Machine Learning Part V

Anomaly Detection/ Recommender System

Anomaly detection example

- Fraud detection:
  - $x^{(i)}$ = features of user i's activities
  - Model $p(x)$ from data.
  - Identify unusual users by checking which have $p(x) < \varepsilon$
- Manufacturing
- Monitoring computers in a data center.
  - $x^{(i)}$ = features of machine i

    x_1 = memory use, x_2 = number of disk accesses/sec,

    x_3 = CPU load, x_4 = CPU load/network traffic.

    ...

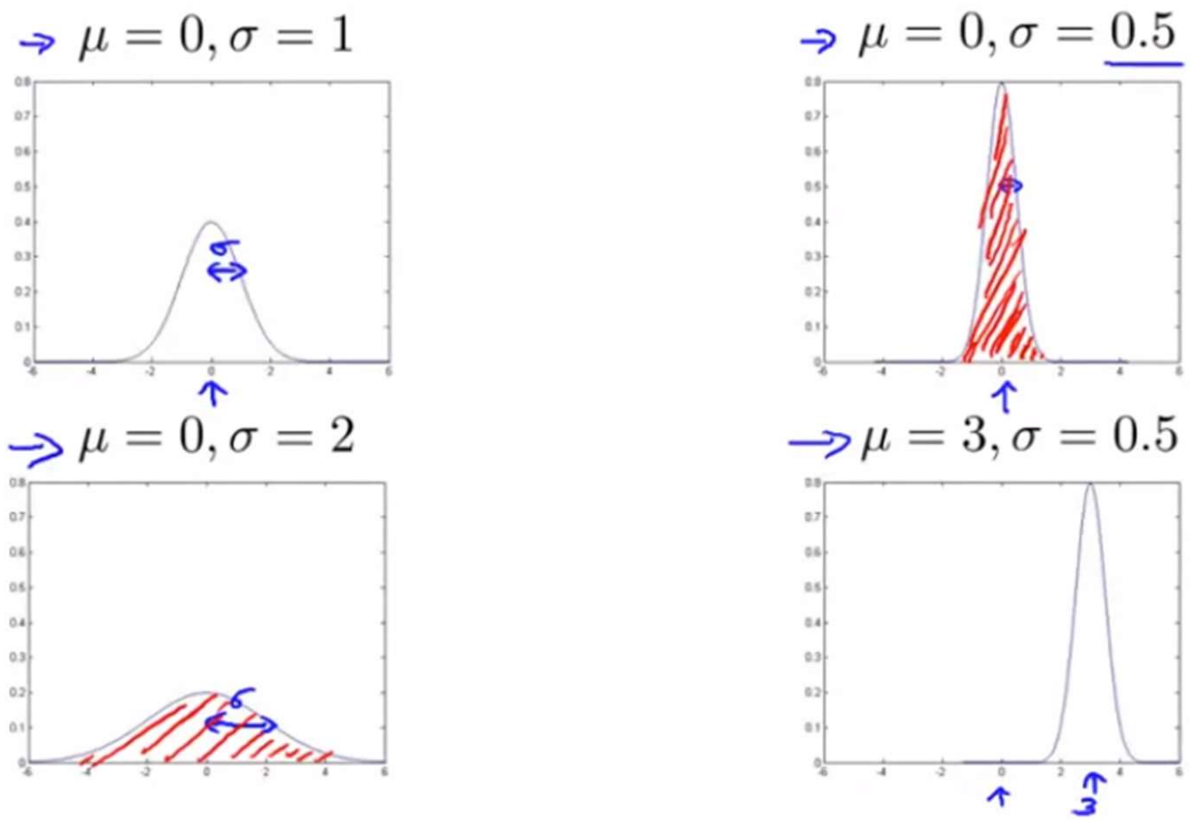
x_1
x_2
x_3
x_4

$p(x)$

$p(x) < \varepsilon$

Gaussian (Normal) Distribution

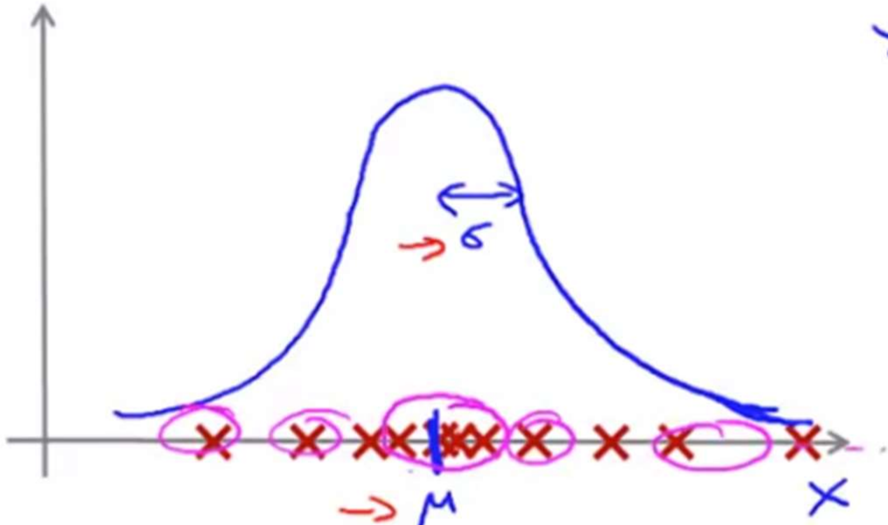
- Probability distribution (add up to 1)



Parameter estimation

- Dataset: $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$ $\quad x^{(i)} \in \mathbb{R}$

