OPF model

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1 Optimal Power Flow Formulation

1.1 Decision Variables

$$
\begin{aligned}
v_i &: \quad \text{Voltage magnitude at bus } i,\ v_i \geq 0, \\
\theta_i &: \quad \text{Voltage phase angle at bus } i,\ -\frac{\pi}{2} \leq \theta_i \leq \frac{\pi}{2}, \\
P_g, Q_g &: \quad \text{Active and reactive power generation at generator } g.
\end{aligned}
$$

1.2 Objective Function

The objective in the model minimizes the total active power generation:

$$\min\ f = \sum_{g=1}^{N_G} P_g. \tag{1}$$

Remark: In this version, the objective is simply the sum of real power generation, equivalent to minimizing total generation dispatch without explicit cost coefficients.

1.3 Active Power Balance

For each bus $i \in \{1, \ldots, N_B\}$,

$$\sum_{g \in \mathcal{G}(i)} P_g - P_{d,i} = v_i \sum_{j \in \mathcal{N}(i)} v_j \Big(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \Big), \tag{2}$$

where $\mathcal{G}(i)$ is the set of generators connected to bus i, and $\mathcal{N}(i)$ is the set of buses adjacent to bus i.

1.4 Reactive Power Balance

For each bus $i \in \{1, \ldots, N_B\}$,

$$\sum_{g \in \mathcal{G}(i)} Q_g - Q_{d,i} = v_i \sum_{j \in \mathcal{N}(i)} v_j \Big(G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \Big). \tag{3}$$

1.5 Voltage Magnitude Limits

$$V_i^{\min} \leq v_i \leq V_i^{\max}, \qquad \forall i \in \{1, \ldots, N_B\}. \tag{4}$$

1.6 Generator Operating Limits

For each generator $g \in \{1, \ldots, N_G\}$,

$$P_g^{\min} \leq P_g \leq P_g^{\max}, \tag{5}$$

$$Q_g^{\min} \leq Q_g \leq Q_g^{\max}. \tag{6}$$

1.7 Branch Flow Limits

For each branch $\ell \in \{1, \ldots, N_L\}$ connecting buses i and j, the apparent power limit is modeled as:

$$P_\ell^2 + Q_\ell^2 \leq I_{\ell,\max}^2, \tag{7}$$

where P_ℓ and Q_ℓ denote the active and reactive power flow of branch ℓ, and $I_{\ell,\max}$ is the current limit.

1.8 Slack Bus Reference

To eliminate the rotational degree of freedom in voltage angles, the slack bus angle is fixed:

$$\theta_{i_{\text{slack}}} = 0. \tag{8}$$

1.9 Complete Model

$$\begin{aligned} \min_{v_i, \theta_i, P_g, Q_g} \; & \sum_{g=1}^{N_G} P_g \\ \text{s.t. } & (2) - (8). \end{aligned}$$

1.10 Notation Summary

- G_{ij}, B_{ij}: real and imaginary parts of the bus admittance matrix.
- $P_{d,i}, Q_{d,i}$: active and reactive power demand at bus i.
- $V_i^{\min}, V_i^{\max}$: lower and upper voltage magnitude limits.
- $I_{\ell,\max}$: branch current limit.
- $\mathcal{G}(i)$: set of generators connected to bus i.
- $\mathcal{N}(i)$: set of neighbor buses of bus i.

References