

# Untitled

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```
library(traveltimeCLT)
library(data.table)
```

```
## Warning:  'data.table' R 4.3.3
```

```
library(dplyr)
```

```
##
```

```
##  'dplyr'
```

```
## The following objects are masked from 'package:data.table':
```

```
##
```

```
##  between, first, last
```

```
## The following objects are masked from 'package:stats':
```

```
##
```

```
##  filter, lag
```

```
## The following objects are masked from 'package:base':
```

```
##
```

```
##  intersect, setdiff, setequal, union
```

```
source("sde_sim.R")
```

```
##
```

```
##  'reshape2'
```

```
## The following objects are masked from 'package:data.table':
```

```
##
```

```
##  dcast, melt
```

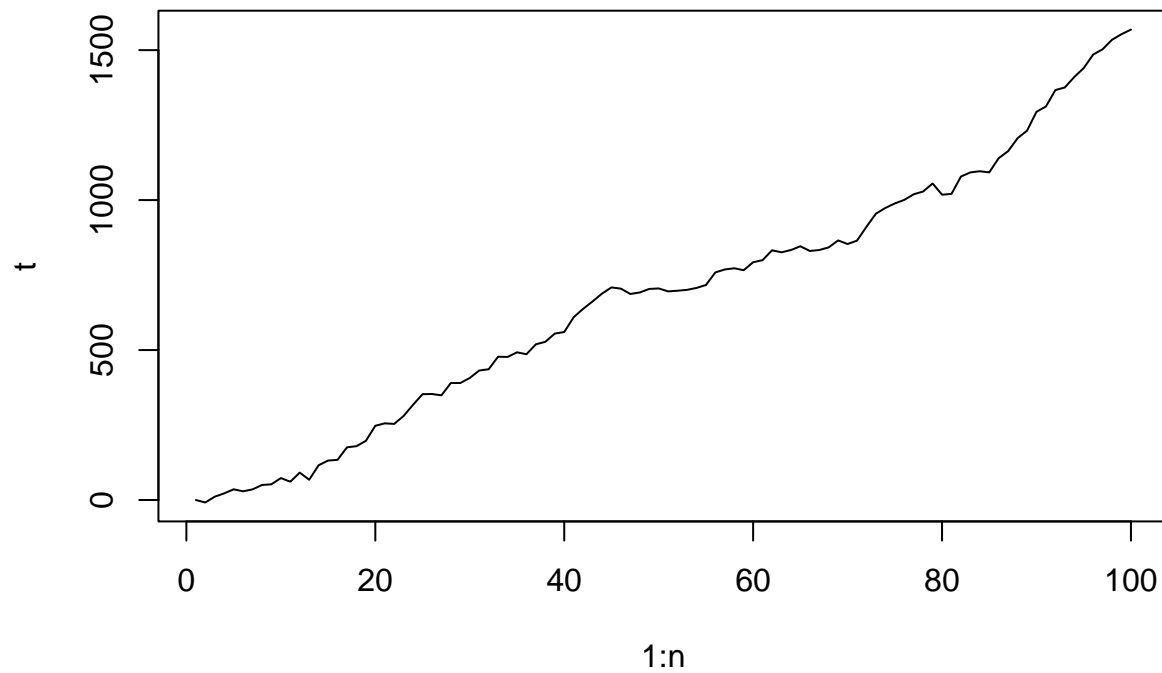
```
## Warning in sqrt(t[i - 1]):  NaNs
```

```
## Warning in sqrt(t[i - 1]):  NaNs
```

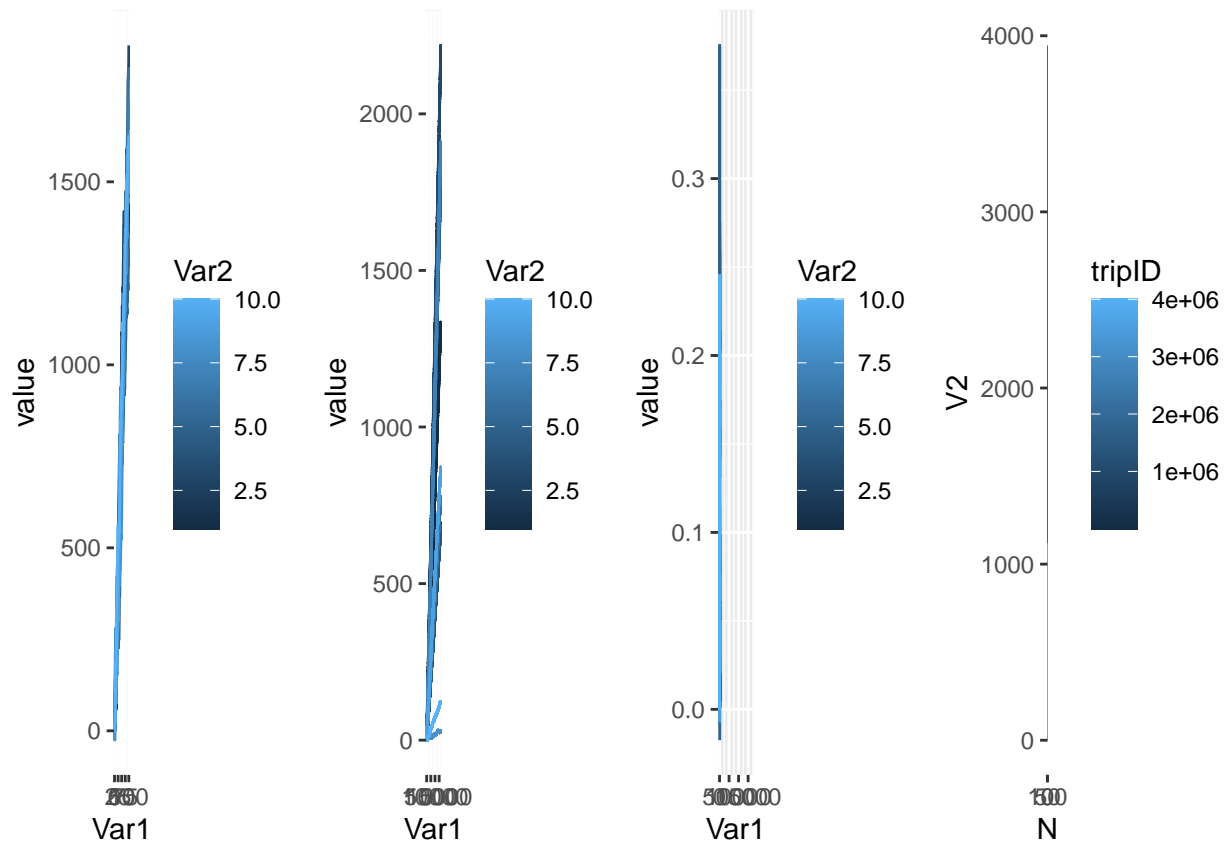
```
## Warning in sqrt(t[i - 1]):  NaNs
```

