1. (a) $S(p) = \frac{T(1)}{T(p)} = \frac{n-1}{\frac{n}{p} - 1 + 11\log p}$

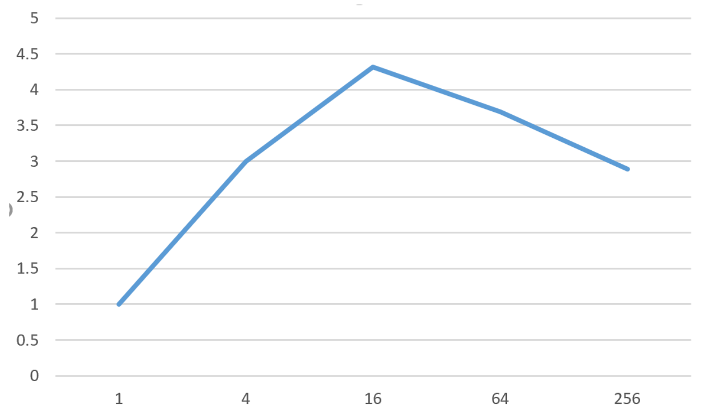
$S(1) = \frac{255}{255+11\log 1} = 1$

$S(4) = \frac{255}{\frac{256}{4} - 1 + 11\log 4} = 3$

$S(16) = \frac{255}{\frac{256}{16} - 1 + 11\log 16} = 4.32$

$S(64) = \frac{255}{\frac{256}{64} - 1 + 11\log 64} = 3.69$

$S(256) = \frac{255}{\frac{256}{256} - 1 + 11\log 256} = 2.89$



(b) $S_s(p) = \frac{pW}{T_p(pW,p)} = \frac{p(n-1)}{\frac{np}{p} - 1 + 11\log p}$

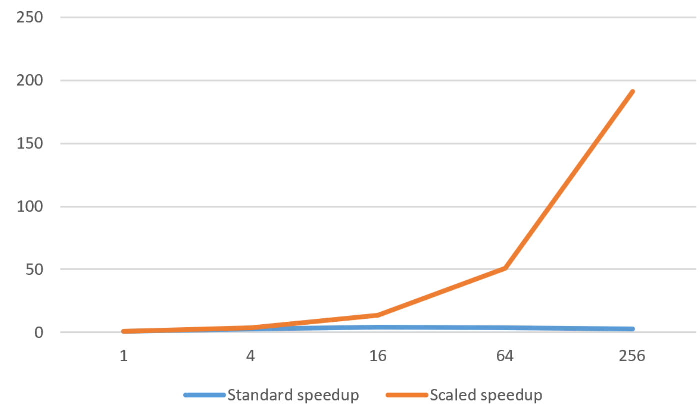
$S_s(1) = 1$

$S_s(4) = \frac{255\times 4}{255+11\log 4} = 3.68$

$S_s(16) = \frac{255\times 16}{255+11\log 16} = 13.64$

$S_s(64) = \frac{255\times 64}{255+11\log 64} = 50.84$

$S_s(256) = \frac{255\times 256}{255+11\log 256} = 190.32$



2. $S(p) = \frac{T(1)}{T(p)} = \frac{n-1}{\frac{n}{p} - 1 + 11\log p}$

$E(p) = \frac{S(p)}{p} = \frac{n-1}{n - p + 11 p\log p} \geqslant 50\%$

$p(11\log p - 1) \leq 254$

When $p = 1, 2, \cdots, 7$, the efficiency will be no less than 50%.

3. two matrix $n\times n$, each block has $\frac{n}{\sqrt{p}} \times \frac{n}{\sqrt{p}}$.

Calculate time: $t_1 = [\underbrace{2(\frac{n}{\sqrt{p}})^3}_{\text{calculate in each block}} + \underbrace{(\frac{n}{\sqrt{p}})^2}_{\text{add}}]\underbrace{(\sqrt{p}-1)}_{\text{steps}} + 2(\frac{n}{\sqrt{p}})^3$

Communication time: $t_2 = (2t_s + 2(\frac{n}{\sqrt{p}})^2 t_w)(\sqrt{p}-1)$

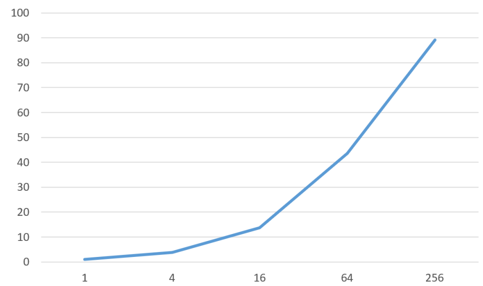
$t = (\sqrt{p}-1)\left[2(\frac{n}{\sqrt{p}})^3 + (\frac{n}{\sqrt{p}})^2 + 2t_s + 2(\frac{n}{\sqrt{p}})^2 t_w\right] + 2(\frac{n}{\sqrt{p}})^3$

