

# **CSP and LP models of IQ Twist and Cube Puzzler - Go**

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**A thesis submitted for the course  
COMP8755 Individual Computing Project  
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The Australian National University**

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# Declaration

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This thesis is an account of research undertaken between July 2020 and November 2020 at The College of Engineering and Computer Science, Faculty of Computing Science, The Australian National University, Canberra, Australia.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

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Mingzhen Ao  
November, 2020

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# Acknowledgements

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I would like to thank my lucky stars, and the cat, for not eating me.

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# Abstract

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This thesis tells a great story about what I achieved in my research project. The abstract is short, but informative. it makes clear the general area in which I worked, and what I achieved.

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# Introduction

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# IQ twist

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## 2.1 CSP model

In this part, I'd like to use CSP to model the IQ twist game. Above all, I'd like to define all the initial state for each piece:



initial state of Yellow 1



initial state of Yellow 2



initial state of Blue 1



initial state of Blue 2



initial state of Green 1



initial state of Green 2

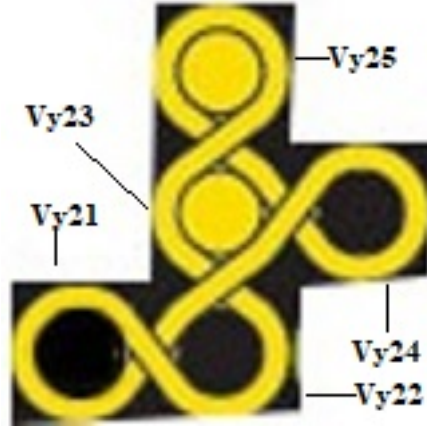


initial state of Red 1



initial state of Red 2

For this method, there are some rules. Firstly, all the variables correspond to the initial states. Such as the Yellow 2, I use  $V_{y21}$  to represent the left and bottom unit. For other variables, name them as  $V_{y22}$ ,  $V_{y23}$ ,  $V_{y24}$  and  $V_{y25}$  follow the order from left to right and bottom up.



naming rules example for yellow 2

### 2.1.1 Variables

$$V_{units} = \{V_{y11}, V_{y12}, V_{y13}, V_{y21}, V_{y22}, V_{y23}, V_{y24}, V_{y25}, \\ V_{b11}, V_{b12}, V_{b13}, V_{b14}, V_{b15}, V_{b21}, V_{b22}, V_{b23}, \\ V_{b24}, V_{g11}, V_{g12}, V_{g13}, V_{g14}, V_{g21}, V_{g22}, V_{g23}, \\ V_{r11}, V_{r12}, V_{r13}, V_{r14}, V_{r21}, V_{r22}, V_{r23}, V_{r24}\}$$

$$V_{pegs} = \{V_{py1}, V_{py2}, V_{pb1}, V_{pb2}, V_{pg1}, V_{pg2}, V_{pr}\}$$

$$V = V_{units} \cup V_{pegs}.$$

### 2.1.2 Domains

$$\text{For all } v \in V_{units}, D(v) = \{(i, j) \in \mathbb{Z} \times \mathbb{Z} \mid 0 < i \leq 8, 0 < j \leq 4\}$$

$$\text{For all } v \in V_{pegs}, D(v) = \{(0, 0) \cup (i, j) \in \mathbb{Z} \times \mathbb{Z} \mid 0 < i \leq 8, 0 < j \leq 4\}$$

### 2.1.3 Constrains

- ① For each pair of variables –  $V_m$  and  $V_n, V_m \in V_{units}, V_n \in V_{units}$ ,



$$C_{v_m, v_n} = \{((a, b), (c, d)) \in D(v_m) \times D(v_n) \mid a \neq c \text{ or } b \neq d\}$$