$x^{(i)} \sim \mathcal{N}(\mu, \sigma^2)$



$$\mu = \frac{1}{m}\sum_{i=1}^{m} x^{(i)} \qquad \sigma^2 = \frac{1}{m}\sum_{i=1}^{m} \left(x^{(i)} - \mu\right)^2$$

Anomaly detection algorithm

1. Choose features x_i that you think might be indicative of anomalous examples.

   $\{x^{(1)}, \ldots, x^{(m)}\}$

2. Fit parameters $\mu_1, \ldots, \mu_n, \sigma_1^2, \ldots, \sigma_n^2$

$$\boxed{\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}}$$

$$\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m}(x_j^{(i)} - \mu_j)^2$$

$p(x_j; \mu_j, \sigma_j^2)$

$\mu_1, \mu_2, \ldots, \mu_n$

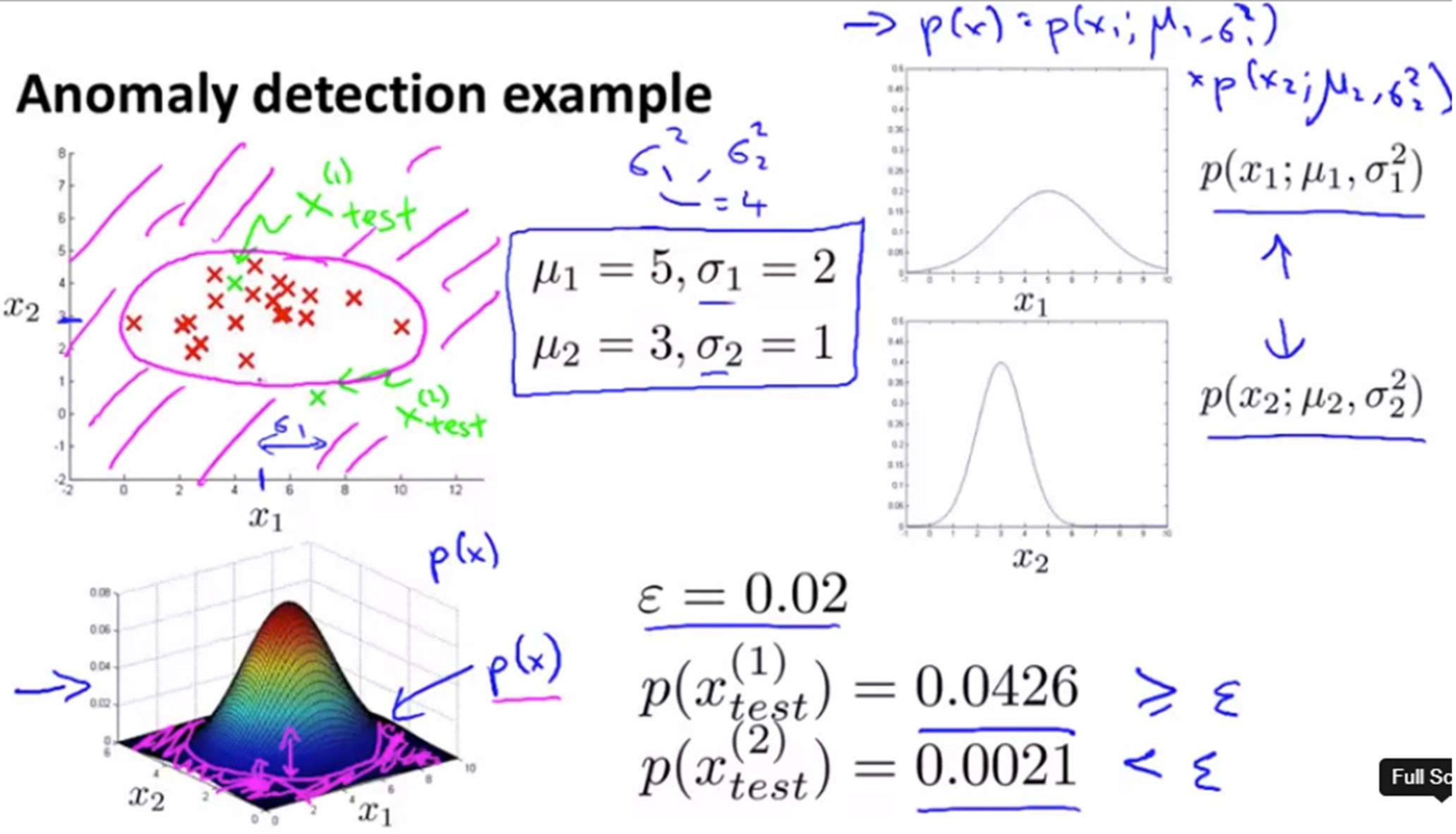
$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix} = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$$

