Unit 5: Distance

IPM Text Analysis

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Cluster analysis / Clustering

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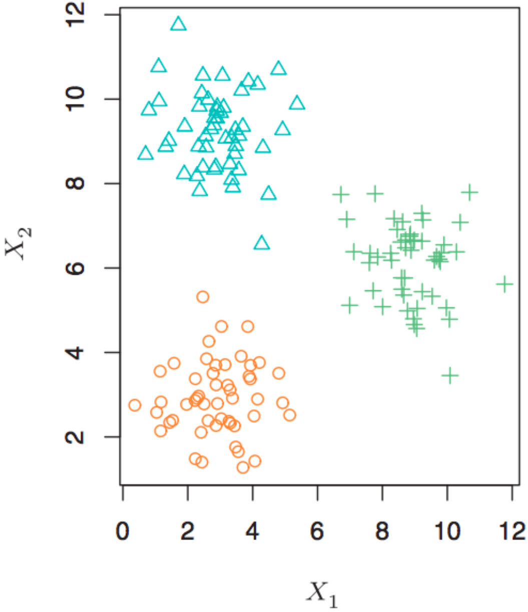
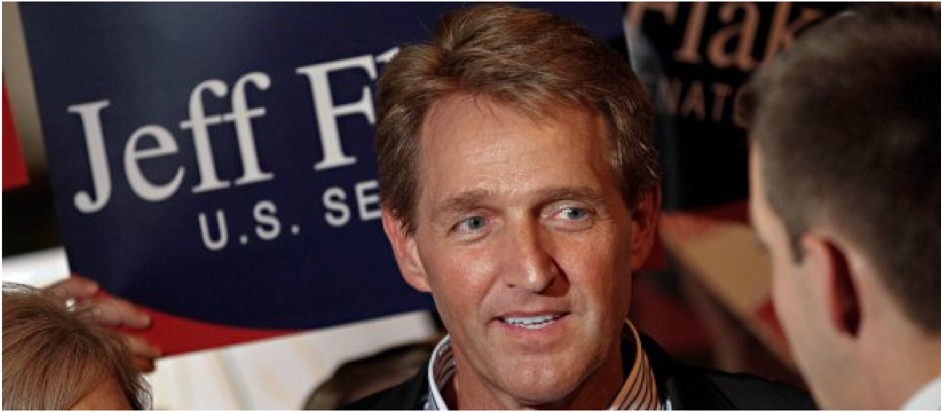
- These groups are interesting because the may correspond to some category or quantity of interest.


12
10
8
6
4
2
X_2
0
2
4
6
8
10
12
X_1


Jeff F
U.S. SE

Today: Cluster Jeff Flake's press releases
Goal: partition documents such that:

- similar documents are together
- dissimilar documents are apart

Method: Clustering methods
Game Plan:

1) What makes two data points (i.e. documents) similar?
2) How do we find a good partition?
3) How do we interpret the clusters?

Key Terms:

- (Multidimensional) Space
- Distance
- Euclidean Distance
- Cosine Distance
- Cluster Analysis / Clustering
- K-means
- Centroid

What makes two documents similar?

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Similar $=$ Geometrically Close

Dissimilar $=$ Geometrically Distant

Texts and Geometry

Consider a document-term matrix

$$\boldsymbol{X} = \begin{pmatrix} 1 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 3 \end{pmatrix}$$

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- Natural notions of distance and similarity

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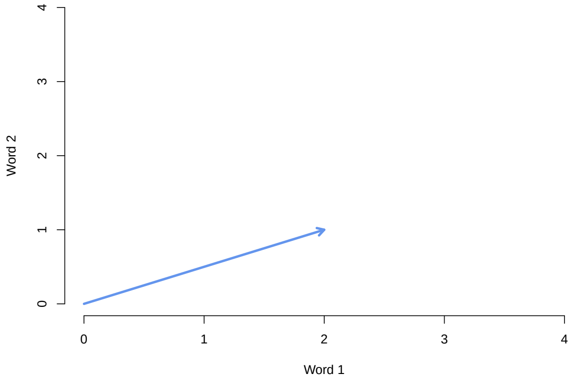
By transforming our text into a word count vector, we are representing it as a point in a multidimensional space

- Provides a geometry
- Natural notions of distance and similarity
- Tools from linear algebra to calculate distances mathematically.

