An Euler Angle Calculation Method for Tailsitter UAV

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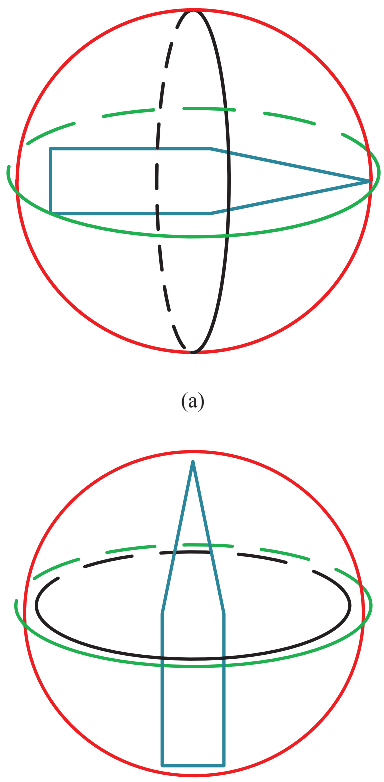
***Abstract*—The tailsitter UAV has very broad application prospects. However, it needs to switch between flat and vertical flights, so the pitch angle will exceed 90°. Unfortunately, the traditional Euler angle calculation method is not only singular (gimbal lock) but is also not compatible with the navigation calculations in the two flight modes. Therefore, a new Euler angle calculation method for a UAV is proposed. This method only avoids the singularity in the flight mode switching, but is also compatible with the navigation calculations in the two flight modes.**

Keywords—Tailsitter; Attitude calculation; Euler angles

I. INTRODUCTION

At present, the overall efficiency of the traditional fixed-wing UAV is very high, but it requires special take-off and landing sites. The rotor UAV does not require special take-off and landing sites, but its flight efficiency is very low and its endurance is also very less. Therefore, taking into account the advantages of both fixed- wing UAVs and rotor UAVs, a tailsitter UAV was designed [1]. This UAV does not require a special take-off site and also has high flight efficiency [2] [3].

However, because of the different flight mode of the tailsitter UAV, the conventional attitude solver is not very compatible with the tailsitter UAV [4]-[6], particularly in the vertical flight mode. The traditional calculation method causes a gimbal lock. When the gimbal lock occurs, the computation becomes singular wherein the attitude angle is not accurate or the attitude angle undergoes a step change, which causes the attitude controller to fail. The gimbal lock is a phenomenon involving the loss of one degree of freedom when two axes of three gimbals coincide. When Euler angles are used to represent the attitude kinematic equations of aircraft, irrespective of the rotation order, there will always be a stance angle. Singular points are encountered at large angles, thereby resulting in a gimbal lock state. In the traditional attitude calculation[7], the rotation order of Z-Y-X is usually used, and the rotation sequence of the gimbal is shown in Figure 1 (a). Z (green) is the parent of Y (red), and Y is the parent of X (black). In this rotation order, singularities occur when the pitch angle is ± 90°, as shown in Figure 1 (B).



(a)

(b)

Fig.1. Sketch map of Z-Y-X rotation order. (a) Z-Y-X rotary gimbal. (b) Pitch angle is 90°.

The traditional attitude calculation method chooses the Z-Y-X rotation order because traditional aircraft will not use a high elevation angle in the flight process. However, in a tailsitter UAV, in the flight-mode switching process, there will be pitch angle greater than 90°, so the Z-Y-X rotation order cannot be used for the attitude calculation of the tailsitter UAV.

At present, there are some methods to solve the singularity problem in the attitude angle calculation of tailsitter UAVs. Beach used resolved tilt-twist [8], but the computation is complex, and there was no real reaction problem. To avoid singularities, many people choose quaternion for attitude calculation similar to Jung and Nathan, who used a computationally efficient adaptive quaternion control algorithm [9] [10]. Further, Meenu used unit quaternions for rotations and derived an algorithm to point a camera [11], mounted with a two-axis (roll-pitch) gimbal on a micro aerial vehicle, toward a target at a known location on the ground. The use of unit quaternions makes the algorithm more computationally efficient and free of mathematical singularities. However, the physical meaning of the quaternion is not intuitive, which is not conducive to the preparation of the control law. In addition, there are some methods such as Liu's method, which involves changing the rotation order [12] so that the aircraft does not appear singular during flight mode transitions. However, those methods have a common disadvantage at present. When the aircraft is in vertical flight, the yaw angle cannot be used for heading determination. To overcome the drawbacks of Liu's method, Kuang used two sets of airframe coordinate systems [13] and selected different airframe coordinate systems, based on the flight mode. However, there is the problem of coordinating the system switching and ensuring a smooth transition of the Euler angles. Moreover, the calculation is tedious, and the computational complexity is high. Lu also selected the quaternion algorithm, but he selected the positive and inverse Euler angles according to the pitch angle values [14]. This algorithm is not only computationally expensive but also has the disadvantage of Liu's method.

