

# Using a Directed Graph Model and Greedy Algorithm to Determine the Maximum Allowable Current in a Reconfigurable Battery System

30577<sup>1</sup>, Binghui Xu<sup>1†</sup>, Guangbin Hua<sup>1†</sup>, Cheng Qian<sup>1\*</sup>, Quan Xia<sup>1,2</sup>, Bo Sun<sup>1</sup>, Yi Ren<sup>1</sup>, and Zili Wang<sup>1</sup>

<sup>1</sup>School of Reliability and Systems Engineering, Beihang University, Beijing, 100191, China

<sup>2</sup>School of Aeronautic Science and Engineering at Beihang University, Beijing, China

\*Address correspondence to: cqian@buaa.edu.cn

<sup>†</sup>These authors contributed equally to this work.

## Abstract

Reconfigurable ~~Battery Systems~~ battery systems (RBSs) present a promising alternative to traditional battery systems due to their flexible and dynamically changeable topological structure ~~subjected to that can be adapted to different~~ battery charging and discharging strategies. During ~~the operation of the RBS~~, ~~the Maximum Allowable Current~~ RBS operation, the maximum allowable current (MAC) of the system that ensures that each battery's current remains within a safe range ~~is~~ is a critical indicator to guide the system's ~~reconfiguring control~~ reconfiguration, ensuring its safety and reliability. ~~In this paper, we firstly propose a calculation method for~~ This paper proposes a method to calculate the MAC of arbitrary ~~RBS~~ RBSs using a greedy algorithm in conjunction with a directed graph model of the RBS. By introducing the shortest path of the battery, the greedy algorithm transforms the enumeration of switch states in the brute-force algorithm into the combination of the shortest paths, which greatly increases the efficiency with which the MAC is determined. The directed graph model, based on the equivalent circuit, provides a specific method for calculating the MAC of a given structure. The proposed method is validated on two published 4-battery-RBSs and one with a more complex structure. The results are the same as those of the brute-force algorithm, but the proposed method significantly improves the computational efficiency ( $N_s 2^{N_s - N_b} \log_{10} N_b$  times faster than the brute force algorithm for a RBS with  $N_b$  batteries and  $N_s$  switches, theoretically). The main advantage of the proposed method is its ability to calculate the MAC of RBSs with arbitrary structures, even in scenarios with random isolated batteries.

# 1 Introduction

Battery ~~Energy Storage Systems~~ energy storage systems (BESSs) are extensively ~~employed~~ used in various applications, such as wind power plants and space power systems, to store and release high-quality electrical energy [1–5]. Typically, a BESS consists of numerous batteries interconnected by series-parallel circuitry to provide the required capacity storage. However, traditional BESSs, in which the batteries are connected in a fixed topology, ~~exhibit~~ suffer from a significant weakness in their worst battery due to the so-called cask effect. Moreover, if ~~this~~ the worst battery fails during operation, it ~~can~~ is highly likely to exacerbate the degradation of ~~other batteries with a high possibility~~ the other batteries, leading to reliability and safety issues [6–8]. These problems have become significant technical barriers in the development of new-generation space vehicles and need to be addressed [9].

Reconfigurable ~~Battery System (RBS)~~ battery systems (RBSs), which can dynamically switch as required to different circuit ~~topology configurations as required, is~~ topologies, are expected to solve ~~the above problem~~ this problem [10]. The ~~ability of~~ switching circuit helps to isolate unhealthy batteries, thereby improving the safety and reliability of the battery system. To illustrate the working principle of an RBS, we consider a typical RBS structure developed by Visairo [11] (Fig. 1a), which is taken as an example to show the reconfiguration process. In this structure, the batteries can be connected not only in series when the switches  $S_1$ ,  $S_5$ ,  $S_6$ ,  $S_7$ ,  $S_8$ ,  $S_9$ , and  $S_{13}$  are closed (Figure 1b), see Fig. 1b) but also in parallel when  $S_1$ ,  $S_2$ ,  $S_3$ ,  $S_4$ ,  $S_5$ ,  $S_9$ ,  $S_{10}$ ,  $S_{11}$ ,  $S_{12}$ , and  $S_{13}$  are closed (Figure Fig. 1c). Furthermore, when an unhealthy battery, for instance, the orange one  $B_3$  in Figure Fig. 1d, appears in the RBS, it can be isolated by opening its two adjacent switches (i.e.,  $S_4$  and  $S_{11}$ ), ensuring ~~the system still remains that the system remains in~~ a reliable working mode.

Recently, various types of RBSs with different flexibility and reconfigurability have been designed to meet application requirements. For example, Ci et al. [12] proposed an RBS structure that dynamically adjusts the battery discharge rate to fully exploit the available capacity of each battery. Jan's [13, 14] structures reconfigure structures with variant batteries in series to reach the (constantly changing) voltage requirements during electric vehicle charging. As shown in Fig. 1a, the structure proposed by Visairo et al. [11] changes the system's output voltage based on the load conditions, thereby reducing the power loss of the voltage regulator during the power supply process and improving the efficiency of energy use. Also, to enhance the energy efficiency of the system, Lawson et al. [15] and He et al. [16] proposed simplified structures that have fewer switches than Visairo's design. Kim et al. [17] improved the system's ability to recover from battery failures by introducing multiple ports into the structure.

The complex structure between batteries and switches gives RBSs flexibility but also creates challenges in the design and control of the system. Thus, several approaches to analyze the RBS structure and performance have been proposed to tackle these challenges. For instance, Han et al. [18] derived an analytical expression for the maximum switch current during battery system reconfiguration for a specific RBS structure. This helps guide the selection of switches and supports the design of RBS hardware. Chen et al. [19] proposed a systematic approach based on sneak circuit theory to fundamentally avoid the short-circuit problem of RBSs: They thoroughly analyzed

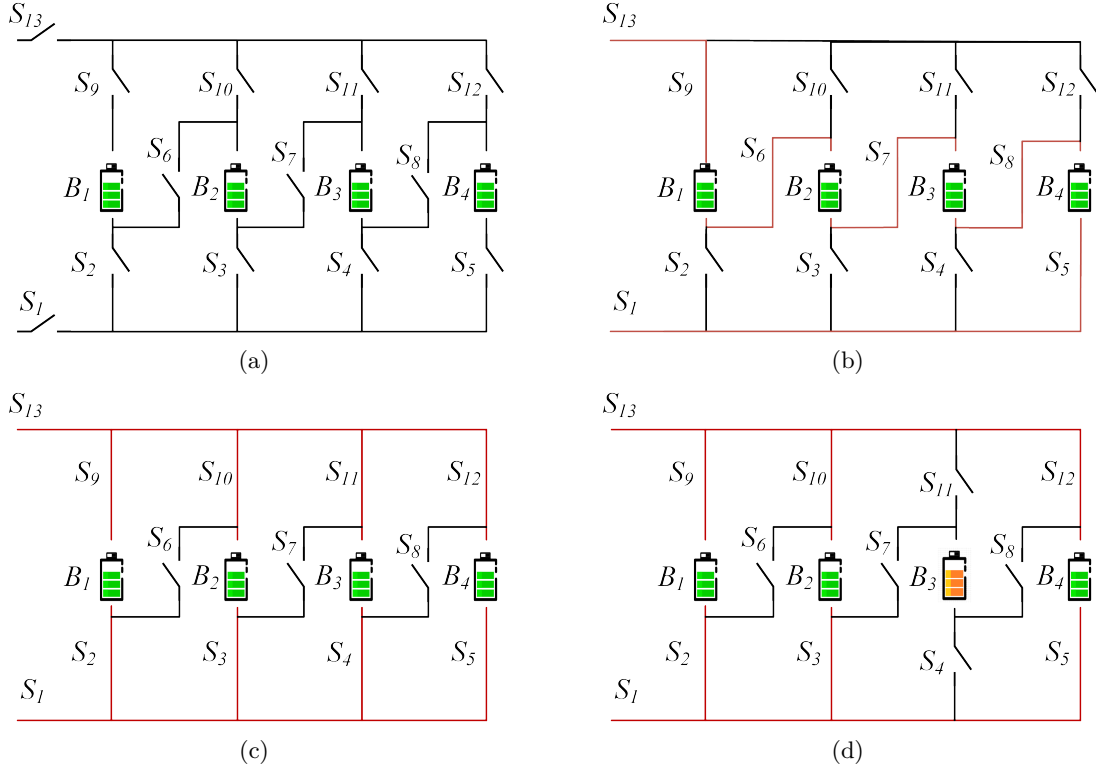


Figure 1: (a) The RBS structure proposed by Visairo[11], with all batteries in (b) series connection, (c) parallel connection, and (d) battery  $B_3$  isolated.

all paths between the cathode and anode of each battery in the RBS and identified paths that only contain switches as short-circuit paths for pre-checking before system reconfiguration.

