on Robotics and Automation (ICRA)*, vol. 3, Washington, D.C., U.S.A., May 2002, pp. 2222–2227.

[4] D. M. Wilson, "Insect walking," *Annual Review of Entomology*, vol. 11, pp. 103 – 122, 1966.

[5] H. Cruse, "What mechanisms coordinate leg movement in walking arthropods?" *Trends in Neurosciences*, vol. 13, pp. 15 – 21, 1990.

[6] R. A. Brooks, "A Robot That Walks; Emergent Behaviors from a Carefully Evolved Network," MIT AI Lab," Memo 1091, February 1989.

[7] K. S. Espenschied, R. D. Quinn, H. J. Chiel, and R. D. Beer, "Leg coordination mechanisms in stick insect applied to hexapod robot locomotion," *Adaptive Behavior*, vol. 1, no. 4, pp. 455 – 468, 1993.

[8] D. Wettergreen, H. Thomas, and C. Thorpe, "Planning strategies for the ambler walking robot," in *IEEE Int. Conf. on Systems Engineering*, August 1990, pp. 198 – 203.

[9] J. Chestnutt, M. Lau, G. Cheung, J. Kuffner, J. K. Hodgins, and T. Kanade, "Footstep planning for the honda asimo humanoid," in *IEEE Int. Conf. on Robotics and Automation (ICRA)*, April 2005.

[10] J. D. Weingarten, G. A. D. Lopes, M. Buehler, R. E. Groff, and D. E. Koditschek, "Automated gait adaptation for legged robots," in *IEEE Int. Conf. on Robotics and Automation (ICRA)*, vol. 3, 2004, pp. 2153 – 2158.

[11] S. Schaal and C. G. Atkeson, "Open Loop Stable Strategies for Robot Juggling," in *International Conference on Robotics and Automation*, vol. 3, GA, Atlanta, 1993, pp. 913–918.

[12] J. D. Weingarten, R. E. Groff, and D. E. Koditschek, "A framework for the coordination of legged robot gaits," in *IEEE Int. Conf. on Robotics, Automation and Mechatronics (RAM)*, vol. 2, 2004, pp. 679 – 686.