CSP and LP models of IQ Twist and Cube Puzzler - Go

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Declaration

This thesis is an account of research undertaken between July 2020 and November 2020 at The College of Engineering and Computer Science, Faculty of Computing Science, The Australian National University, Canberra, Australia.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

Mingzhen Ao November, 2020

Acknowledgements

I would like to thank my lucky stars, and the cat, for not eating me.

Abstract

This thesis tells a great story about what I achieved in my research project. The abstract is short, but informative. it makes clear the general area in which I worked, and what I achieved.

Contents

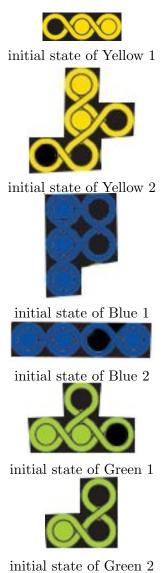
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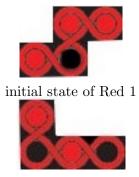
Introduction

IQ twist

CSP model 2.1

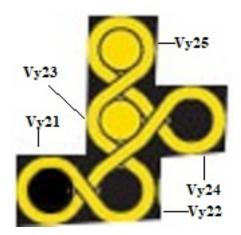
In this part, I'd like to use CSP to model the IQ twist game. Above all, I'd like to define all the initial state for each piece:





initial state of Red 2

For this method, there are some rules. Firstly, all the variables coprespond to the initial states. Such as the Yellow 2, I use V_{y21} to represent the left and bottom unit. For other variables, name them as V_{y22} , V_{y23} , V_{y24} and V_{y25} follow the order from left to right and bottom up.



naming rules example for yellow 2

2.1.1 Variables

$$\begin{split} V_{units} &= \{V_{y11}, V_{y12}, V_{y13}, V_{y21}, V_{y22}, V_{y23}, V_{y24}, V_{y25}, \\ V_{b11}, V_{b12}, V_{b13}, V_{b14}, V_{b15}, V_{b21}, V_{b22}, V_{b23}, \\ V_{b24}, V_{g11}, V_{g12}, V_{g13}, V_{g14}, V_{g21}, V_{g22}, V_{g23}, \\ V_{r11}, V_{r12}, V_{r13}, V_{r14}, V_{r21}, V_{r22}, V_{r23}, V_{r24} \} \end{split}$$

$$V_{pegs} = \{V_{py1}, V_{py2}, V_{pb1}, V_{pb2}, V_{pg1}, V_{pg2}, V_{pr}\}$$

$$V = V_{units} \cup V_{pegs}$$
.

2.1.2 Domains

For all
$$v \in V_{units}$$
, $D(v) = \{(i, j) \in \mathbb{Z} \times \mathbb{Z} \mid 0 < i \le 8, 0 < j \le 4\}$
For all $v \in V_{pegs}$, $D(v) = \{(0, 0) \cup (i, j) \in \mathbb{Z} \times \mathbb{Z} \mid 0 < i \le 8, 0 < j \le 4\}$

2.1.3 Constrains

1 For each pair of variables $-V_m$ and $V_n, V_m \in V_{units}, V_n \in V_{units}$

```
C_{v_m,v_n} = \{((a,b),(c,d)) \in D(v_m) \times D(v_n) \mid a \neq c \text{ or } b \neq d\}
```

(2) For each pair of variables $-V_m$ and $V_n, V_m \in V_{pegs}, V_n \in V_{pegs}$,

```
C_{v_m,v_n} = \{((0,0),(0,0)) \cup ((a,b),(c,d)) \in D(v_m) \times D(v_n) \mid a \neq c \text{ or } b \neq d\}
```

