Authors' Response to Reviewer 1

Comment 1:

Grammatical and spelling errors are still observed. Please check and fix carefully.

Response:

We apologize for our oversight regarding the grammatical and spelling errors, and we have thoroughly reviewed the manuscript and rectified them. Additionally, the manuscript has undergone revisions by a professional organization to ensure the accuracy of grammar and spelling.

Comment 2:

Since authors stated that there is no existing works on the MAC determination of RBSs that they could compare their solution with. So how the authors validate their works and their results.

Response:

To address the concern raised by the reviewer, we have incorporated a comprehensive discussion in the Case study section, specifically titled "Result validation". In this part, we thoroughly examine the correctness of the results obtained through the proposed greedy algorithm from two distinct perspectives: circuit analysis and validation against the brute-force algorithm. Additionally, we have included a comparison between the proposed greedy algorithm and two heuristic algorithms, namely simulated annealing and genetic algorithm, in the revised manuscript. There is a reference (No. 35 in the manuscript) claiming that the path selection problem under consideration may be NP-hard. Therefore, it is reasonable to compare the performance of the proposed greedy algorithm with these heuristic approaches, which are commonly employed to tackle NP-hard problems. Remarkably, our results demonstrate that the proposed

greedy algorithm consistently achieves the same or superior outcomes compared to the heuristic algorithms.

Here is the specific modification we made in the "Discussion":

> The correctness of the outcomes provided by the proposed greedy algorithm will now be discussed from two perspectives: circuit analysis and validation against the brute-force algorithm.The result of the four-battery RBS structure shown in Fig. 4c is determined as an example.When B_1 and B_2 or B_3 and B_4 are connected in parallel, the RBS produces the maximum current, which is $\eta = 2$ (i.e., twice the current output of a single battery in the RBS).Adding more batteries to the main circuit only creates a series structure and does not improve the MAC.Therefore, the switch-control scheme provided in Tab. 4 maximizes the RBS output current.The brute-force method, which examines all possible switch states, yields the same η.This indicates that the proposed greedy algorithm successfully identifies the MAC among all the potential reconfigured structures.

And here is one of the comparisons between the proposed greedy algorithm and the heuristic algorithms in the "Result" section:

> To verify and compare the proposed greedy algorithm, we also used the brute-force algorithm, which iterates through all possible switch states, and the heuristic algorithms (SA and GA) to calculate the MACs of the same RBSs.The final results of the brute-force algorithm are the same as those of the greedy algorithm and are shown in Tabs. 2, 3, and 4.However, the brute-force algorithm counts all possible switch states, which equates to 2^{15}, 2^{13}, and 2^{19} structures, respectively.The temporal evolutions of the objective values of the two heuristic algorithms during the iteration process are shown in Figs. 7a, 7b, and 7c, respectively, and compared with the proposed greedy algorithm.Compared with the SA and GA, the proposed greedy algorithm identifies the correct results within fewer iteration steps.

For more information on the other compared results and details, please refer to the revised "Case Study" section.

We hope that the reviewer is satisfied with the modifications made to the manuscript.

> **Comment 3:**
>
> Refer to the mentioned article as previously stated in comment 8 of the first revision.

Response:

We have reconsidered and modified the corresponding content about the analysis on MAC problem in the introduction. The mentioned article becomes important to this paper, therefore, we have cited it in the introduction and accept the reviewer's suggestion.

We have reevaluated and revised the relevant content regarding the analysis of the MAC problem in the introduction. The mentioned article holds significance in this paper, hence we have referenced it in the introduction and acknowledged the suggestion made by the reviewer.

Here is the specific modification we made in the introduction:

> However, few studies have directly determined the MACs of RBSs, primarily due to the complexity arising from reconfiguration.In the field of computer science, there is a similar problem with scheduling tasks on dynamically reconfigurable hardware with limited resources and task interdependencies. This problem is analogous to the determination of the MAC and a corresponding solution has been proposed [33, 34].However, dealing with the structural characteristics and circuit equations of RBSs is challenging for this method.

Authors' Response to Reviewer 2

Comment 1:

The latest related works need to be reviewed carlfully please, expecially the works published in recent year 2022,2023.