Therefore, this paper aims to propose a new attitude calculation method for tailsitter UAVs by analyzing the requirements of the attitude calculation for a UAV to ensure that the tailsitter UAV is controllable in all its flight modes. The calculation method presented in this paper is not only simple but is also compatible with the control and navigation of the two flight modes.

II. Demand Analysis

Before discussing the problem, we first define the navigation coordinate system and body coordinate system. In this paper, the navigation coordinate system considered is the NED (North East Down) coordinate system, and the body coordinate system is defined in the traditional way, as shown in figure 1.

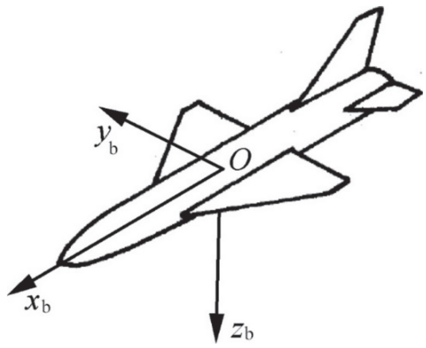


Fig.2. Body coordinate system.

To effectively solve the attitude problem of the tailsitter UAV and satisfy the navigation calculation, first, it is necessary to conduct a thorough analysis of the flight requirements of the tailsitter UAV. It is essential to consider the fact that the flight mode switch may cause singularity problems. Further, it is also important to consider different flight missions in two flight modes to avoid the singularity problem and the navigation algorithm should be compatible with the two flight modes.

Demand 1: In the flight-mode-switching process, the aircraft rotates mainly around the Y axis of the airframe; therefore, when the aircraft revolves around the Y axis of the airframe, it is necessary to avoid the gimbal lock.

Demand 2: In the vertical flight mode, the flight vehicle rotates mainly around the X axis of the airframe; therefore, when the aircraft is in the vertical flight mode, the aircraft revolves around the X axis of the airframe. Hence, it is necessary to avoid the gimbal lock around the X axis.

Demand 3: In the horizontal flight mode, the flight vehicle rotates mainly around the Z axis of the airframe; therefore, when the aircraft is in the horizontal flight mode, the aircraft revolves around the Z axis of the airframe, it is necessary to avoid the gimbal lock;

Demand 4: The attitude calculation of the flight vehicle in the two flight modes can be effectively applied to the following navigation calculation. Thus, the attitude calculation should be consistent with the fixed-wing aircraft attitude calculation in horizontal flight. Further, the attitude calculation for vertical flight should be consistent with the attitude calculation for four-rotor aircraft. When the aircraft flies horizontally, the rotation angle around the Z axis of the airframe is the yaw angle. The rotation angle around the Y axis of the airframe is the pitch angle, and the rotation angle around the X axis of the airframe is the roll angle. Further, it is essential to ensure that the yaw and roll angles remain unchanged during flight-mode switching so that, irrespective of the flight mode, the flight attitude angles are accurate and a smooth, seamless transition occurs.

From the above demand analysis, it is observed that it is impossible to meet all requirements at the same time. Therefore, the second best method was selected. First, it is necessary to ensure that the aircraft can achieve a large pitching angle at any heading angle without the singularity associated with the attitude angle calculation. Second, to ensure that the aircraft is in the vertical flight state, it is essential that the whole angle around the X axis rotation and small angle around the body Z axis rotation do not present a singularity problem. Finally, it is necessary to ensure that when the aircraft is in horizontal flight, the whole angle around the Z axis rotation and small angle around the body X axis rotation do not present a singularity problem.