In spite of the maximum switch current mentioned above, the maximum allowable current (MAC), defined as the maximum allowed current under the constraints of the battery cell, is another critical indicator of RBSs that needs to be evaluated during the design or control of the system. The MAC helps the designers assess whether the RBS meets the output current requirements and contributes to the formulation of appropriate and safe management strategies for the battery management system. Unfortunately, few studies have analyzed the RBS structure to determine the RBS MAC. An intuitive and straightforward method is to enumerate all possible switch states and calculate the output current of the system under each reconfigured structure. However, this method is inefficient and time-consuming, especially for RBSs with a large number of switches.

To solve this issue, this paper proposes an efficient method to evaluate the MAC of RBSs. First, a greedy algorithm is designed to efficiently search the possible circuit topology of RBSs with MAC. Moreover, the method provides an improved directed graph model that considers the voltage, the internal resistance, the MAC of the battery, and the external load. The method obtains the accurate current of the RBS under a specific circuit topology. The main contributions of this paper can be summarized as follows:

- An efficient method is proposed to determine the MAC of RBSs with arbitrary structures, including scenarios with isolated batteries.
- The greedy algorithm is applied to solve the MAC problem, the computational complexity of which is greatly reduced compared with the brute-force algorithm.
- An improved directed graph model is introduced; it considers the voltage, the internal resistance, the MAC of the battery, and the external load to analyze the current of the RBS.

The remainder of this paper is organized as follows: Section II presents the framework and details of the proposed directed graph model and the greedy algorithm. Section III demonstrated discusses a case study of using that uses the proposed method to determine the MAC of a novel and complex structure. The calculation results, the algorithm's computational complexity, and scenarios such as batteries isolation also are battery isolation are also discussed. Finally, the concluding remarks are drawn in Section presented in Sec. IV.

## 2 Methodology

The central principle of this method is to make connect the batteries in RBS connected in parallel as much as an RBS in parallel to the extent possible, thereby maximizing the output current of the RBS. To achieve this universally and automatically achieve this, the overall process is divided into four steps, as shown in Figure 2. Firstly the four steps shown in Fig. 2. First, a directed graph model is established for subsequent computations. The model not only contains the connected relationships between batteries and switches, but also retains the performance parameters of the batteries. Subsequently, based on the equivalent circuit, the MAC problem is transformed into

specific objective functions and constraints. ~~Then, the~~ The shortest paths (SPs, where additional batteries and switches on the path are penalized as distance) for the batteries are ~~obtained then~~ obtained by using the Dijkstra algorithm to ~~guide connect~~ the batteries in the RBS ~~connect~~ in parallel. Finally, a greedy algorithm is ~~employed used~~ to organize the switches, allowing the batteries to connect via their SPs while satisfying the constraints, resulting in the MAC of the RBS.

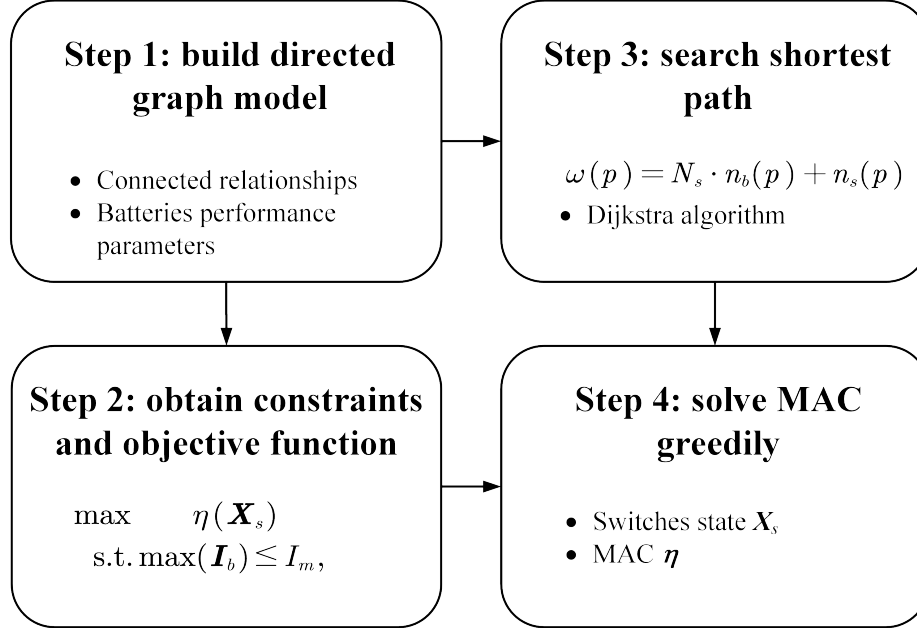


Figure 2: Diagram of this method, which contains four main steps.

## 2.1 Directed graph ~~Model~~model

He et al. [20] ~~once~~ proposed an abstracted directed graph model for an RBS, where the nodes ~~represented represent~~ the batteries, the edges ~~represented represent~~ the configuration flexibility, and the weight of each vertex ~~corresponded corresponds~~ to the battery voltage (~~Figure Fig.~~ 3a). The model ~~effectively captured captures~~ all potential system configurations and ~~offered offers~~ a direct metric for configuration flexibility, but it ~~did does~~ not specify the physical implementation of the connectivity between batteries, meaning that one graph might correspond to multiple RBS structures. We previously proposed a ~~novel~~ directed graph model that ~~differs completely from~~ He’s model ~~used by using~~ nodes to represent the connections between batteries and switches ~~and~~ directed edges to represent batteries and switches (~~Figure Fig.~~ 3b), allowing for a one-to-one correspondence between the RBS structure and the directed graph model. This model ~~was able to~~ accurately and comprehensively ~~represent represents~~ the RBS topological structure but ~~could not~~ cannot be used for quantitative MAC calculations ~~due to the lack of consideration because it does not consider the voltage, internal resistance, and MAC of the battery~~. To address this ~~an issue~~, we improve our previous model by adding electromotive force and resistance attributes on the edges based ~~equivalent circuit on its equivalent circuits~~. The model also considers the external load as an

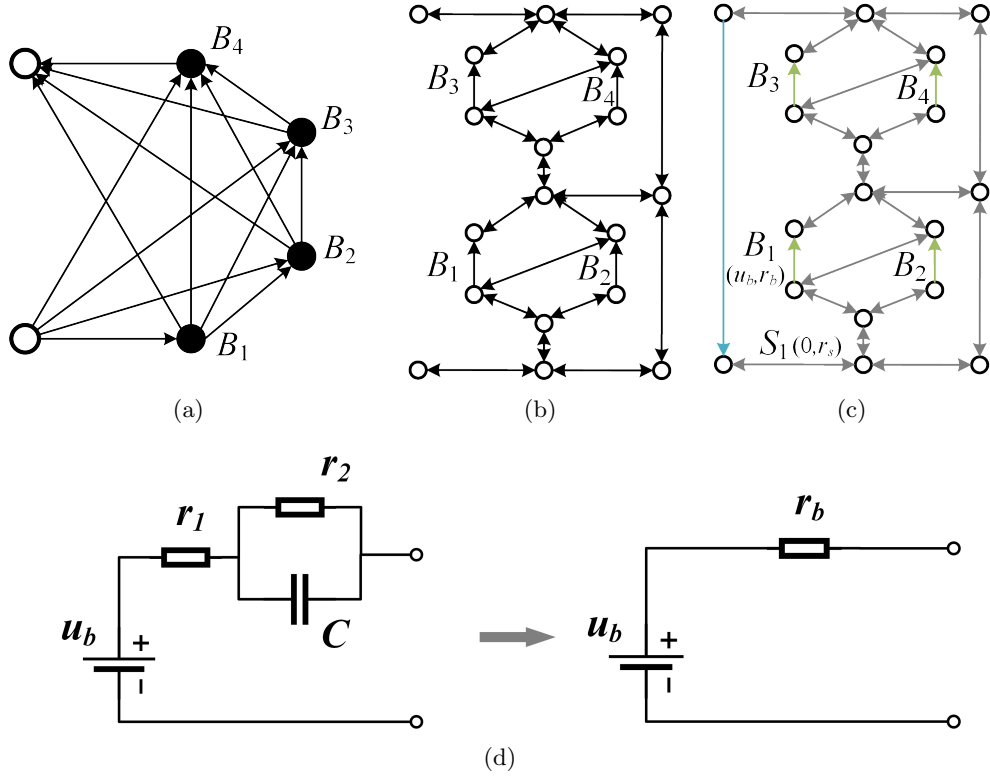


Figure 3: Directed graph models used in (a) He's work [20], (b) our previous work, and (c) [the improved model in](#) this paper. (d) The equivalent circuit of a battery in this method.

equivalent resistance and ~~integrate~~ integrates it into the analysis, making it a complete circuit model for later circuit analysis. ~~The following will provide analyses.~~ Fig. 3c shows the improved directed graph model used in this paper. ~~The following provides~~ a detailed explanation of the method for equating ~~the components in RBS~~ components in RBSs and constructing the directed graph model.