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(3) piece Yellow1
C_{v_{y11},v_{y12}} = \{((a,b),(c,d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a+1, d = b+0\},\
C_{v_{y11},v_{y13}} = \{((a,b),(c,d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a+2, d = b+0\},\
C_{v_{v11},v_{v12}} = \{((a,b),(c,d)) \in D(v_{v11}) \times D(v_{v12}) \mid c = a - 0, d = b + 1\},
C_{v_{y11},v_{y13}} = \{((a,b),(c,d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a - 0, d = b + 2\},\
C_{v_{u11},v_{u12}} = \{((a,b),(c,d)) \in D(v_{u11}) \times D(v_{u12}) \mid c = a-1, d = b-0\},
C_{v_{y11},v_{y13}} = \{((a,b),(c,d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a-2, d = b-0\},
C_{v_{u11},v_{u12}} = \{((a,b),(c,d)) \in D(v_{u11}) \times D(v_{u12}) \mid c = a+0, d = b-1\},
C_{v_{y11},v_{y13}} = \{((a,b),(c,d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a+0, d = b-2\},\
C_{v_{u11},v_{u12}} = \{((a,b),(c,d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a-1 \ , \ d = b+0\} \ ,
C_{v_{u11},v_{u13}} = \{((a,b),(c,d)) \in D(v_{u11}) \times D(v_{u13}) \mid c = a-2, d = b+0\},
C_{v_{y11},v_{y12}} = \{((a,b),(c,d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a - 0, d = b - 1\},
C_{v_{y11},v_{y13}} = \{((a,b),(c,d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a - 0, d = b - 2\},\
C_{v_{y11},v_{y12}} = \{((a,b),(c,d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a+1, d = b-0\},\
C_{v_{y11},v_{y13}} = \{((a,b),(c,d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a+2, d = b-0\},\
\bigcup
C_{v_{y11},v_{y12}} = \{((a,b),(c,d)) \in D(v_{y11}) \times D(v_{y12}) \mid c = a+0, d = b+1\},
C_{v_{y11},v_{y13}} = \{((a,b),(c,d)) \in D(v_{y11}) \times D(v_{y13}) \mid c = a+0, d = b+2\},
(4) piece Yellow2
C_{v_{y21},v_{y22}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y22}) \mid c = a+1, d = b+0\},
C_{v_{u21},v_{u23}} = \{((a,b),(c,d)) \in D(v_{u21}) \times D(v_{u23}) \mid c = a+1, d = b+1\},
C_{v_{y21},v_{y24}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y24}) \mid c = a+2, d = b+1\},
C_{v_{y21},v_{y25}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y25}) \mid c = a+1, d = b+2\},\
C_{v_{u21},v_{u22}} = \{((a,b),(c,d)) \in D(v_{u21}) \times D(v_{u22}) \mid c = a - 0, d = b + 1\},
C_{v_{y21},v_{y23}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y23}) \mid c = a-1, d = b+1\},
C_{v_{y21},v_{y24}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y24}) \mid c = a-1, d = b+2\},\
C_{v_{y21},v_{y25}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y25}) \mid c = a-2, d = b+1\},
C_{v_{u21},v_{u22}} = \{((a,b),(c,d)) \in D(v_{u21}) \times D(v_{u22}) \mid c = a-1, d = b-0\},
C_{v_{u21},v_{u23}} = \{((a,b),(c,d)) \in D(v_{u21}) \times D(v_{u23}) \mid c = a-1, d = b-1\},
C_{v_{u21},v_{u24}} = \{((a,b),(c,d)) \in D(v_{u21}) \times D(v_{u24}) \mid c = a-2, d = b-1\},
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C_{v_{y21},v_{y23}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y23}) \mid c = a+1, d = b-1\},
C_{v_{y21},v_{y24}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y24}) \mid c = a+1, d = b-2\},
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C_{v_{y21},v_{y25}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y25}) \mid c = a+2, d = b-1\},\
C_{v_{y21},v_{y22}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y22}) \mid c = a-1, d = b+0\},\
C_{v_{u21},v_{u23}} = \{((a,b),(c,d)) \in D(v_{u21}) \times D(v_{u23}) \mid c = a-1, d = b+1\},
C_{v_{u21},v_{u24}} = \{((a,b),(c,d)) \in D(v_{u21}) \times D(v_{u24}) \mid c = a-2, d = b+1\},
C_{v_{u21},v_{u25}} = \{((a,b),(c,d)) \in D(v_{u21}) \times D(v_{u25}) \mid c=a-1, d=b+2\},
C_{v_{y21},v_{y22}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y22}) \mid c = a - 0, d = b - 1\},
C_{v_{y21},v_{y23}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y23}) \mid c = a-1, d = b-1\},\
C_{v_{y21},v_{y24}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y24}) \mid c = a-1, d = b-2\},
C_{v_{y21},v_{y25}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y25}) \mid c = a-2, d = b-1\},
