

Question 1 (Exercise 13.2-1, Cormen et al.)

Write pseudo code for RIGHT-ROTATE.

```
RIGHT-ROTATE(T, y)
    x = y.left           // set x
    y.left = x.right     // turn y's left subtree into x's right subtree
    IF x.right ≠ T.nil
        x.right.p = y
    x.p = y.p           // Link y's parent to x
    IF y.p == T.nil
        T.root = x
    ELSEIF y == y.p.right
        y.p.right = x
    ELSE y.p.left = x
    x.right = y          // Put y on x's right
    y.p = x
```

Question 2 (Exercise 13.2-2, Cormen et al.)

Argue that in every n -node binary search tree, there are exactly $(n - 1)$ possible rotations.

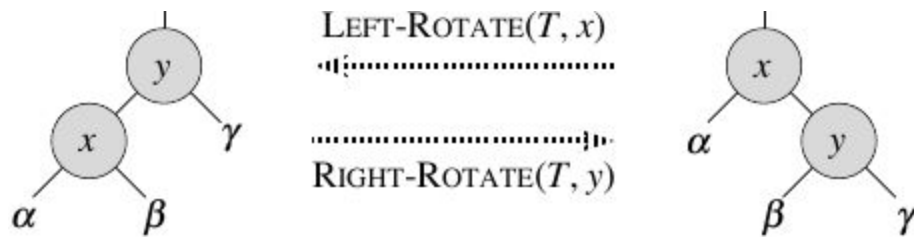
All nodes have a left and right child and therefore, at max there will always be 2 nodes that share a parent. Except for the root node. It doesn't share a parent with any other node.

Therefore, we can rotate all the other nodes using left and right rotates except this one node. $(n - 1)$

Question 3 (Exercise 13.2-3, Cormen et al.)

How do the depths of nodes in a BST change when a rotation is performed?

To analyze how the depths changes we can use the example from Cormen et al.



As we can see, when we `left rotate`, x decreases in depth while its right child y increases in depth as well as y 's right child, γ .

While if we `right rotate`, y decreases in depth while its left child, x , increases in depth as well as its left child, α .

In both cases though, β , doesn't change in depth. During `left rotation` it is the left child of y and during `right rotation` it is the right child of x .

Question 4 (Exercise 13.3-2, Cormen et al.)

Write down or illustrate the red-black trees that result after successively inserting the keys 41; 38; 31; 12; 19; 8 into an initially empty red-black tree.

Cases:

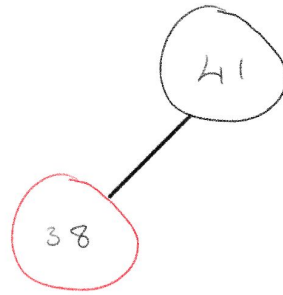
1. $Z = \text{root} \rightarrow$ color black
2. $Z.\text{uncle} = \text{red} \rightarrow$ recolor grandparent, parent, and uncle
3. $Z.\text{uncle} = \text{black (triangle)} \rightarrow$ rotate $Z.\text{parent}$
4. $Z.\text{uncle} = \text{black(line)} \rightarrow$ rotate $Z.\text{grandparent}$ and recolor grandparent, parent, and uncle

Inserting 41, 38, 31, 12, 19, 8

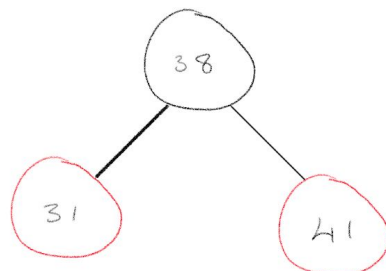
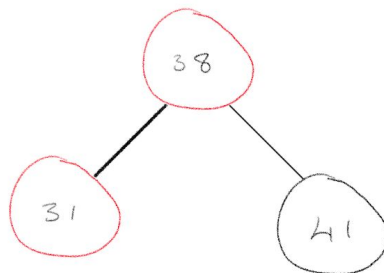
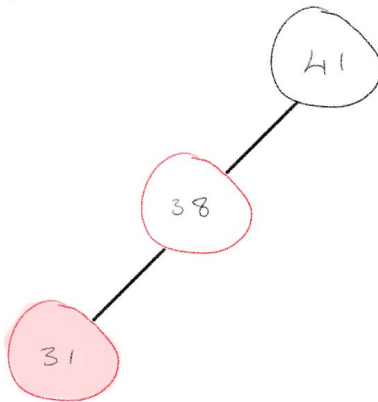


After inserting 41 case 0 occurs.

38

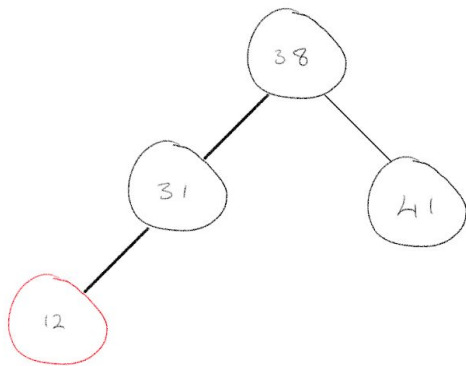
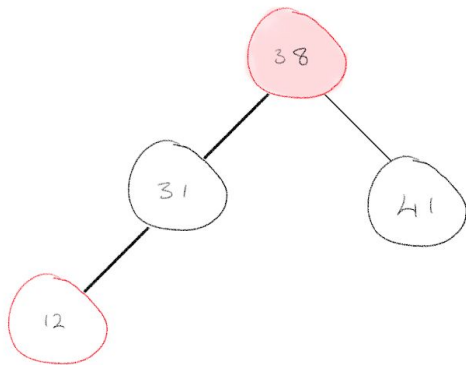
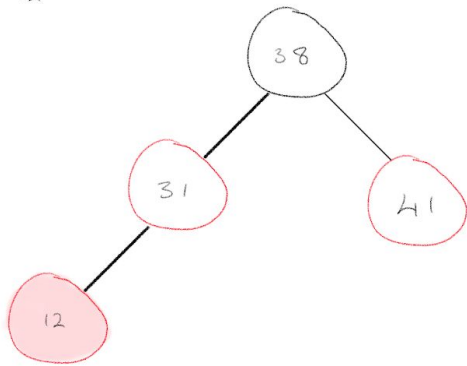