~~In order to~~ To use circuit analysis methods to solve the MAC of the RBS, the components in the RBS are equated to ideal circuit elements. ~~s shown in Figure~~ For instance, as shown in Fig. 3d, the battery in the RBS can be represented as a black-box circuit consisting of two resistors ~~(i.e.,~~  $r_1$  and  $r_2$  ~~)~~ and a capacitor ~~(i.e.,~~  $C$  ~~)~~, known as the Thevenin model [21, 22]. With an emphasis on the stable output of the RBS, the capacitor in the Thevenin model can be considered as an open circuit without affecting the steady-state current. Therefore, ~~the~~ battery  $i$  in the RBS can be simplified as ~~the a~~ series connection between a constant voltage source  $u_i$  and a resistor  $r_i$ . Furthermore, the state of switch  $j$  in the RBS is represented by a binary variable  $x_j$ , where 0 is ~~for~~ ON and 1 is ~~for~~ OFF, respectively OFF. When the switch is closed, ~~it the circuit~~ can be regarded as a resistor with a very small resistance ~~value~~  $r_j$ . ~~Lastly~~ Finally, the external load is considered as a resistor with ~~a value of resistance~~  $R_o$ .

For a given RBS structure, ~~its directed graph model~~  $G(V, E)$  is constructed as follows:

1. Nodes: The nodes in the directed graph correspond to the connection points of components in the actual RBS. Assuming there are a total of  $N$  nodes in the RBS, for the sake of convenience, the anode of the RBS is denoted as  $v_1$  and the cathode as  $v_N$ .
2. Edges: The edges in the directed graph correspond to the batteries, switches, and external electrical loads in the actual RBS. Therefore, there are three types of directed edges. For a battery  $B_i$ , its directed edge  $e_i$  is drawn from the cathode to the anode ~~, as the battery because the battery in operation~~ only allows current to flow in one direction ~~when in operation~~. For a switch  $S_j$ , since it is allowed to work under bi-directional currents, it is represented by a pair of directed edges with two-way directions. Regarding the external electronic load, ~~as because~~ it is connected to the anode and cathode of the RBS, a directed edge from  $v_N$  to  $v_1$  ~~is used to represent~~ represents it. In conclusion, for a given RBS structure with  $N_b$  batteries and  $N_s$  switches, the ~~total~~ number of directed edges is  $N_b + 2N_s + 1$ , where 1 refers to the external electrical load.
3. ~~Edges' attributes~~ Attributes of edges: Each edge is assigned two attributes, voltage difference and resistance, based on the equivalent method mentioned above. The values for ~~the~~ battery  $B_i$ , switch  $S_j$ , and external loads correspond to  $(u_i, r_i)$ ,  $(0, r_j)$ , and  $(0, R_o)$ , respectively.

## 2.2 Constraints and ~~Objective Function~~ objective function

~~For a given RBS, determining its MAC~~ involves maximizing the RBS output current while ensuring that the ~~sum of the~~ currents of all batteries ~~do~~ does not exceed the batteries' ~~maximum allowable current.~~ In this subsection, ~~MAC.~~ This subsection establishes the constraints and objective function to ~~solve~~ determine the RBS's MAC ~~will be established~~ through circuit analysis ~~, based on~~ based on the directed graph model provided in the previous section.

First, the topology in the directed graph model is represented in matrix form  $\mathbf{A}$ , known as the incidence matrix, ~~in Equation 1.~~ and defined as follows:

$$a_{kl} = \begin{cases} 1, & \text{edge } l \text{ leaves node } k, \\ -1, & \text{edge } l \text{ enters node } k, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

For a directed graph consisting of  $N$  nodes and  $N_b + 2N_s + 1$  directed edges, its incidence matrix  $\mathbf{A}$  is an  $N \times (N_b + 2N_s + 1)$  matrix. In this matrix, the rows and columns represent the nodes and edges of the directed graph, respectively. By distinguishing the components in the RBS corresponding to each column,  $\mathbf{A}$  can be rewritten as  $\div$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_b & \mathbf{A}_s & \mathbf{A}_o \end{bmatrix}, \quad (2)$$

where  $\mathbf{A}_b$ ,  $\mathbf{A}_s$ , and  $\mathbf{A}_o$  are the sub-matrices corresponding to the batteries, switches, and external electrical load, respectively. To ~~alleviate~~ reduce the computational complexity, the dimensions of matrix  $\mathbf{A}$  ~~undergoes dimensionality reduction~~ are reduced. Since each directed edge has one node to leave and one to enter, the ~~sum of the~~ values in every column of  $\mathbf{A}$  is sum to zero. Therefore ~~removing~~ , removing the last row will not result in a loss of information. ~~On the other hand~~ Conversely, since each switch in the RBS is represented by a pair of directed edges with two-way directions, the two columns corresponding to the switch are mutually opposite. Thus, for the ~~sub-matrix~~ submatrix  $\mathbf{A}_s$ , only one column is retained for each pair of columns representing the same switch. As a result,  $\mathbf{A}$  can be reduced to ~~a~~ an  $(N - 1) \times (N_b + N_s + 1)$  matrix, denoted ~~as~~  $\tilde{\mathbf{A}}$ , for further calculation of current and voltage. Similar to ~~Equation 2~~ Eq. (2),  $\tilde{\mathbf{A}}$  can be rewritten as  $\div$

$$\tilde{\mathbf{A}} = \begin{bmatrix} \tilde{\mathbf{A}}_b & \tilde{\mathbf{A}}_s & \tilde{\mathbf{A}}_o \end{bmatrix}. \quad (3)$$

After obtaining the incidence matrix, the currents of all batteries and output in the RBS are determined by solving the circuit equations. According to ~~Kirchhoffs~~ Kirchhoff's law, we have

$$\begin{cases} \tilde{\mathbf{A}}\mathbf{I} = \mathbf{0}, \\ \mathbf{U} = \tilde{\mathbf{A}}^T \mathbf{U}_n, \end{cases} \quad (4)$$

where  $\mathbf{I}$  and  $\mathbf{U}$  indicate the current and voltage difference arrays of the  $N_b + N_s + 1$  edges, respectively; and  $\mathbf{U}_n$  is the voltage array of the  $N - 1$  nodes. These directed edges are treated as generalized branches and expressed in matrix form as follows:

$$\mathbf{I} = \mathbf{Y}\mathbf{X}\mathbf{U} - \mathbf{Y}\mathbf{X}\mathbf{U}_s + \mathbf{I}_s, \quad (5)$$

where  $\mathbf{U}_s$  and  $\mathbf{I}_s$  denote the source voltage and source current of the generalized branches, respectively. Because all batteries have been equivalent to voltage sources rather than current sources in the previous ~~subsection~~ section, all elements of the array  $\mathbf{I}_s$  are ~~0,~~ while zero, whereas the elements



of the array  $\mathbf{U}_s$  are equal to the first attribute of the corresponding edges in the directed graph. The matrix  $\mathbf{Y}$  in Eq. (5) is the admittance matrix of the circuit, and is defined as the inverse of the impedance matrix. The elements on the diagonal of matrix  $\mathbf{Y}$  are equal to the reciprocal of the resistance, which is the second attribute of the corresponding edges in the directed graph, and the off-diagonal elements are 0. The of  $\mathbf{Y}$  are zero.  $\mathbf{X}$  is the state matrix, which describes that determines whether the RBS batteries and switches are allowed to can pass current. It is defined as

$$\mathbf{X} = \text{diag}(\underbrace{1, 0, \dots, 1}_{N_b \text{ of } 0/1}, \underbrace{1, 0, \dots, 1}_{N_s \text{ of } 0/1}, 1) = \begin{bmatrix} \mathbf{X}_b & & \\ & \mathbf{X}_s & \\ & & 1 \end{bmatrix}, \quad (6)$$

Where the elements where element  $x_i$  of the matrix  $\mathbf{X}_b$  represent whether the indicates whether battery  $i$  has been removed from the circuit, with  $x_i = 1$  indicating removal and  $x_i = 0$  indicating that it battery  $i$  is still available to supply power. When all batteries are healthy and capable of providing current to the external load,  $\mathbf{X}_b$  is an the identity matrix. The elements  $x_j$  of the matrix  $\mathbf{X}_s$  represent whether the determine whether switch  $j$  is closed, with  $x_j = 1$  indicating closure a closed switch and  $x_j = 0$  indicating disconnection an open switch, which is consistent with the previous subsectionsection.

