CSP and LP models of IQ Twist and Cube Puzzler - Go

Mingzhen Ao

A thesis submitted for the course
COMP8755 Individual Computing Project
Supervised by: Pascal Bercher and Florian Geisser
The Australian National University

November, 2020

*Declaration

This thesis is an account of research undertaken between July 2020 and November 2020 at The College of Engineering and Computer Science, Faculty of Computing Science, The Australian National University, Canberra, Australia.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

Mingzhen Ao
November, 2020

*Acknowledgements
I would like to thank my lucky stars, and the cat, for not eating me.

*Abstract

This thesis tells a great story about what I achieved in my research project. The abstract is short, but informative. it makes clear the general area in which I worked, and what I achieved.

*Contents

1 Introduction

2 IQ twist

2.1 CSP model

In this part, I'd like to use CSP to model the IQ twist game. Above all, I'd like to define all the initial state for each piece:

initial state of Yellow 1

initial state of Yellow 2

initial state of Blue 1

initial state of Blue 2

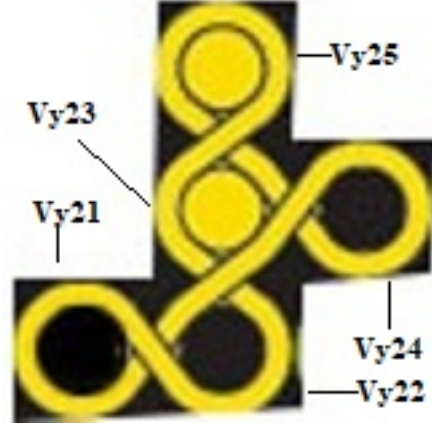initial state of Green 1

initial state of Green 2

initial state of Red 1

initial state of Red 2

For this method, there are some rules. Firstly, all the variables correspond to the initial states. Such as the Yellow 2, I use Vy_{21} to represent the left and bottom unit. For other

variables, name them as Vy_{22}, Vy_{23}, Vy_{24} and Vy_{25} follow the order from left to right and bottom up.

naming rules example for yellow 2

2.1.1 Variables

$V = (V_{y11}, V_{y12}, V_{y13}, V_{y21}, V_{y22}, V_{y23}, V_{y24}, V_{y25}, V_{b11}, V_{b12}, V_{b13}, V_{b14},$
$V_{b15}, V_{b21}, V_{b22}, V_{b23}, V_{b24}, V_{g11}, V_{g12}, V_{g13}, V_{g14}, V_{g21}, V_{g22}, V_{g23}, V_{r11},$
$V_{r12}, V_{r13}, V_{r14}, V_{r21}, V_{r22}, V_{r23}, V_{r24}, V_{ry1}, V_{ry2}, V_{rb1}, V_{rb2}, V_{rg1}, V_{rg2}, V_{rr})$

it consists of two parts:

$Vunits = (V_{y11}, V_{y12}, V_{y13}, V_{y21}, V_{y22}, V_{y23}, V_{y24}, V_{y25}, V_{b11}, V_{b12}, V_{b13}, V_{b14},$
$V_{b15}, V_{b21}, V_{b22}, V_{b23}, V_{b24}, V_{g11}, V_{g12}, V_{g13}, V_{g14}, V_{g21}, V_{g22}, V_{g23}, V_{r11},$
$V_{r12}, V_{r13}, V_{r14}, V_{r21}, V_{r22}, V_{r23}, V_{r24})$

$Vpegs = (V_{ry1}, V_{ry2}, V_{rb1}, V_{rb2}, V_{rg1}, V_{rg2}, V_{rr})$

2.1.2 Domains

For all $v \in Vunit$, Dv=$\{(i,j), (i,j) \in Z \times Z : 0 < i \leq 8 \ and \ 0 < j \leq 4\}$

For all $v \in Vregs$, Dv=$\{None \ or \ (i,j), (i,j) \in Z \times Z : 0 < i \leq 8 \ and \ 0 < j \leq 4\}$