② For each pair of variables –  $V_m$  and  $V_n, V_m \in V_{\text{pegs}}, V_n \in V_{\text{pegs}},$

$$C_{v_m, v_n} = \{((0, 0), (0, 0)) \cup ((a, b), (c, d)) \in D(v_m) \times D(v_n) \mid a \neq c \text{ or } b \neq d\}$$

③ piece Yellow1

$$C_{v_{y11}, v_{y12}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a + 1, d = b + 0\},$$

$$C_{v_{y11}, v_{y13}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a + 2, d = b + 0\},$$

$\cup$

$$C_{v_{y11}, v_{y12}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a - 0, d = b + 1\},$$

$$C_{v_{y11}, v_{y13}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a - 0, d = b + 2\},$$

$\cup$

$$C_{v_{y11}, v_{y12}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a - 1, d = b - 0\},$$

$$C_{v_{y11}, v_{y13}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a - 2, d = b - 0\},$$

$\cup$

$$C_{v_{y11}, v_{y12}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a + 0, d = b - 1\},$$

$$C_{v_{y11}, v_{y13}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a + 0, d = b - 2\},$$

$\cup$

$$C_{v_{y11}, v_{y12}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a - 1, d = b + 0\},$$

$$C_{v_{y11}, v_{y13}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a - 2, d = b + 0\},$$

$\cup$

$$C_{v_{y11}, v_{y12}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a - 0, d = b - 1\},$$

$$C_{v_{y11}, v_{y13}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a - 0, d = b - 2\},$$

$\cup$

$$C_{v_{y11}, v_{y12}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a + 1, d = b - 0\},$$

$$C_{v_{y11}, v_{y13}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a + 2, d = b - 0\},$$

$\cup$

$$C_{v_{y11}, v_{y12}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a + 0, d = b + 1\},$$

$$C_{v_{y11}, v_{y13}} = \{((a, b), (c, d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a + 0, d = b + 2\},$$

$\cup$

$$C_{v_{y21}, v_{y22}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y22}) \mid c = a + 1, d = b + 0\},$$

$$C_{v_{y21}, v_{y23}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y23}) \mid c = a + 1, d = b + 1\},$$

$$C_{v_{y21}, v_{y24}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y24}) \mid c = a + 2, d = b + 1\},$$

$$C_{v_{y21}, v_{y25}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y25}) \mid c = a + 1, d = b + 2\},$$

$\cup$

$$C_{v_{y21}, v_{y22}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y22}) \mid c = a - 0, d = b + 1\},$$

$$C_{v_{y21}, v_{y23}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y23}) \mid c = a - 1, d = b + 1\},$$

$$C_{v_{y21}, v_{y24}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y24}) \mid c = a - 1, d = b + 2\},$$

$$C_{v_{y21}, v_{y25}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y25}) \mid c = a - 2, d = b + 1\},$$

$\cup$

$$C_{v_{y21}, v_{y22}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y22}) \mid c = a - 1, d = b - 0\},$$

$$C_{v_{y21}, v_{y23}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y23}) \mid c = a - 1, d = b - 1\},$$

$$C_{v_{y21}, v_{y24}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y24}) \mid c = a - 2, d = b - 1\},$$

$$C_{v_{y21}, v_{y25}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y25}) \mid c = a - 1, d = b - 2\},$$

$\cup$

$$C_{v_{y21}, v_{y22}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y22}) \mid c = a + 0, d = b - 1\},$$

$$C_{v_{y21}, v_{y23}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y23}) \mid c = a + 1, d = b - 1\},$$

$$C_{v_{y21}, v_{y24}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y24}) \mid c = a + 1, d = b - 2\},$$

$\cup$

$$C_{v_{y21}, v_{y22}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y22}) \mid c = a + 0, d = b - 1\},$$

$$C_{v_{y21}, v_{y23}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y23}) \mid c = a + 1, d = b - 1\},$$

$$C_{v_{y21}, v_{y24}} = \{((a, b), (c, d)) \in D(v_{y21}) \times D(v_{y24}) \mid c = a + 1, d = b - 2\},$$