3. Given new example x, compute $p(x)$:

$$p(x) = \prod_{j=1}^{n} p(x_j; \mu_j, \sigma_j^2) = \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left(-\frac{(x_j - \mu_j)^2}{2\sigma_j^2}\right)$$

Anomaly if $p(x) < \varepsilon$

Visualization



Evaluation (supervised learning)

When developing a learning algorithm (choosing features, etc.), making decisions is much easier if we have a way of evaluating our learning algorithm.

- Assume we have some labeled data, of anomalous and non-anomalous examples. ($y = 0$ if normal, $y = 1$ if anomalous).
- Training set: $x^{(1)}, x^{(2)}, \dots, x^{(m)}$ (assume normal examples/not anomalous)
- Cross validation set: $(x_{cv}^{(1)}, y_{cv}^{(1)}), \dots, (x_{cv}^{(m_{cv})}, y_{cv}^{(m_{cv})})$
- Test set: $(x_{test}^{(1)}, y_{test}^{(1)}), \dots, (x_{test}^{(m_{test})}, y_{test}^{(m_{test})})$

Aircraft engines motivating example

$\boxed{10000}$ good (normal) engines

$\boxed{20}$ flawed engines (anomalous) 2-50

$y=1$

$\mu_1, \sigma_1^2, \ldots, \mu_n, \sigma_n^2$

Training set: $\boxed{6000}$ good engines $(y=0)$ $p(x) = p(x_1; \mu_1, \sigma_1^2) \cdots p(x_n; \mu_n, \sigma_n^2)$

CV: $\boxed{2000}$ good engines ($y = 0$), $\boxed{10}$ anomalous ($y = 1$)

Test: $\boxed{2000}$ good engines ($y = 0$), $\boxed{10}$ anomalous ($y = 1$)

Algorithm evaluation

- Fit model $p(x)$ on training set $\{x^{(1)}, \dots, x^{(m)}\}$
- On a cross validation/test example x, predict

$$y = \begin{cases} 1 & \text{if } p(x) < \varepsilon \text{ (anomaly)} \\ 0 & \text{if } p(x) \geq \varepsilon \text{ (normal)} \end{cases}$$

Possible evaluation metrics:

- True positive, false positive, false negative, true negative
- Precision/Recall
- F_1-score

Can also use cross validation set to choose parameter ε

Also, use validation set to decide what features to include (square co?)

Anomaly detection vs. Supervised learning

Anomaly detection

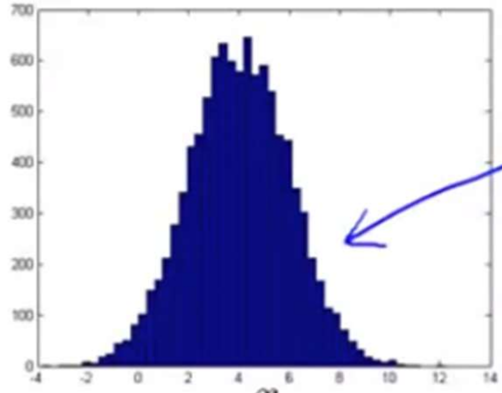
- Very small number of positive examples ($y = 1$). (0-20 is common).
- Large number of negative ($y = 0$) examples. $\boxed{p(x)}$
- Many different "types" of anomalies. Hard for any algorithm to learn from positive examples what the anomalies look like;
- future anomalies may look nothing like any of the anomalous examples we've seen so far.