Theoretically, the output current  $I_o$  and the currents of each battery  $\mathbf{I}_b$  in the RBS can be determined by solving Equations 4, 5, and 6 Eqs. (4)–(6) under any given state  $\mathbf{X}$ . In order to To obtain specific constraint conditions and objective functions, it is further assumed that all batteries have the same electromotive force and internal resistance, denoted as which are denoted  $u_b$  and  $r_b$ , respectively. This allows for the derivation of us to derive explicit expressions for  $I_o$  and  $\mathbf{I}_b$ . After derivation and simplification, the output current  $I_o$  and the currents of each battery  $\mathbf{I}_b$  are ultimately represented as Equations 7 and 2.2, respectively.

$$I_o = \frac{1}{R_o r_b} \tilde{\mathbf{A}}_o^T \mathbf{Y}_n^{-1}(\mathbf{X}) \tilde{\mathbf{A}}_b \mathbf{U}_b,$$

Eqs. (7) and (2.2), respectively:

$$\tilde{I}_o \equiv \frac{1}{R_o r_b} \tilde{\mathbf{A}}_o^T \mathbf{Y}_n^{-1}(\mathbf{X}) \tilde{\mathbf{A}}_b \mathbf{U}_b, \quad (7)$$

$$\tilde{\mathbf{I}}_b \equiv \frac{1}{r_b^2} [\tilde{\mathbf{A}}_b^T \mathbf{Y}_n^{-1}(\mathbf{X}) \tilde{\mathbf{A}}_b \mathbf{U}_b - r_b \mathbf{U}_b], \quad (8)$$

$$\mathbf{I}_b = \frac{1}{r_b^2} [\tilde{\mathbf{A}}_b^T \mathbf{Y}_n^{-1}(\mathbf{X}) \tilde{\mathbf{A}}_b \mathbf{U}_b - r_b \mathbf{U}_b],$$

where  $\mathbf{U}_b$  is an  $N_b \times 1$  array with all elements equaling equal to  $u_b$ , and  $\mathbf{Y}_n$  is the equivalent

admittance matrix of the circuit ~~and is~~ defined as

$$\mathbf{Y}_n(\mathbf{X}) = \frac{1}{R_o} \tilde{\mathbf{A}}_o \tilde{\mathbf{A}}_o^T + \frac{1}{r_b} \tilde{\mathbf{A}}_b \mathbf{X}_b \tilde{\mathbf{A}}_b^T + \frac{1}{r_s} \tilde{\mathbf{A}}_s \mathbf{X}_s \tilde{\mathbf{A}}_s^T. \quad (9)$$

To characterize the current output capacity of the RBS structure under different switching states, an indicator  $\eta$  is defined by the ratio of  $I_o$  ~~and to~~  $\max(\mathbf{I}_b)$  ~~shown in Equation 10:~~

$$\eta = \frac{I_o}{\max(\mathbf{I}_b)}. \quad (10)$$

Finally the problem of ~~solving finding the~~ MAC can be formulated as

$$\max \eta(\mathbf{X}_s) \quad (11)$$

$$\text{s.t.s.t.} \max(\mathbf{I}_b) \leq I_m, \quad (12)$$

where  $I_m$  is the ~~maximum allowable current-MAC~~ of the battery.

However, it ~~is remains~~ computationally difficult to solve ~~11 because of the Eq. (11) because of~~  $\mathbf{Y}_n^{-1}$ . On one hand, ~~due to~~ the introduction of nonlinear terms by  $\mathbf{Y}_n^{-1}$ , ~~many effective renders many~~ methods in linear optimization ~~are not suitable for this unsuitable for this problem~~. On the other hand, the rank of  $\mathbf{Y}_n$  is proportional to the number of batteries and switches, which can be very large for a large RBS ~~system~~, leading to a significant computational burden. ~~Therefore As a result,~~ intelligent algorithms that rely on ~~evolving evolution~~ by iteration may face efficiency ~~problems~~ when dealing with ~~large RBS system. In order to a large RBS. To~~ address this issue, the problem should be considered from the perspective of guiding the RBS to reconstruct as many parallel structures as possible. Consequently, a greedy algorithm based on the shortest path is proposed. The detailed implementation ~~process of this algorithm~~ is presented in the following two ~~subsections~~ sections.

### 2.3 Shortest ~~Path~~ path

The path  $p$  used in this method is defined as the complete route that passes through one battery (or a consecutive series of batteries) and closed switches, connecting the anode  $v_1$  to the cathode  $v_N$  of the RBS. By applying a penalty to the series-connected batteries on the path, where additional batteries imply a ~~longer-greater~~ distance, the algorithm encourages the RBS to form parallel structures ~~as much as possible. Meanwhile to the extent possible. In addition,~~ to reduce the number of switches controlled during the reconstruction process, a penalty is also applied to the total number of switches on the path ~~while ensuring the minimum number of batteries~~. Therefore, the distance  $\omega$  of ~~the~~ path  $p$  is ~~defined by the following equation:-~~

$$\omega(p) = N_s \cdot n_b(p) + n_s(p), \quad (13)$$

where  $N_s$  is the total number of switches in the system, ~~and~~  $n_b(p)$  and  $n_s(p)$  are number of batteries and switches in ~~the~~ path  $p$ , respectively. Moreover, the shortest path  $SP_i$  is defined as the path with

the minimum  $\omega$  for battery  $i$ , as shown in the following equation:

$$SP_i = \arg \min_{p \in P_i} \omega(p), \quad (14)$$

where  $P_i$  is the set of all paths from  $v_1$  to  $v_N$  which pass through the that pass through directed edge  $i$ .

The  $SP_i$  can be solved by the Dijkstra algorithm. The Dijkstra algorithm is a graph-search method that finds the shortest path between two given nodes in a weighted graph, efficiently solving the single-source shortest-path problem. Assuming that shortest-path problem. Denoting the cathode and anode of battery  $i$  are denoted as  $v_i^-$  and  $v_i^+$  respectively, the then path  $p$  of battery  $i$  can be divided into three segments:  $v_1 \rightarrow v_i^-$ ,  $v_i^+ \rightarrow v_N$ , and  $v_i^- \rightarrow v_i^+$ . The  $v_i^- \rightarrow v_i^+$  is the directed edge corresponding to battery  $i$ . With the Dijkstra algorithm, shortest paths for the  $v_1 \rightarrow v_i^-$  and  $v_i^+ \rightarrow v_N$  can be calculated under the weights given in Equation 13, denoted as Eq. (13) and denoted  $SP(v_i^- \rightarrow v_i^+)$  and  $SP(v_i^+ \rightarrow v_N)$ , respectively. Finally, the  $SP_i$  for battery  $i$  is formed by the complete path with, which consists of  $SP(v_1 \rightarrow v_i^-)$ ,  $v_i^- \rightarrow v_i^+$ , and  $SP(v_i^+ \rightarrow v_N)$ .

## 2.4 Greedy Algorithm

From the perspective of series vs parallel connections, integrating more batteries into the circuit through their shortest paths ( $SP$ s) results in a larger number of more batteries connected in parallel, thereby increasing the total output current of the RBS. However, conflicts may arise between the  $SP$ s of different batteries. For instance, the  $SP$ s of two batteries might form a short-circuited short-circuit RBS structure, which is not allowed. To address this issue, a greedy algorithm is employed to incorporate incorporates as many  $SP$ s as possible while satisfying the reconstruction requirements.

The algorithm, as illustrated in Figure 4, can be (see pseudo-code in Algorithm 1) is illustrated in Fig. 4 and is summarized as follows, with the corresponding pseudo-code presented in Algorithm 1. First, the shortest paths ( $SP$ s) are obtained using Equations 13 and 14 are obtained by using Eqs. (13) and (14) in conjunction with Dijkstra Search the Dijkstra search. Next, the matrix  $\mathbf{A}$  is calculated using Equation 1 Eq. (1), and the initial  $N_{set}$   $N_{set}$  is set to  $N_b$ . The algorithm iteratively checks uses a dichotomy method to iteratively check until convergence different combinations of  $c_b$  batteries from  $N_b$  and updates  $N_{set}$  using a dichotomy method until convergence is reached  $N_{set}$ . For each combination, the algorithm constructs an effective solution if possible, and calculates the currents  $I_o$  and  $I_b$  using Equations 7 and 2.2 by using Eqs. (7) and (2.2). If the maximum current  $I_b$  is less than or equal to  $I_m$ , the  $\eta$  is calculated using Equation 10 by using Eq. (10), and the maximum  $\eta$  is updated accordingly. Finally, the algorithm outputs the maximum  $\eta$  once  $N_{set}$   $N_{set}$  converges.