C_{v_{y21},v_{y22}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y22}) \mid c = a+1, d = b-0\},\
C_{v_{u21},v_{u23}} = \{((a,b),(c,d)) \in D(v_{u21}) \times D(v_{u23}) \mid c = a+1, d = b-1\},
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C_{v_{y21},v_{y23}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y23}) \mid c = a+1, d = b+1\},
C_{v_{u21},v_{u24}} = \{((a,b),(c,d)) \in D(v_{u21}) \times D(v_{u24}) \mid c = a+1, d = b+2\},
C_{v_{y21},v_{y25}} = \{((a,b),(c,d)) \in D(v_{y21}) \times D(v_{y25}) \mid c = a+2, d = b+1\},
(5) piece Blue1
C_{v_{b11},v_{b12}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b12}) \mid c = a+0, d = b+1\},
C_{v_{b11},v_{b13}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b13}) \mid c = a+1, d = b+1\},
C_{v_{b11},v_{b14}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{y14}) \mid c = a+0, d = b+2\},
C_{v_{b11},v_{b15}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b15}) \mid c = a+1, d = b+2\},\
C_{v_{b11},v_{b12}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b12}) \mid c = a-1, d = b+0\},
C_{v_{b11},v_{b13}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b13}) \mid c = a-1, d = b+1\},
C_{v_{b11},v_{b14}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{y14}) \mid c = a-2, d = b+0\},
C_{v_{b11},v_{b15}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b15}) \mid c = a-2, d = b+1\},
C_{v_{b11},v_{b12}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b12}) \mid c = a - 0, d = b - 1\},
C_{v_{b11},v_{b13}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b13}) \mid c = a-1, d = b-1\},
C_{v_{b11},v_{b14}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{y14}) \mid c = a - 0, d = b - 2\},
C_{v_{b11},v_{b15}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b15}) \mid c=a-1, d=b-2\},
C_{v_{b11},v_{b12}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b12}) \mid c = a+1, d = b-0\},\
C_{v_{b11},v_{b13}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b13}) \mid c = a+1, d = b-1\},
C_{v_{b11},v_{b14}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{y14}) \mid c = a+2, d = b-0\},
C_{v_{b11},v_{b15}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b15}) \mid c = a+2, d = b-1\},
C_{v_{b11},v_{b12}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b12}) \mid c = a-0 \ , \ d = b+1\} \ ,
C_{v_{b11},v_{b13}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b13}) \mid c = a-1, d = b+1\},
C_{v_{b11},v_{b14}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{y14}) \mid c = a - 0, d = b + 2\},\
C_{v_{b11},v_{b15}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b15}) \mid c = a-1, d = b+2\},
C_{v_{b11},v_{b12}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b12}) \mid c = a-1, d = b-0\},
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C_{v_{b11},v_{b13}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b13}) \mid c = a-1, d = b-1\},
C_{v_{b11},v_{b14}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{y14}) \mid c = a-2, d = b-0\},
C_{v_{b11},v_{b15}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b15}) \mid c = a-2, d = b-1\},
C_{v_{b11},v_{b12}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b12}) \mid c = a+0, d = b-1\},
C_{v_{b11},v_{b13}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b13}) \mid c = a+1, d = b-1\},
C_{v_{b11},v_{b14}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{y14}) \mid c = a+0, d = b-2\},\
C_{v_{b11},v_{b15}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b15}) \mid c = a+1, d = b-2\},
C_{v_{b11},v_{b12}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b12}) \mid c = a+1, d = b+0\},
C_{v_{b11},v_{b13}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b13}) \mid c = a+1, d = b+1\},
C_{v_{b11},v_{b14}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{y14}) \mid c = a+2, d = b+0\},
C_{v_{b11},v_{b15}} = \{((a,b),(c,d)) \in D(v_{b11}) \times D(v_{b15}) \mid c = a+2, d = b+1\},
(6) piece Blue2
C_{v_{b21},v_{b22}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b22}) \mid c = a+1, d = b+0\},