2.1.3 Constrains

① For each pairs of variables-V_m and V_n, $V_m \in$ V units, $V_n \in$ V units

$$Cv_m, v_n = \{((a,b),(c,d)) \in D \times D : a \neq c \ or \ b \neq d\}$$

② For each pairs of variables-V_m and V_n, $V_m \in$ V regs, $V_n \in$ V regs,
if $V_n \neq None \ and \ V_n \neq None, Cv_m, v_n = \{((a,b),(c,d)) \in D \times D : a \neq c \ or \ b \neq d\}$

③ piece Yellow1
$Cv_{y11}, v_{y12} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 0\}$,
$Cv_{y11}, v_{y13} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b + 0\}$,
or

$Cv_{y11}, v_{y12} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 1\}$,
$Cv_{y11}, v_{y13} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 2\}$,
or
$Cv_{y11}, v_{y12} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 0\}$,
$Cv_{y11}, v_{y13} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b - 0\}$,
or
$Cv_{y11}, v_{y12} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 1\}$,
$Cv_{y11}, v_{y13} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 2\}$,
or
$Cv_{y11}, v_{y12} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 0\}$,
$Cv_{y11}, v_{y13} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b + 0\}$,
or
$Cv_{y11}, v_{y12} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 1\}$,
$Cv_{y11}, v_{y13} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 2\}$,
or
$Cv_{y11}, v_{y12} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 0\}$,
$Cv_{y11}, v_{y13} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b - 0\}$,
or
$Cv_{y11}, v_{y12} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 1\}$,
$Cv_{y11}, v_{y13} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 2\}$,
④ piece Yellow2
$Cv_{y21}, v_{y22} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 0\}$,
$Cv_{y21}, v_{y23} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 1\}$,
$Cv_{y21}, v_{y24} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b + 1\}$,
$Cv_{y21}, v_{y25} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 2\}$,
or
$Cv_{y21}, v_{y22} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 1\}$,
$Cv_{y21}, v_{y23} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 1\}$,
$Cv_{y21}, v_{y24} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 2\}$,
$Cv_{y21}, v_{y25} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b + 1\}$,
or
$Cv_{y21}, v_{y22} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 0\}$,
$Cv_{y21}, v_{y23} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 1\}$,
$Cv_{y21}, v_{y24} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b - 1\}$,
$Cv_{y21}, v_{y25} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 2\}$,
or
$Cv_{y21}, v_{y22} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 1\}$,
$Cv_{y21}, v_{y23} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 1\}$,
$Cv_{y21}, v_{y24} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 2\}$,
$Cv_{y21}, v_{y25} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b - 1\}$,
or
$Cv_{y21}, v_{y22} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 0\}$,
$Cv_{y21}, v_{y23} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 1\}$,
$Cv_{y21}, v_{y24} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b + 1\}$,
$Cv_{y21}, v_{y25} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 2\}$,
or
$Cv_{y21}, v_{y22} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 1\}$,
$Cv_{y21}, v_{y23} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 1\}$,