```
## Warning in sqrt(t[i - 1]):  NaNs
```

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```



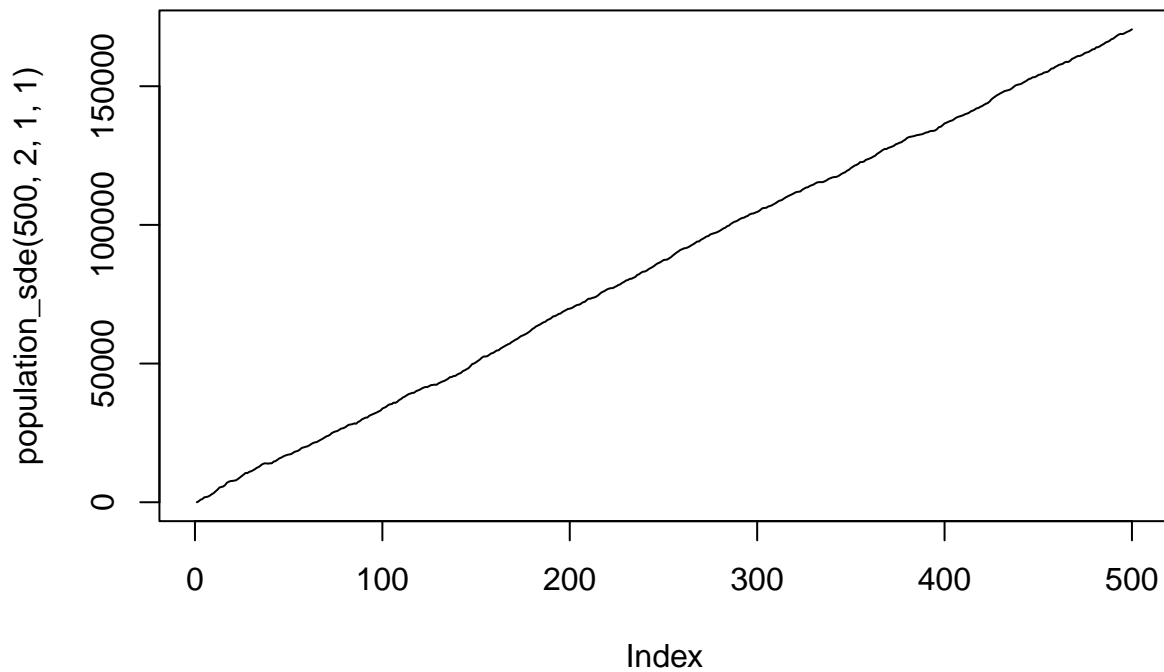
```
## Warning: Removed 169750 rows containing missing values (`geom_line()`).
```



##Study the random trip simulators

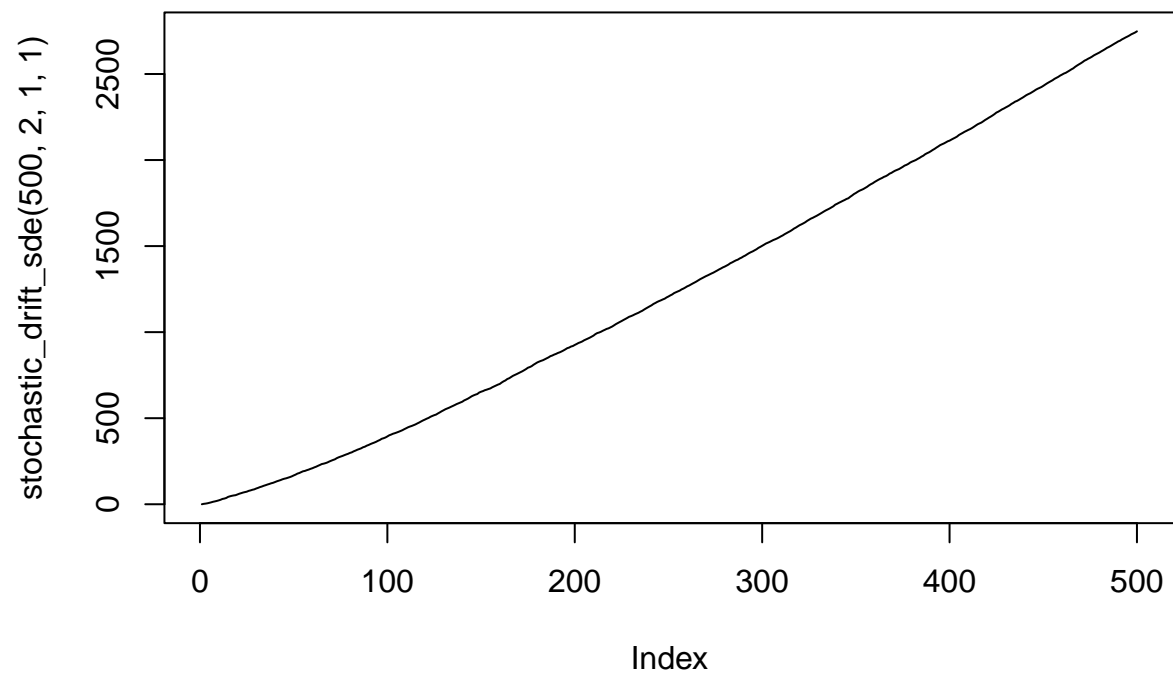
population\_sde simulate the arrival time at every edge. The result may look close to a straight line points top right. Input : n is # of edges, mu sigma are the global estimated mean and standard error of the speed. Delta is the difference in standard error estimation.

```
plot(population_sde(500,2,1,1),type='l')
```

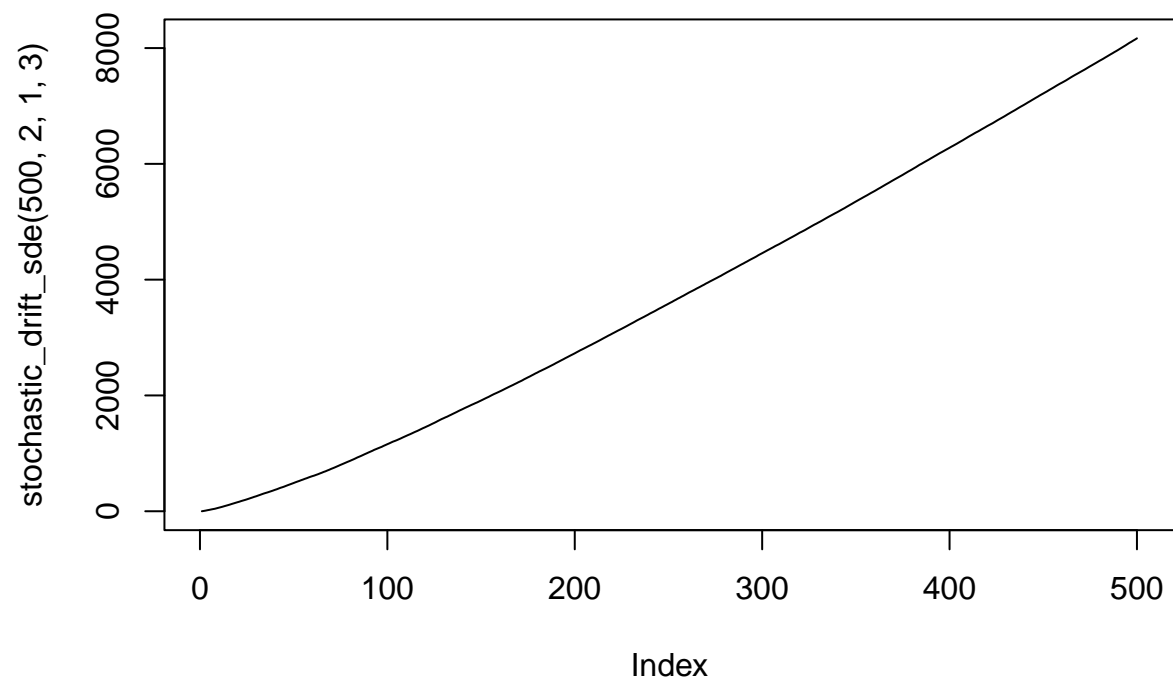


stochastic\_drift\_sde simulates the arrival time at every edge. When  $n$  is large,  $(1+t[i-1]/(1+i*\delta))$  will be more close to 1. the new  $t$  will depends less on the previous simulated value  $t$ . The direction of the line will be more close to a steady slop.

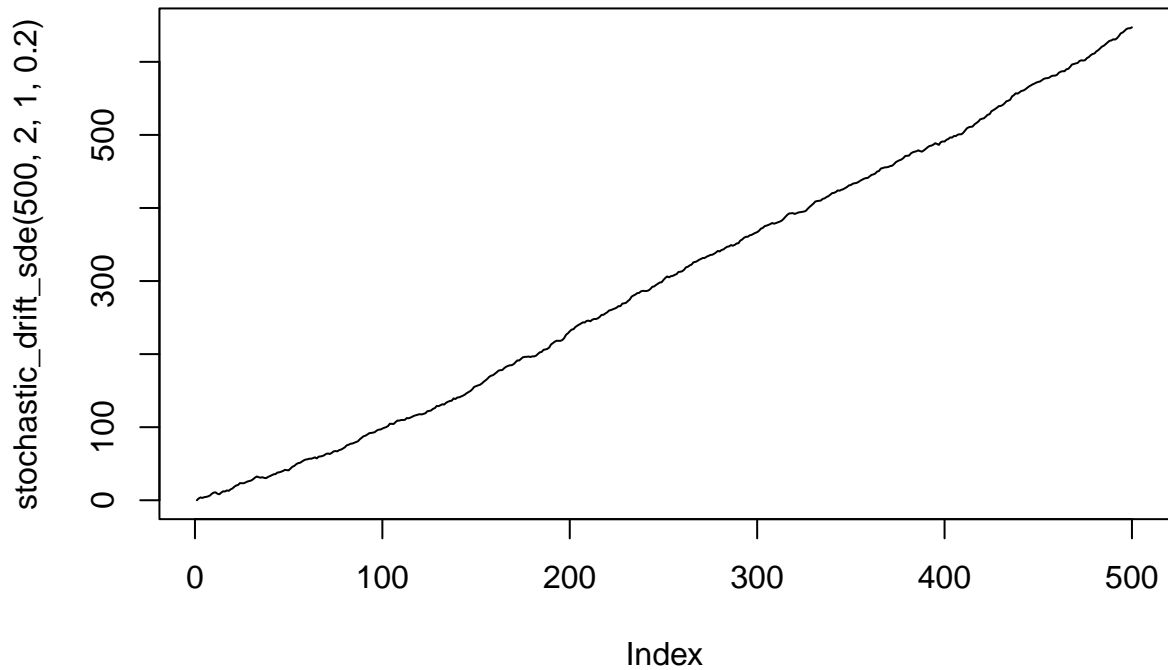
```
plot(stochastic_drift_sde(500,2,1,1),type='l')
```