III. ATTITUDE CALCULATION

Based on the above analysis, it was determined that the gimbal Z (green) is the parent of X (black), and X is the parent of Y (red), as shown in Figure 3. The resulting directional cosine matrices are computed in the order of Z-X-Y rotation, as shown in equation (1), and the Euler angles are computed as shown in equation (2). In this rotation order, we need to redefine the attitude angle: ψ is the yaw angle, which is between the projection of the body Y axis on the XY plane of the navigation coordinate system and the Y axis of the navigation coordinate system. The Y axis of the body rotates clockwise around the Z axis of the navigation coordinate system the yaw angle is positive, conversely it is negative. ϕ is the roll angle, which is between the Y axis of the airframe and the XY plane of the navigation coordinate system. If the rotation of the Y axis of the body is in the positive direction of the Z axis of the navigation coordinate system, the roll angle is positive, otherwise it is negative. θ is the pitch angle, which is between the Z axis of the body and the vertical plane which is through the Y axis of the body. If the X axis of the body rotates up, the pitch angle is positive, otherwise it is negative. And the coordinates rotate in the order of Z-X-Y which is shown in figure 4.

$c\psi = \cos\psi; s\psi = \sin\psi; c\theta = \cos\theta; s\theta = \sin\theta; c\phi = \cos\phi; s\phi = \sin\phi.$

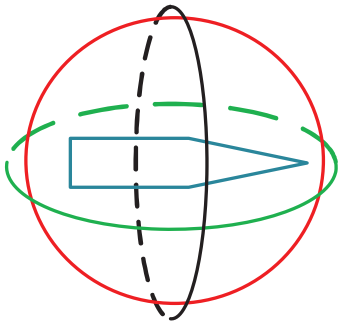


Fig.3. Z-X-Y rotary gimbal.

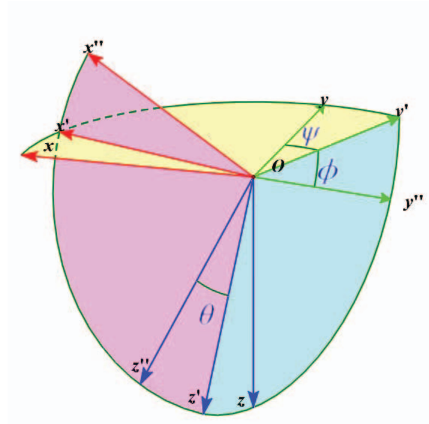


Fig.4. Z-X-Y rotation order of the coordinate.

$$C_b^n = \begin{bmatrix} c\theta c\psi - s\theta s\phi s\psi & -c\phi s\psi & c\psi s\theta + c\theta s\phi s\psi \\ c\theta s\psi + c\psi s\theta s\phi & c\phi c\psi & s\theta s\psi - c\theta c\psi s\phi \\ -c\phi s\theta & s\phi & c\theta c\phi \end{bmatrix} \quad (1)$$

$$\begin{cases} \psi = \text{atan2}(-C_b^n(1,2), C_b^n(2,2)) \\ \theta = \text{atan2}(-C_b^n(3,1), C_b^n(3,3)) \\ \phi = \text{asin}(C_b^n(3,2)) \end{cases} \quad (2)$$

According to a study involving Euler angle calculation [15], we can see that, irrespective of the flight mode, the three independent directional cosine matrices, which are respectively rotated around X, Y, and Z axes, are invariant, as shown in equations (3)-(5).

$$C_{\theta x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta_x & s\theta_x \\ 0 & -s\theta_x & c\theta_x \end{bmatrix} \quad (3)$$

$$C_{\theta y} = \begin{bmatrix} c\theta_y & 0 & -s\theta_y \\ 0 & 1 & 0 \\ s\theta_y & 0 & c\theta_y \end{bmatrix} \quad (4)$$

$$C_{\theta z} = \begin{bmatrix} c\theta_z & s\theta_z & 0 \\ -s\theta_z & c\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

When the vehicle is in a vertical flight mode, it is assumed that the pitch angle is 90°, the roll angle is 0°, and the yaw angle is any arbitrary angle. Therefore, the direction cosine matrix can be simplified to equation (6).

$$C_b^n = \begin{bmatrix} 0 & -\mathrm{s}\psi & \mathrm{c}\psi \\ 0 & \mathrm{c}\psi & \mathrm{s}\psi \\ -1 & 0 & 0 \end{bmatrix} \tag{6}$$

At this point, the aircraft rotates around the X axis of the airframe at θ_x degrees, and the direction cosine matrix obtained is $C_b^{n'}$, as shown in equation (7). According to equation (2), the Euler angles are computed, and the results are shown in equation (8).