Supervised learning

- Large number of positive and negative examples.
- Enough positive examples for algorithm to get a sense of what positive examples are like, future positive examples likely to be similar to ones in training set.

Spam

Non-gaussian features



hist

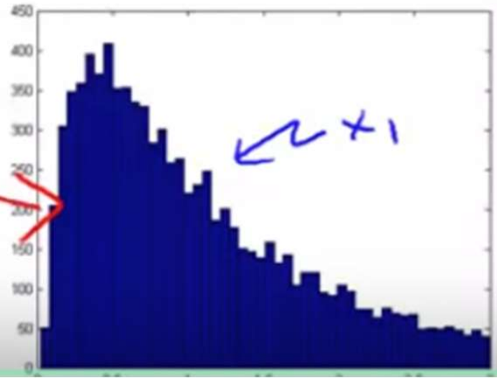
$$p(x_i; \mu_i, \sigma_i^2)$$

$$x_1 \leftarrow \log(x_1)$$

$$x_2 \leftarrow \log(x_2 + 1) \qquad \log(x_2 + c)$$

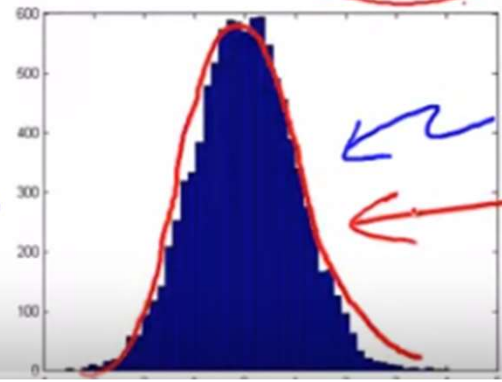
$$x_3 \leftarrow \sqrt{x_3} = x_3^{\frac{1}{2}}$$

$$x_4 \leftarrow x_4^{\frac{1}{3}}$$



x_1

$\xrightarrow{\log(x)}$



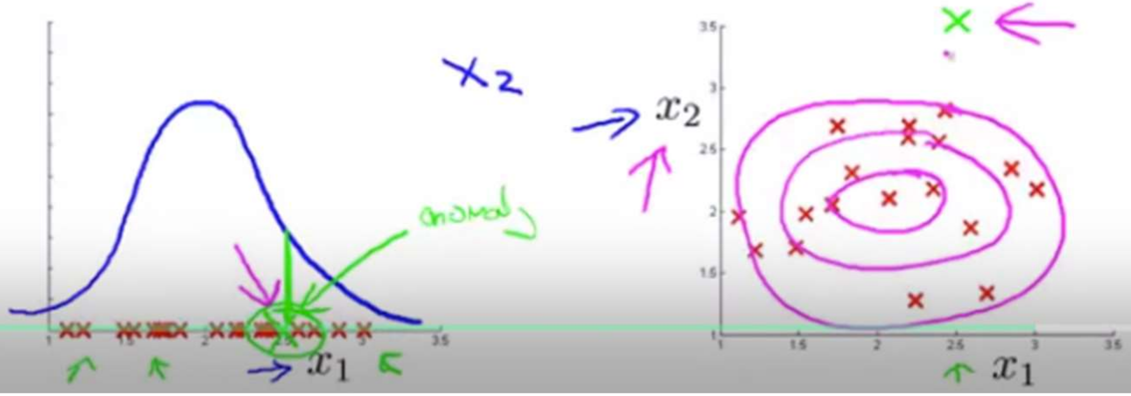
Error analysis for anomaly detection

Want $p(x)$ large for normal examples x.

$p(x)$ small for anomalous examples x.

