BLP Method Algorithm–Modified Version of Rasmusen (2007)

Mingyi Li

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Abstract

This document describes the algorithm used to estimate a BLP (Berry, Levinsohn, and Pakes) model, which is commonly used in empirical industrial organization to model consumer choice among differentiated products.

Algorithm

Algorithm 1: BLP Model Algorithm

Input: Initial values for δ, (Π, Σ), (α, β), and random draws (v_i, D_i) for $i = 1, \ldots, ns$.
Output: Estimated parameters $(\hat{\alpha}, \hat{\beta})$, $(\hat{\Pi}, \hat{\Sigma})$, and $\hat{\Phi}$.

Initialization:

1. Select arbitrary values for δ, (Π, Σ), and (α, β) as starting points.
2. Draw random values for (v_i, D_i) for $i = 1, \ldots, ns$ from the distribution $P_v^*(v)$ and $\hat{P}_D^*(D)$ for a sample of size ns (the bigger you pick ns the more accurate your estimate will be).

Iteration:

1. Using the starting values and the random values, approximate the integral for market share that results from aggregating across i:

$$s_{jt}(p_t, x_t, \delta_t, P_{ns}; \theta_2) = \frac{1}{ns} \sum_{i=1}^{ns} s_{jti}$$

2. Keeping (Π, Σ) fixed, find values of δ by the contraction mapping iterative process:

$$\delta_t^{h+1} = \delta_t^h + \ln S_t - \ln s(\delta_t^h; p_t, x_t, \delta_t, P_{ns}; \theta_2)$$

   (a) Coming up with estimates for δ

3. Calculate the error term ω_{jt} and the value of the moment expression:

   (a) Calculate the error term:

$$\omega_{jt} = \delta_{jt} - (-\alpha p_{jt} + x_{jt}\beta)$$

   (b) Calculate the moment expression:

$$\omega' Z \Phi^{-1} Z' \omega$$

      i. Ideally, the weighting matrix Φ^{-1} is

$$\Phi^{-1} = \left(E(Z'\omega\omega' Z)\right)^{-1}$$

      ii. In practice, we use a consistent estimator of Φ^{-1}.
      iii. For the moment, we just use $\Phi^{-1} = (Z'Z)^{-1}$ as the starting point

4. Compute better estimates of all the parameters:

   (a) Find an estimate of the common parameters (α, β) using the standard GMM estimator:

$$(\hat{\alpha}, \hat{\beta}) = (X' Z \Phi^{-1} Z' X)^{-1} X' Z \Phi^{-1} Z' \delta$$

      i. This is a linear estimator that can be found analytically by multiplying matrices without any need for numerically minimizing something with search algorithm.

   (b) Estimate the value of the error term $\hat{\omega}$ and then the moment expression using the improved estimates $(\hat{\alpha}, \hat{\beta})$:

$$\hat{\omega}_{jt} = \delta_{jt} - (-\hat{\alpha} p_{jt} + x_{jt}\hat{\beta})$$

   (c) Update the value of the weighting matrix $\Phi = Z'\omega\omega' Z$ using the $\hat{\omega}$ just calculated.

   (d) Use a search algorithm to find new values for (Π, Σ). Take the new values and return to step 1. Keep iterating, searching for parameter estimates that minimize the moment expression Equation 3b, until the moment expression is less than the tolerance level.