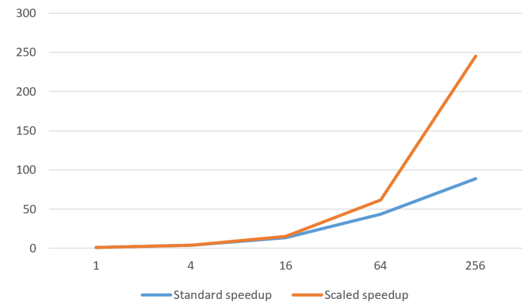
$$S(p)=\frac{T(1)}{T(p)}=\frac{2n^3}{T(n,p)}=\frac{2n^3}{(\sqrt{p}-1)\left[2\left(\frac{n}{\sqrt{p}}\right)^3+\left(\frac{n}{\sqrt{p}}\right)^2+20+2\left(\frac{n}{\sqrt{p}}\right)^2\right]+2\left(\frac{n}{\sqrt{p}}\right)^3}$$

$S(1)=1,\ S(4)=3.8,\ S(16)=13.8,\ S(64)=43.6,\ S(256)=89.1$

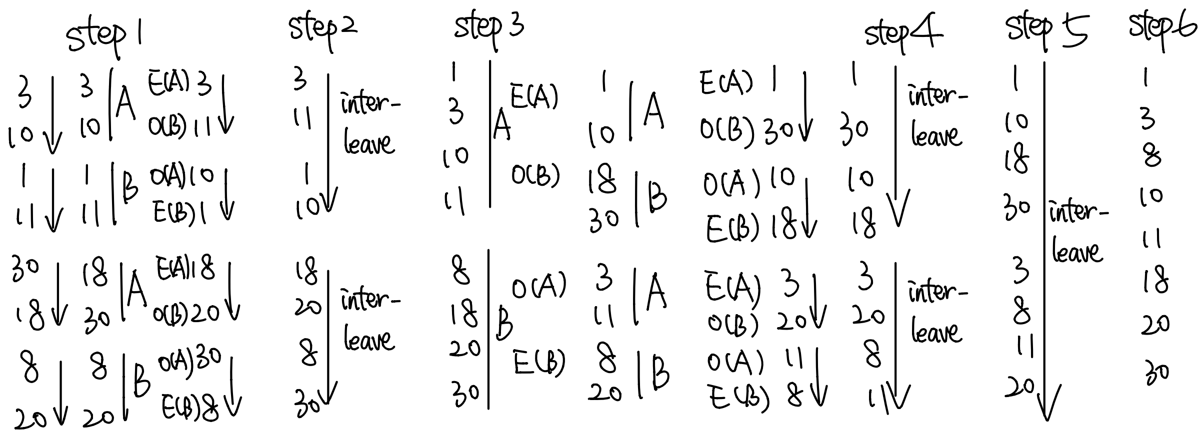
$$S_{\text{scaled}}(p)=\frac{T(n_0,1)\cdot(\sqrt{p})^3}{T(n_0\sqrt{p},p)}=\frac{2n_0^3(\sqrt{p})^3}{(\sqrt{p}-1)\left[2n_0^3+n_0^2+20+2n_0^2\right]+2n_0^3}$$

$S(1)=1,\ S(4)=3.9,\ S(16)=15.5,\ S(64)=61.4,\ S(256)=245.2$

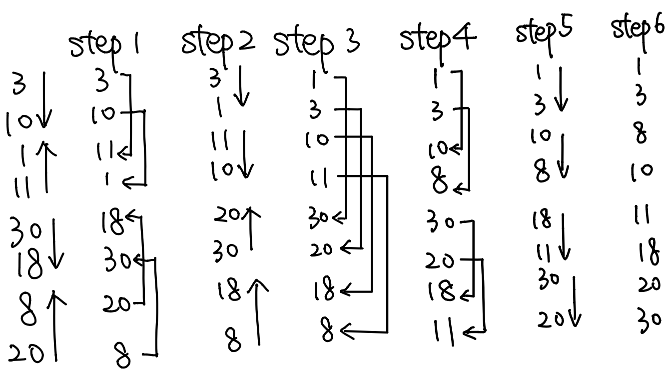




4. (a)



(b)