Texts in Space



Doc1 = "Wait? No wait." $\rightsquigarrow (2, 1)$

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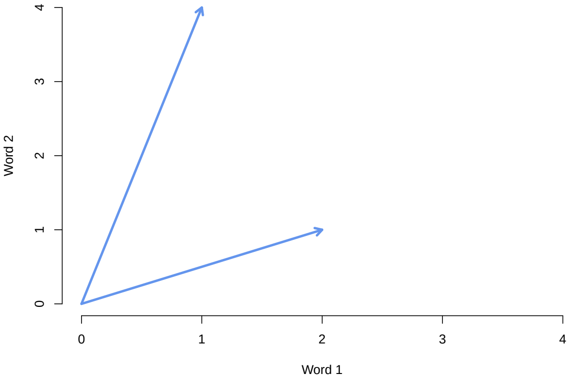

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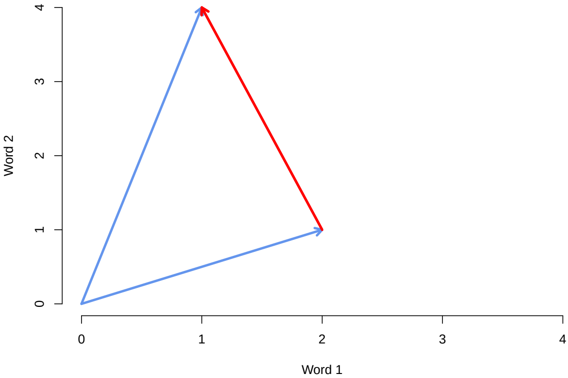

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Suppose $\boldsymbol{X}_1 = (1, 4)$ and $\boldsymbol{X}_2 = (2, 1)$.

The Euclidean distance (aka norm) between $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ (or from $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$) is the length of the line segment connecting them.

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$$d(\boldsymbol{X}_1, \boldsymbol{X}_2) = d(\boldsymbol{X}_2, \boldsymbol{X}_1) \quad = \quad \sqrt{(x_{1,1} - x_{2,1})^2 + (x_{1,2} - x_{2,2})^2}$$

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Test your knowledge

The Euclidean distance between any documents $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ is:

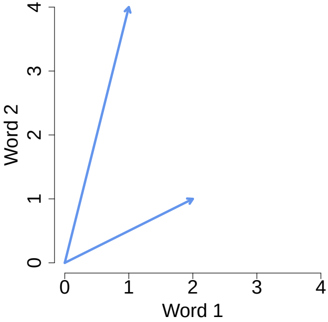
$$d(\boldsymbol{X}_1, \boldsymbol{X}_2) = \sqrt{\sum_{p=1}^{P} (x_{1p} - x_{2p})^2}$$

Suppose

- $\boldsymbol{X}_1 =$ Oh na na na.
- $\boldsymbol{X}_2 =$ Oh, me? Na.

Calculate the euclidean distance between these two documents.

Problem(?) with Euclidean Distance



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\boldsymbol{X}_1 &= (2,1) \\
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&= \sqrt{10}
\end{aligned}
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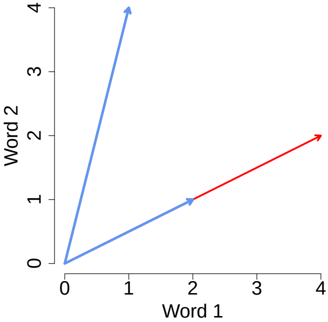
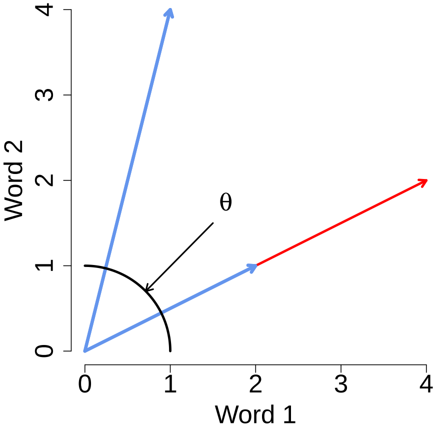


$$
\begin{aligned}
\boldsymbol{X}_1 &= (2,1) \\
\boldsymbol{X}_2 &= (1,4) \\
\boldsymbol{X}_3 &= 2\boldsymbol{X}_1 = (4,2) \\
d(\boldsymbol{X}_3, \boldsymbol{X}_2) &= \sqrt{(4-1)^2 + (2-4)^2} \\
&= \sqrt{13}
\end{aligned}
$$

Euclidean distance depends on document-length.

