Variational Auto-Encoder (VAE)

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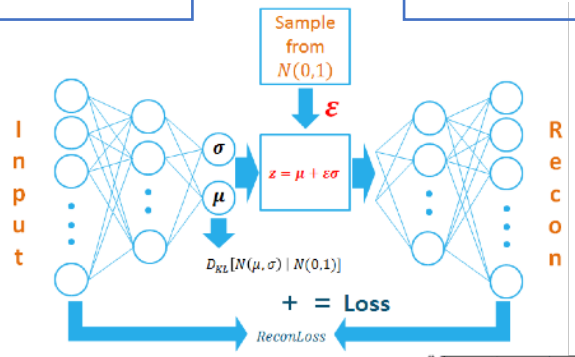
INTRODUCTION TO VAE (1)

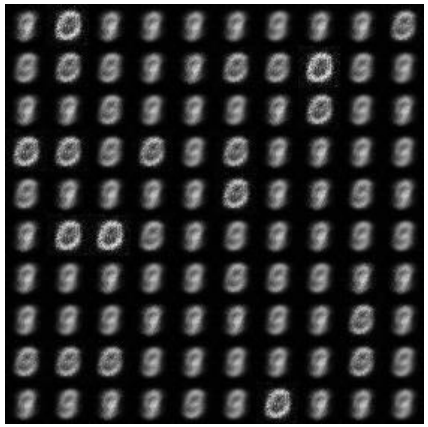
Loss Function

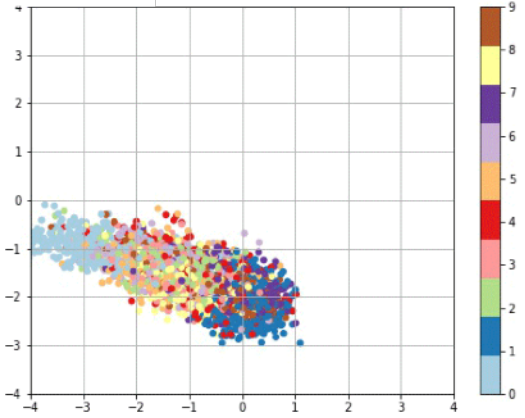
- Variational Inference

Neural network architecture

- Auto encoder
- Mixture Density Network







INTRODUCTION TO VAE (2)

- **Characteristics**
  - **Scalable** generative model
    - **Amortized** variational inference
    - Simple network architecture
    - Stochastic gradient descent (Back-propagation)
      - Scales to huge datasets
  - **Inference** based on probability graphical model
    - Continuous Latent Space
    - Data reduction / Data Imputation
      - Liable to noisy

This Seminar

- **Paper "Auto-encoding variational bayes"**
  - "VAE" is a example of experiment results in this paper.
- **Introduce the rest of this paper without "VAE"**
  - Stochastic Gradient Variational Bayes(SGVB) estimator
    - Auto-encoding Variational Bayes algorithm
  - Reparameterization trick
  - Appendix
    - Full Variational Bayes
    - Marginal Likelihood estimator
- **Research trend of VAE**

PRELIMINARY

TRADITIONAL VARIATIONAL INFERENCE

Bayesian Inference

- **General expression**
  - A random variable $x \in X$
  - A set $\{y_1, \cdots, y_N\}$ of i.i.d observations

$$p(x, y_1, \cdots, y_N) = p_0(x) \prod_{\{n=1\}}^{N} p(y_n \mid x)$$

  - Given prior $p_{0(x)}$ and observations, posterior can be evaluated.

$$p(x \mid y_1, \cdots, y_N) = \frac{p_0(x) \prod_{\{n=1\}}^{N} p(y_n \mid x)}{Z} := \frac{\bar{p}(x)}{Z}$$

$$Z := \int_X \bar{p}(x) dx$$

VARIATIONAL INFERENCE

- **Variational Inference**
  - Approximate the target distribution $p(x)$ by using a tractable distribution $q(x)$ in Q

$$q^* = argmin_{q \in Q} KL(q||p)$$

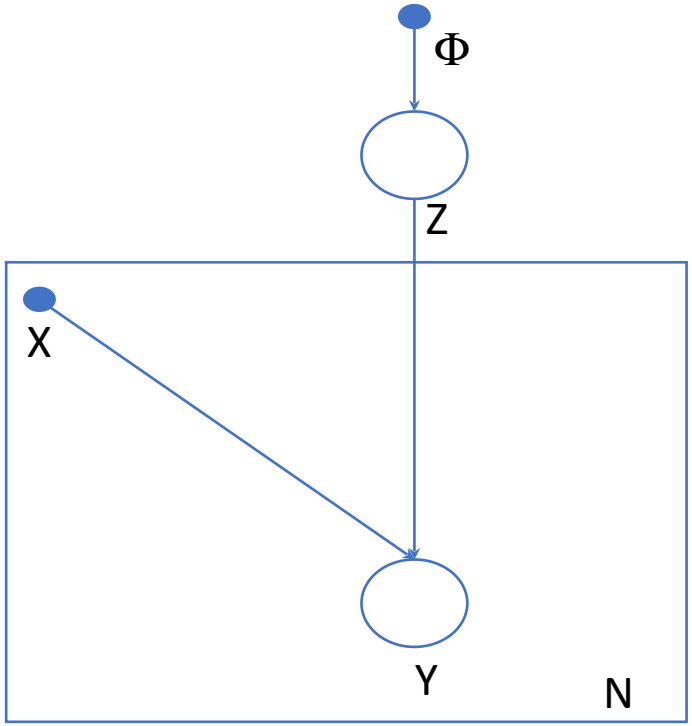
- **Variational Bound**

$$logp(x) = \int logp(x) \cdot q(z)dz = \int log\frac{p(x,z)}{p(z|x)} \cdot q(z)d$$

$$= \int log\frac{p(x,z)}{q(z)} \cdot q(z)dz - \int log\frac{q(z)}{p(z|x)} \cdot q(z)dz$$

