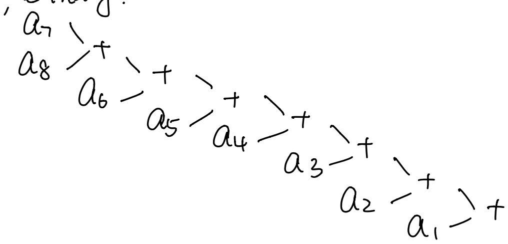
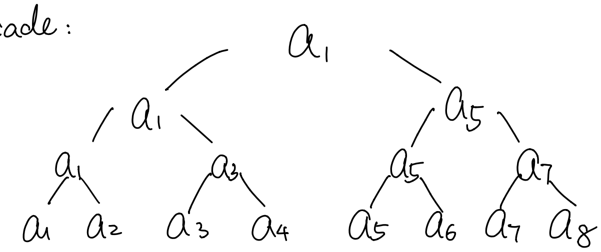
1. When $n=8$, Binary:



Cascade:



Shortest Height:

If $\log_2 n$ is an integer, shortest height is $\log_2 n$. Otherwise, shortest height is $[\log_2 n]+1$.

Smallest Number of Time Units: $\log_2 n$ when using cascade algorithm.

2. (a)

```
for(stride=1; stride<n; stride*=2)
{
        //Parallel loop
        for(i=stride; i+stride<=n; i+=2*stride)
        {
            for(j=0; j<stride; j++)
            {
                x(i+j) = x(i-1) + x(i+j);
            }
        }
}
```

When $x=[1,2,-1,4,-1,6,-1,8]$, first, update $x(1)$, $x(3)$, $x(5)$, $x(7)$.

Stride=1, $i=1,3,5,7$, $j=0$. $x(1)=x(0)+x(1)=3$, $x(3)=x(2)+x(3)=3$, $x(5)=x(4)+x(5)=5$, $x(7)=x(6)+x(7)=7$.

Then, update $x(2), x(3), x(6), x(7)$. Stride $=2$, $i=2,6$, $j=0,1$. $x(2)=x(1)+x(2)=2$, $x(3)=x(1)+x(3)=6$, $x(6)=x(5)+x(6)=4$, $x(7)=x(5)+x(7)=12$.

Finally, update $x(4), x(5), x(6), x(7)$. Stride $=4$, $i=4$, $j=0,1,2,3$.

$x(4)=x(3)+x(4)=5$, $x(5)=x(3)+x(5)=11$, $x(6)=x(3)+x(6)=10$, $x(7)=x(3)+x(7)=18$.

Therefore, we can get the result $y=[1,3,2,6,5,11,10,18]$.

Time: $(\log_2 n)\tau$

Memory access protocol: CREW

Processors: $\frac{n}{2}$

(b) $x=[0,1,0,1,0,1,1,0]$

$y=[0,1,0,3,0,5,6,0]$ First, we find the index when $x(i)=1$ and then record it in y.

$y=[0,1,0,3,0,5,6,6]$ Then, update $y(1), y(3), y(5), y(7)$. And $y(7)$ needs to be changed as $y(7)=y(6)$.

$y=[0,1,1,3,0,5,6,6]$ Then, update $y(2), y(3), y(6), y(7)$. And still need to change $y(2)$ as $y(2)=y(1)$.

$y=[0,1,1,3,3,5,6,6]$ Finally, update $y(4), y(5), y(6), y(7)$. Change $y(4)$ as $y(4)=y(3)$.

Processors: $\frac{n}{2}=4$.

Time: $o(\log_2 n)$

3. (a) MFLOPS: $\frac{n}{t(n)10^6} = \frac{n}{(10^{-6}+\frac{n}{10^8})10^6} = \frac{n}{1+\frac{n}{100}}$

When $n\to+\infty$, reach peak, $\frac{n}{\sigma+\frac{n}{\rho}} = \frac{1}{\frac{\sigma}{n}+\frac{1}{\rho}} \approx \rho = 100$MFLOPS

When length $=1000$, MFLOPS rate $=\frac{1000}{1+10}=90.9$ MFLOPS

When length $=100$, MFLOPS rate $=\frac{100}{1+1}=50$ MFLOPS

When length $=10$, MFLOPS rate $=\frac{10}{1+\frac{1}{10}}=9.09$ MFLOPS

(b) $\dfrac{n}{t(n)} = \dfrac{n}{\sigma + \frac{n}{\rho}} = \dfrac{\rho n}{\sigma\rho + n}$