Cosine Similarity



Cosine Similarity

- Takes into consideration documents length.

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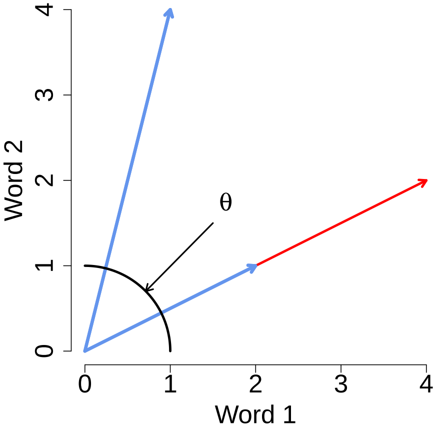

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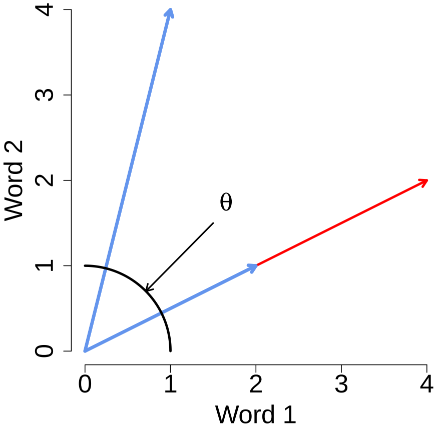

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- Measure of similarity (rather than distance) ranging between 0 and 1.

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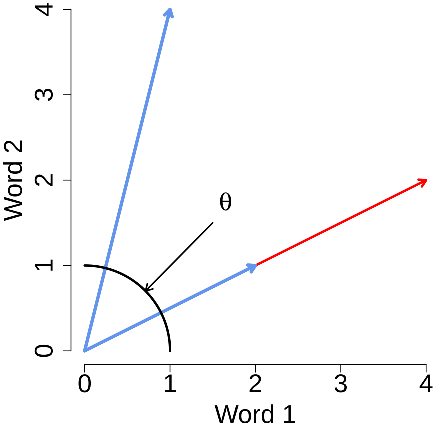

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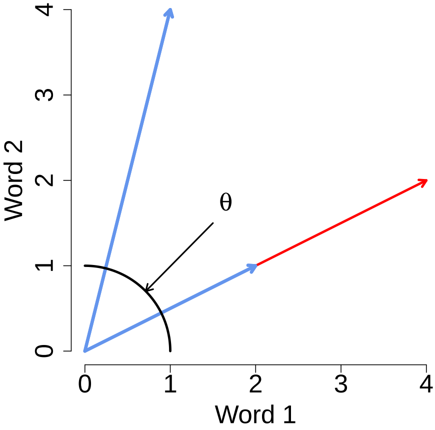

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- Distances $\rightsquigarrow$ clustering.
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Next Up:

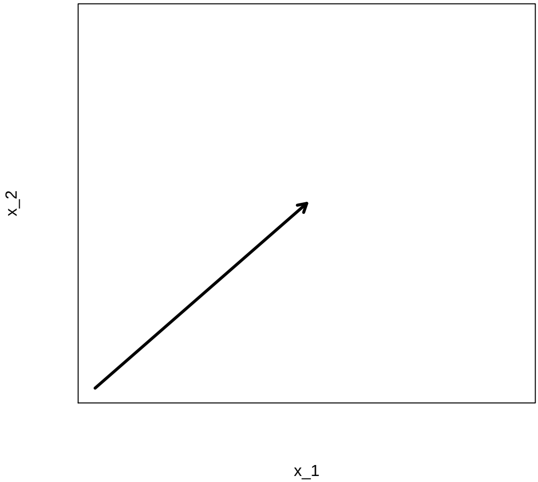
- How do we find a good partition?
- How do we interpret the clusters?

To the `R` code!

Bonus Slides

For those who heart math.