$$C_b^{n'} = C_b^n C_{\theta x}^T = \begin{bmatrix} 0 & -\mathrm{s}\psi\,\mathrm{c}\theta_x + \mathrm{c}\psi\,\mathrm{s}\theta_x & \mathrm{s}\psi\,\mathrm{s}\theta_x + \mathrm{c}\psi\,\mathrm{c}\theta_x \\ 0 & \mathrm{c}\psi\,\mathrm{c}\theta_x + \mathrm{s}\psi\,\mathrm{s}\theta_x & -\mathrm{c}\psi\,\mathrm{s}\theta_x + \mathrm{s}\psi\,\mathrm{c}\theta_x \\ -1 & 0 & 0 \end{bmatrix} \tag{7}$$

$$\begin{cases} \psi = \psi - \theta_x \\ \theta = 90° \\ \phi = 0° \end{cases} \tag{8}$$

At this point, the aircraft rotates at θ_x around the X axis of the airframe, equivalent to the $-\theta_x$ rotation around the Z axis of the navigation coordinate system, as shown in figure 5. This means the yaw angle is reduced by $-\theta_x$, and the calculated heading value is the correct heading value.

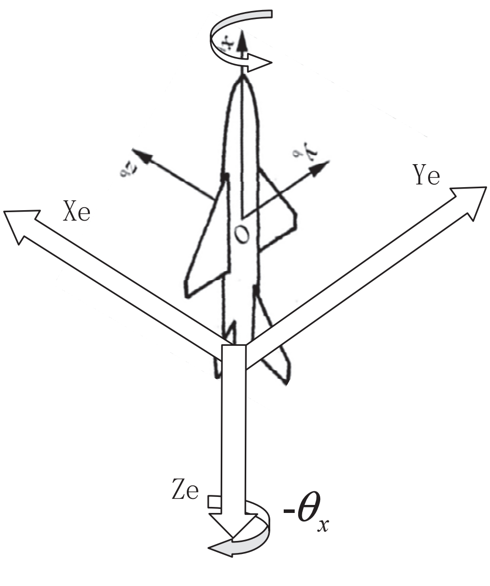


Fig.5. The vertical mode vehicle rotation around the X axis of the airframe.

In the vertical mode, the aircraft rotates at θ_y around the Y axis of the airframe, and the direction cosine matrix obtained is $C_b^{n''}$, as shown in equation (9). The Euler angles were computed using equation (2), and the results are shown in equation (10).

$$C_b^{n''} = C_b^n C_{\theta y}^T = \begin{bmatrix} -\mathrm{c}\psi\,\mathrm{s}\theta_y & -\mathrm{s}\psi & \mathrm{c}\psi\,\mathrm{c}\theta_y \\ -\mathrm{s}\psi\,\mathrm{s}\theta_y & \mathrm{c}\psi & \mathrm{s}\psi\,\mathrm{c}\theta_y \\ -\mathrm{c}\theta_y & 0 & -\mathrm{s}\theta_y \end{bmatrix} \tag{9}$$

$$\begin{cases} \psi = \psi \\ \theta = 90° + \theta_y \\ \phi = 0° \end{cases} \tag{10}$$

In the vertical mode, the aircraft rotates at a small angle (θ_z) is around the Z axis of the airframe, and the direction cosine matrix obtained is $C_b^{n'''}$, as shown in formula (11). As θ_z is a small angle, equation (11) can be approximately represented by equation (12). The Euler angles are computed according to equation (2), and the results are shown in equation (13).

$$C_b^{n'''} = C_b^n C_{\theta z}^T = \begin{bmatrix} -\mathrm{s}\psi\,\mathrm{s}\theta_z & -\mathrm{s}\psi\,\mathrm{c}\theta_z & \mathrm{c}\psi \\ \mathrm{c}\psi\,\mathrm{s}\theta_z & \mathrm{c}\psi\,\mathrm{c}\theta_z & \mathrm{s}\psi \\ -\mathrm{c}\theta_z & \mathrm{s}\theta_z & 0 \end{bmatrix} \tag{11}$$

$$C_b^{n'''} = \begin{bmatrix} -\mathrm{s}\psi\,\mathrm{s}\theta_z & -\mathrm{s}\psi\,\mathrm{c}\theta_z & \mathrm{c}\psi \\ -\mathrm{c}\psi\,\mathrm{s}\theta_z & \mathrm{c}\psi\,\mathrm{c}\theta_z & \mathrm{s}\psi \\ -1 & \mathrm{s}\theta_z & 0 \end{bmatrix} \tag{12}$$

$$\begin{cases} \psi = \psi \\ \theta = 90° \\ \phi = \theta_z \end{cases} \tag{13}$$