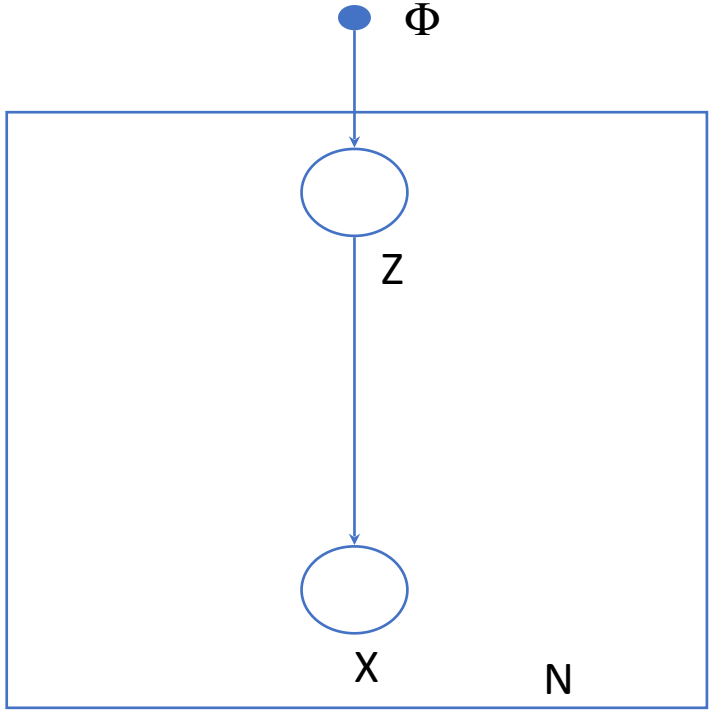
Probability Graphical Model

- **Supervised Learning**



e.g) Bayesian Logistic regression

- **Unsupervised Learning**
  (Latent Variable Model)



e.g) EM algorithm for
Gaussian Mixture Model

PROBABILITY GRAPHICAL MODEL

- **Supervised learning**

  e.g) Bayesian logistic regression

$$p\left(x \middle| w, y\right) \cdot p(w) \propto p\left(w, x \middle| y\right)$$

$$p\left(x \middle| w, y\right) : \prod_{\{i=1\}}^{n} \sigma\left(w^T x\right)^{y_i} \cdot \left(1 - \sigma\left(w^T x\right)^{1-y_i}\right) \rightarrow$$ Approximate Gaussian Dist.

- **Latent Variable Model**

  e.g) Gaussian Mixture Model (EM algorithm)

  → tractable conditional pdf : $p(z \mid x)$

$$q(z) \approx p\left(z \middle| x,\ \theta^{old}\right)$$

$$\theta^* = argmax_\theta \sum_z p\left(z \middle| x, \theta^{old}\right) \cdot lnp(x, z \mid \theta)$$

TRADITIONAL VARIATIONAL INFERENCE

- **Mean field assumption**
  - If $p(z \mid x)$ is intractable ?

$$q(\boldsymbol{z}) = \prod_i q_i(z_i)$$

$$ln q_j^*\left(z_j\right) = E_{i \neq j}\left[lnp(\boldsymbol{x}, \boldsymbol{z})\right] + const$$

$$q_j^*\left(z_j\right) = \frac{\exp(E_{i \neq j}[lnp(\boldsymbol{x}, \boldsymbol{z})]}{\int \exp(E_{i \neq j}\left[lnp(\boldsymbol{x}, \boldsymbol{z})\right] dz}$$

- Under specific probabilistic graphical model, approximate distribution can be evaluated by expectation of $E_{i \neq j}[lnp(\boldsymbol{x}, \boldsymbol{z})]$
- Depending on each problem, we have to derive the form of $E_{i \neq j}[lnp(\boldsymbol{x}, \boldsymbol{z})]$
- Through sequential optimization, All of $q_j(z_j)$ can be updated in order until converged.

Traditional Variational Inference (2)

- **VI with Stochastic gradient descent**

$$L = \int log \frac{p(x,z)}{q_\phi(z)} \cdot q_\phi(z) dz$$

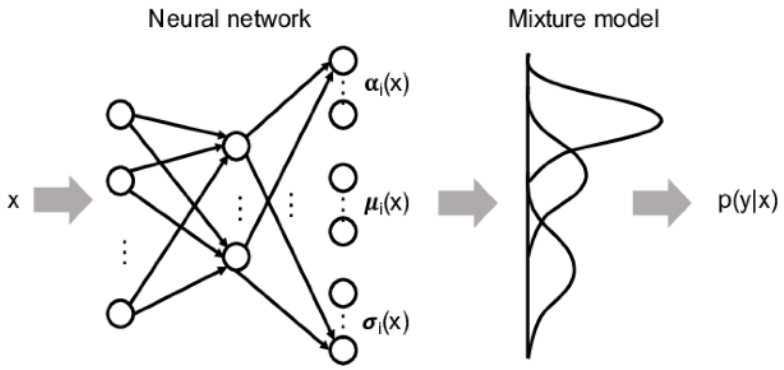
$$\nabla_\phi L = \int lnp(x,z) \cdot \nabla_\phi lnq(z) \cdot q(z) dz = E_{x\sim q}[lnp(x,z) \cdot \nabla_\phi lnq(z)]$$

$$\nabla_\phi L = E_{x\sim q}\Big[lnp(x,z) \cdot \nabla_\phi lnq(z)\Big] \approx \frac{1}{n}\sum_i lnp\big(x_i, z\big) \cdot \nabla_\phi lnq(z)$$

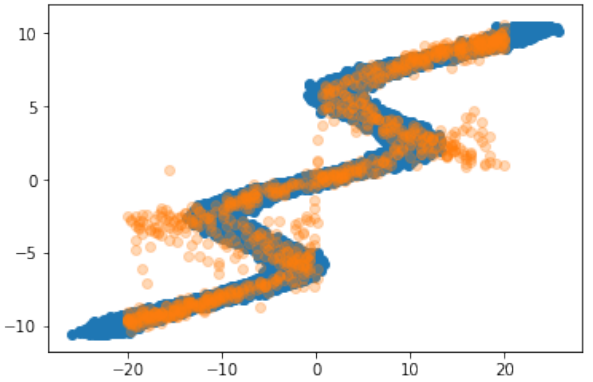
  - In order to avoid exact derivation of expectation and sequential optimization, stochastic gradient descent method was introduced.
  - This method only requires $logp(x,z)$, $q_\phi(z)$ and its derivative
  - All components can be easily handed
    - $logp(x,z)$ can be derived from probabilistic graphical model
    - $q_\phi(z)$ and $\nabla_\phi q_\phi(z)$ are decided by users.
  - However, since this estimator has a huge variance, it also requires many samples.