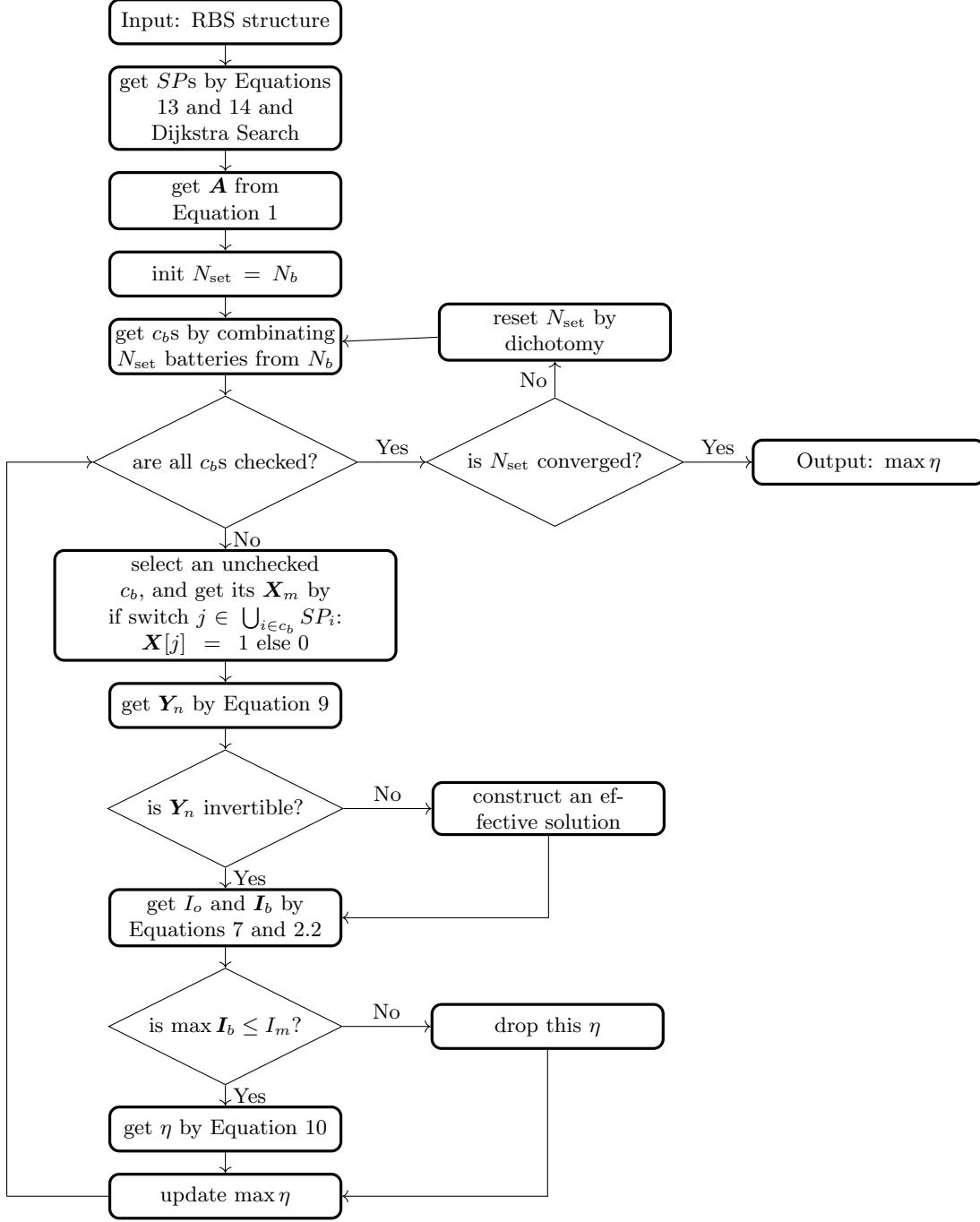


Figure 4: The computational flowchart of the MAC for a given RBS.

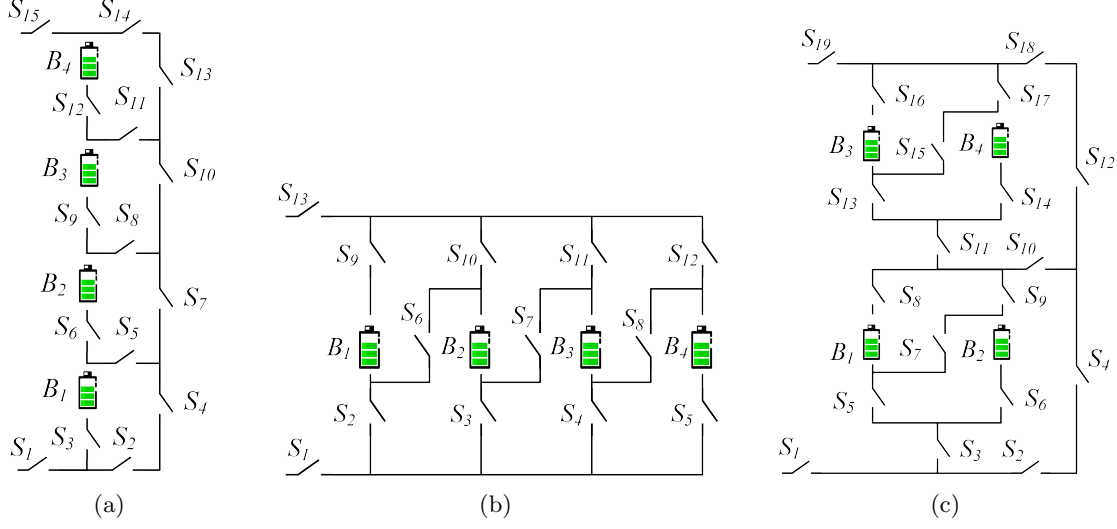


Figure 5: The ~~4-battery-four-battery~~ RBS structures proposed by (a) Lawson [15], (b) Visairo [11], and (c) this paper.

### 3 Case Study

#### 3.1 Structures

Currently, two types of RBS structures have been proposed by Visairo et al. [11] and Lawson et al. [15], both of which have ~~been seen real use~~. The primary goal of Visairo's structure (~~Figure 5b~~) ~~was to achieve dynamic adjustment of RBS output~~; however ~~Fig. 5b~~ is to dynamically adjust the ~~RBS output power~~. However, the isolation of unhealthy batteries ~~was is~~ not sufficiently addressed ~~in their work~~. Lawson et al. ~~This structure has the advantage of easily isolating batteries, but designed the RBS structure shown in Fig. 5a to isolate batteries. Although this structure easily isolates batteries,~~ it cannot dynamically adjust the output current of ~~the~~ RBS. Based on the structures of Visairo and Lawson, this paper ~~presents a new structure, as shown in Figure 5c~~ ~~proposes the structure shown in Fig. 5c~~. By integrating the Visairo RBS structure into the Lawson RBS structure, the ~~new-proposed~~ structure not only ~~allows has~~ the flexibility to switch the batteries between series, parallel, and mixed series-parallel modes ~~, but also easily enables but also allows~~ the isolation of highly degraded batteries from the RBS. ~~And their variations Their variation~~ in output current under battery isolation ~~conditions~~ will be studied. This RBS structure ~~will be is~~ used to validate the ~~effectiveness of the~~ proposed method for calculating the MAC ~~, and be compared with the and is compared with~~ Lawson's and Visairo's structure to ~~illustrate its advantage on demonstrate its advantages for~~ battery isolation.

#### 3.2 Result

As shown in ~~Figure Fig.~~ 5c, the new RBS structure consists of ~~4-four~~ batteries and 19 switches. ~~The Figure 6a shows the~~ corresponding directed graph ~~is depicted in Figure 6a~~, which is composed of ~~a~~

total of 18 nodes and 43 edges. Batteries  $B_1$ ,  $B_2$ ,  $B_3$ , and  $B_4$  are denoted by green directed edges in the graph, while the 19 switches are represented by gray directed edges with bi-directional arrows. The external electrical load is treated as a directed edge from the cathode of the RBS (i.e., node 18) to the anode (i.e., node 1), as indicated by the blue directed edge in the graph. Utilizing Equation 13 and the Dijkstra algorithm, the  $SP$ s of the four batteries in the RBS structure of Figure 5c are highlighted by red in Figures 6b–in red in Figs. 6b and 6e. Finally, the MAC calculation results—calculated MACs of the structure in Figure 5c are shown as Fig. 5c are listed in Table 1 and Figure 6f, Fig. 6f, as obtained by the greedy algorithm 1. Table 1 contains the switches—states of the switches, the output current  $I_o$ , the battery current  $I_b$  and, and the ratio  $\eta$  of the RBS structure with all batteries in good health when the RBS output reaches the MAC. Figure 6f presents the corresponding circuit, with the red highlight indicating that the current is flowing through the respective branches.