$Cv_{y21}, v_{y24} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 2\}$,
$Cv_{y21}, v_{y25} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b - 1\}$,
or
$Cv_{y21}, v_{y22} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 0\}$,
$Cv_{y21}, v_{y23} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 1\}$,
$Cv_{y21}, v_{y24} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b - 1\}$,
$Cv_{y21}, v_{y25} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 2\}$,
or
$Cv_{y21}, v_{y22} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 1\}$,
$Cv_{y21}, v_{y23} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 1\}$,
$Cv_{y21}, v_{y24} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 2\}$,
$Cv_{y21}, v_{y25} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b + 1\}$,
(5) piece Blue1
$Cv_{b11}, v_{b12} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 1\}$,
$Cv_{b11}, v_{b13} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 1\}$,
$Cv_{b11}, v_{b14} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 2\}$,
$Cv_{b11}, v_{b15} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 2\}$,
or
$Cv_{b11}, v_{b12} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 0\}$,
$Cv_{b11}, v_{b13} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 1\}$,
$Cv_{b11}, v_{b14} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b + 0\}$,
$Cv_{b11}, v_{b15} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b + 1\}$,
or
$Cv_{b11}, v_{b12} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 1\}$,
$Cv_{b11}, v_{b13} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 1\}$,
$Cv_{b11}, v_{b14} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 2\}$,
$Cv_{b11}, v_{b15} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 2\}$,
or
$Cv_{b11}, v_{b12} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 0\}$,
$Cv_{b11}, v_{b13} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 1\}$,
$Cv_{b11}, v_{b14} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b - 0\}$,
$Cv_{b11}, v_{b15} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b - 1\}$,
or
$Cv_{b11}, v_{b12} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 1\}$,
$Cv_{b11}, v_{b13} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 1\}$,
$Cv_{b11}, v_{b14} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 2\}$,
$Cv_{b11}, v_{b15} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 2\}$,
or
$Cv_{b11}, v_{b12} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 0\}$,
$Cv_{b11}, v_{b13} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 1\}$,
$Cv_{b11}, v_{b14} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b - 0\}$,
$Cv_{b11}, v_{b15} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b - 1\}$,
or
$Cv_{b11}, v_{b12} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 1\}$,
$Cv_{b11}, v_{b13} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 1\}$,
$Cv_{b11}, v_{b14} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 2\}$,
$Cv_{b11}, v_{b15} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 2\}$,
or

$Cv_{b11}, v_{b12} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 0\}$,
$Cv_{b11}, v_{b13} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 1\}$,
$Cv_{b11}, v_{b14} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b + 0\}$,
$Cv_{b11}, v_{b15} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b + 1\}$,
(6) piece Blue2
$Cv_{b21}, v_{b22} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 0\}$,
$Cv_{b21}, v_{b23} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b + 0\}$,
$Cv_{b21}, v_{b24} = \{((a,b),(c,d)) \in D \times D : c = a + 3 \ and \ d = b + 0\}$,
or
$Cv_{b21}, v_{b22} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 1\}$,
$Cv_{b21}, v_{b23} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 2\}$,
$Cv_{b21}, v_{b24} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 3\}$,
or
$Cv_{b21}, v_{b22} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 0\}$,
$Cv_{b21}, v_{b23} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b - 0\}$,
$Cv_{b21}, v_{b24} = \{((a,b),(c,d)) \in D \times D : c = a - 3 \ and \ d = b - 0\}$,
or
$Cv_{b21}, v_{b22} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 1\}$,
$Cv_{b21}, v_{b23} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 2\}$,
$Cv_{b21}, v_{b24} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 3\}$,
or
$Cv_{b21}, v_{b22} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 0\}$,
$Cv_{b21}, v_{b23} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b + 0\}$,
$Cv_{b21}, v_{b24} = \{((a,b),(c,d)) \in D \times D : c = a - 3 \ and \ d = b + 0\}$,
or
$Cv_{b21}, v_{b22} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 1\}$,
$Cv_{b21}, v_{b23} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 2\}$,
$Cv_{b21}, v_{b24} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 3\}$,
or
$Cv_{b21}, v_{b22} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 0\}$,
$Cv_{b21}, v_{b23} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b - 0\}$,
$Cv_{b21}, v_{b24} = \{((a,b),(c,d)) \in D \times D : c = a + 3 \ and \ d = b - 0\}$,
or
$Cv_{b21}, v_{b22} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 1\}$,
$Cv_{b21}, v_{b23} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 2\}$,
$Cv_{b21}, v_{b24} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 3\}$,
(7) piece Green1
$Cv_{g11}, v_{g12} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 0\}$,
$Cv_{g11}, v_{g13} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b + 0\}$,
$Cv_{g11}, v_{g14} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 1\}$,
or
$Cv_{g11}, v_{g12} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 1\}$,
$Cv_{g11}, v_{g13} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 2\}$,
$Cv_{g11}, v_{g14} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 1\}$,
or
$Cv_{g11}, v_{g12} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 0\}$,
$Cv_{g11}, v_{g13} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b - 0\}$,
$Cv_{g11}, v_{g14} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 1\}$,