$\cup$

④ piece Yellow2

[illegible]

[illegible]

[illegible]

[illegible]

$$\begin{aligned}
& C_{v_{r21}, v_{r23}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a - 2, d = b - 0\}, \\
& C_{v_{r21}, v_{r24}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a - 0, d = b - 1\}, \\
& \cup \\
& C_{v_{r21}, v_{r22}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a + 0, d = b - 1\}, \\
& C_{v_{r21}, v_{r23}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a + 0, d = b - 2\}, \\
& C_{v_{r21}, v_{r24}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a + 1, d = b - 0\}, \\
& \cup \\
& C_{v_{r21}, v_{r22}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a - 1, d = b + 0\}, \\
& C_{v_{r21}, v_{r23}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a - 2, d = b + 0\}, \\
& C_{v_{r21}, v_{r24}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a - 0, d = b + 1\}, \\
& \cup \\
& C_{v_{r21}, v_{r22}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a - 0, d = b - 1\}, \\
& C_{v_{r21}, v_{r23}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a - 0, d = b - 2\}, \\
& C_{v_{r21}, v_{r24}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a - 1, d = b - 0\}, \\
& \cup \\
& C_{v_{r21}, v_{r22}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a + 1, d = b - 0\}, \\
& C_{v_{r21}, v_{r23}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a + 2, d = b - 0\}, \\
& C_{v_{r21}, v_{r24}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a + 0, d = b - 1\}, \\
& \cup \\
& C_{v_{r21}, v_{r22}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a + 0, d = b + 1\}, \\
& C_{v_{r21}, v_{r23}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a + 0, d = b + 2\}, \\
& C_{v_{r21}, v_{r24}} = \{((a, b), (c, d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a + 1, d = b + 0\}, \\
& \textcircled{11} \text{ Yellow peg1} \\
& C_{v_{py1}} = \{V_{py1} = V_{y11} \cup V_{py1} = V_{y21} \cup V_{py1} = V_{y22} \cup V_{py1} = V_{y24} \cup V_{py1} = (0, 0)\} \\
& \textcircled{12} \text{ Yellow peg2} \\
& C_{v_{py2}} = \{V_{py2} = V_{y11} \cup V_{py2} = V_{y21} \cup V_{py2} = V_{y22} \cup V_{py2} = V_{y24} \cup V_{py2} = (0, 0)\} \\
& \textcircled{13} \text{ Blue peg1} \\
& C_{v_{pb1}} = \{V_{pb1} = V_{b13} \cup V_{pb1} = V_{b15} \cup V_{pb1} = V_{b23} \cup V_{pb1} = (0, 0)\} \\
& \textcircled{14} \text{ Blue peg2} \\
& C_{v_{pb2}} = \{V_{pb2} = V_{b13} \cup V_{pb2} = V_{b15} \cup V_{pb2} = V_{b23} \cup V_{pb2} = (0, 0)\} \\
& \textcircled{15} \text{ Green peg1} \\
& C_{v_{pg1}} = \{V_{pg1} = V_{g13} \cup V_{pg1} = V_{g14} \cup V_{pg1} = V_{g22} \cup V_{pg1} = V_{g23} \cup V_{pg1} = (0, 0)\} \\
& \textcircled{16} \text{ Green peg2} \\
& C_{v_{pg2}} = \{V_{pg2} = V_{g13} \cup V_{pg2} = V_{g14} \cup V_{pg2} = V_{g22} \cup V_{pg2} = V_{g23} \cup V_{pg2} = (0, 0)\} \\
& \textcircled{17} \text{ Red peg} \\
& C_{v_{pr}} = \{V_{pr} = V_{r12} \cup V_{pr} = V_{r21} \cup V_{pr} = V_{r23} \cup V_{pr} = (0, 0)\}
\end{aligned}$$

## 2.2 LP model

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# Cube Puzzler- Go

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