C_{v_{b21},v_{b23}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b23}) \mid c = a+2, d = b+0\},
C_{v_{b21},v_{b24}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b24}) \mid c = a+3, d = b+0\},\
C_{v_{b21},v_{b22}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b22}) \mid c = a - 0, d = b + 1\},
C_{v_{b21},v_{b23}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b23}) \mid c = a - 0, d = b + 2\},
C_{v_{b21},v_{b24}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b24}) \mid c = a - 0, d = b + 3\},
C_{v_{b21},v_{b22}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b22}) \mid c = a-1, d = b-0\},
C_{v_{b21},v_{b23}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b23}) \mid c = a-2, d = b-0\},\
C_{v_{b21},v_{b24}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b24}) \mid c = a-3, d = b-0\},\
C_{v_{b21},v_{b22}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b22}) \mid c = a+0 , d = b-1\},
C_{v_{b21},v_{b23}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b23}) \mid c = a+0, d = b-2\},
C_{v_{b21},v_{b24}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b24}) \mid c = a+0, d = b-3\},
\bigcup
C_{v_{b21},v_{b22}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b22}) \mid c = a-1, d = b+0\},\
C_{v_{b21},v_{b23}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b23}) \mid c = a-2 \ , \ d = b+0\} \ ,
C_{v_{b21},v_{b24}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b24}) \mid c = a-3, d = b+0\},
C_{v_{b21},v_{b22}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b22}) \mid c = a - 0, d = b - 1\},
C_{v_{b21},v_{b23}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b23}) \mid c = a - 0, d = b - 2\},\
C_{v_{b21},v_{b24}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b24}) \mid c = a - 0, d = b - 3\},
C_{v_{b21},v_{b22}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b22}) \mid c = a+1, d = b-0\},\
C_{v_{b21},v_{b23}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b23}) \mid c = a+2, d = b-0\},\
C_{v_{b21},v_{b24}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b24}) \mid c = a+3, d = b-0\},
C_{v_{b21},v_{b22}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b22}) \mid c = a+0, d = b+1\},
C_{v_{b21},v_{b23}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b23}) \mid c = a+0, d = b+2\},
C_{v_{b21},v_{b24}} = \{((a,b),(c,d)) \in D(v_{b21}) \times D(v_{b24}) \mid c = a+0, d = b+3\},
(7) piece Green1
C_{v_{g11},v_{g12}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g12}) \mid c = a+1, d = b+0\},
C_{v_{g11},v_{g13}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g13}) \mid c = a+2, d = b+0\},
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C_{v_{q11},v_{q14}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g14}) \mid c=a+1 \ , \ d=b+1\} \ ,
C_{v_{g11},v_{g12}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g12}) \mid c = a - 0, d = b + 1\},
C_{v_{g11},v_{g13}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g13}) \mid c = a - 0, d = b + 2\},
C_{v_{g11},v_{g14}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g14}) \mid c = a-1, d = b+1\},
C_{v_{q11},v_{q12}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g12}) \mid c = a-1, d = b-0\},\
C_{v_{q11},v_{q13}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g13}) \mid c = a-2, d = b-0\},\
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C_{v_{g11},v_{g14}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g14}) \mid c = a+1, d = b-1\},\
C_{v_{q11},v_{q12}} = \{((a,b),(c,d)) \in D(v_{q11}) \times D(v_{q12}) \mid c = a-1, d = b+0\},
C_{v_{q11},v_{q13}} = \{((a,b),(c,d)) \in D(v_{q11}) \times D(v_{q13}) \mid c = a-2, d = b+0\},
C_{v_{g11},v_{g14}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g14}) \mid c = a-1, d = b+1\},\
C_{v_{q11},v_{q12}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g12}) \mid c = a - 0, d = b - 1\},
C_{v_{g11},v_{g13}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g13}) \mid c = a - 0, d = b - 2\},
C_{v_{g11},v_{g14}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g14}) \mid c = a-1, d = b-1\},\
