Minecraft Pool Datapack Models and Equations

Squid Workshop

1. Rolling of the ball on table

We have kinetic energy dissipation

$$\frac{dE(v,\omega)}{dt} = \boldsymbol{F} \cdot \boldsymbol{v} \qquad 1.1$$

Assume coefficient of friction is constant, 1.1 yields the simple relation

$$\frac{d\boldsymbol{v}}{dt} = -\frac{5}{7}\mu g \hat{\boldsymbol{v}} \qquad 1.2$$

2. Sliding of the ball on table

Starting from equations of motion

$$\frac{d\boldsymbol{v}}{dt} = \frac{\boldsymbol{F}}{m} \qquad 2.1$$

$$\frac{d\boldsymbol{\omega}}{dt} = -\frac{r(\hat{\boldsymbol{n}} \times \boldsymbol{F})}{I} \qquad 2.2$$

Where $\boldsymbol{F}$ is derived from relative velocity $\boldsymbol{v}'$ between table and the ball

$$\boldsymbol{F} = -mg\mu\widehat{\boldsymbol{v}'} \qquad 2.3$$

$$\boldsymbol{v}' = \boldsymbol{v} - r(\boldsymbol{\omega} \times \hat{\boldsymbol{n}}) \qquad 2.4$$

Taking time derivative of 2.4, we find out $\boldsymbol{F}$ and $\boldsymbol{v}'$ does not change their directions. Integration yields the expression of position $\boldsymbol{r}$ and $\boldsymbol{\omega}$

$$\boldsymbol{r} = \boldsymbol{r}_0 + t\boldsymbol{v_0} - \frac{1}{2}g\mu t^2 \widehat{\boldsymbol{v}'}_{\boldsymbol{0}} \qquad 2.5$$

$$\boldsymbol{\omega} = \boldsymbol{\omega_0} - \frac{5}{7r}\left(\widehat{\boldsymbol{v}'}_{\boldsymbol{0}} \times \hat{\boldsymbol{n}}\right) \qquad 2.6$$

In reality $\boldsymbol{F}$ is often proportional to the relative velocity, however the simple model is used to reach an analytical solution that well describes the behavior.

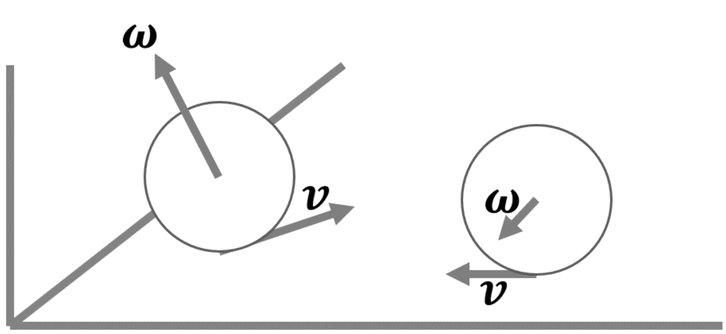

Figure 1: A rolling ball (right) and a sliding ball (left)

3. Collision between two balls

If the two balls have the same physical properties, given the initial conditions of the two balls and their relative positions, the resulting conditions follow conservation of momentum and kinetic energy.

$$\boldsymbol{P_{a0}} + \boldsymbol{P_{b0}} = \boldsymbol{P_{a1}} + \boldsymbol{P_{b1}} \qquad 3.1$$

$$v_{a0}^2 + v_{b0}^2 = v_{a1}^2 + v_{b1}^2 \qquad 3.2$$

The following calculation solves the equations by using relative velocity

$$\boldsymbol{v_{a1}} = \left(\hat{\boldsymbol{r}} \times (\hat{\boldsymbol{v}}_{\boldsymbol{r}} \times \hat{\boldsymbol{r}})\right) * v_r * \sin\theta + \boldsymbol{v_{b0}} \qquad 3.3$$

$$\boldsymbol{v_{b1}} = \hat{\boldsymbol{r}} * v_r * \cos\theta + \boldsymbol{v_{b0}} \qquad 3.4$$

Where

$$\boldsymbol{v_r} = \boldsymbol{v_{a0}} - \boldsymbol{v_{b0}} \qquad 3.5$$

$$\theta = \arccos\frac{\boldsymbol{v_r} \cdot \hat{\boldsymbol{r}}}{v_r} \qquad 3.6$$

The above expression is generalized to 3D collisions. It is conceptually easier to calculate than the velocity component exchange approach. On a pool table the expression can be further simplified. Note that if $\theta \geq \pi/2$ then the balls will not collide.

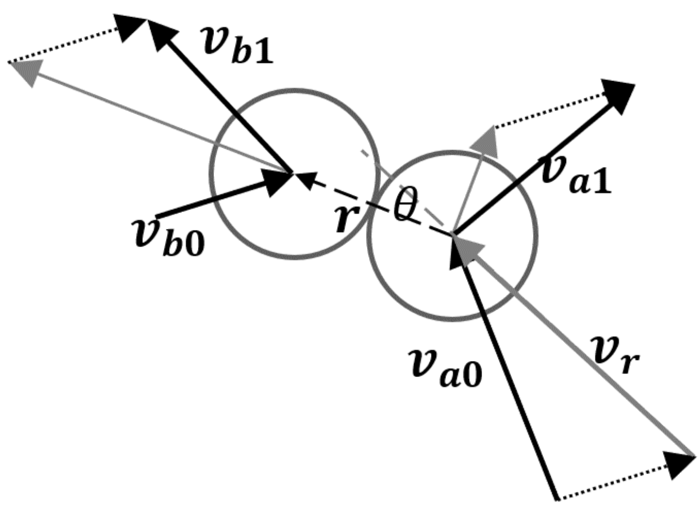

Figure 2: Schematics of 2-ball collision

4. Collision between the ball and the rail

Here a very simplified model is constructed to quickly address the change of speed and spin of the ball after the collision. Starting from the assumption that there is no change of magnitude of velocity normal the cushion (perfect reflection)

$$\boldsymbol{v_{n1}} = -\boldsymbol{v_{n0}} \tag{4.1}$$

We can calculate the normal force by the change of momentum component

$$\boldsymbol{N} = \frac{\Delta \boldsymbol{P}}{\Delta t} = \frac{-2m\boldsymbol{v_{n0}}}{\Delta t} \tag{4.2}$$