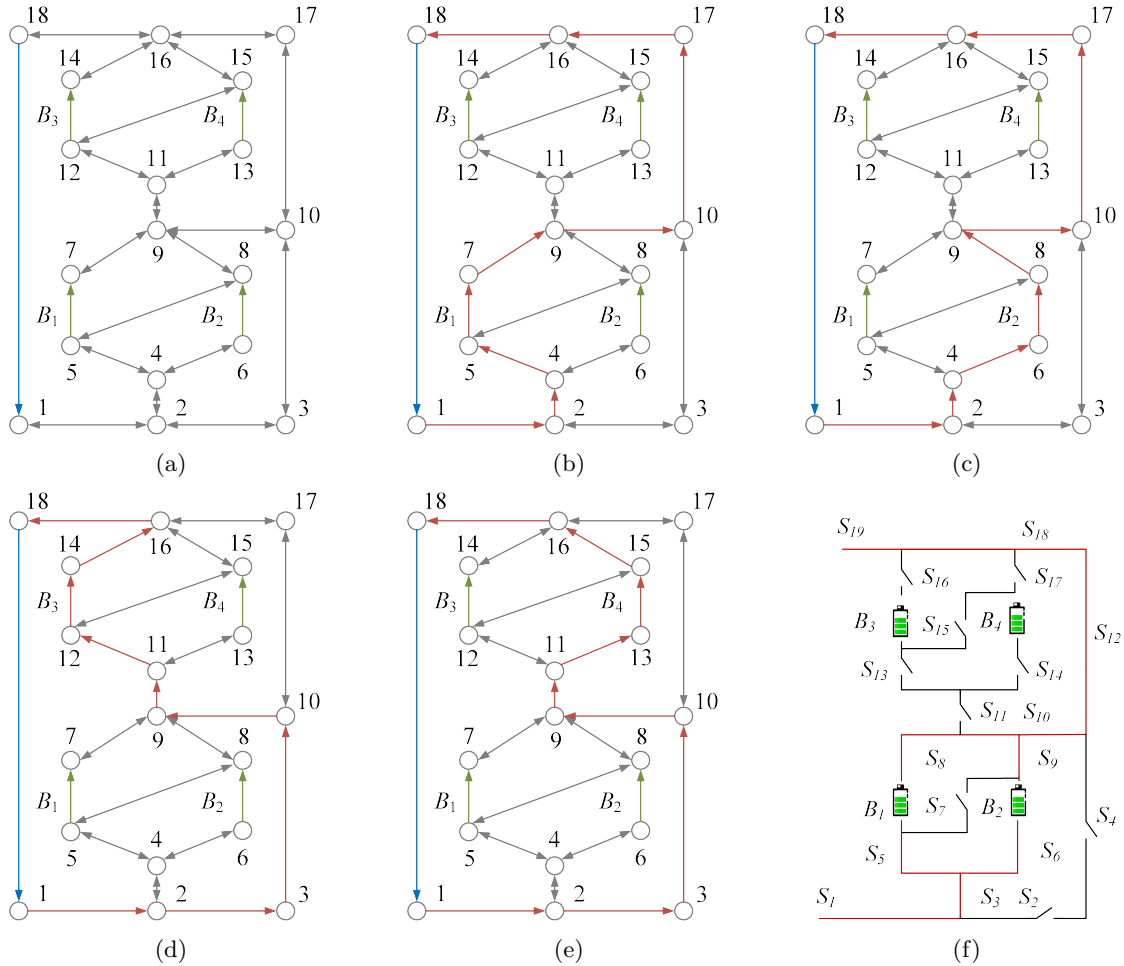


Figure 6: For the RBS structure in Figure 5c, (a) its directed graph and the  $SP$ s (highlighted in red) of battery (b)  $B_1$ , (c)  $B_2$ , (d)  $B_3$ , and (e)  $B_4$ . (f) The circuit of the RBS with its output reaching the MAC.

Table 1: ~~Calculated MAC Calculating result of the 4-battery for four-battery~~ RBS structure in ~~Figure Fig.~~ 5c.

Structure	Figure 5c with <del>4</del> four batteries and 19 switches
Switch <del>ON</del> on	$S_1, S_3, S_5, S_6, S_8, S_9, S_{10}, S_{12}, S_{18}, S_{19}$
$I_o$	$2u_b/(2R_o + r_b)$
$I_b$	$[u_b/(2R_o + r_b), u_b/(2R_o + r_b), 0, 0]$
<del><math>\eta</math></del> max $\eta$	2

Similarly, the ~~MAC calculation~~ results of the ~~structures in Figures~~ ~~MAC calculation for the~~ structures in Figs. 5a and 5b are ~~shown as Table 2 and Table~~ listed in Tables 2 and 3, respectively. To verify and compare the results from the greedy algorithm, we also used a brute-force algorithm that iterates through all possible switch states to calculate the MAC of the same three RBSs. For a given RBS structure with  $N_s$  switches, the final max $\eta$  is the maximum  $\eta$  from all  $2^{N_s}$  reconfigured structures. The final results are the same as the results shown in Tables 1–3. The method uses the greedy algorithm to calculate 7, 11, and 1 reconfigured structures for the RBS structure in Figs. 5c, 5a, and 5b, respectively. For the same RBS, the method counts all possible switch states, which equates to  $2^{19}$ ,  $2^{15}$ , and  $2^{13}$  structures, respectively.

Table 2: MAC Calculating result of the 4-battery RBS structure in ~~Figure Fig.~~ 5a.

Structure	Figure 5a with 4 batteries and 15 switches
Switch ON	$S_1, S_3, S_5, S_7, S_{10}, S_{13}, S_{14}, S_{15}$
$I_o$	$u_b/(R_o + r_b)$
$I_b$	$[u_b/(R_o + r_b), 0, 0, 0]$
<del><math>\eta</math></del> max $\eta$	1

Table 3: MAC Calculating result of the 4-battery RBS structure in ~~Figure Fig.~~ 5b.

Structure	Figure 5b with 4 batteries and 13 switches
Switch ON	$S_1, S_2, S_3, S_4, S_5, S_9, S_{10}, S_{11}, S_{12}, S_{13}$
$I_o$	$4u_b/(4R_o + r_b)$
$I_b$	$[u_b/(4R_o + r_b), u_b/(4R_o + r_b), u_b/(4R_o + r_b), u_b/(4R_o + r_b)]$
<del><math>\eta</math></del> max $\eta$	4

Furthermore, the RBS ~~with~~ isolated batteries is taken into consideration and calculated. The MAC calculation results for the three structures under study, with varying numbers of isolated batteries, are presented in Table 4. Figures 7a–7d illustrate the corresponding ~~switch-control~~ ~~switch-control~~ schemes for the new structure proposed in this paper under different ~~isolated battery~~ ~~conditions~~ ~~conditions of isolated batteries~~.

### 3.3 Discussion

Table 4: ~~The variation~~ Variation of MAC with the number of isolated batteries for different RBS structures, including the structure proposed by Lawson et al., Visairo et al., and the structure proposed in this paper.

Number of isolated batteries	$\eta$ of RBS structure		
	<del>our</del> <u>This paper</u>	Visairo <del>'s</del>	Lawson <del>'s</del>
0	2	4	1
1	2	3	1
2	2 <sup>a</sup> or 1 <sup>b</sup>	2	1
3	1	1	1

<sup>a</sup> Isolate two batteries within the same substructure, as shown in Fig. 7b.

<sup>b</sup> Isolate one battery in each of the two substructures, as shown in Fig. 7c.

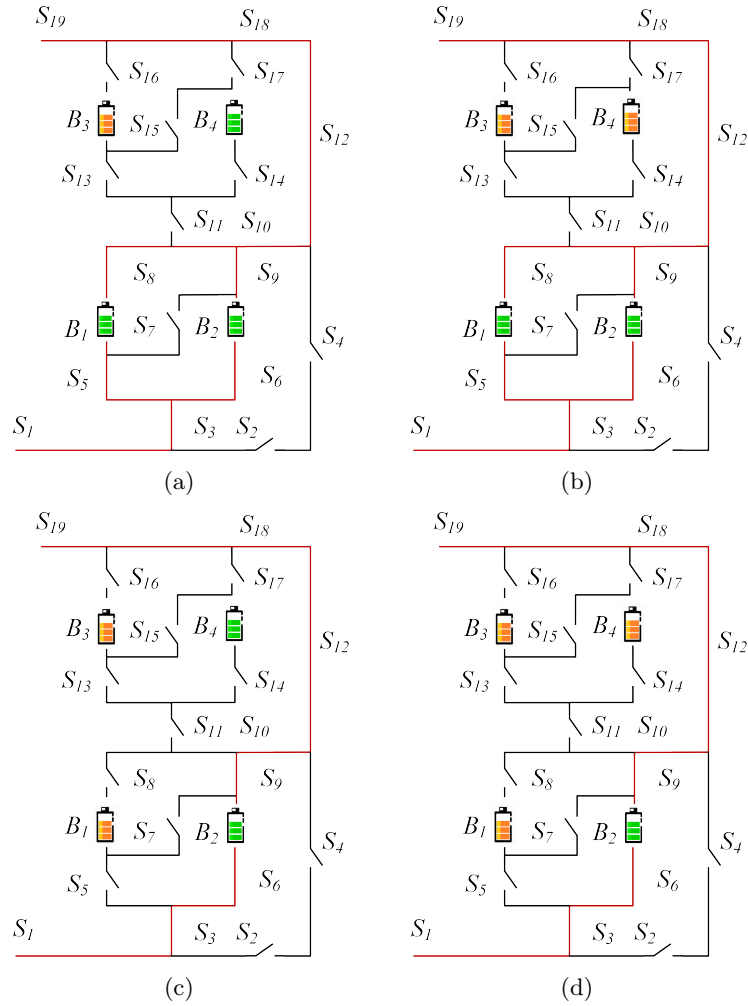


Figure 7: ~~The circuit~~ Circuit states of MACs when isolating (a) one, (b) two (best case), (c) two (worst case), and (d) three batteries for the structure in ~~Figure Fig.~~ Fig. 5c.