```
plot(stochastic_drift_sde(500,2,1,3),type='l')
```



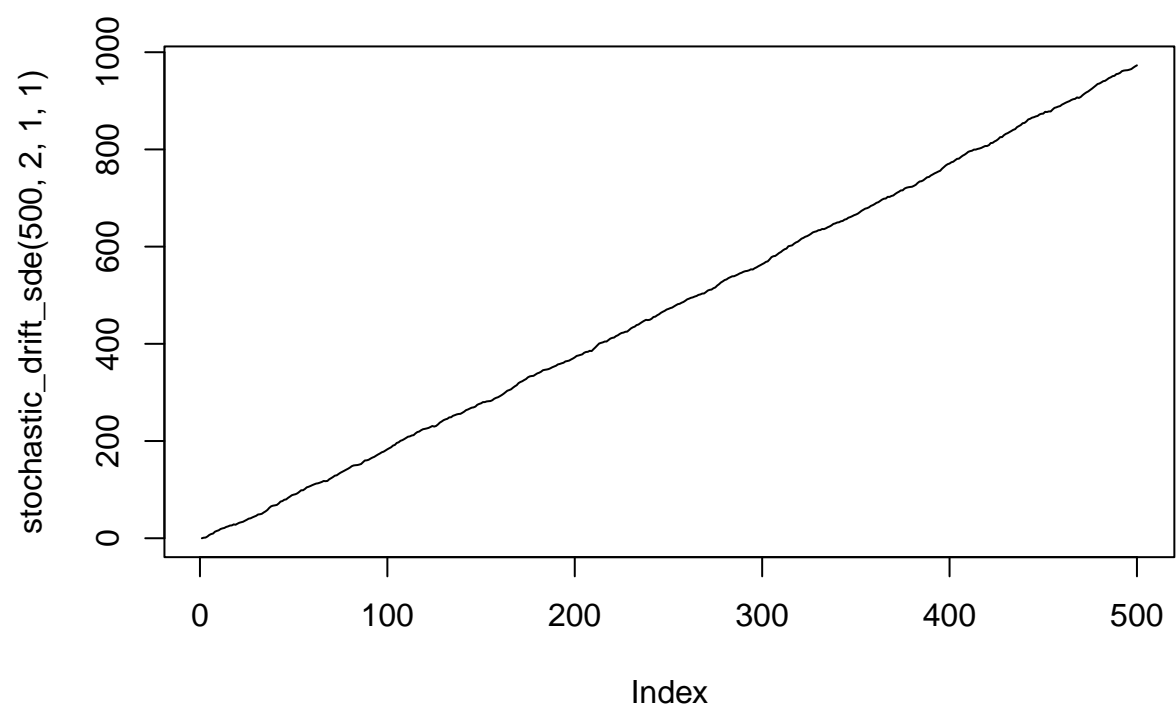
```
plot(stochastic_drift_sde(500,2,1,0.2),type='l')
```



There is another version of `stochastic_drift_sde`. When `n` is large, `lambda` will be more close to 1. the new `t` will depend on the previous simulated value `t`. When `delta` is large, this simulator behave like exponential functions.

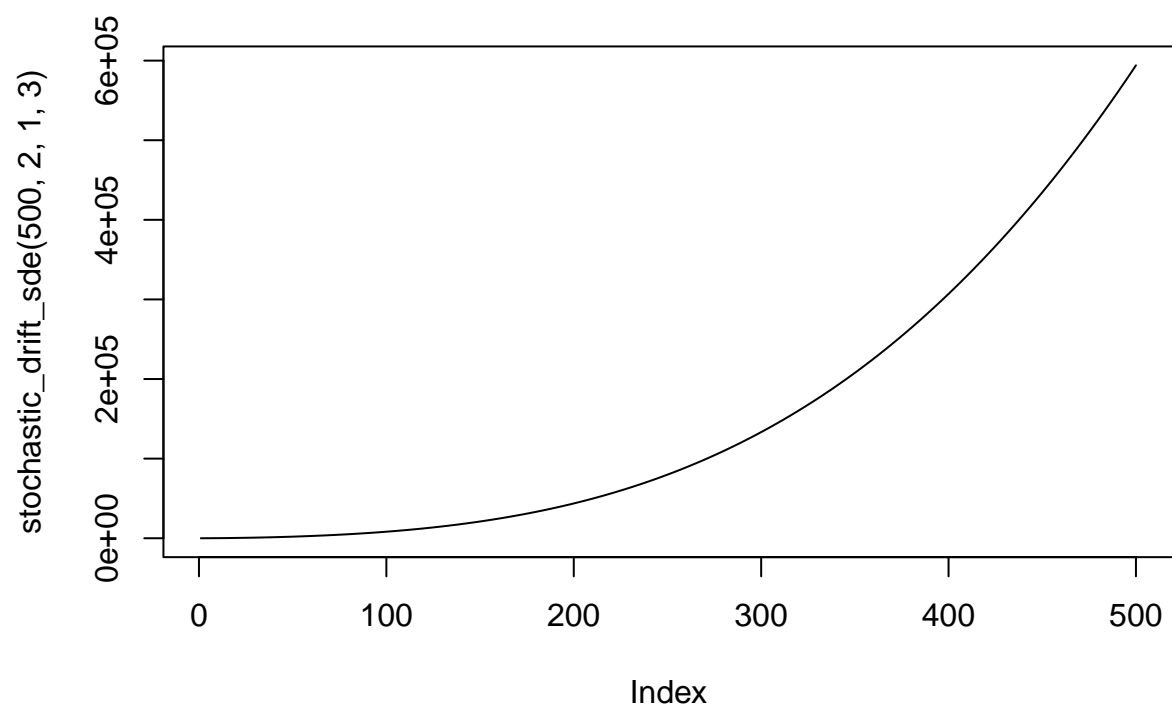
```
stochastic_drift_sde <- function(n,
                                mu=mu,
                                sigma=sigma,
                                delta=delta) {
  B = rnorm(n , 0, sqrt(delta))
  t = numeric(n)
  for(i in 2:n) {
    lambda = exp(0.01*i)/(1+exp(0.01*i))
    mu_avrg = mu * (1-lambda) + (t[i-1]/(i-1)) * lambda
    t[i] = t[i-1] + mu_avrg * delta + sigma * B[i]
  }
  t
}
```