Then, further assume the friction $\boldsymbol{F}$ is proportional to the magnitude of $\boldsymbol{N}$ and relative velocity $\boldsymbol{v}'$ between the ball and the cushion. Here the relation between friction and relative velocity is computationally simple and satisfies the boundary conditions.

$$\boldsymbol{F} = -kN\boldsymbol{v}' \tag{4.3}$$

$$\boldsymbol{v}' = \boldsymbol{v_p} - r(\boldsymbol{\omega} \times \hat{\boldsymbol{n}}) \cdot \hat{\boldsymbol{v}}_{\boldsymbol{p}} \tag{4.4}$$

We can then write the change of parallel speed and vertical spin by the friction during Δt

$$\Delta \boldsymbol{v_p} = \frac{\boldsymbol{F}\Delta t}{m} = -2kv_{n0}\boldsymbol{v}' \tag{4.5}$$

$$\Delta \boldsymbol{\omega} = \frac{\boldsymbol{\tau}\Delta t}{I} = -\frac{5}{r}kv_{n0}(\boldsymbol{v}' \times \hat{\boldsymbol{n}}) \tag{4.6}$$

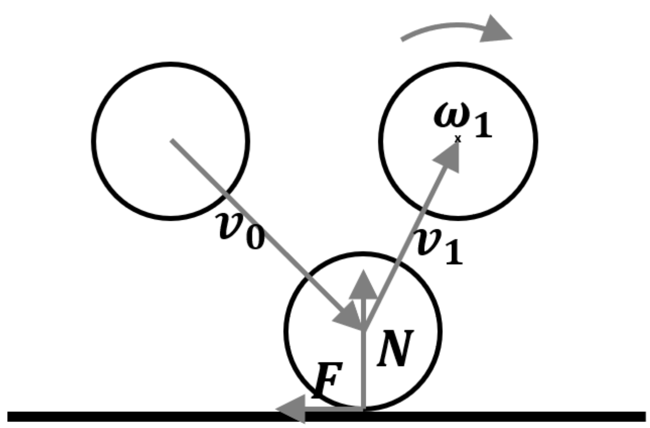

Figure 3: Schematics of ball-rail collision

5. Collision between the tip of cue stick and the cue ball

Similar to part 4, but since two components are free to move, the kinetic energy conservation relation is used to solve for impulse in an idealized situation.

$$E_0 = E_1 \tag{5.1}$$

$$E_0 = \frac{1}{2} m_s v_{s0}^2 \tag{5.2}$$

$$E_1 = \frac{1}{2} m_s v_{s1}^2 + \frac{1}{2} m_b v_{b1}^2 + \frac{1}{2} I_s \omega_{s1}^2 + \frac{1}{2} I_b \omega_{b1}^2 \tag{5.3}$$

With the relation between friction and normal force that depends on incident angle, approximated as a sine function

$$F = \mu N = \mu_0 \sin\theta\, N \tag{5.4}$$

Expanding and solving 5.1 gives the magnitude of impulse $\boldsymbol{J}$ and the final state of cue ball

$$J = \frac{v_{s0}(\cos\theta + \mu\sin\theta)}{\frac{1+\mu^2}{2m_s} + \frac{1+\mu^2}{2m_b} + \frac{l_s^2(\mu\cos\theta - \sin\theta)^2}{2I_s} + \frac{r^2\mu^2}{2I_b}} \tag{5.5}$$

$$\boldsymbol{v_{b1}} = \boldsymbol{\hat{v}_{s0}} \frac{J}{m_b}(\cos\theta + \mu\sin\theta) + \boldsymbol{\hat{F}} \times \boldsymbol{\hat{N}} \times \boldsymbol{\hat{v}_{s0}} \frac{J}{m_b}(\sin\theta - \mu\cos\theta) \tag{5.6}$$

$$\boldsymbol{\omega_{b1}} = \boldsymbol{\hat{v}_{s0}} \times \boldsymbol{\hat{N}} \frac{J}{I_b} \mu r \tag{5.7}$$

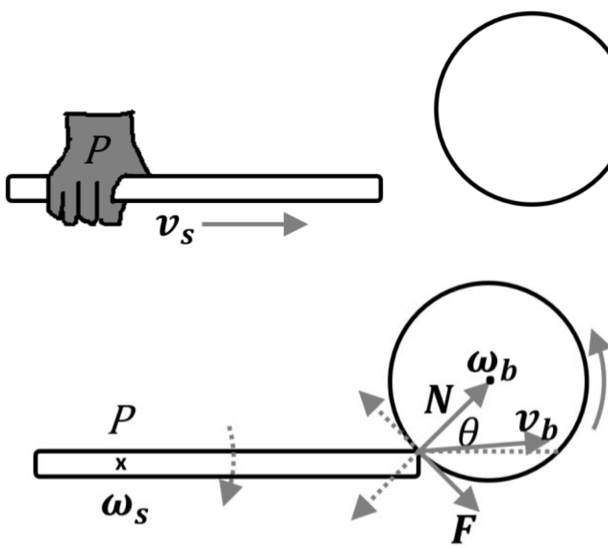

Figure 4: Schematics of stick-ball collision