C_{v_{q11},v_{q12}} = \{((a,b),(c,d)) \in D(v_{q11}) \times D(v_{q12}) \mid c = a+1, d = b-0\},
C_{v_{q11},v_{q13}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g13}) \mid c = a+2, d = b-0\},
C_{v_{q11},v_{q14}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g14}) \mid c = a+1, d = b-1\},
C_{v_{q11},v_{q12}} = \{((a,b),(c,d)) \in D(v_{q11}) \times D(v_{q12}) \mid c = a+0, d = b+1\},
C_{v_{q11},v_{q13}} = \{((a,b),(c,d)) \in D(v_{q11}) \times D(v_{q13}) \mid c = a+0, d = b+2\},
C_{v_{g11},v_{g14}} = \{((a,b),(c,d)) \in D(v_{g11}) \times D(v_{g14}) \mid c = a+1, d = b+1\},
(8) piece Green2
C_{v_{q21},v_{q22}} = \{((a,b),(c,d)) \in D(v_{q21}) \times D(v_{q22}) \mid c = a+1, d = b+0\},
C_{v_{g21},v_{g23}} = \{((a,b),(c,d)) \in D(v_{g21}) \times D(v_{g23}) \mid c = a+1, d = b+1\},
\bigcup
C_{v_{q21},v_{q22}} = \{((a,b),(c,d)) \in D(v_{q21}) \times D(v_{q22}) \mid c = a - 0, d = b + 1\},
C_{v_{g21},v_{g23}} = \{((a,b),(c,d)) \in D(v_{g21}) \times D(v_{g23}) \mid c = a-1, d = b+1\},\
\bigcup
C_{v_{q21},v_{q22}} = \{((a,b),(c,d)) \in D(v_{q21}) \times D(v_{q22}) \mid c = a-1, d = b-0\},\
C_{v_{g21},v_{g23}} = \{((a,b),(c,d)) \in D(v_{g21}) \times D(v_{g23}) \mid c = a-1 \ , \ d = b-1\} \ ,
C_{v_{q21},v_{q22}} = \{((a,b),(c,d)) \in D(v_{q21}) \times D(v_{q22}) \mid c=a+0, d=b-1\},
C_{v_{g21},v_{g23}} = \{((a,b),(c,d)) \in D(v_{g21}) \times D(v_{g23}) \mid c = a+1 \ , \ d = b-1\} \ ,
C_{v_{g21},v_{g22}} = \left\{ ((a,b),(c,d)) \in D(v_{g21}) \times D(v_{g22}) \mid c = a-1 \ , \ d = b+0 \right\} \, ,
C_{v_{q21},v_{q23}} = \{((a,b),(c,d)) \in D(v_{q21}) \times D(v_{q23}) \mid c = a-1, d = b+1\},
C_{v_{q21},v_{q22}} = \{((a,b),(c,d)) \in D(v_{g21}) \times D(v_{g22}) \mid c = a - 0, d = b - 1\},
C_{v_{g21},v_{g23}} = \{((a,b),(c,d)) \in D(v_{g21}) \times D(v_{g23}) \mid c = a-1 \ , \ d = b-1\} \ ,
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C_{v_{q21},v_{q22}} = \{((a,b),(c,d)) \in D(v_{g21}) \times D(v_{g22}) \mid c = a+1, d = b-0\},\
C_{v_{g21},v_{g23}} = \{((a,b),(c,d)) \in D(v_{g21}) \times D(v_{g23}) \mid c = a+1, d = b-1\},\
C_{v_{q21},v_{q22}} = \{((a,b),(c,d)) \in D(v_{q21}) \times D(v_{q22}) \mid c = a+0, d = b+1\},
C_{v_{g21},v_{g23}} = \{((a,b),(c,d)) \in D(v_{g21}) \times D(v_{g23}) \mid c=a+1 \ , \ d=b+1\} \ ,
(9) piece Red1
C_{v_{r11},v_{r12}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r12}) \mid c = a+1, d = b+0\},
C_{v_{r11},v_{r13}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r13}) \mid c = a+1, d = b+1\},
C_{v_{r11},v_{r14}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r14}) \mid c = a+2, d = b+1\},
C_{v_{r11},v_{r12}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r12}) \mid c = a - 0, d = b + 1\},
C_{v_{r11},v_{r13}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r13}) \mid c = a-1, d = b+1\},
C_{v_{r11},v_{r14}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r14}) \mid c = a-1, d = b+2\},
\bigcup
C_{v_{r11},v_{r12}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r12}) \mid c = a-1, d = b-0\},
C_{v_{r11},v_{r13}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r13}) \mid c=a-1 \ , \ d=b-1\} \ ,
C_{v_{r11},v_{r14}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r14}) \mid c = a-2, d = b-1\},\
C_{v_{r11},v_{r12}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r12}) \mid c = a+0, d = b-1\},
C_{v_{r11},v_{r13}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r13}) \mid c = a+1, d = b-1\},
C_{v_{r11},v_{r14}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r14}) \mid c = a+1, d = b-2\},\
\bigcup
C_{v_{r11},v_{r12}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r12}) \mid c = a-1, d = b+0\},
C_{v_{r11},v_{r13}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r13}) \mid c = a-1, d = b+1\},
C_{v_{r11},v_{r14}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r14}) \mid c = a-2, d = b+1\},
C_{v_{r11},v_{r12}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r12}) \mid c = a - 0, d = b - 1\},\
C_{v_{r11},v_{r13}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r13}) \mid c = a-1, d = b-1\},