Most common problem:

$p(x)$ is comparable (say, both large) for normal and anomalous examples



Adding features that can distinguish normal and anomalous samples

Multivariate Gaussian (Normal) distribution

$x \in \mathbb{R}^n$. Don't model $p(x_1), p(x_2), \ldots$, etc. separately.
Model $p(x)$ all in one go.
Parameters: $\mu \in \mathbb{R}^n, \Sigma \in \mathbb{R}^{n \times n}$ (covariance matrix)

$$p(x; \mu, \Sigma) =$$

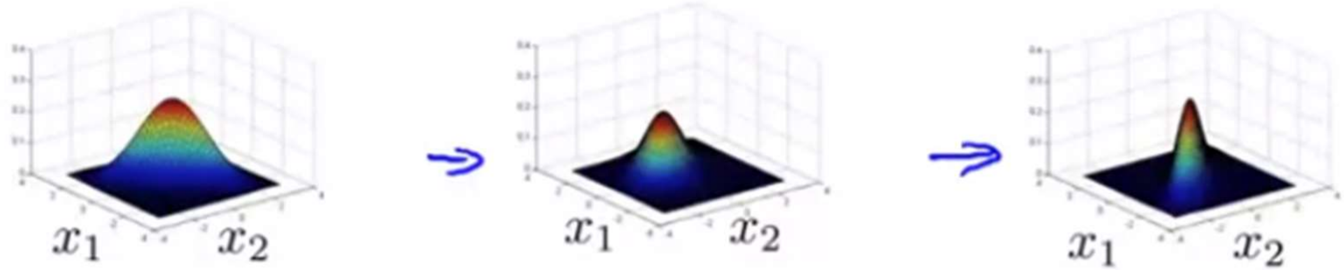
$$\frac{1}{(2\pi)^{n/2} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

$|\Sigma|$ = determinant of Σ | det(Sigma)

Multivariate Gaussian (Normal) distribution

Parameters μ, Σ $\quad \mu \in \mathbb{R}^n \quad \Sigma \in \mathbb{R}^{n \times n}$

$$p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$



Parameter fitting:

Given training set $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$ $\quad x \in \mathbb{R}^n$

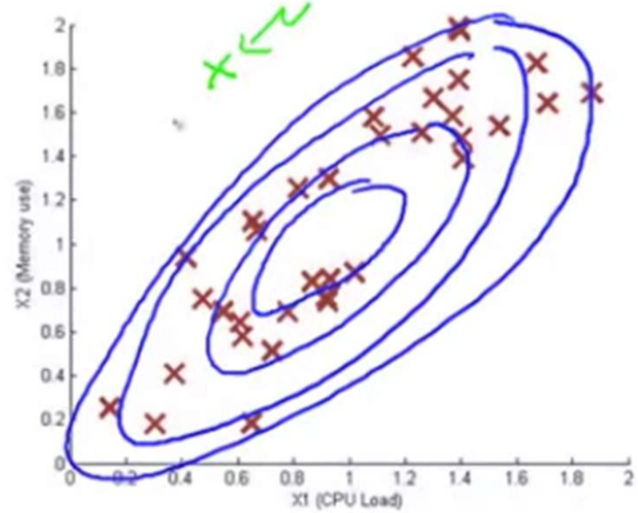
$$\boxed{\mu} = \frac{1}{m}\sum_{i=1}^{m} x^{(i)} \qquad \boxed{\Sigma} = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)}-\mu)(x^{(i)}-\mu)^T$$

Anomaly detection with the multivariate Gaussian

1. Fit model $p(x)$ by setting

$$\mu = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$$

$$\Sigma = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)} - \mu)(x^{(i)} - \mu)^T$$



2. Given a new example x, compute

$$p(x) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}}\exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

Flag an anomaly if $p(x) < \varepsilon$

Original model vs. Multivariate Gaussian

$p(x_1; \mu_1, \sigma_1^2) \times \cdots \times p(x_n; \mu_n, \sigma_n^2)$

Manually create features to capture anomalies where x_1, x_2 take unusual combinations of values.

$\boxed{x_3 = \frac{x_1}{x_2} = \frac{\text{CPU load}}{\text{memory}}}$

Computationally cheaper (alternatively, scales better to large n)

$n = 10{,}000$, $n = 100{,}000$

OK even if m (training set size) is small

$p(x; \mu, \Sigma) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$

Cool!

Automatically captures correlations between features

$\Sigma \in \mathbb{R}^{n \times n}$ $\quad \Sigma^{-1}$

Computationally more expensive

$\Sigma \quad \sim \frac{n^2}{2}$

$x_1 = \cancel{x_2}$

$\cancel{x_3} = x_4 + x_5$

Must have $\boxed{m > n}$ or else Σ is non-invertible.