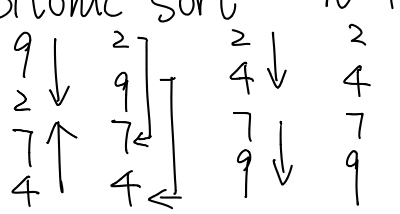


"↑", "↓" means the direction of sort

"↓" asending order

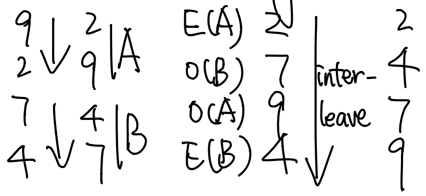
"↑" descending order

5. Bitonic sort $n=4$



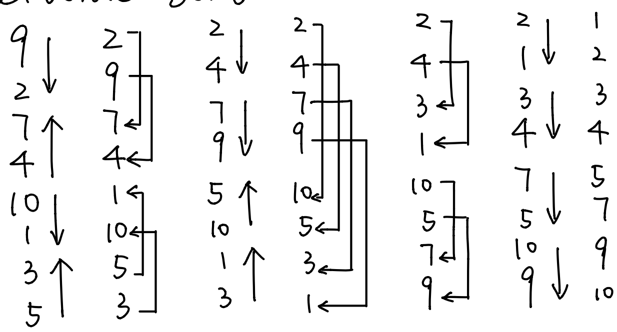
6

Odd-even mergesort $n=4$



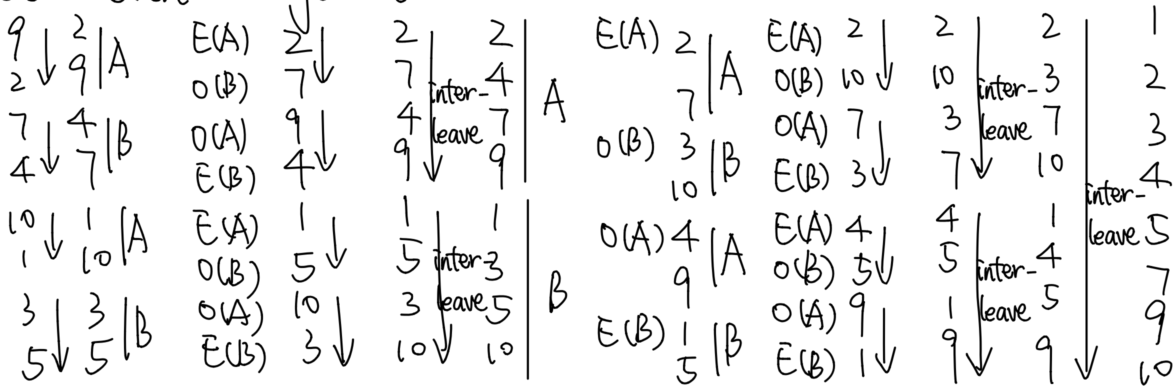
6

Bitonic sort $n=8$



comparators: $4 \times 6 = 24$

Odd-even mergesort $n=8$



comparators: $4 \times 7 = 24$

6. $OEMerge(\frac{n}{2}, \frac{n}{2}) = 2\, OEMerge(\frac{n}{4}, \frac{n}{4}) + \frac{n}{2}$

comparasion:

$$= 2\left[2\, OEMerge(\tfrac{n}{8}, \tfrac{n}{8}) + \tfrac{n}{4}\right] + \tfrac{n}{2}$$

$$= 2^2\, OEMerge(\tfrac{n}{8}, \tfrac{n}{8}) + 2 \times \tfrac{n}{2}$$

$$= 2^2\left[2\, OEMerge(\tfrac{n}{16}, \tfrac{n}{16}) + \tfrac{n}{8}\right] + 2 \times \tfrac{n}{2}$$

$$= 2^3\, OEMerge(\tfrac{n}{16}, \tfrac{n}{16}) + 3 \times \tfrac{n}{2}$$

$$\cdots$$

$$= 2^{k-1}\, OEMerge(1,1) + (k-1)\tfrac{n}{2}$$

$$= 2^{k-1} + (k-1)\tfrac{n}{2}$$

$$= \frac{n \log_2 n}{2}$$

Time: $\frac{n}{2} OEMerge(1,1) + \frac{n}{2^2} OEMerge(2,2) + \cdots + OEMerge(\frac{n}{2}, \frac{n}{2})$

$$= \frac{n}{2} \times 1 + \frac{n}{2^2} \times 2 \log_2 4 + \frac{n}{2^3} \times 4 \log_2 8 + \cdots + \frac{n}{2} \log_2 n$$

$$= \frac{n}{2} \times (1 + 2 + \cdots + \log_2 n)$$

$$= \frac{n}{2} \cdot \frac{\log_2 n (\log_2 n + 1)}{2}$$