Consider first the results shown in Fig. 6 and Table 1. When  $B_1$  and  $B_2$  or  $B_3$  and  $B_4$  are connected in parallel, the RBS can output the maximum current, which is  $\eta = 2$  (i.e., twice the current output of a single battery in the RBS). Adding more batteries to the main circuit can only form a series structure and will not improve the MAC. Therefore, the switches state of the switches given in Table 1 can make the RBS output current reach the maximum. The MAC obtained by the brute-force algorithm is identical to the one obtained by the greedy algorithm.

The literature contains no report on an algorithm for calculating the MAC of an RBS. The brute-force algorithm, which goes through all possible switch states, is the most straightforward way to determine the MAC and is used as a benchmark for the proposed greedy algorithm. If an RBS has  $N_b$  batteries and  $N_s$  switches and the corresponding directed graph has  $N$  nodes,  $2^{N_s}$  iterations are required to traverse all reconfigured structures. Calculating each reconfigured structure using Eqs. (7)–(10) requires matrix inversion and matrix multiplication, which has a time complexity of  $O(N^3 + 2N^2N_b + N^2N_s + NN_b^2)$ . Therefore, the time complexity of the brute-force algorithm is  $O(2^{N_s}(N^3 + 2N^2N_b + N^2N_s + NN_b^2))$ . The greedy algorithm proposed in this paper requires that  $SP$  be found for each battery, which requires  $N_b$  iterations. Each  $SP$  can be obtained by several applications of Dijkstra's algorithms. Therefore, the total time complexity for calculating all  $SP$ s is  $O(2N_b(N_b + 2N_s)\log_{10} N)$ . According to Appendix 1, the RBS can reconfigure  $C_{N_b}^{N_{set}}$  structures by selecting  $N_{set}$  batteries from  $N_b$  batteries, which gives  $\sum_{N_{set}=1}^{N_b} C_{N_b}^{N_{set}}/N_b \approx 2^{N_b}/N_b$  on average. Thus, with the bisection method, the time complexity of the greedy algorithm is  $O(2^{N_b}/N_b(N^3 + 2N^2N_b + N^2N_s + NN_b^2)\log_{10} N_b + 2N_b(N_b + 2N_s)\log_{10} N)$  [i.e.,  $O(2^{N_b}/N_b(N^3 + 2N^2N_b + N^2N_s + NN_b^2)\log_{10} N_b + 2N_b(N_b + 2N_s)\log_{10} N)$ ]. Based on currently proposed RBS structures [23–28], the number  $N_b$  of batteries,  $N_s$  of switches, and  $N$  of nodes are quantitatively related as follows:  $N_s \approx (3-5)N_b$ ,  $N \approx N_s$ . After simplifying, the time complexity of the greedy algorithm is  $O(2^{N_b}N_s^2\log N_b)$ , while it is  $O(2^{N_s}N_s^3)$  for brute force algorithm. Therefore, as the RBS grows, especially in the number of switches, the greedy algorithm gains an advantage over the brute-force algorithm.

It is important to note that This is confirmed by the number of structures required to determine the MAC in the previous section. Compared with the brute-force algorithm, the method based on the greedy algorithm is 3000 to 75 000 times more efficient, which is theoretically  $N_s 2^{N_s - N_b} \log_{10} N_b$  times according to the above time-complexity analysis. Of the three RBS structures, the largest is the RBS structure with 19 switches (Fig. 5c). This benefits from two key points:

- (1) The  $SP$ s guide the RBS to reconfigure reasonable structures rather than blindly going through all possible structures. This reduces the complexity from  $2^{N_s}$  to  $2^{N_b}$ , which is the main reason for the improvement in efficiency.
- (2) The bisection method further accelerates this process. However, the greedy algorithm proposed in this paper still contains exponential terms in the time complexity, which means it may not be able to handle extremely large RBS structures.

Note that  $\eta$  is used as the objective function instead of  $I_o$  in solving for the MAC. This choice makes the result of resulting MAC more reasonable. As shown in Table 1,  $I_o$  and  $I_b$  are functions of  $R_o$ ,  $u_b$ , and  $r_b$ . However, when  $I_o$  is used as the objective function, even for the same RBS structure,

the MAC ~~and corresponding switches state solution and corresponding switch states~~ could change due to different external electrical appliances. ~~It~~ This would increase the difficulty and uncertainty ~~in RBS structuredesign.~~ of designing the RBS structure. To eliminate this problem, the ratio  $\eta = I_o / \max I_b$  is adopted as the objective function in our research. Recall that  $\eta$  ~~solely reflects the maximum output current capability reflects only the MAC~~ of the RBS structure. Assuming that the ~~maximum allowed current~~ MAC of batteries in the RBS is  $I_m$ , the maximum output current of the RBS structure can be calculated as  $\eta I_m$  by determining the  ~~$\eta$  of value of  $\eta$  for~~ the structure.

The method proposed in this paper ~~is significant for facilitates~~ the design of RBSs in the following ~~aspects. Most of the ways: Most~~ currently proposed RBS structures [23–28] ~~exhibit have~~ simple topological characteristics, ~~and the calculation of so calculating the~~ MACs is relatively straightforward, even intuitive. However, these simple structures do not always fully satisfy the requirements of complex applications, such as dynamically adapting the circuit to variable and random operating conditions ~~and or~~ actively equalizing differences ~~among the between~~ batteries in the RBS. Moreover, ~~isolating~~ the batteries disrupts the original regularity and symmetry of the topology, which complicates the otherwise simple structure, and the maximum output current of the system becomes more challenging to obtain. ~~In contrast,~~ the proposed method ~~can be employed to calculate calculates~~ the MAC of arbitrary RBS structures, ~~notably the complex and flexible RBS structures.~~

To illustrate this point, the MACs of three RBS structures mentioned above are calculated after ~~batteries are isolated isolating one or more of the batteries~~, as shown in Table 4. Specifically, for the structure presented in ~~Figure Fig.~~ 5c, the corresponding circuit states ~~of MACs isolating for the~~ MACs ~~when isolating one to three~~ batteries are depicted in ~~Figures 7a–Figs. 7a–7d.~~ This structure has two cases ~~of isolating two batteries in which two batteries are isolated:~~ one is to isolate two batteries within the same substructure (~~Figure Fig.~~ 7b), in which case  $\eta = 2$ ; the other is to isolate one battery in each of the two substructures (~~Figure Fig.~~ 7c), in which case  $\eta = 1$ . ~~From the results~~ ~~it can be observed The results in Figs. 7a–7d show~~ that the proposed method provides reasonable outcomes for isolating ~~batteries with any number and any number of batteries in any~~ position.

Furthermore, the output current for the three ~~RBS RBSs with isolated~~ batteries is also shown in Table 4. For the structure proposed by Lawson et al., the MAC ~~or is independent of the~~ number of isolated batteries. However, for Visairo’s structure, the MAC decreases ~~upon increasing the number of isolated batteries. Nevertheless,~~ the MAC of the structure proposed in this work ~~is positioned between falls between the MACs of these two structures.~~ This ~~result~~ indicates that the structure proposed in this paper ~~compared to Lawson’s structure,~~ has a larger MAC ~~under than~~ Lawson’s ~~for the same number of batteries~~ ~~a wider output current regulation range. and has a wider range of regulation of the output current.~~

## 4 Conclusion

This paper proposes a pervasive and ~~automatic method for computing automated method to efficiently compute~~ the MAC of ~~given RBS. a RBS.~~ The method is implemented by a greedy algorithm combined with a directed graph model that considers the voltage, internal resistance, and MAC of the batteries, as well as the external load. The main advantage of this method is its ability

to calculate the MAC of RBSs with arbitrary structures. Even in scenarios with random isolated batteries, the proposed method remains effective. The proposed method is more computationally efficient than the brute-force algorithm for the same calculation results, which is achieved by two key points when constructing the greedy algorithm:

- (1) Calculate the shortest path of each battery by using the Dijkstra algorithm to obtain the  $SP$  for each battery.
- (2) Determine the maximum number of available batteries by bisection to reduce the computational complexity.

This method helps to fully tap the current output potential of the RBS, guide the RBS structure design and optimization in the design stage, and assist in evaluating the current-overload risk of the system in practical applications.