$\boxed{m \geq 10n}$

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Recommender Systems

$n_u = 4$, $n_m = 5$

$x_0 = 1$

$x^{(1)} = \begin{bmatrix} 1 \\ 0.9 \\ 0 \end{bmatrix}$

Content-based recommender systems

Movie	Alice (1) $\theta^{(1)}$	Bob (2) $\theta^{(2)}$	Carol (3) $\theta^{(3)}$	Dave (4) $\theta^{(4)}$	x_1 (romance)	x_2 (action)
$x^{(1)}$ Love at last 1	5	5	0	0	0.9	0
$x^{(2)}$ Romance forever 2	5	?	?	0	1.0	0.01
$x^{(3)}$ Cute puppies of love 3	? 4.95	4	0	?	0.99	0
Nonstop car chases 4	0	0	5	4	0.1	1.0
$x^{(5)}$ Swords vs. karate 5	0	0	5	?	0	0.9

$n = 2$

For each user j, learn a parameter $\theta^{(j)} \in \mathbb{R}^3$. Predict user j as rating movie i with $(\theta^{(j)})^T x^{(i)}$ stars.

$\theta^{(j)} \in \mathbb{R}^{n+1}$

$x^{(3)} = \begin{bmatrix} 1 \\ 0.99 \\ 0 \end{bmatrix} \longleftrightarrow \theta^{(1)} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$ $\quad (\theta^{(1)})^T x^{(3)} = 5 \times 0.99$

$= 4.95$

Content Based Recommendations

- content means we have the features to describe the product
- It is essentially a linear regression problem, only that we train a set of parameters for each user.

Optimization objective:

To learn $\theta^{(j)}$ (parameter for user j):

$$\min_{\theta^{(j)}} \frac{1}{2} \sum_{i:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{k=1}^{n} (\theta_k^{(j)})^2$$

To learn $\theta^{(1)}, \theta^{(2)}, \dots, \theta^{(n_u)}$:

$$\min_{\theta^{(1)},\dots,\theta^{(n_u)}} \frac{1}{2} \boxed{\sum_{j=1}^{n_u}} \underbrace{\sum_{i:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^{n} (\theta_k^{(j)})^2}_{\theta^{(1)}, \dots, \theta^{(n_u)}}$$

Optimization algorithm:

$$\min_{\theta^{(1)},\ldots,\theta^{(n_u)}} \underbrace{\frac{1}{2}\sum_{j=1}^{n_u}\sum_{i:r(i,j)=1}\left((\theta^{(j)})^T x^{(i)} - y^{(i,j)}\right)^2 + \frac{\lambda}{2}\sum_{j=1}^{n_u}\sum_{k=1}^{n}(\theta_k^{(j)})^2}_{J(\theta^{(1)},\ldots,\theta^{(n_u)})}$$

Gradient descent update:

$$\theta_k^{(j)} := \theta_k^{(j)} - \alpha \sum_{i:r(i,j)=1}((\theta^{(j)})^T x^{(i)} - y^{(i,j)})x_k^{(i)} \quad (\text{for } k = 0)$$

$$\theta_k^{(j)} := \theta_k^{(j)} - \alpha \underbrace{\left(\sum_{i:r(i,j)=1}((\theta^{(j)})^T x^{(i)} - y^{(i,j)})x_k^{(i)} + \lambda\theta_k^{(j)}\right)}_{\frac{\partial}{\partial \theta_k^{(j)}} J(\theta^{(1)},\ldots,\theta^{(n_u)})} \quad (\text{for } k \neq 0)$$

$\cancel{\frac{1}{m^{(j)}}}$

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Collaborative Filtering

Given $x^{(1)}, \ldots, x^{(n_m)}$ (and movie ratings),

can estimate $\theta^{(1)}, \ldots, \theta^{(n_u)}$

Given $\theta^{(1)}, \ldots, \theta^{(n_u)}$,

can estimate $x^{(1)}, \ldots, x^{(n_m)}$

Guess $\theta \rightarrow x \rightarrow \theta \rightarrow x \rightarrow \theta \rightarrow x \rightarrow \cdots$

Each movie is a vector of genre features

Each audience is also a vector of preference features

Collaborative filtering optimization objective

$(i,j) : r(i,j) = 1$

$\cancel{x_0 = 1}$ $\cancel{\theta_0}$ $\quad x \in \mathbb{R}^n \quad \theta \in \mathbb{R}^n \quad \cancel{x \in \mathbb{R}^{n+1}} \quad x_1 = 1$

→ Given $x^{(1)}, \dots, x^{(n_m)}$, estimate $\theta^{(1)}, \dots, \theta^{(n_u)}$:

$$\min_{\theta^{(1)},\dots,\theta^{(n_u)}} \boxed{\frac{1}{2} \sum_{\boxed{j=1}}^{n_u} \sum_{\boxed{i:r(i,j)=1}} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2} + \boxed{\frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^{n} (\theta_k^{(j)})^2}$$