NETWORK FOR LEARNING PARAMETERS

- **Mixture density network**



[Structure of Mixture density Network]



[Data point and result by Mixture Density Network]

$$E(w) = -\sum_{\{n=1\}}^{N} \ln\{ \sum_{\{k=1\}}^{K} \pi_k(x_n, w) N(t_n \mid u_k(x_n, w), \sigma_k^2(x_n, w))\}$$

Mixture density network presents that artificial neural network trains hyper-parameters for learning parameters of distributions

Chapter Summary

- **Variational Inference**
  - It provide the evidence in which mathematical analysis can be conducted
  - Before emerging amortized VI, Most researches had a interest in efficient usage of VI using stochastic gradient descent and improving performance of approximate distribution for classification model

    : Basic example : Bayesian logistic regression.
  - Especially, VI with stochastic gradient descent influenced that “Auto-encoding variational bayes” were published.

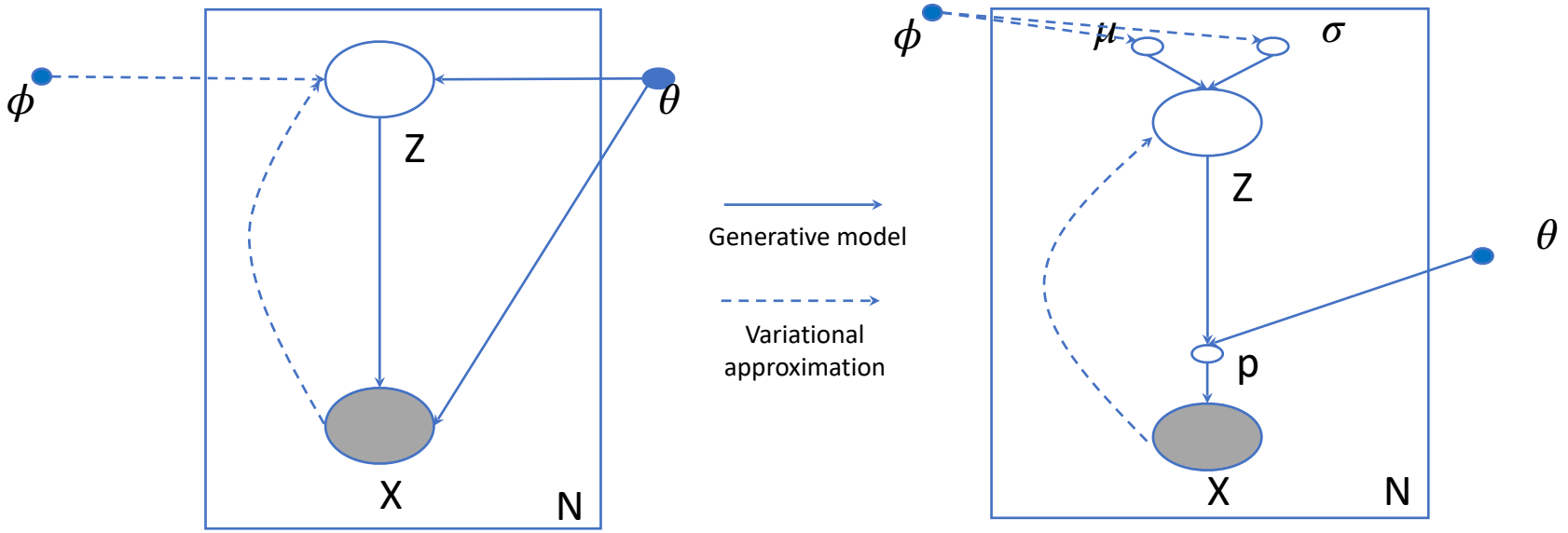
- **Mixture Density Network**
  - Mixture Density Network is known to learn parameters of specific distributions using NN
  - It is starting point to derive an approach to bayesian neural network

Auto-encoding Variational Bayes

Probabilistic Graphical Model
Stochastic variational Gradient bayes Estimator

PROBABILISTIC GRAPHICAL MODEL

- **Amortized generative model**



[AEVB algorithm]

[VAE]

- ϕ : hyper-parameters in recognition network
- θ : hyper-parameters in generative network

Problems Scenario

- **Traditional Latent variable model**
  - Intractable : $p(z|x)$
  - A large data set : Sampling based EM algorithm → Too slow

- **This paper has interested in**
  - Using stochastic gradient descent
  - Efficient approximate posterior inference of latent variable z

- **This paper introduce** (The reason why this method referred "Variational")
  - $q_\phi(z|x)$ *:* recognition model / approximate posterior distribution *to* $p_\theta(z|x)$
    - Figure out parameters of approximate distribution
  - $p_\theta(x|z)$ *:* generative model (defined as graphical model)

  - Learning the recognition model parameter ϕ with generative model parameter θ using reparameterization trick

Variational Bound

- Developing variational Bound

$$\int log\frac{p(x,z)}{q(z)}\cdot q(z)dz = \int log\frac{p(x\,|\,z)\cdot p(z)}{q(z)}\cdot q(z)dz$$

$$\int \Big(logp(x\,|\,z) + logp(z) - logq(z)\Big)\cdot q(z)dz$$

$$= \int logp(x\,|\,z)\cdot q(z)dz + \int logp(z) - logq(z)dz$$

$$= \int logp(x\,|\,z)\cdot q(z)dz \; - D_{KL}(q(z)\,||\,p(z)$$

STOCHASTIC GRADIENT VARIATIONAL BAYES ESTIMATOR (1)

- **Reparametrization Trick**
  - Random variable z → Function of variables (Deterministic)
    - Capable of using neural network
    - $z \sim q(z \mid x)$ → $\bar{z} = g_\phi(x, \epsilon), \epsilon \sim p(\epsilon)$
      - Assume that $p(\epsilon)$ is defined ($p(\epsilon) \sim N(0,1)$)
      - The function g_θ is determined depending on $q(z \mid x)$
        (In subsequent chapter, the forms of function g_θ is represented)
  - Form of function g_θ
    - Tractable inverse CDF : similar to finding PDF
      - $F(x) = p,\ F^{-1}(p) = x\ (unique\ x)$ / $g_\phi(\epsilon, x) = \mathrm{F}_\phi^{-1}(\epsilon; x) = \bar{Z}$
    - Location Scale Model :
      - $g_\phi(\epsilon, x)$ = location + scale $\cdot\ \epsilon$

Stochastic Gradient Variational Bayes Estimator (2)