```
plot(stochastic_drift_sde(500,2,1,1),type='l')
```

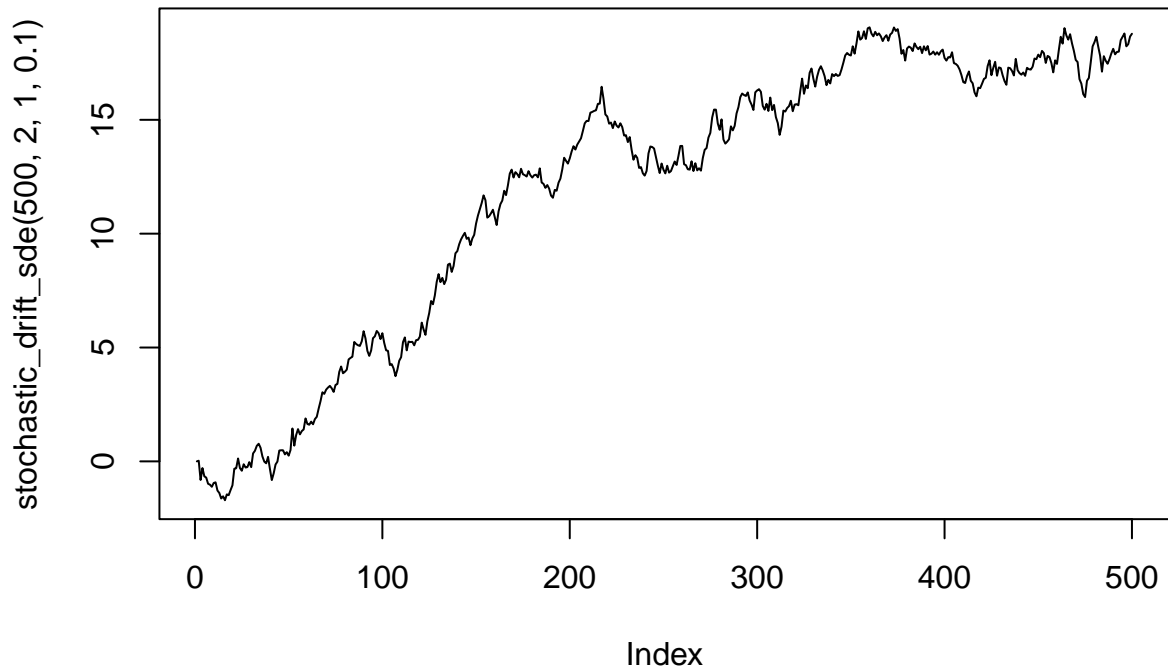


```
plot(stochastic_drift_sde(500,2,1,3),type='l')
```



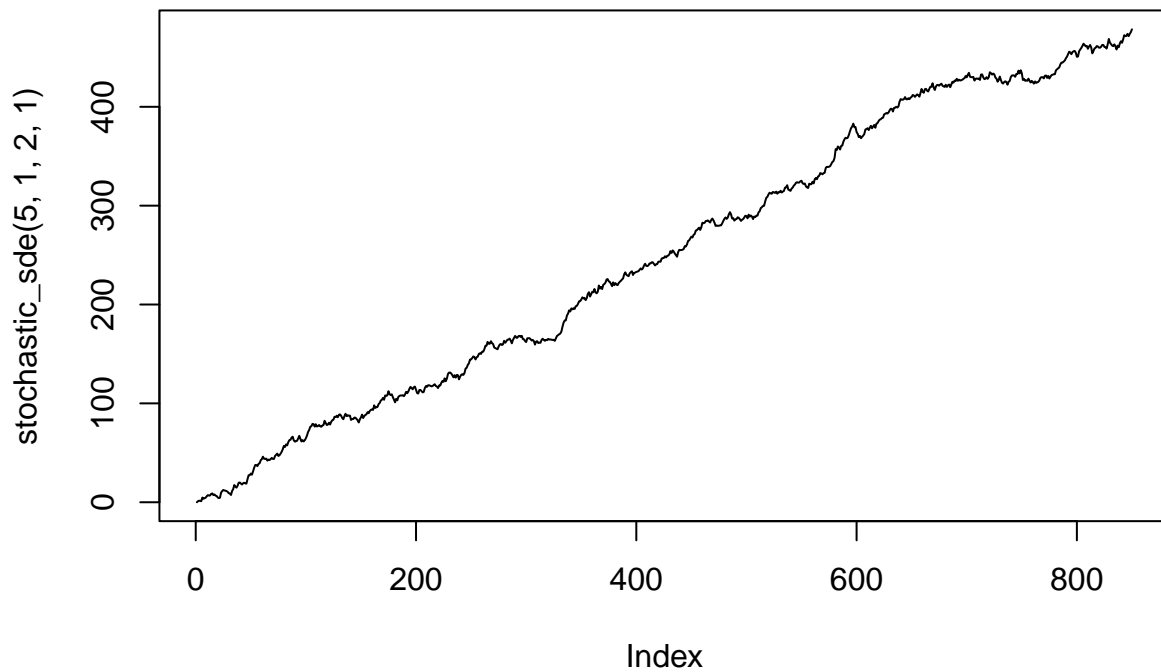


```
plot(stochastic_drift_sde(500,2,1,0.1),type='l')
```



This simulator will jump to the  $\mu$  in the second observation, and never become lower than  $\mu$ . Every edges are expected to increase by half of the  $\mu$ . Delta does not make any change. The resulting length is multiplied by 170, as the `avg_seg_length` in `sde_sim.R` is set by 170. I suggest to modify this process slightly on the line “ $t[i] = \max(t[i-1] + (t[i-1])/(i) + \text{sigma} * B[i], \mu)$ ”, the new one ensures there is no backward movement. Also, I added a result vector to make sure the resulting length is the same as  $n$ .