## 5 Appendix

---

**Algorithm 1:** Get the max available currents of a certain RBS

---

**Data:** Directed graph model  $G(V, E)$  of the RBS  
**Result:**  $\max \eta$

```

1 for  $i \in E_b$  do
2    $P_i \leftarrow \{path | \text{starts at } v_1 \text{ and ends at } v_n\}$ ;
3    $SP_i \leftarrow p_i$  which has the minimum  $\omega(p_i)$  among all  $p_i \in P_i$ .
4 end
5 get  $A$  by Equation 1;
6 while not yet determine  $\max \eta$  do
7    $N_{set} \leftarrow N_{set}$  number of selected  $SP$ s calculated by dichotomy;
8    $C_b \leftarrow$  set of all combinations of  $N_{set}$  batteries from  $N_b$ ;
9   for  $c_b \in C_b$  do
10     $x_s \leftarrow$  list of all switches' state:  $x_s[j] = 1$  if  $j \in \bigcup_{i \in c_b} SP_i$  else 0;
11     $X \leftarrow diag[1, 1, \dots, 1, x_s]$ ;
12    get  $Y_n$  by Equation 9;
13    if  $Y_n$  is invertible then
14      pass
15    else
16      construct an effective solution
17    end
18    get  $I_o$  by Equation 7;
19    get  $I_b$  by Equation 2.2;
20    if  $\max(I_b) \leq I_m$  then
21       $\eta \leftarrow I_o / \max(I_b)$ ;
22    else
23      break
24    end
25  end
26 end

```

---

## Acknowledgments

### Author Contributions

B. Xu conceived the main idea, formulated the overarching research goals and aims, designed the algorithm, and reviewed and revised the manuscript. G. Hua developed and analyzed the model, implemented the code and supporting algorithms, and wrote the initial draft. C. Qian provided critical review, commentary, and revisions. Q. Xia contributed to shaping the research, analysis, and manuscript. B. Sun conducted the research and investigation process. Y. Ren secured the funding and supervised the project. Z. Wang verified the results and provided necessary resources.

### Funding

This work was supported by the National Natural Science Foundation of China (NSFC, No.52075028).

### Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

### Data Availability

This work does not require any data to be declared or publicly disclosed.

## References

1. de Siqueira LMS and Peng W. Control Strategy to Smooth Wind Power Output Using Battery Energy Storage System: A Review. *Journal of Energy Storage* 2021;35:102252.
2. Yang Y, Bremner S, Menictas C, and Kay M. Battery Energy Storage System Size Determination in Renewable Energy Systems: A Review. *Renewable and Sustainable Energy Reviews* 2018;91:109–25.
3. Cho J, Jeong S, and Kim Y. Commercial and Research Battery Technologies for Electrical Energy Storage Applications. *Progress in Energy and Combustion Science* 2015;48:84–101.
4. Zhang L. Development and Prospect of Chinese Lunar Relay Communication Satellite. *Space: Science & Technology* 2021;2021.
5. Schwanbeck E and Dalton P. International Space Station Lithium-ion Batteries for Primary Electric Power System. In: *2019 European Space Power Conference (ESPC)*. IEEE, 2019:1–1. DOI: 10.1109/ESPC.2019.8932009.
6. Yang N, Zhang X, Shang B, and Li G. Unbalanced Discharging and Aging Due to Temperature Differences among the Cells in a Lithium-Ion Battery Pack with Parallel Combination. *Journal of Power Sources* 2016;306:733–41.

7. Feng F, Hu X, Hu L, Hu F, Li Y, and Zhang L. Propagation Mechanisms and Diagnosis of Parameter Inconsistency within Li-Ion Battery Packs. *Renewable and Sustainable Energy Reviews* 2019;112:102–13.
8. Jeevarajan JA and Winchester C. Battery Safety Qualifications for Human Ratings. *Interface magazine* 2012;21:51–5.
9. Pombo DV. A Hybrid Power System for a Permanent Colony on Mars. *Space: Science & Technology* 2021;2021.
10. Han W, Wik T, Kersten A, Dong G, and Zou C. Next-Generation Battery Management Systems: Dynamic Reconfiguration. *IEEE Industrial Electronics Magazine* 2020;14:20–31.
11. Visairo H and Kumar P. A Reconfigurable Battery Pack for Improving Power Conversion Efficiency in Portable Devices. In: *2008 7th International Caribbean Conference on Devices, Circuits and Systems*. IEEE, 2008:1–6. DOI: 10.1109/ICDCS.2008.4542628.
12. Ci S, Zhang J, Sharif H, and Alahmad M. A novel design of adaptive reconfigurable multicell battery for power-aware embedded networked sensing systems. In: *IEEE GLOBECOM 2007-IEEE Global Telecommunications Conference*. IEEE. 2007:1043–7.
13. Engelhardt J, Gabderakhmanova T, Rohde G, and Marinelli M. Reconfigurable Stationary Battery with Adaptive Cell Switching for Electric Vehicle Fast-Charging. In: *2020 55th International Universities Power Engineering Conference (UPEC)*. 2020:1–6. DOI: 10.1109/UPEC49904.2020.9209774.
14. Engelhardt J, Zepter JM, Gabderakhmanova T, Rohde G, and Marinelli M. Double-string battery system with reconfigurable cell topology operated as a fast charging station for electric vehicles. *Energies* 2021;14:2414.
15. Lawson B. A Software Configurable Battery. *EVS26 International Battery, Hybrid and Fuel Cell Electric Vehicle Symposium* 2012.
16. He L, Kong L, Lin S, et al. Reconfiguration-assisted charging in large-scale lithium-ion battery systems. In: *2014 ACM/IEEE International Conference on Cyber-Physical Systems (ICCPS)*. IEEE. 2014:60–71.
17. Kim H and Shin KG. On dynamic reconfiguration of a large-scale battery system. In: *2009 15th IEEE Real-Time and Embedded Technology and Applications Symposium*. IEEE. 2009:87–96.
18. Han W, Kersten A, Zou C, Wik T, Huang X, and Dong G. Analysis and estimation of the maximum switch current during battery system reconfiguration. *IEEE Transactions on Industrial Electronics* 2021;69:5931–41.
19. Chen SZ, Wang Y, Zhang G, Chang L, and Zhang Y. Sneak Circuit Theory Based Approach to Avoiding Short-Circuit Paths in Reconfigurable Battery Systems. *IEEE Transactions on Industrial Electronics* 2021;68:12353–63.
20. He L, Gu L, Kong L, Gu Y, Liu C, and He T. Exploring Adaptive Reconfiguration to Optimize Energy Efficiency in Large-Scale Battery Systems. In: *2013 IEEE 34th Real-Time Systems Symposium*. 2013:118–27. DOI: 10.1109/RTSS.2013.20.

21. He H, Xiong R, Zhang X, Sun F, and Fan J. State-of-Charge Estimation of the Lithium-Ion Battery Using an Adaptive Extended Kalman Filter Based on an Improved Thevenin Model. *IEEE Transactions on Vehicular Technology* 2011;60:1461–9.
22. Mousavi G. S and Nikdel M. Various Battery Models for Various Simulation Studies and Applications. *Renewable and Sustainable Energy Reviews* 2014;32:477–85.
23. Ci S, Zhang J, Sharif H, and Alahmad M. A Novel Design of Adaptive Reconfigurable Multicell Battery for Power-Aware Embedded Networked Sensing Systems. In: *IEEE GLOBECOM 2007-2007 IEEE Global Telecommunications Conference*. 2007:1043–7. DOI: 10.1109/GLOCOM.2007.201.
24. Alahmad M, Hess H, Mojarradi M, West W, and Whitacre J. Battery Switch Array System with Application for JPL’s Rechargeable Micro-Scale Batteries. *Journal of Power Sources* 2008;177:566–78.
25. Kim H and Shin KG. Dependable, Efficient, Scalable Architecture for Management of Large-Scale Batteries. In: *Proceedings of the 1st ACM/IEEE International Conference on Cyber-Physical Systems*. ICCPS ’10. New York, NY, USA: Association for Computing Machinery, 2010:178–87. DOI: 10.1145/1795194.1795219.
26. Kim Y, Park S, Wang Y, et al. Balanced Reconfiguration of Storage Banks in a Hybrid Electrical Energy Storage System. In: *2011 IEEE/ACM International Conference on Computer-Aided Design (ICCAD)*. 2011:624–31. DOI: 10.1109/ICCAD.2011.6105395.
27. Kim T, Qiao W, and Qu L. A Series-Connected Self-Reconfigurable Multicell Battery Capable of Safe and Effective Charging/Discharging and Balancing Operations. In: *2012 Twenty-Seventh Annual IEEE Applied Power Electronics Conference and Exposition (APEC)*. 2012:2259–64. DOI: 10.1109/APEC.2012.6166137.
28. He L, Kong L, Lin S, et al. Reconfiguration-Assisted Charging in Large-Scale Lithium-ion Battery Systems. In: *2014 ACM/IEEE International Conference on Cyber-Physical Systems (ICCPS)*. 2014:60–71. DOI: 10.1109/ICCPS.2014.6843711.