C_{v_{r11},v_{r14}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r14}) \mid c = a-1, d = b-2\},\
C_{v_{r11},v_{r12}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r12}) \mid c = a+1, d = b-0\},\
C_{v_{r11},v_{r13}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r13}) \mid c = a+1, d = b-1\},
C_{v_{r11},v_{r14}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r14}) \mid c = a+2, d = b-1\},
C_{v_{r11},v_{r12}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r12}) \mid c = a+0, d = b+1\},
C_{v_{r11},v_{r13}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r13}) \mid c = a+1, d = b+1\},
C_{v_{r11},v_{r14}} = \{((a,b),(c,d)) \in D(v_{r11}) \times D(v_{r14}) \mid c = a+1, d = b+2\},
(10) piece Red2
C_{v_{r21},v_{r22}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a+1, d = b+0\},\
C_{v_{r21},v_{r23}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a+2, d = b+0\},
C_{v_{r21},v_{r24}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a+0, d = b+1\},
C_{v_{r21},v_{r22}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a - 0, d = b + 1\},\
C_{v_{r21},v_{r23}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a - 0, d = b + 2\},\
C_{v_{r21},v_{r24}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a-1, d = b+0\},\
C_{v_{r21},v_{r22}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a-1, d = b-0\},\
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C_{v_{r21},v_{r23}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a-2, d = b-0\},
C_{v_{r21},v_{r24}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a - 0, d = b - 1\},
C_{v_{r21},v_{r22}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a+0, d = b-1\},
C_{v_{r21},v_{r23}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a+0, d = b-2\},\
C_{v_{r21},v_{r24}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a+1, d = b-0\},
C_{v_{r21},v_{r22}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a-1, d = b+0\},\
C_{v_{r21},v_{r23}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a-2 \ , \ d = b+0\} \ ,
C_{v_{r21},v_{r24}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a - 0, d = b + 1\},
\bigcup
C_{v_{r21},v_{r22}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a - 0, d = b - 1\},
C_{v_{r21},v_{r23}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a - 0, d = b - 2\},
C_{v_{r21},v_{r24}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a-1, d = b-0\},
C_{v_{r21},v_{r22}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a+1, d = b-0\},
C_{v_{r21},v_{r23}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a+2, d = b-0\},
C_{v_{r21},v_{r24}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a+0, d = b-1\},
C_{v_{r21},v_{r22}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r22}) \mid c = a+0, d = b+1\},
C_{v_{r21},v_{r23}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r23}) \mid c = a+0, d = b+2\},
C_{v_{r21},v_{r24}} = \{((a,b),(c,d)) \in D(v_{r21}) \times D(v_{r24}) \mid c = a+1, d = b+0\},
(11) Yellow peg1
\widetilde{C_{v_{py1}}} = \{V_{py1} = V_{y11} \cup V_{py1} = V_{y21} \cup V_{py1} = V_{y22} \cup V_{py1} = V_{y24} \cup V_{py1} = (0,0)\}
(12) Yellow peg2
\widetilde{C_{v_{py2}}} = \{V_{py2} = V_{y11} \ \cup \ V_{py2} = V_{y21} \ \cup \ V_{py2} = V_{y22} \ \cup \ V_{py2} = V_{y24} \ \cup \ V_{py2} = (0,0)\}
(13) Blue peg1
C_{v_{pb1}} = \{V_{pb1} = V_{b13} \cup V_{pb1} = V_{b15} \cup V_{pb1} = V_{b23} \cup V_{pb1} = (0,0)\}
(14) Blue peg2
\bar{C_{v_{pb2}}} = \{V_{pb2} = V_{b13} \cup V_{pb2} = V_{b15} \cup V_{pb2} = V_{b23} \cup V_{pb2} = (0,0)\}
(15) Green peg1
\widecheck{C_{v_{pg1}}} = \{V_{pg1} = V_{g13} \ \cup \ V_{pg1} = V_{g14} \ \cup \ V_{pg1} = V_{g22} \ \cup \ V_{pg1} = V_{g23} \ \cup \ V_{pg1} = (0,0)\}
(16) Green peg2

\widetilde{C_{v_{pq2}}} = \{V_{pg2} = V_{g13} \cup V_{pg2} = V_{g14} \cup V_{pg2} = V_{g22} \cup V_{pg2} = V_{g23} \cup V_{pg1} = (0,0)\}

(17) \operatorname{Red} \operatorname{peg}

\widetilde{C_{v_{pr}}} = \{V_{pr} = V_{r12} \cup V_{pr} = V_{r21} \cup V_{pr} = V_{r23} \cup V_{pr} = (0,0)\}
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2.2 LP model

Cube Puzzler- Go