```
plot(stochastic_sde(5,1,2,1),type='l')
```



```
length(stochastic_sde(5,2,1,1))
```

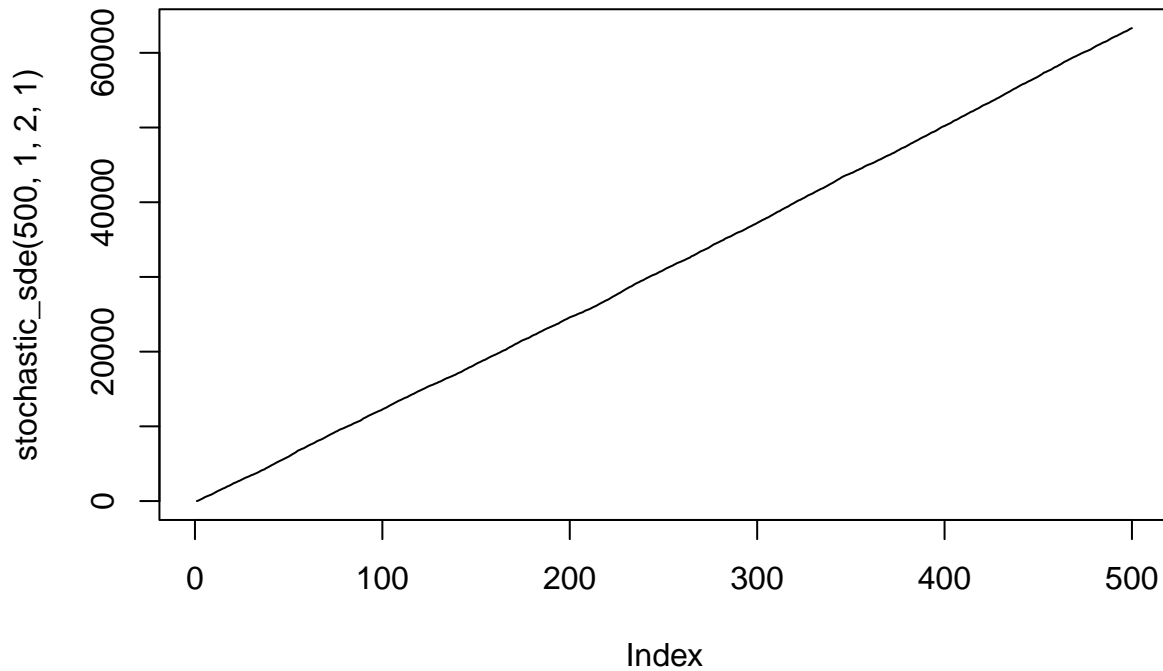
```
## [1] 850
```

```
stochastic_sde <- function(n,
                           mu=mu,
                           sigma=sigma,
                           delta=delta) {
  m = floor(avg_seg_length) * n-1
  mu <- mu*avg_seg_length/2
  B = rnorm(m , 0, 1)
  t = numeric(m)
  result = c(0)
  for(i in 2:m) {
    t[i] = t[i-1] +(t[i-1])/(i) + sigma * B[i]
    if(i%%floor(avg_seg_length)==0&
      result[length(result)]<=t[i]-mu)result<-c(result,t[i])
    else if (i%%floor(avg_seg_length)==0){
      result<-c(result,result[length(result)]+mu)
      t[i]<-result[length(result)]
    }
  }
  result
}
```

```

}
plot(stochastic_sde(500,1,2,1),type='l')

```



I modified this simulator, as the sqrt function cannot accept negative input so sometime produce NaN. I added a absolute value to make sure it works. The variance increases according to t, meaning no matter where the t is, the probability of t back to small persists. This simulator seems more suitable for financial data.

```

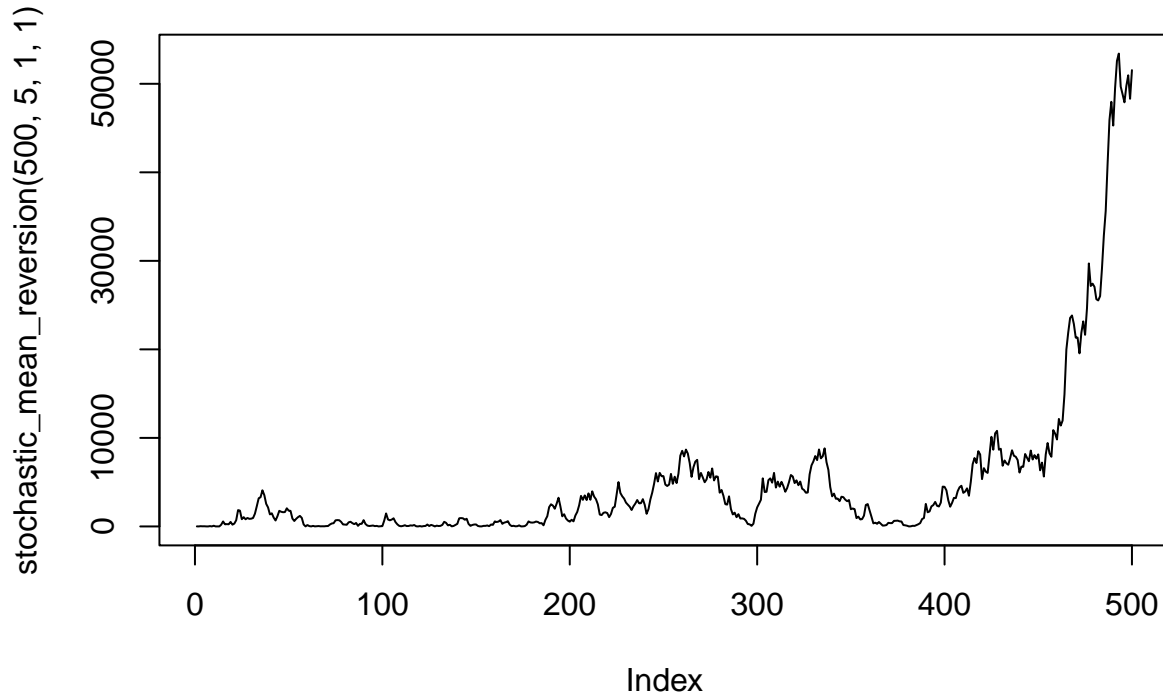
stochastic_mean_reversion <- function(n,
                                     mu=mu,
                                     sigma=sigma,
                                     delta=delta) {

  m = floor(avg_seg_length * n)
  B = rnorm(m , 0, 1)
  t = numeric(m)
  result = c(mu)
  mu_function<-function(x,y, alpha=0.1) {
    alpha * (x - y)
  }

  for(i in 2:(m-1)) {
    #t[i] = max(t[i-1] + mu_function(mu, t[i-1]/i, 0.05) + sigma * sqrt(t[i-1]) * B[i], mu)
    t[i] = t[i-1] + mu_function(mu, t[i-1]/i, 0.05) + sigma * sqrt(abs(t[i-1])) * B[i]
    if(i%%floor(avg_seg_length)==0)result<-c(result,t[i])
  }
  result
}

```

