

# Euler Angle Based Attitude Estimation Avoiding the Singularity Problem

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**Abstract:** Precise control of UAV (Unmanned Aerial Vehicle) or MAV (Micro Aerial Vehicle), requires the accurate determination of vehicle orientation. Attitude and heading reference systems (AHRS) are popularly used for vehicle orientation determination. In these systems, Euler angle estimation is more appropriate than quaternion estimation because the accelerometer measurement and magnetometer measurement are separated. Furthermore, the accelerometer and magnetometer measurements have independent effects on the tilt and heading angles. An Euler based attitude estimation system that uses two-stage extended Kalman filtering is proposed. To avoid singularity of Euler angle, a new heading estimation parameter is introduced and a filter mode switching algorithm is proposed. The experimental results indicated that the proposed system performed better than a quaternion algorithm based on magnetometer disturbance. Furthermore, the results indicated that the proposed algorithm was immune to the singularity problem.

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## 1. INTRODUCTION

Precise UAV (Unmanned Aerial Vehicle) or MAV (Micro Aerial Vehicle) control requires the accurate determination of vehicle orientation. Attitude heading reference systems (AHRS) have been used to estimate the attitudes of those vehicles. General AHRS is comprised of a three-axis gyro, a three-axis accelerometer, and a three-axis magnetometer. The gyros measure the angular rate of the vehicle, the accelerometers measure the specific forces of the vehicle, and the magnetometers measure the magnetic vector.

These inertial sensors independently estimate the attitude without external signals. In conventional inertial navigation systems, the attitude is computed by integrating the angular rate obtained from the gyro outputs. However, this is not an appropriate method for calculating the attitude of the UAV because the gyro bias error causes the attitude error to diverge. Thus, the attitude is calculated by using accelerometer outputs and magnetometer outputs as well as the gyro outputs. Tilt angle can be estimated by accelerometers using the gravitational force, and magnetometers provide the heading information.

Although the outputs of accelerometers and magnetometers provide long term attitude stability, vehicle movement and magnetic disturbances can temporarily influence the attitude calculation obtained by these sensors. The attitude estimation must then rely on the gyro signal in this case. A number of attitude estimation algorithms have been developed to integrate the gyro, accelerometer and the magnetometer outputs in order to estimate the attitude.

The quaternion and DCM (direction cosine matrix) model have been widely used to calculate the attitude through angular rate integration. Alternatively, quaternion has been

widely used to estimate the attitude of the AHRS using sensor data fusion of the angular rate and acceleration.

Although the direction cosine matrix is generally used in inertial navigation systems, this attitude representation method possesses too many parameters. Thus, it cannot be used for attitude estimation. On the other hand, the Euler angle uses only 3 parameters, which makes the singularity on the pitch 90 degrees. Therefore, the Euler angle cannot be applied to a system exhibiting complicated motion. However, the Euler angle provides advantageous separation between tilt angle and heading angle errors. In aircraft control, the tilt angle estimation is important for heading estimation because it relates to the survivability of the vehicle, which is significant when magnetic disturbances exist. In this situation, the heading error should not be propagated to the tilt angle when attitude estimation is based on the Euler angle.

Quaternion consists of four parameters. The quaternion method is a non-singular parametric attitude representation method, widely used in spacecraft or robotics fields. Most AHRS attitude estimation algorithms use the quaternion method. However using this method make the AHRS impossible to separate the tilt angle to heading angle. Thus, with the quaternion method, unwanted heading errors can propagate toward the tilt angle. This characteristic can make a bad effect on stability of an aircraft.

In this paper, Euler angle estimation is proposed for AHRS. Furthermore, the AHRS algorithm for tilt and heading angle estimations is also proposed. Tilt angle estimation based on the modified Euler angle was proposed in a former paper. Finally, the singular avoidance technique is also purposed to estimate the attitude in a singular case.

This paper focused on a new attitude model for AHRS, thus the sensor data fusion algorithm to remove disturbance of acceleration was not considered.

## 2. MODIFIED EULER EQUATIONS

The DCM from body to reference frame is given by

$$C_n^b = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \quad (1)$$

$$= \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

The transpose provides the inverse transformation.

$$C_n^b = C_b^{nT} \quad (2)$$

Using the third row of the DCM, the modified Euler angle is defined by  $X_1$ ,  $X_2$ ,  $X_3$ , and yaw  $\psi$ .