or
$Cv_{g11}, v_{g12} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 1\},$
$Cv_{g11}, v_{g13} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 2\},$
$Cv_{g11}, v_{g14} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 1\},$
or
$Cv_{g11}, v_{g12} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 0\},$
$Cv_{g11}, v_{g13} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b + 0\},$
$Cv_{g11}, v_{g14} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 1\},$
or
$Cv_{g11}, v_{g12} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 1\},$
$Cv_{g11}, v_{g13} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 2\},$
$Cv_{g11}, v_{g14} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 1\},$
or
$Cv_{g11}, v_{g12} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 0\},$
$Cv_{g11}, v_{g13} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b - 0\},$
$Cv_{g11}, v_{g14} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 1\},$
or
$Cv_{g11}, v_{g12} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 1\},$
$Cv_{g11}, v_{g13} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 2\},$
$Cv_{g11}, v_{g14} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 1\},$
⑧ piece Green2
$Cv_{g21}, v_{g22} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 0\},$
$Cv_{g21}, v_{g23} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 1\},$
or
$Cv_{g21}, v_{g22} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 1\},$
$Cv_{g21}, v_{g23} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 1\},$
or
$Cv_{g21}, v_{g22} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 0\},$
$Cv_{g21}, v_{g23} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 1\},$
or
$Cv_{g21}, v_{g22} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 1\},$
$Cv_{g21}, v_{g23} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 1\},$
or
$Cv_{g21}, v_{g22} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 0\},$
$Cv_{g21}, v_{g23} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 1\},$
or
$Cv_{g21}, v_{g22} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 1\},$
$Cv_{g21}, v_{g23} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 1\},$
or
$Cv_{g21}, v_{g22} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 0\},$
$Cv_{g21}, v_{g23} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 1\},$
or
$Cv_{g21}, v_{g22} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 1\},$
$Cv_{g21}, v_{g23} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 1\},$
⑨ piece Red1
$Cv_{r11}, v_{r12} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 0\},$
$Cv_{r11}, v_{r13} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 1\},$
$Cv_{r11}, v_{r14} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b + 1\},$

or
$Cv_{r11}, v_{r12} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 1\}$,
$Cv_{r11}, v_{r13} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 1\}$,
$Cv_{r11}, v_{r14} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 2\}$,
or
$Cv_{r11}, v_{r12} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 0\}$,
$Cv_{r11}, v_{r13} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 1\}$,
$Cv_{r11}, v_{r14} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b - 1\}$,
or
$Cv_{r11}, v_{r12} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 1\}$,
$Cv_{r11}, v_{r13} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 1\}$,
$Cv_{r11}, v_{r14} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 2\}$,
or
$Cv_{r11}, v_{r12} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 0\}$,
$Cv_{r11}, v_{r13} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 1\}$,
$Cv_{r11}, v_{r14} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b + 1\}$,
or
$Cv_{r11}, v_{r12} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 1\}$,
$Cv_{r11}, v_{r13} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 1\}$,
$Cv_{r11}, v_{r14} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 2\}$,
or
$Cv_{r11}, v_{r12} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 0\}$,
$Cv_{r11}, v_{r13} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 1\}$,
$Cv_{r11}, v_{r14} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b - 1\}$,
or
$Cv_{r11}, v_{r12} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 1\}$,
$Cv_{r11}, v_{r13} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 1\}$,
$Cv_{r11}, v_{r14} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 2\}$,
(10) piece Red2
$Cv_{r21}, v_{r22} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 0\}$,
$Cv_{r21}, v_{r23} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b + 0\}$,
$Cv_{r21}, v_{r24} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 1\}$,
or
$Cv_{r21}, v_{r22} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 1\}$,
$Cv_{r21}, v_{r23} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 2\}$,
$Cv_{r21}, v_{r24} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 0\}$,
or
$Cv_{r21}, v_{r22} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 0\}$,
$Cv_{r21}, v_{r23} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b - 0\}$,
$Cv_{r21}, v_{r24} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 1\}$,
or
$Cv_{r21}, v_{r22} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 1\}$,
$Cv_{r21}, v_{r23} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 2\}$,
$Cv_{r21}, v_{r24} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 0\}$,
or
$Cv_{r21}, v_{r22} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b + 0\}$,
$Cv_{r21}, v_{r23} = \{((a,b),(c,d)) \in D \times D : c = a - 2 \ and \ d = b + 0\}$,
$Cv_{r21}, v_{r24} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b + 1\}$,