31



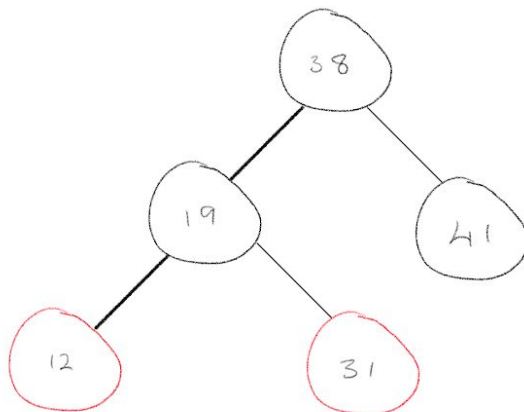
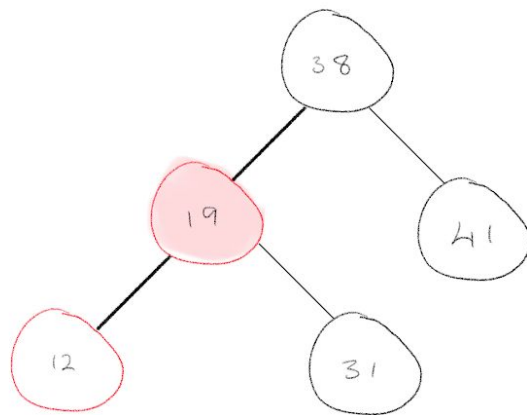
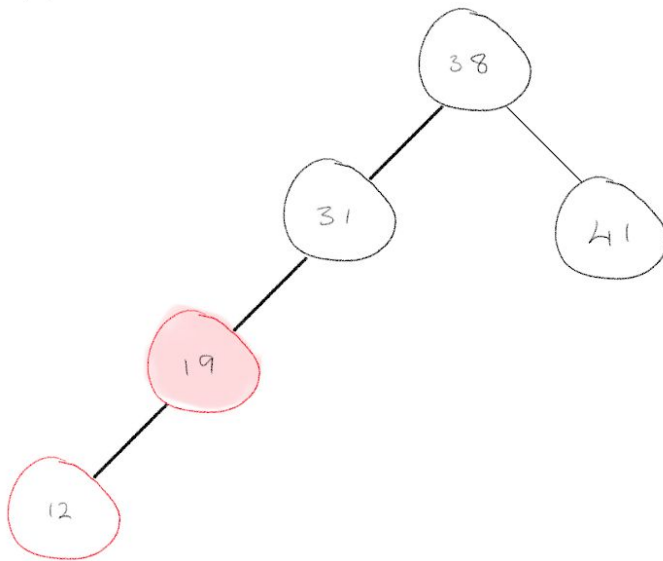
After inserting 31, case 3 occurs

12



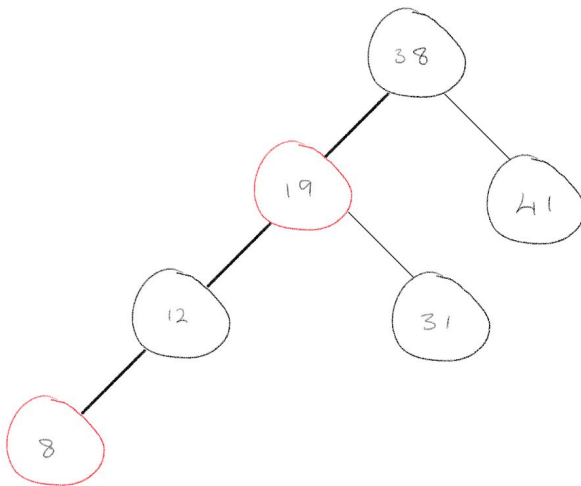
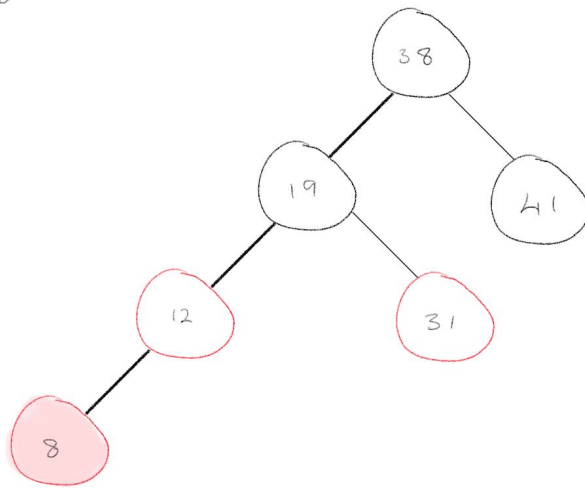
After inserting 12, case 1 occurs followed by case 0

19



After inserting 19 case 2 occurs

8



After inserting 8, case 1 occurs