```
}
plot(stochastic_mean_reversion(500,5,1,1),type='l')
```

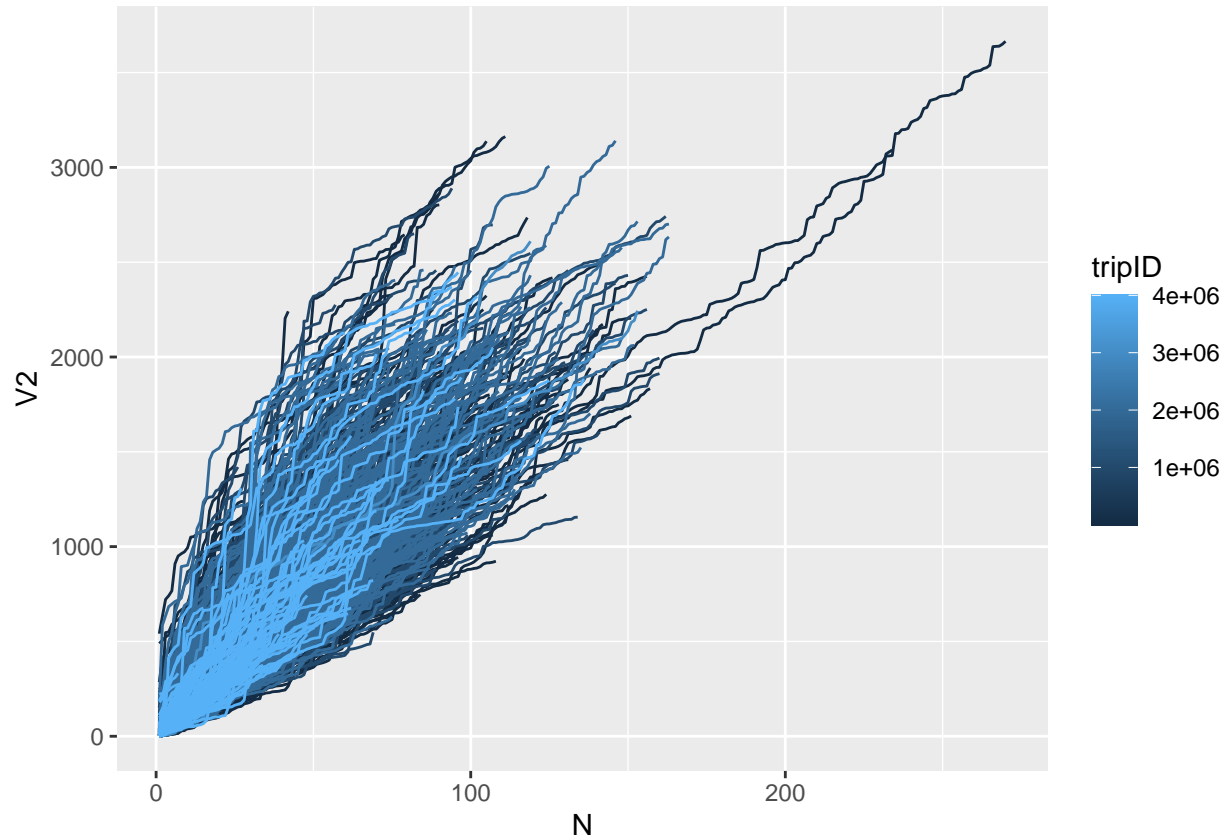


```
## Calculate trip parameters.
```

First, randomly selected 1000 trips, then calculate the estimated global mean  $\hat{\mu}$  and its unconditional variance  $\hat{V}(n^{-1}\tau_{\rho})$ . Their formulas are given by formula (16) and formula (19) of the paper Predictive Inference of Travel Time on Transportation Networks.

```
id = sample(unique(trips$tripID),1000)
sampled_trips = trips[tripID %in% id]
a = sampled_trips[, .(N=1:.N, cumsum(duration_secs)),tripID]
groupmean <- sampled_trips[, mean(duration_secs),tripID]$V1
muhat = mean(groupmean)
vhat = var(groupmean)
trip_length <- sampled_trips[, length(duration_secs),tripID]$V1
```

```
ggplot(a, aes(x = N, y = V2)) +
  geom_line(aes(group=tripID,color=tripID))
```



```
#b = trips[, .(N=1:.N, duration_secs),tripID]
#ggplot(b, aes(x = N, y = duration_secs)) +
#  theme_light() +
#  geom_line(aes(group=tripID),color="grey")
c = trips[, .(N=1:.N, cummean(duration_secs)),tripID]
c=c[, ave_time := mean(cummean(V2)[1:.N]), N]
ggplot(c, aes(x = N, y = V2)) +
  theme_light() +
  geom_line(aes(group = tripID, linetype = "single trips (time average)",color="single trips (time average)"),size=0.6) +
  geom_line(aes(y = ave_time, linetype = "average over trips per edge"),size=0.6) +
  labs(y = "Average travel time per edge (seconds)",
       x = "Number of edges(n)") +
  geom_hline(aes(yintercept = muhat, linetype = "mu"),size=0.6) +
  coord_cartesian(xlim = c(0, 120), ylim = c(0, 200)) +
  scale_color_manual(name = "Legend",values = c("mu" = "black","single trips (time average)" = "grey","average over trips per edge" = "blue")) +
  scale_linetype_manual(
    name = "Random sample from real data",
    values = c("mu" = "solid",
               "single trips (time average)" = "dotted",
               "average over trips per edge" = "dashed"),
    labels = c("mu" = expression(hat(mu)),
               "single trips (time average)" = "single trips (time average)",
               "average over trips per edge" = "average over trips per edge")) +
  theme(legend.position = c(0.95, 0.95),
        legend.justification = c(1, 1),
        legend.text.align = 0,
```

```

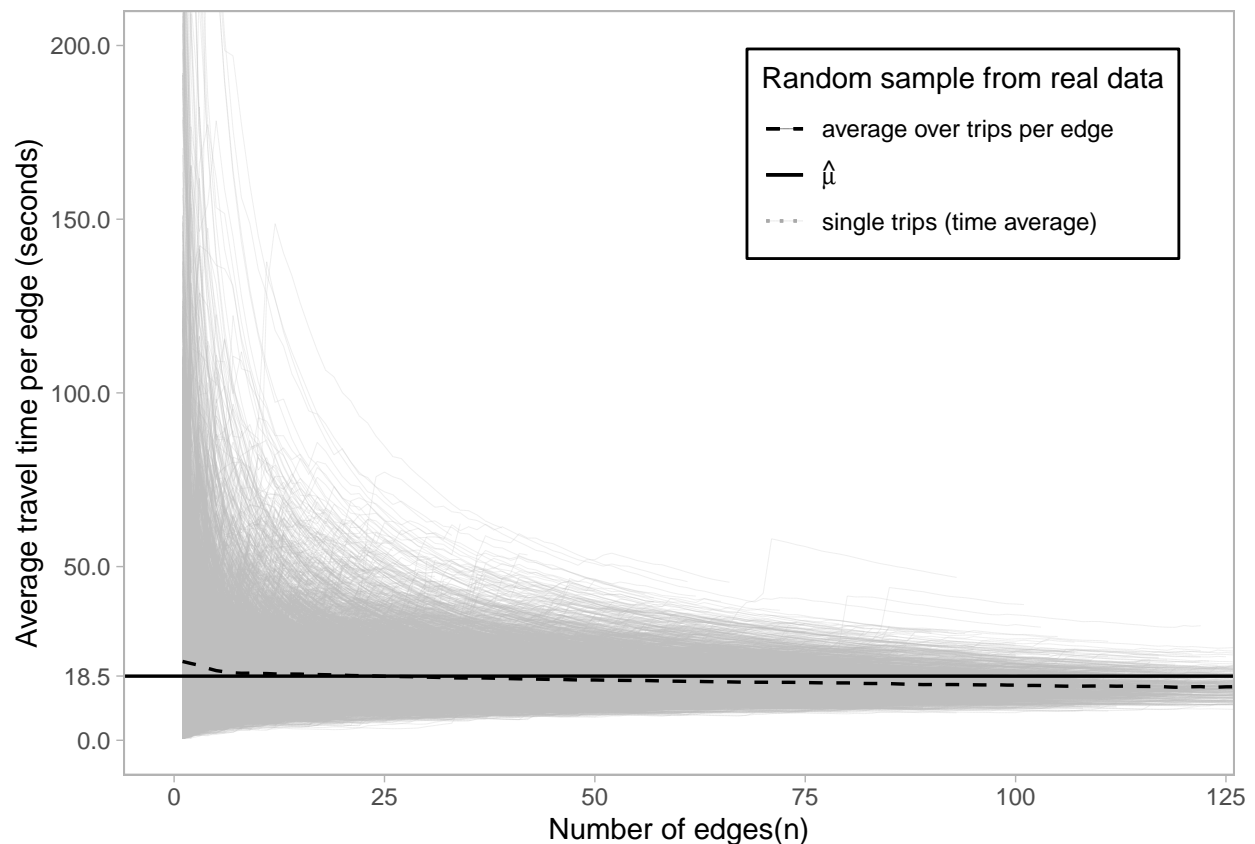
legend.background = element_rect(color = "black", fill = "white"),
panel.grid.major = element_blank(),
panel.grid.minor = element_blank()) +
scale_y_continuous(breaks=c(seq(0,200,50),round(muhat,1)))+
guides(linetype = guide_legend(
  override.aes = list(color = c("black", "black", "darkgrey"),
    size = c(0.6, 0.6, 0.4))))

```

```

## Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
## i Please use `linewidth` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.

```



simulation using pop\_sed. the data contains 1000 trips, trips length are the same as the real data. The trip simulator use the same estimated mu and variance in real sampled data.