or

$Cv_{r21}, v_{r22} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 1\}$,

$Cv_{r21}, v_{r23} = \{((a,b),(c,d)) \in D \times D : c = a - 0 \ and \ d = b - 2\}$,

$Cv_{r21}, v_{r24} = \{((a,b),(c,d)) \in D \times D : c = a - 1 \ and \ d = b - 0\}$,

or

$Cv_{r21}, v_{r22} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b - 0\}$,

$Cv_{r21}, v_{r23} = \{((a,b),(c,d)) \in D \times D : c = a + 2 \ and \ d = b - 0\}$,

$Cv_{r21}, v_{r24} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b - 1\}$,

or

$Cv_{r21}, v_{r22} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 1\}$,

$Cv_{r21}, v_{r23} = \{((a,b),(c,d)) \in D \times D : c = a + 0 \ and \ d = b + 2\}$,

$Cv_{r21}, v_{r24} = \{((a,b),(c,d)) \in D \times D : c = a + 1 \ and \ d = b + 0\}$,

(11) Yellow reg1

$if \ V_{ry1} \neq None, Cv_{ry1} = \{V_{ry1} = V_{y11} \ or \ V_{ry1} = V_{y21} \ or \ V_{ry1} = V_{y22} \ or \ V_{ry1} = V_{y24}\}$

(12) Yellow reg2

$if \ V_{ry2} \neq None, Cv_{ry2} = \{V_{ry2} = V_{y11} \ or \ V_{ry2} = V_{y21} \ or \ V_{ry2} = V_{y22} \ or \ V_{ry2} = V_{y24}\}$

(13) Blue reg1

$if \ V_{rb1} \neq None, Cv_{rb1} = \{V_{rb1} = V_{b13} \ or \ V_{rb1} = V_{b15} \ or \ V_{rb1} = V_{b23}\}$

(14) Blue reg2

$if \ V_{rb2} \neq None, Cv_{rb2} = \{V_{rb2} = V_{b13} \ or \ V_{rb2} = V_{b15} \ or \ V_{rb2} = V_{b23}\}$

(15) Green reg1

$if \ V_{rg1} \neq None, Cv_{rg1} = \{V_{rg1} = V_{g13} \ or \ V_{rg1} = V_{g14} \ or \ V_{rg1} = V_{g22} \ or \ V_{rg1} = V_{g23}\}$

(16) Green reg2

$if \ V_{rg2} \neq None, Cv_{rg2} = \{V_{rg2} = V_{g13} \ or \ V_{rg2} = V_{g14} \ or \ V_{rg2} = V_{g22} \ or \ V_{rg2} = V_{g23}\}$

(17) Red reg

$if \ V_{rr} \neq None, Cv_{rr} = \{V_{rr} = V_{r12} \ or \ V_{rr} = V_{r21} \ or \ V_{rr} = V_{r23}\}$

2.2 LP model