$\dfrac{n}{t(n_{\frac{1}{2}})} = \dfrac{n_{\frac{1}{2}}}{\sigma + \frac{n_{\frac{1}{2}}}{\rho}} = \dfrac{\rho n_{\frac{1}{2}}}{\sigma\rho + n_{\frac{1}{2}}} = \frac{1}{2}\rho$

$n_{\frac{1}{2}} = \sigma\rho$

$\because t(n) = \sigma + \dfrac{n}{\rho}$

$\therefore t(n) = \dfrac{n_{\frac{1}{2}} + n}{\rho}$

(c) $t(n) = 8.5\times10^{-9} \times 10 = 8.5\times10^{-8}$.

Peak performance: $\dfrac{1}{8.5\times10^{-8}} = 11.7$ MFLOPS

Half vector length: $n_{\frac{1}{2}} = \rho\sigma = \dfrac{1}{8.5\times10^{-9}} \times 9 \times 8.5\times10^{-9} = 9$

4. (a) For one professor: time $t_1 = \dfrac{N-1}{\rho}$.

For p professors:

Moving data: $p\beta + \dfrac{N}{\tau}$

Calculating N numbers: $\dfrac{\frac{N}{p}-1}{\rho}$

Do p subsums: $p(\beta + \frac{1}{\tau})$

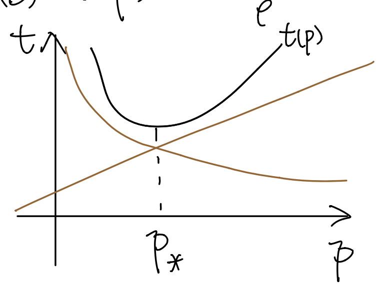
Calculate sums: $\dfrac{p-1}{\rho}$

Total time $t_2 = p\beta + \dfrac{N}{\tau} + \dfrac{\frac{N}{p}-1}{\rho} + p\beta + \dfrac{p}{\tau} + \dfrac{p-1}{\rho}$

$= \dfrac{\frac{N}{p}+p-2}{\rho} + 2\rho\beta + \dfrac{N+p}{\tau}$

$t_1 < t_2 \quad \therefore$ No speeds-up.

(b) $t(p) = \dfrac{\frac{N}{p}+p-2}{\rho} + 2\rho\beta + \dfrac{N+p}{\tau} = \dfrac{N}{\rho p} + (\frac{1}{\rho} + \frac{1}{\tau} + 2\beta)p + \dfrac{N}{\tau} - \dfrac{2}{\rho}$



$\dfrac{N}{\rho P_*} = (\frac{1}{\rho} + \frac{1}{\tau} + 2\beta)P_*$

$P_*^2 = \dfrac{N}{\rho(\frac{1}{\rho} + \frac{1}{\tau} + 2\beta)}$

$P_* = \sqrt{\dfrac{N}{1 + \frac{\rho}{\tau} + 2\rho\beta}}$

(c) $P_* = \left[\sqrt{\dfrac{10000}{1+\frac{\rho}{\tau}+2\rho\cdot 50}}\right] = \left[\sqrt{\dfrac{10000}{1+10+100}}\right] = 10$

$$\sim 1 + \frac{\ }{\frac{p}{10}} + 2p \cdot \frac{\ }{p} \sim \ldots$$

$$\text{Time} = \frac{\frac{10000}{10} + 10 - 2}{p} + 2 \times p \times \frac{50}{p} + \frac{10000 + 10}{\frac{p}{60}}$$

$$= \frac{101108}{p} + 100$$

5. Suppose the size of vector is n.

$$\frac{2n-1}{\frac{2n-1}{8}\left(2 \times \frac{100}{1\text{GHz}} + 8 \times \frac{1}{1\text{GHz}}\right)} = 38.5 \text{ MFLOPS}$$

Peak: ignore memory cycle $\frac{2n-1}{\frac{2n-1}{8}\left(2 \times \frac{100}{1\text{GHz}}\right)} = 40 \text{ MFLOPS}$