- **SGVB Estimator**

$$E_{q_{\phi_{(z|x^i)}}}\left[f(z)\right] = E_{p(\epsilon)}\left[f\left(g_\phi\left(\epsilon, x^i\right)\right)\right] \approx \frac{1}{L}\sum_i f\left(g_\phi\left(\epsilon^l, x^i\right)\right) where,\ \epsilon^l \sim p(\epsilon)$$

  - $f(z) = log p(x \mid z) + q(z) - p(z)$
    - $Given\ x,\ \epsilon\ sampled\ from\ p(\epsilon),\ z \approx \bar{z} = g_\phi\left(\epsilon, x^i\right),\ f(z)$ can be evaluated.
  - The form of $\frac{1}{L}\sum_i f\left(g_\phi\left(\epsilon^l, x^i\right)\right)$
    - Stochastic gradient method can be introduced w.r.t ϕ , θ
      (Back propagation)

Stochastic Gradient Variational Bayes Estimator (3)

- **SGVB Estimator A**

$$\tilde{L}^{A}(\theta, \phi; x^i) = \frac{1}{L}\sum_{l} log\, p_\theta\left(x^i, z^{i,l}\right) - log q_\phi(z^{i,l} \mid x^i) where,\ g_\phi\left(\epsilon, x^i\right),\ \epsilon^l \sim p(\epsilon)$$

  - $log\, p_\theta\left(x^i, z^{i,l}\right)$ is defined by probabilistic graphical model
  - $log q_\phi(z^{i,l} \mid x^i)$ can be determined $g_\phi\left(\epsilon, x^i\right),\ \epsilon^l \sim p(\epsilon)$

- **SGVB Estimator B**

$$\tilde{L}^{B}(\theta, \phi; x^i) = \frac{1}{L}\sum_{l} log\, p_\theta\left(x^i \mid z^{i,l}\right) - D_{KL}(q_\phi(z \mid x^i) \mid\mid p_\theta(z))$$

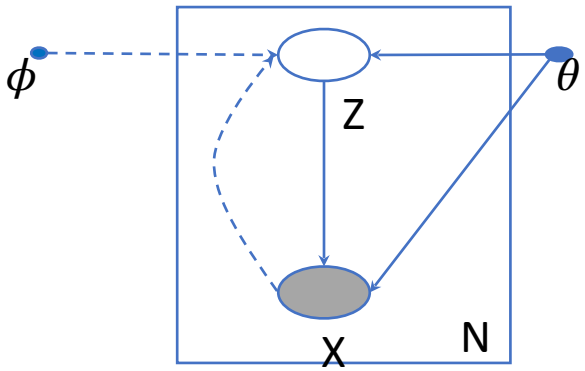
$$where,\ g_\phi\left(\epsilon, x^i\right),\ \epsilon^l \sim p(\epsilon)$$

  - $D_{KL}(q_\phi(z \mid x^i) \mid\mid p_\theta(z))$ is analytically evaluated (Gaussian Dist.)
  - $D_{KL}(q_\phi(z \mid x^i) \mid\mid p_\theta(z))$ doesn't require sample *z,* only require parameter of approximate dist.

AUTO ENCODING VARIATIONAL BAYES ALGORITHM(1)

Algorithm 1 Minibatch version of the Auto-Encoding VB (AEVB) algorithm. Either of the two SGVB estimators in section 2.3 can be used. We use settings $M = 100$ and $L = 1$ in experiments.

$\boldsymbol{\theta}, \boldsymbol{\phi} \leftarrow$ Initialize parameters
repeat
  $\mathbf{X}^M \leftarrow$ Random minibatch of M datapoints (drawn from full dataset)
  $\boldsymbol{\epsilon} \leftarrow$ Random samples from noise distribution $p(\boldsymbol{\epsilon})$
  $\mathbf{g} \leftarrow \nabla_{\boldsymbol{\theta},\boldsymbol{\phi}} \widetilde{\mathcal{L}}^M(\boldsymbol{\theta}, \boldsymbol{\phi}; \mathbf{X}^M, \boldsymbol{\epsilon})$ (Gradients of minibatch estimator (8))
  $\boldsymbol{\theta}, \boldsymbol{\phi} \leftarrow$ Update parameters using gradients $\mathbf{g}$ (e.g. SGD or Adagrad [DHS10])
until convergence of parameters $(\boldsymbol{\theta}, \boldsymbol{\phi})$
return $\boldsymbol{\theta}, \boldsymbol{\phi}$



$$lnp_\theta(X, Z) = \sum_i lnp_\theta(x^i \mid z^i)$$

$$lnq_\phi(Z) = \sum_i lnq_\phi(z^i)$$

$$lnp_\theta(Z) = \sum_i lnp_\theta(z^i)$$

AUTO ENCODING VARIATIONAL BAYES ALGORITHM (2)

- **AEVB for SGVB estimator**

$$\frac{1}{M}\sum_i \tilde{L^A}(\theta,\ \phi; x^i) = \frac{1}{M}\sum_i \frac{1}{L}\sum_l logp_\theta\left(x^i, z^{i,l}\right) - logq_\phi(z^{i,l} \mid x^i)$$

$$\frac{1}{M}\sum_i \tilde{L^B}(\theta,\ \phi; x^i) = \frac{1}{M}\sum_i \frac{1}{L}\sum_l logp_\theta\left(x^i \mid z^{i,l}\right) - D_{KL}(q_\phi(z \mid x^i) \mid\mid p_\theta(z))$$

Chapter Summary

- **SGVB Estimator**
  - This estimator was developed to use an approach of neural network for learning probabilistic latent variable model
  - Reparameterization trick is the key point to derive SGVB estimator
- **AEVB algorithm**
  - It can train a neural network using gradient of loss function (SGVB estimator) w.r.t hyper-parameters in recognition and generative network at once.
  - This gradient can be evaluated by back-propagation

Variational Auto Encoder

Framework
Network Architecture
Full Variational Bayes
Marginal Likelihood estimator

FRAMEWORK (1)

- **Loss function** (Bernoulli Case)
  - $p(z) \sim N(0, I)$
  - $p_\theta(x \mid z) \sim N\left(u_\theta(z), \sigma_\theta(z)\right)$ *or* $Bern(\mathrm{p}_\theta(z))$
  - $q_\phi(z \mid x) \sim N(u_\phi(x), diag_\phi(x))$