→ Given $\theta^{(1)}, \dots, \theta^{(n_u)}$, estimate $x^{(1)}, \dots, x^{(n_m)}$:

$$\min_{x^{(1)},\dots,x^{(n_m)}} \boxed{\frac{1}{2} \sum_{\boxed{i=1}}^{n_m} \sum_{\boxed{j:r(i,j)=1}} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2} + \boxed{\frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^{n} (x_k^{(i)})^2}$$

Minimizing $x^{(1)}, \dots, x^{(n_m)}$ and $\theta^{(1)}, \dots, \theta^{(n_u)}$ simultaneously:

$$J(x^{(1)}, \dots, x^{(n_m)}, \theta^{(1)}, \dots, \theta^{(n_u)}) = \boxed{\frac{1}{2} \sum_{(i,j):r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2} + \boxed{\frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^{n} (x_k^{(i)})^2} + \boxed{\frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^{n} (\theta_k^{(j)})^2}$$

$$\boxed{\min_{\substack{x^{(1)},\dots,x^{(n_m)} \\ \theta^{(1)},\dots,\theta^{(n_u)}}} J(x^{(1)}, \dots, x^{(n_m)}, \theta^{(1)}, \dots, \theta^{(n_u)})}$$

$\theta \to x \to \theta \to x \to \dots$

5:54 / 8:26

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Learn x and θ in simultaneously!

$\cancel{x_0 = 1}$ $\quad x \in \mathbb{R}^n$, $\theta \in \mathbb{R}^n$

Collaborative filtering algorithm

1. Initialize $x^{(1)}, \ldots, x^{(n_m)}, \theta^{(1)}, \ldots, \theta^{(n_u)}$ to small random values.
2. Minimize $J(x^{(1)}, \ldots, x^{(n_m)}, \theta^{(1)}, \ldots, \theta^{(n_u)})$ using gradient descent (or an advanced optimization algorithm). E.g. for every $j = 1, \ldots, n_u, i = 1, \ldots, n_m$:

$\cancel{\theta_0}$ θ_1 $\ldots$ θ_n

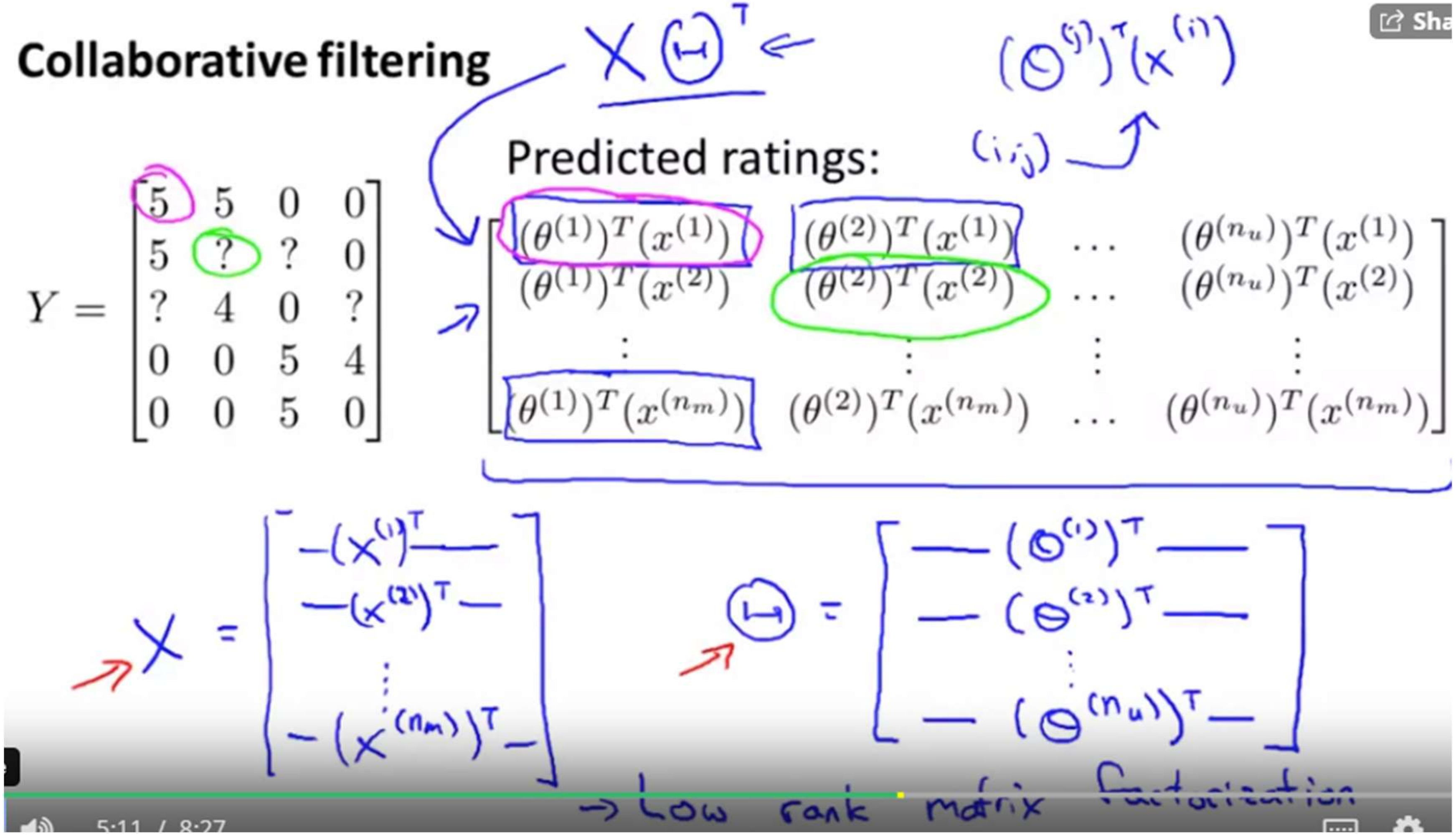
$$x_k^{(i)} := x_k^{(i)} - \alpha \underbrace{\left(\sum_{j:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)}) \theta_k^{(j)} + \lambda x_k^{(i)} \right)}_{\frac{\partial}{\partial x_k^{(i)}} J(\ldots)}$$