When the flight vehicle is in a horizontal flight mode, it is assumed that the pitch angle is 0°, the roll angle is 0°, and the yaw angle is any arbitrary angle. Therefore, the direction cosine matrix can be simplified to equation (14).

$$C_b^n = \begin{bmatrix} \mathrm{c}\psi & -\mathrm{s}\psi & 0 \\ \mathrm{s}\psi & \mathrm{c}\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

At this point, the aircraft rotates at a small angle (θ_x) around the X axis of the airframe, and the direction cosine matrix obtained is $C_b^{n'}$, as shown in formula (11). Because θ_x is a small angle, equation (15) can be approximately represented by the formula in equation (16). The Euler angles were computed using equation (2), and the results are shown in equation (17).

$$C_b^{n'} = C_b^n C_{\theta x}^T = \begin{bmatrix} c\psi & -s\psi\, c\theta_x & s\psi\, s\theta_x \\ 0 & c\psi\, c\theta_x & -c\psi\, s\theta_x \\ 0 & s\theta_x & c\theta_x \end{bmatrix} \quad (15)$$

$$C_b^{n'} = \begin{bmatrix} c\psi & -s\psi\, c\theta_x & -s\psi\, s\theta_x \\ 0 & c\psi\, c\theta_x & c\psi\, s\theta_x \\ 0 & s\theta_x & 1 \end{bmatrix} \quad (16)$$

$$\begin{cases} \psi = \psi \\ \theta = 90^\circ \\ \phi = \theta_z \end{cases} \quad (17)$$

In the horizontal mode, the aircraft rotates at θ_y around the Y axis of the airframe, and the direction cosine matrix obtained is $C_b^{n''}$, as shown in equation (18). The Euler angles were computed according to equation 2, and the results are shown in formula (19).

$$C_b^{n''} = C_b^n C_{\theta y}^T = \begin{bmatrix} c\psi\, c\theta_y & -s\psi & s\theta_y\, c\psi \\ s\psi\, c\theta_y & c\psi & s\theta_y\, s\psi \\ -s\theta_y & 0 & c\theta_y \end{bmatrix} \quad (18)$$

$$\begin{cases} \psi = \psi \\ \theta = \theta_y \\ \phi = 0^\circ \end{cases} \quad (19)$$

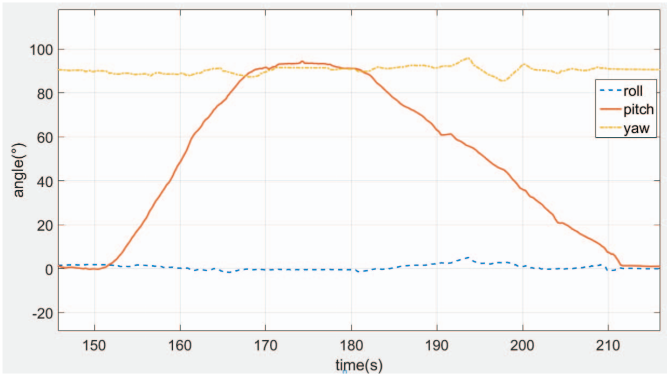
In the horizontal mode, the aircraft rotates at θ_z around the Y axis of the airframe, and the direction cosine matrix obtained is $C_b^{n'''}$, as shown in equation (20). The Euler angles were computed using equation (2), and the results are shown in equation (21).

$$C_b^{n'''} = C_b^n C_{\theta z}^T = \begin{bmatrix} c\psi\, c\theta_z - s\psi\, s\theta_z & -(c\psi\, s\theta_z + s\psi\, c\theta_z) & 0 \\ s\psi\, c\theta_z + c\psi\, s\theta_z & c\psi\, c\theta_z - s\psi\, s\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (20)$$

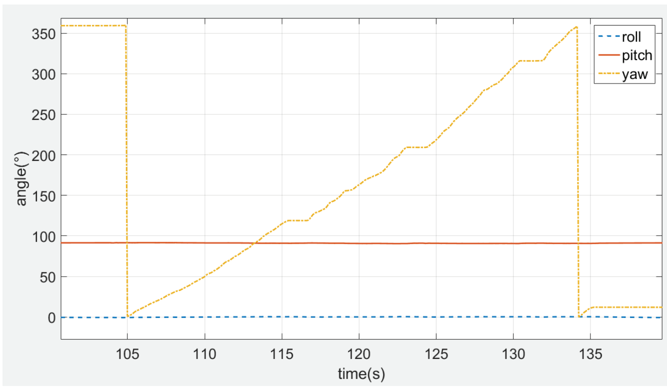
$$\begin{cases} \psi = \psi + \theta_z \\ \theta = 0^\circ \\ \phi = 0^\circ \end{cases} \quad (21)$$