- **Estimator (Loss function)**

$$\tilde{L}^B(\theta, \phi; x^i) = \frac{1}{L} \sum_l log p_\theta\left(x^i \mid z^{i,l}\right) - D_{KL}(q_\phi(z \mid x^i) \mid\mid p_\theta(z))$$

$$where\ g_\phi\left(\epsilon, x^i\right) = u_\theta(x) + \epsilon \cdot \sigma_\theta(x)$$

$$\epsilon \sim N(0, I),\ \ l = 1$$

  - Since $l = 1$, neural network architecture can be similar to “auto-encoder”

FRAMEWORK (2)

- **Loss function**
  - $p_\theta(x \mid z) \sim Bern(\mathrm{p}_\theta(z))$

    → $log p_\theta(x \mid z) \sim \sum_i x_i log \mathrm{p}_\theta(z_i) + (1 - x_i) log(1 - \mathrm{p}_\theta(z_i))$

  - $D_{KL}(q_\phi(z \mid x^i) \mid\mid p_\theta(z))$
    - $p(z) \sim N(0, I),\ q_\phi(z \mid x) \sim N(u_\phi(x), diag_\phi(x))$
    - $D_{KL}(q_\phi(z \mid x) \mid\mid p_\theta(z)) =$

$$\frac{1}{2}\{\left(0 - u_\phi(x)\right) I^{-1} \left(0 - u_\phi(x)\right) + trace(I^{-1} diag_\phi(x)) - k + ln \frac{\mid I \mid}{\mid diag_\phi(x) \mid}$$

$$= \sum_j u_\phi(x)^2_{(j)} - 1 + \sigma^2_\phi(x)_{(j)} - ln\sigma^2_\phi(x)_{(j)}$$

$where\ j = index\ of\ dimension\ of\ latent\ variable\ z$

Framework (3)

- **Loss Function**

$$\therefore \sum_{i} x_i log \mathrm{p}_{\theta}\left(z_i\right) + \left(1 - x_i\right) log(1 - \mathrm{p}_{\theta}(z_i)) +$$

$$\sum_{i} \sum_{j} u_{\phi}\left(x_i\right)_{(j)}^{2} - 1 + \sigma_{\phi}^{2}\left(x_i\right)_{(j)} - ln\sigma_{\phi}^{2}\left(x_i\right)_{(j)}$$

Structure of VAE(1)

- **Network Architecture**
  - Recognition network (like density network)

$$u_\phi(x_i) = w_2 tanh(w_1 x_i + b_1) + b_2$$

$$\sigma_\phi(x_i) = w_4 tanh(w_3 x_i + b_3) + b_4$$

  - Generative network
    - Bernoulli Case

$$\mathrm{p}_\theta(z_i) = f_a\left(w_2 \tanh(w_1 z_i + b_1) + b_2\right)$$

$$where\ f_a(\cdot): element - wise\ sigmoid\ function$$

$$\theta = \{w_1, b_1, w_2, b_2\}$$

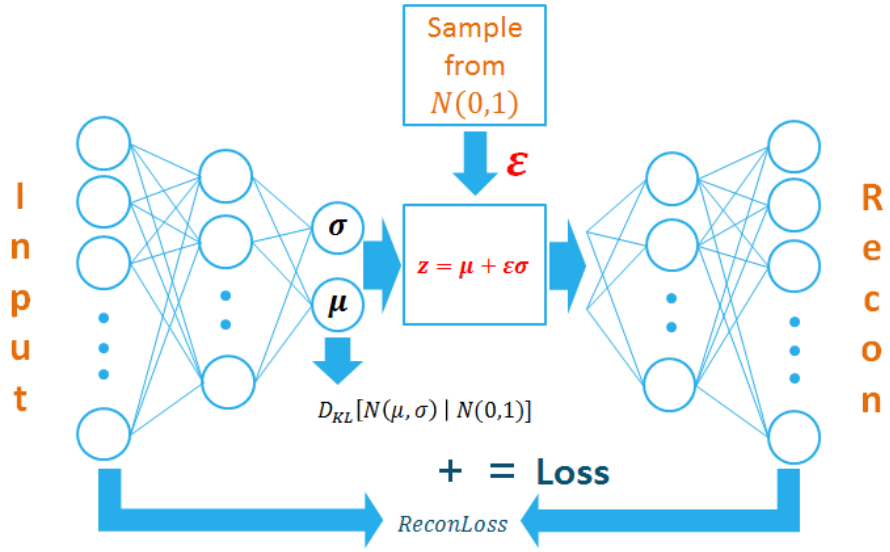
    - Gaussian Case

$$u_\theta(z_i) = w_4(tanh(w_3 z + b_3) + b_4$$

$$\sigma_\theta(z_i) = w_5(tanh(w_3 z + b_3) + b_5$$

$$\theta = \{w_3, w_4, w_5,\ b_3, b_4, b_5\}$$

VAE



- **Probabilistic modeling**
  - All data should be mapped to latent space defined by probability graph model
  - Robust against some noises
    - Key difference between VAE and general AE

Marginal Likelihood Estimator (1)

- **Derivation**
  - Marginal Likelihood : $p_\theta(x)$
  - Marginal likelihood estimator

$$\frac{1}{p_\theta(x_i)} = \int \frac{q(z)}{p_\theta(x_i)} dz = \frac{\int q(z) \cdot \frac{p_\theta(x_i, z)}{p_\theta(x_i, z)} dz}{p_\theta(x_i)} = \int \frac{p_\theta(x_i, z)}{p_\theta(x_i)} \cdot \frac{q(z)}{p_\theta(x_i, z)} dz$$

$$= \int p_\theta(z \mid x_i) \frac{q(z)}{p_\theta(x_i, z)} dz \approx \frac{1}{L} \sum_l \frac{q_\phi(z^{(l)})}{p_\theta(z) p_\theta(x_i \mid z^{(l)})}$$

  - In VAE, marginal likelihood estimator cannot be evaluated because it requires $l > 1$
  - If it was forced to evaluate the marginal likelihood in VAE, the result is almost same as loss function, it doesn't give any special meaning.