```

n=trip_length
index =1;tripID=c();duration=c()
while (index<=1000) {
  if(n[index]<2)n[index]<-2
  tripID<-c(tripID,rep(index,(n[index])))
  duration<-c(duration,diff(population_sde(n[index]+1,muhat/avg_seg_length,vhat/avg_seg_length,1)))
  index<-index+1
}

```

```

simulated_trips <- data.table(tripID,duration)
d = simulated_trips[, .(N=1:.N, cummean(duration)),tripID]
d=d[, ave_time := mean(cummean(V2)[1:.N]), N]
simulated_groupmean <- simulated_trips[, mean(duration),tripID]$V1
simulated_mu = mean(simulated_groupmean)

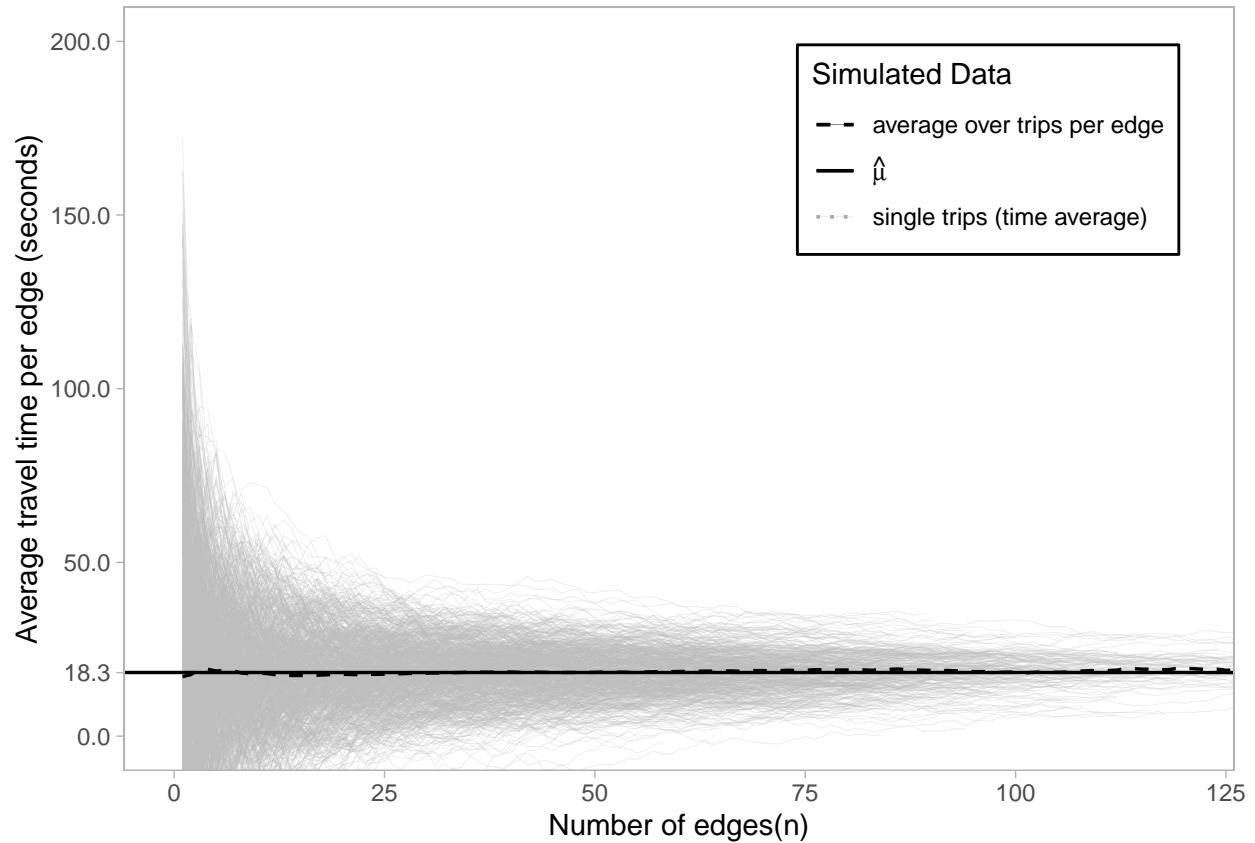
```

```

dataploter<-function(data,title="Simulated Data"){
  ggplot(data, aes(x = N, y = V2)) +
    theme_light() +
    geom_line(aes(group = tripID, linetype = "single trips (time average)",color="single trips (time average)"),size=0.6) +
    geom_line(aes(y = ave_time, linetype = "average over trips per edge"),size=0.6) +
    labs(y = "Average travel time per edge (seconds)",
         x = "Number of edges(n)") +
    geom_hline(aes(yintercept = simulated_mu, linetype = "mu"),size=0.6) +
    coord_cartesian(xlim = c(0, 120), ylim = c(0, 200)) +
    scale_color_manual(name = "Legend",values = c("mu" ="black","single trips (time average)" ="grey","average over trips per edge" ="darkgrey")) +
    scale_linetype_manual(
      name = title,
      values = c("mu" = "solid",
                 "single trips (time average)" = "dotted",
                 "average over trips per edge" = "dashed"),
      labels = c("mu" = expression(hat(mu)),
                 "single trips (time average)" = "single trips (time average)",
                 "average over trips per edge" = "average over trips per edge")) +
    theme(legend.position = c(0.95, 0.95),
          legend.justification = c(1, 1),
          legend.text.align = 0,
          legend.background = element_rect(color = "black", fill = "white"),
          panel.grid.major = element_blank(),
          panel.grid.minor = element_blank()) +
    scale_y_continuous(breaks=c(seq(0,200,50),round(simulated_mu,1)))+
    guides(linetype = guide_legend(
      override.aes = list(color = c("black", "black", "darkgrey"),
                             size = c(0.6, 0.6, 0.4)))))}
dataploter(d)

```

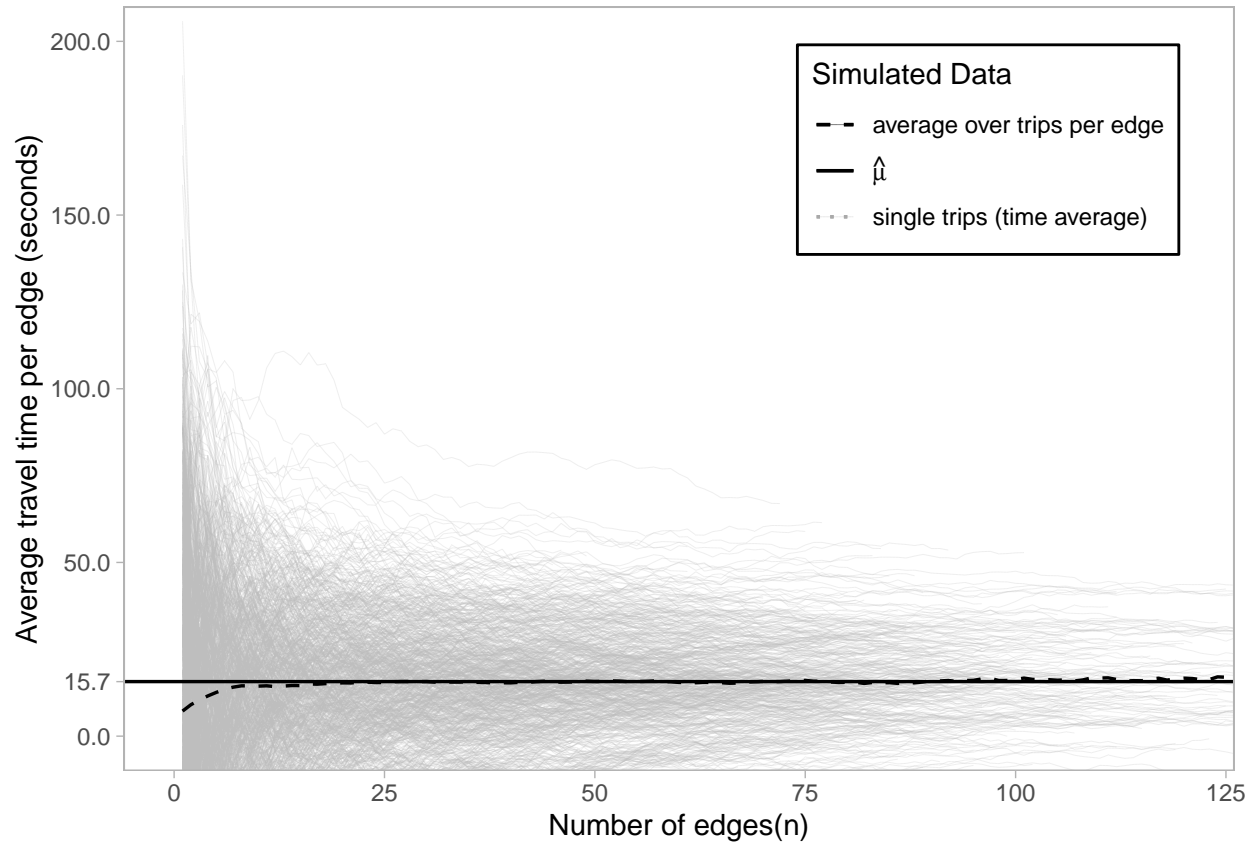