Response:

We concur with the reviewer's assertion that it is important for our paper to undergo a thorough review of the most recent relevant literature. We have carefully scrutinized the pertinent works from the past five years and made appropriate revisions to the manuscript. In the "Introduction" section, we provide an overview of the challenges posed by the complex RBS structures in hardware design. Furthermore, the estimation and control of the system state of RBSs have been receiving increasing attention in recent time. Consequently, we have chosen several corresponding methodologies aimed at optimizing the system's performance.

Here is the specific modification in the introduction:

These complex structures between batteries and switches provide flexibility to RBSs but also pose challenges in hardware design.During the reconfiguration process, current deviation and fluctuation may occur. Specifically, when the system switches from series to parallel connection, a circulating current between parallel cells can be triggered due to a voltage imbalance [20]. Failure to fully consider this issue during the design of RBSs can result in damage to the batteries, switches, and wires.For example, Engelhardt et al. [21] applied an RBS to a fast-charging scenario with adaptive cell switching to balance cell states while adhering to voltage requests. However, the switching of batteries leads to intolerable current variations.To address this problem, Han et al. [22] derived an analytical expression for the maximum switch current during battery system reconfiguration. This analytical expression aids in the selection of switches and supports general hardware design.Recently, increasing attention has been paid to the estimation

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and control of RBS system states, and several approaches have been proposed to optimize the performance of these systems.State estimation, which is an essential technology in traditional battery management systems, serves as the foundation for system control and holds great potential in the context of RBSs [23].Couto et al. [24] introduced a partition-based unscented Kalman filter to estimate the state of a large-scale RBS, utilizing an enhanced reduced-order electrochemical model.Kersten et al. [25] utilized the balancing current of neighboring cells in parallel operation to determine the battery impedance, thereby obtaining information about the state of health and power capability of the RBS.Schmid et al. [26] further leveraged the reconfigurable nature of the system to actively diagnose faults, employing an algorithm that changes the system structure to enhance the fault isolability.Another active research area is the development of effective control strategies for RBSs to achieve optimal performance, including improved stability [27] and efficiency [28].Han et al. [29] proposed a near-fastest battery balancing algorithm to minimize the time required for battery charge equalization.Liu et al. [30] also proposed a scheme for maximizing capacity utilization based on a path planning algorithm, aiming to enhance the battery consistency within the system.To break through the bottleneck of the potential short-circuit paths increasing exponentially with the RBS scale, Chen et al. [31] proposed a systematic approach based on sneak circuit theory.They conducted a comprehensive analysis of all paths between the cathode and anode of each battery in the RBS, identifying paths that consist only of switches as short-circuit paths for pre-checking before system reconfiguration.Artificial intelligence has also appeared in RBS management [32].The effectiveness of the deep reinforcement learning method has been validated in real-world RBSs [28].

Comment 2:

The greedy algorithm and the brute force algorithm are compared in this paper, but the advantages and disadvantages of this algorithm compared with other algorithms cannot be determined.

Response:

Thanks to the suggestion of the reviewer! We have revised the "Discussion" subsection and included a specific part to analyze the advantages and disadvantages of the proposed greedy algorithm in comparison to other algorithms. Based on our discussion and comparison, the proposed greedy algorithm demonstrates a significant advantage in terms of its effectiveness and efficiency. It is also capable of handling RBSs with diverse structures. However, this algorithm may encounter challenges when dealing with large-scale problems due to its exponential time complexity. Furthermore, the simplification of the derivation by assuming that all batteries are identical may introduce a slight bias to the MAC due to variations in open-circuit voltage u_b and internal resistance r_b in reality. We have also provided a solution to address this issue.

The relevant content has been added to the "Pros and cons analysis" part within the "Discussion" subsection. Once again, we appreciate this constructive comment from the reviewer.

Comment 3:

This paper mainly applies to the four-battery system, but the usability of other structures of RBS should also be discussed.

Response:

Thanks for the valuable feedback. We have thoroughly considered the comment made by the reviewer. In response, we have supplemented the case study with a series of experiments on RBSs with variant batteries. Overall, the proposed greedy algorithm

has been applied to RBSs with three different structures, variant batteries, and scenarios involving random isolated batteries. The correctness and efficiency of the proposed greedy algorithm have been verified through the comparison with other algorithms.

The relevant content has been added to the "Case study" section. We hope it can address the reviewer's concerns.