Marginal Likelihood Estimator (2)

- **Gradient MCMC**
  - Max Welling and published ”Bayesian learning via stochastic gradient langevin dynamics (2011), ICML2011”

$$\Delta\theta_t = \frac{\epsilon_t}{2}(\nabla log p(\theta_t) + \frac{N}{n}\sum_{i=1}^{n}\nabla log p(x_i \mid \theta_t)) + \eta_t$$

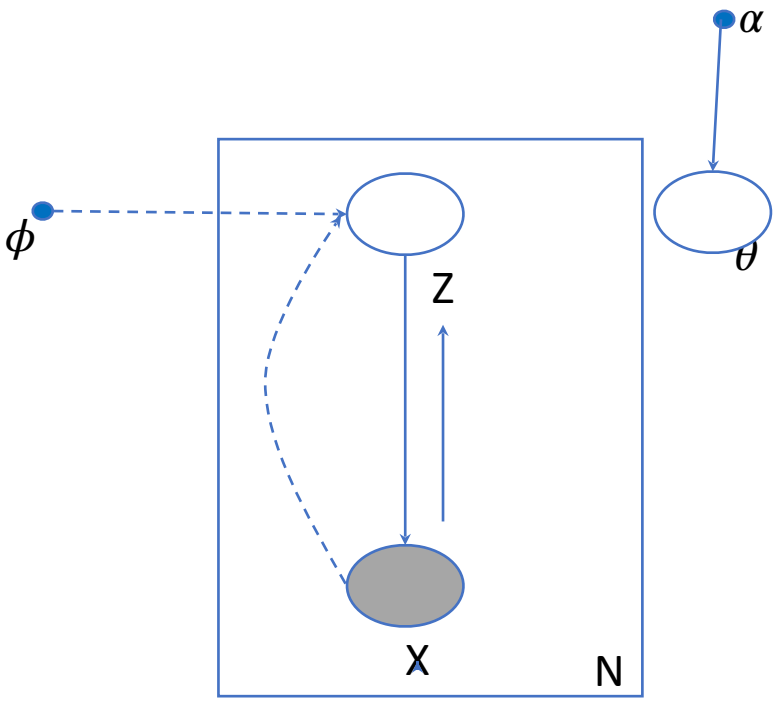
$$where\ \eta_t \sim N(0,\epsilon_t)$$

Update (stochastic gradient descent)

$$\theta = \theta_t + \epsilon_t' \cdot \Delta\theta_t$$

- In this paper, $\nabla \log p(z_t), \nabla \log p(x_i \mid z_t)$ can be evaluated.
- Under this method, marginal likelihood estimator is established.

FULL VARIATIONAL BAYES



[AEVB algorithm]

- **Framework**
  - $p_\theta(z) \sim N(0, I), \quad where\ z \in R^J,\ I \in R^{J \times J}$
  - $p_\alpha(\theta) \sim N(0, I), \quad where\ \theta \in R^M, I \in R^{M \times M}$
  - $p_\theta(x \mid z) \sim N\left(u_\theta(z), \sigma_\theta(z)\right)\ or\ Bern(\mathrm{p}_\theta(z))$
  - $q_\phi(z \mid x) \sim N(u_\phi(x), diag_\phi(x))$

Full variational Bayes

- **Variational Bound**

$$lnp_\theta(X) \approx \int \left(lnP_\alpha(X \mid \theta) + lnP_\alpha(\theta) - lnq_\phi(\theta) \right) \cdot q_\theta(\theta) d\theta$$
$$= \frac{1}{L} \sum_{l=1}^{L} E\left[lnP_\alpha(X \mid \theta) \right] + \frac{1}{2} \sum_{M=1}^{M} \{ 1 + ln\sigma_{\theta_m}^{2(l)} - \mu_{\theta_m}^{2(l)} - \sigma_{\theta_m}^{2\ (l)} \}$$

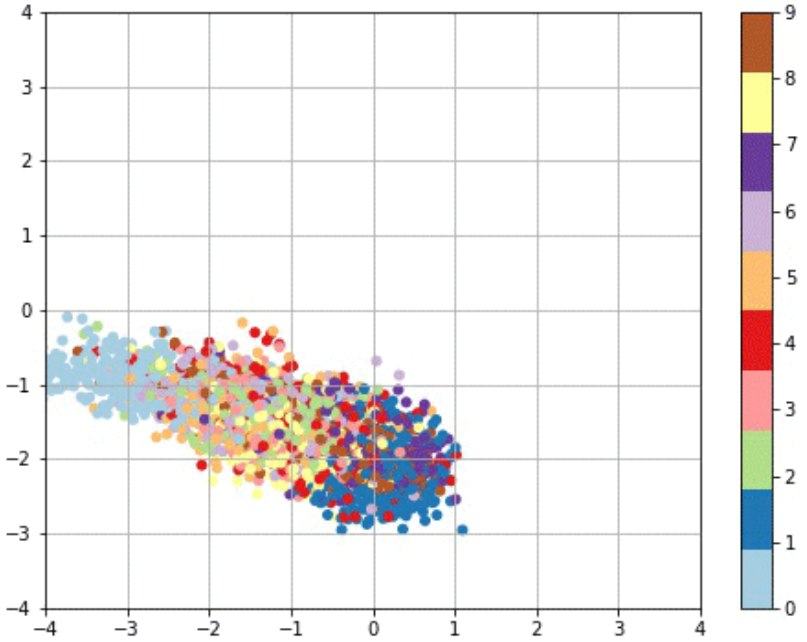
$$lnP_\alpha(X \mid \theta) = \int log \frac{p(x,z)}{q(z)} \cdot q(z) dz - \int log \frac{q(z)}{p(z \mid x)} \cdot q(z) dz$$
$$\approx \frac{1}{K} \sum_{i=1}^{N} (\sum_{k=1}^{K} \ln p_\theta \left(x_i \mid z_{i,j} \right) + \frac{1}{2} \sum_{j=1}^{J} \{ 1 + ln\sigma_{z_{i,j}}^{2(k)} - \mu_{z_{i,j}}^{2(k)} - \sigma_{z_{i,j}}^{2\ (k)} \})$$

- In this paper, $L = K = 1$ and $\ln p_\theta \left(x_i \mid z_{i,j} \right) = lnp_\theta(x \mid z), \forall x_i$

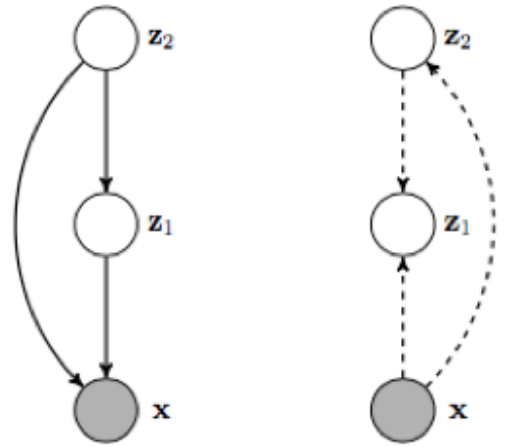
$$\therefore N \cdot lnp(x \mid z) + \frac{N}{2} \cdot \sum_{j=1}^{J} \{ 1 + ln\sigma_{z_{i,j}}^{2'} - \mu_{z_{i,j}}^{2'} - \sigma_{z_{i,j}}^{2\ '} \} + \frac{1}{2} \sum_{M=1}^{M} \{ 1 + ln\sigma_{\theta_m}^{2(l)} - \mu_{\theta_m}^{2(l)} - \sigma_{\theta_m}^{2\ (l)} \}$$