$$\theta_k^{(j)} := \theta_k^{(j)} - \alpha \left(\sum_{i:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)}) x_k^{(i)} + \lambda \theta_k^{(j)} \right)$$

3. For a user with parameters θ and a movie with (learned) features x , predict a star rating of $\theta^T x$.

$(\theta^{(j)})^T (x^{(i)})$

Vectorization



Finding related movies

For each product i, we learn a feature vector $x^{(i)} \in \mathbb{R}^n$.

→ x_1 = romance, x_2 = action, x_3 = comedy, x_4 = ...

How to find movies j related to movie i?

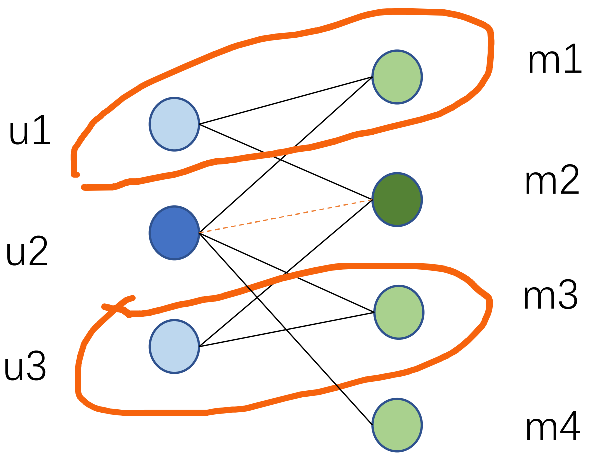
small $\|x^{(i)} - x^{(j)}\|$ → movie j and i are "similar"

5 most similar movies to movie i:

→ Find the 5 movies j with the smallest $\|x^{(i)} - x^{(j)}\|$.

Movie rating to network setting?

- Bipartite network (users, movies)
- Objective: **link weight prediction** (predict user rating for unseen movies).
- common **3 hop neighbors** between a user and a movie



Ex8-2.2.2 of Collaborative Filtering

```
for i = 1:num_movies
    idx = find(R(i, :) == 1);
    Theta_tmp = Theta(idx, :);
    Y_tmp = Y(i, idx);
    X_grad(i, :) = (X(i, :) * Theta_tmp' - Y_tmp) * Theta_tmp;
end


for j = 1:num_users
    idx = find(R(:, j) == 1);
    X_tmp = X(idx, :);
    Y_tmp = Y(idx, j);
    Theta_grad(j, :) = (Theta(j, :) * X_tmp' - Y_tmp') * X_tmp;
end
```