```

n=trip_length
index =1;tripID=c();duration=c()
while (index<=1000) {
  if(n[index]<2)n[index]<-2
  tripID<-c(tripID,rep(index,(n[index])))
  duration<-c(duration,diff(stochastic_drift_sde(n[index]+1,muhat,vhat,1)))
  index<-index+1
}
simulated_trips <- data.table(tripID,duration)
d = simulated_trips[, .(N=1:.N, cummean(duration)),tripID]
d=d[, ave_time := mean(cummean(V2)[1:.N]), N]
simulated_groupmean <- simulated_trips[, mean(duration),tripID]$V1
simulated_mu = mean(simulated_groupmean)
dataploter(d)

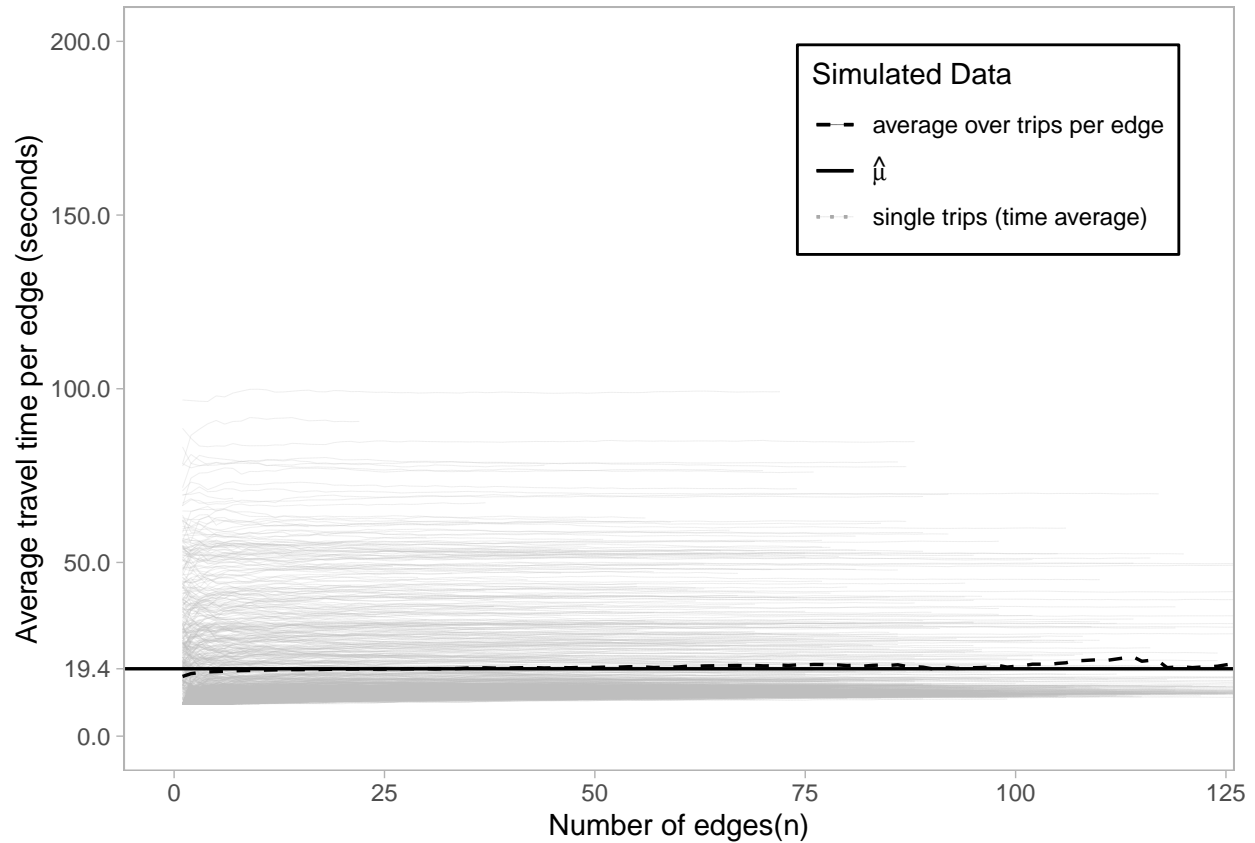
```



```

n=trip_length
index =1;tripID=c();duration=c()
while (index<=1000) {
  if(n[index]<2)n[index]<-2
  tripID<-c(tripID,rep(index,(n[index])))
  duration<-c(duration,diff(stochastic_sde(n[index]+1,muhat/avg_seg_length,vhat/avg_seg_length,1)))
  index<-index+1
}
simulated_trips <- data.table(tripID,duration)
d = simulated_trips[, .(N=1:.N, cummean(duration)),tripID]
d=d[, ave_time := mean(cummean(V2)[1:.N]), N]
simulated_groupmean <- simulated_trips[, mean(duration),tripID]$V1
simulated_mu = mean(simulated_groupmean)
dataploter(d)

```



```

n=trip_length
index =1;tripID=c();duration=c()
while (index<=1000) {
  if(n[index]<2)n[index]<-2
  tripID<-c(tripID,rep(index,(n[index])))
  duration<-c(duration,(stochastic_mean_reversion(n[index],muhat/avg_seg_length,vhat/avg_seg_length,1)))
  index<-index+1
}
simulated_trips <- data.table(tripID,duration)
d = simulated_trips[, .(N=1:.N, cummean(duration)),tripID]
d=d[, ave_time := mean(cummean(V2)[1:.N]), N]
simulated_groupmean <- simulated_trips[, mean(duration),tripID]$V1
simulated_mu = mean(simulated_groupmean)
dataploter(d)

```