IV. Experimental Verification

The Euler angles were computed by using the flight log data, and the correctness of the results obtained in different flight modes was verified. The results are shown in Figure 6. To prove the correctness and universality of the algorithm proposed in this paper, a special yaw angle of approximately 90° was chosen during flight-mode transformation, and the result is shown in Figure 6(a). When flying vertically, the aircraft rotates around the X axis of the airframe at a complete angle (360°), and the calculation results are shown in Figure 6(b). When flying vertically, there is a small rotational angle around the Z axis of the airframe, and the special yaw angle is 90°. The calculation result is shown in Figure 6(c). When flying horizontally, the aircraft rotates around the Z axis of the airframe at a complete angle, and the calculation results are shown in Figure 6(d). When flying horizontally, there is a small angle of rotation around the X axis of the airframe, and the special heading angle is 90°. The result is shown in Figure 6(e).



(a)



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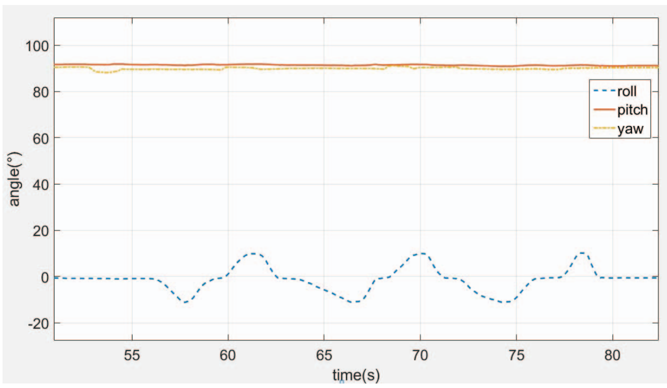

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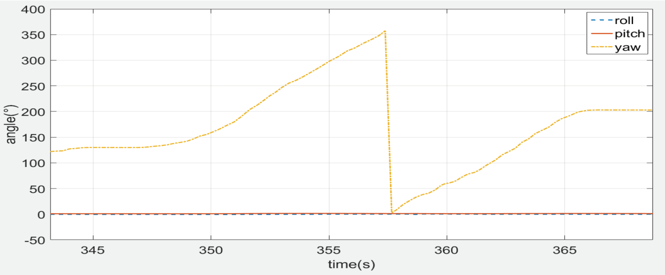

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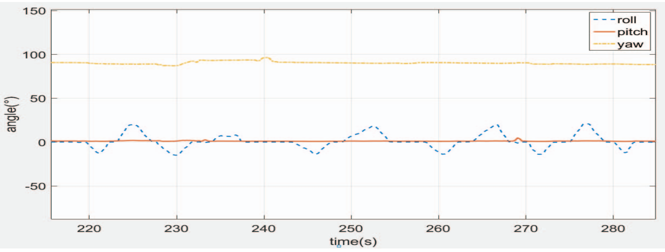

(e)

Fig.5. Experimental result. (a)Flight-mode switching. (b) Vertical mode rotation around the X axis of the airframe at a complete angle. (c) Vertical mode rotation around the Z axis of the airframe at a small angle. (d) Horizontal mode rotation around the Z axis of the airframe at a complete angle.(e) Horizontal mode rotation around the X axis of the airframe.

V. CONCLUSION

This paper proposes the rotation order Y-Z-X for solving obtaining the Euler angles of aircraft. This order not only avoids the singularity problem in all flight modes but also realizes the smooth change of the three attitudes during flight-mode switching, and the entire process is in control. When the aircraft is in a vertical flight mode, the body rotates around the X axis of the airframe and there is a change in the yaw angle. When the aircraft rotated around the Z axis, there is a change in the roll angle. In the horizontal flight mode, when the aircraft rotates around the X axis of the airframe, there is a change in the roll angle; when it rotates around the Z axis, there is a change in the yaw angle. The characteristics of this solution can ensure that the aircraft under the two flight modes of the heading angle can be more direct and simple due to vehicle navigation, and makes the controller design in the future is simpler. Although simple and direct, the method limits the flight of the vehicle to a particular flight attitude, and the vehicle cannot perform acrobatic flights like some other aircraft because abnormal Euler angle values will appear in other flight modes.

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