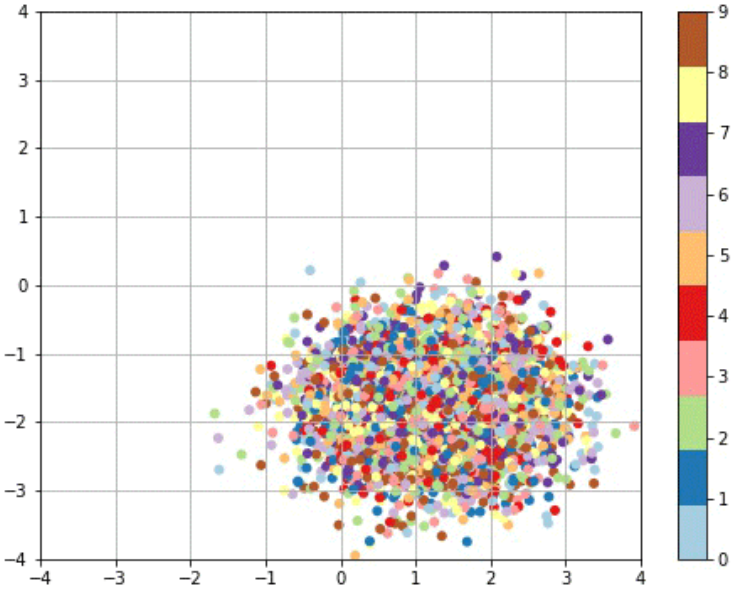
Experimental Result

STANDARD VAE



HIERARCHICAL VAE





$$p_\lambda(\mathbf{z}_1|\mathbf{z}_2) = \mathcal{N}\big(\mathbf{z}_1|\mu_\lambda(\mathbf{z}_2), \mathrm{diag}(\sigma^2_\lambda(\mathbf{z}_2))\big),$$
$$q_\phi(\mathbf{z}_1|\mathbf{x}, \mathbf{z}_2) = \mathcal{N}\big(\mathbf{z}_1|\mu_\phi(\mathbf{x}, \mathbf{z}_2), \mathrm{diag}(\sigma^2_\phi(\mathbf{x}, \mathbf{z}_2$$
$$q_\psi(\mathbf{z}_2|\mathbf{x}) = \mathcal{N}\big(\mathbf{z}_2|\mu_\psi(\mathbf{x}), \mathrm{diag}(\sigma^2_\psi(\mathbf{x}))\big)$$

Experimental Results

- **Table**

Epoch	VAE		Hierarchical VAE	
	Variational Bound	Log Marginal Likelihood	Variational Bound	Log Marginal Likelihood
1	212.87	-210.73	214.06	-214.11
50	152.83	-151.75	152.75	-152.85
100	154.24	-153.59	149.95	-150.15
150	152.32	-152.15	139.21	-139.29
200	138.41	-137.42	132.70	-132.82

What's Next?

Current Research

RESEARCH SUMMARY

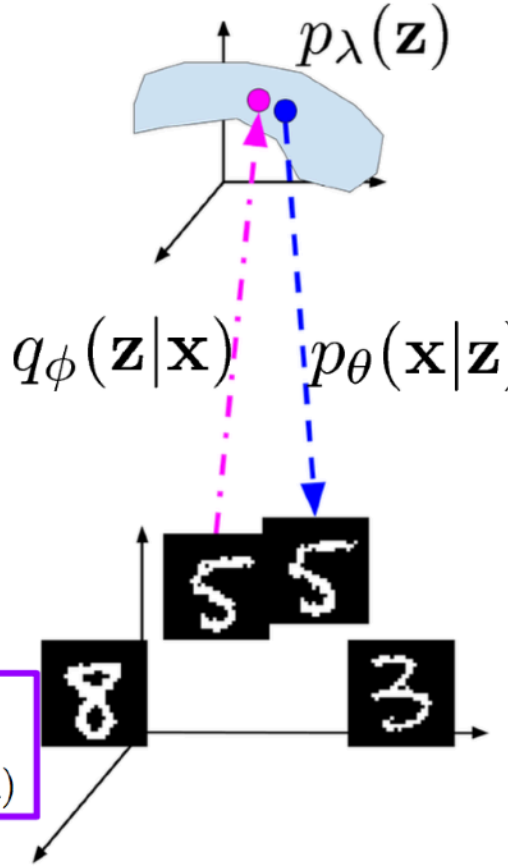
DGM: Variational Auto-Encoder

$$q_\phi(\mathbf{z}|\mathbf{x}) \propto p_\theta(\mathbf{x}|\mathbf{z})\, p_\lambda(\mathbf{z})$$

Normalizing flows
Volume-preserving flows
Gaussian processes
Stein Particle Descent
Operator VI

Feedforward nets
Convolutional nets
PixelCNN
Gated PixelCNN

Auto-regressive Prior (Chen et al.)
Objective Prior (Nalisnick & Smyth)
Stick-Breaking Prior (Nalisnick & Smyth)



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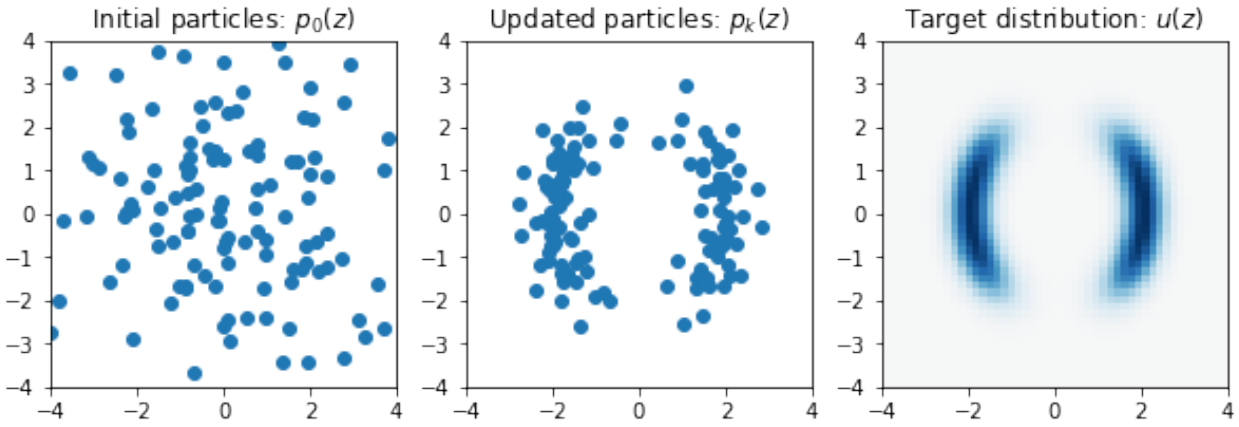
NORMALIZING FLOW / STEIN METHOD