Vector Length



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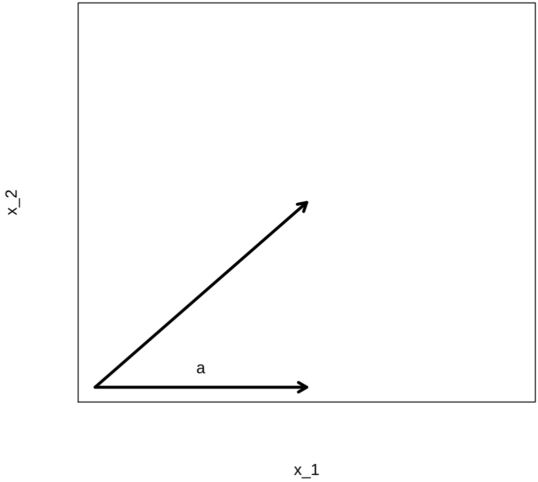

- Pythogorean Theorem: Side with length a

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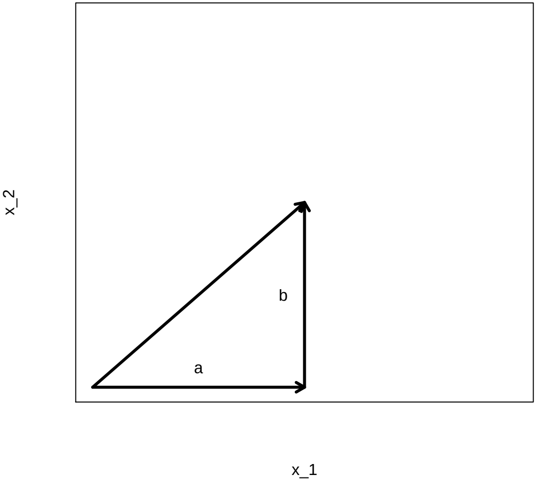

- Pythogorean Theorem: Side with length a
- Side with length b and right triangle

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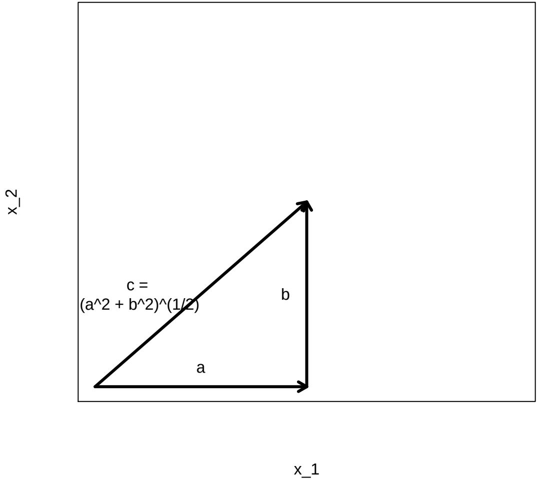

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- $c = \sqrt{a^2 + b^2}$

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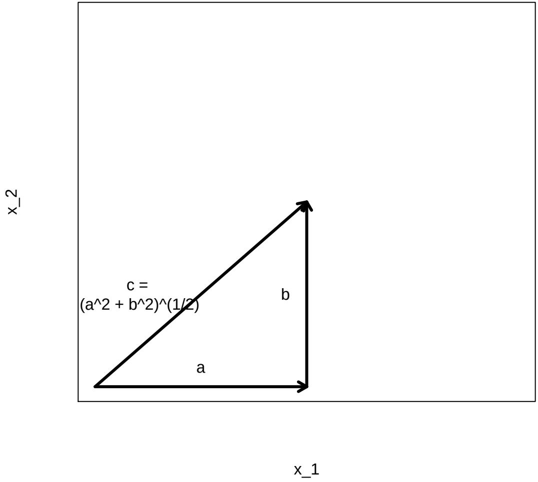

- Pythogorean Theorem: Side with length a
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- $c = \sqrt{a^2 + b^2}$
- Extends beyond 2 dimensions

Vector (Euclidean) Length

Suppose $\boldsymbol{X}_i$ is a document (row from an $N \times K$ document-term matrix).

Then, we will define its length as

$$
\begin{aligned}
||\boldsymbol{X}_i|| &= \sqrt{(\boldsymbol{X}_i \cdot \boldsymbol{X}_i)} \\
&= \sqrt{(X_{i1}^2 + X_{i2}^2 + X_{i3}^2 + \ldots + X_{iK}^2)} \\
&= \sqrt{\sum_{k=1}^{K} X_{ik}^2}
\end{aligned}
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