- **Iterative update : $z_i \sim q_\phi(z_i \mid x_i)$**
  - Based on change of variable
    - Objective : minimizing $KLD(q||p)$
    - Normalizing flow : using tractable determinant of Jacobian
    - Stein method : using the RKHS kernel to describe direction for minimizing KLD

$$T_l^*(x) = x + \epsilon_l \phi_{q_l,p}^*(x).$$

$$q_0 \xrightarrow{T_0^*} q_1 \xrightarrow{T_1^*} q_2 \xrightarrow{T_2^*} \cdots \rightarrow q_\infty = p(\phi_{q_l,p}^* = 0)$$



VAE WITH VAMPPRIOR

- **Main Idea**

  -. Prior Distribution = Aggregate Posterior Distribution

$$P_\lambda(z) = \frac{1}{K}\sum_{k=1}^{K} q(z|x)$$

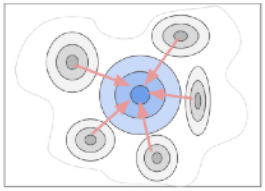
  -. Pseudo inputs

    . Users should select learnable pseudo inputs.

    . If pseudo inputs are randomly chosen, we couldn't expect better performance.

  -. Expectation for this approach

$$\text{min. } \mathbb{H}[q(\mathbf{z})] + KL[q(\mathbf{z})||p_\lambda(\mathbf{z})]$$



Standard prior is too strong and overregularizes the encoder.

What is the "optimal" prior?

VAE WITH VAMPPRIOR

- **Variational Bound**

$$\begin{aligned}\mathbb{E}_{\mathbf{x}\sim q(\mathbf{x})}\big[\ln p(\mathbf{x})\big] \geq & \mathbb{E}_{\mathbf{x}\sim q(\mathbf{x})}\big[\mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})}[\ln p_\theta(\mathbf{x}|\mathbf{z})]\big] + \\ & + \mathbb{E}_{\mathbf{x}\sim q(\mathbf{x})}\big[\mathbb{H}[q_\phi(\mathbf{z}|\mathbf{x})]\big] + \\ & - \mathbb{E}_{\mathbf{z}\sim q(\mathbf{z})}[-\ln p_\lambda(\mathbf{z})]\end{aligned}$$

$$\max_{p_\lambda(\mathbf{z})} -\mathbb{E}_{\mathbf{z}\sim q(\mathbf{z})}[-\ln p_\lambda(\mathbf{z})] + \beta\Big(\int p_\lambda(\mathbf{z})\mathrm{d}\mathbf{z} - 1\Big)$$

$$p_\lambda^*(\mathbf{z}) = \frac{1}{N}\sum_{n=1}^{N} q_\phi(\mathbf{z}|\mathbf{x}_n)$$

VAE WITH VAMPPRIOR

- **Experimental Result**

DATASET	VAE ($L=1$)		HVAE ($L=2$)		CONVHVAE ($L=2$)		PIXELHVAE ($L=2$)	
	standard	VampPrior	standard	VampPrior	standard	VampPrior	standard	VampPrior
staticMNIST	−88.56	**−85.57**	−86.05	**−83.19**	−82.41	**−81.09**	−80.58	**−79.78**
dynamicMNIST	−84.50	**−82.38**	−82.42	**−81.24**	−80.40	**−79.75**	−79.70	**−78.45**
Omniglot	−108.50	**−104.75**	−103.52	**−101.18**	−97.65	**−97.56**	−90.11	**−89.76**
Caltech 101	−123.43	**−114.55**	−112.08	**−108.28**	−106.35	**−104.22**	**−85.51**	−86.22
Frey Faces	4.63	**4.57**	4.61	**4.51**	4.49	**4.45**	4.43	**4.38**
Histopathology	6.07	**6.04**	5.82	**5.75**	5.59	**5.58**	4.84	**4.82**

Table 2: Test LL for static MNIST.

MODEL	LL
VAE ($L=1$) + NF [32]	−85.10
VAE ($L=2$) [6]	−87.86
IWAE ($L=2$) [6]	−85.32
HVAE ($L=2$) + SG	−85.89
HVAE ($L=2$) + MoG	−85.07
HVAE ($L=2$) + VAMPPRIOR *data*	−85.71
HVAE ($L=2$) + VAMPPRIOR	**−83.19**
AVB + AC ($L=1$) [28]	−80.20
VLAE [7]	**−79.03**
VAE + IAF [18]	−79.88
CONVHVAE ($L=2$) + VAMPPRIOR	−81.09
PIXELHVAE ($L=2$) + VAMPPRIOR	−79.78

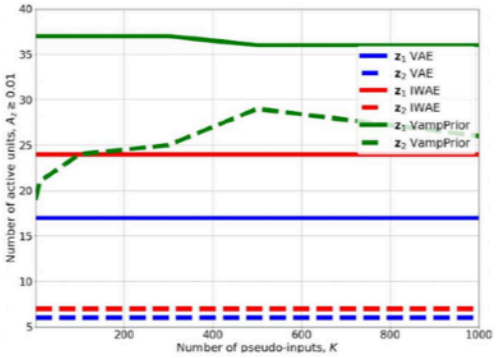


Figure 3: A comparison between two-level VAE and IWAE with the standard normal prior and theirs VampPrior counterpart in terms of number of active units for varying number of pseudo-inputs on static MNIST.

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- Bishop(2006), Pattern recognition and Machine learning
- Rezende, Mohamed and Wiestra(2014) *Stochastic Backpropagation and Approximate Inference in Deep Generative Models*, ICML2014
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Any Questions?