### 2.1 Tilt angle equations

$$X_1 = -\sin\theta \quad (3)$$

$$X_2 = \sin\phi\cos\theta \quad (4)$$

$$X_3 = \cos\phi\cos\theta \quad (5)$$

$X_1$ ,  $X_2$ , and  $X_3$  are derived from the roll and pitch angles; thus  $X_1$ ,  $X_2$ , and  $X_3$  are considered to tilt angles. Because these parameters are components of the direction cosine matrix, the attitude estimation model can be expressed by a linear model.

The system model is derived from differentiating tilt angle states using equations (6) and (7).

$$\dot{\phi} = \omega_x + \omega_y \sin\phi \tan\theta + \omega_z \cos\phi \tan\theta \quad (6)$$

$$\dot{\theta} = \omega_y \cos\phi - \omega_z \sin\phi \quad (7)$$

Then,

$$\dot{X}_1 = -\dot{\theta}\cos\theta = \omega_z X_2 - \omega_y X_3 \quad (8)$$

$$\dot{X}_2 = \dot{\phi}\cos\phi\cos\theta - \dot{\theta}\sin\phi\sin\theta = -\omega_z X_1 + \omega_x X_3 \quad (9)$$

$$\dot{X}_3 = -\dot{\phi}\sin\phi\cos\theta - \dot{\theta}\cos\phi\sin\theta = \omega_y X_1 - \omega_x X_2 \quad (10)$$

These parameters can be represented through a linear matrix :

$$\dot{X} = \begin{bmatrix} 0 & \omega_z + w_z & -\omega_y - w_y \\ -\omega_z - w_z & 0 & \omega_x + w_x \\ \omega_y + w_y & -\omega_x - w_x & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 & X_3 & -X_2 \\ -X_3 & 0 & X_1 \\ X_2 & -X_1 & 0 \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$$

$$= Wx + Gw \quad (11)$$

The yaw angle  $\psi$  does not affect  $X_1$ ,  $X_2$ , and  $X_3$ . Thus the roll and pitch angles are properly separated from heading angle.

The accelerometer measurements can be used for tilt angle estimation. The accelerometer measures the gravitational vector, and it is related to tilt parameters through following:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = C_n^b \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} = -g \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \quad (12)$$

This measurement equation is also linear, and independent of heading angle. Thus the tilt angle states can be estimated by linear Kalman filter independently.

### 2.2 Heading angle equations on non-singular case

Differentiation of yaw angle is given by

$$\dot{\psi} = \omega_y \sin\phi \sec\theta + \omega_z \cos\phi \sec\theta \quad (13)$$

and this equation can be transformed to

$$\dot{\psi} = \frac{X_2}{X_2^2 + X_3^2} \omega_y + \frac{X_3}{X_2^2 + X_3^2} \omega_z \quad (14)$$

This parameter is dependent of the tilt angles. Thus, the tilt angles must be estimated prior to the yaw angle.

When singularity occurs,  $X_2$  and  $X_3$  become zero. Consequently, the yaw angle update equation can not compute the proper heading angle. Therefore, new states and equations are needed for the singular case. These equations will be proposed in section 3.

A magnetometer measurement equation is used to estimate the yaw angle. The local magnetic vector, which consists of the dip angle and declination angle, is defined as  $H_n$ . The magnetometer measurement is expressed as

$$H_b = C_n^b H_n \quad (15)$$

where  $H_n$  signifies the earth magnetic vector on the navigation frame,

$$H_n = [H_N \ H_E \ H_D]^T \quad (16)$$

Using the definition of  $C_n^b$ ,  $H_b$  is derived as

$$H_b = \begin{bmatrix} H_N \cos\theta\cos\psi + H_E \cos\theta\sin\psi - H_D \sin\theta \\ H_N (\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) + H_E (\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi) + H_D \sin\phi\cos\theta \\ H_N (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) + H_E (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) + H_D \cos\phi\cos\theta \end{bmatrix} \quad (17)$$

The magnetometer measurement model is a nonlinear model. Thus, the EKF(Extended Kalman Filter) is needed for estimating the heading angle. This model is dependent on the tilt angles through nonlinear relationships.

### 3. ATTITUDE ESTIMATION ON SINGULAR CASE

For the singular case, the yaw angle cannot be used to estimate the attitude because of singularity. This singularity is explained through the gimbal lock phenomenon as shown in Fig. 1.

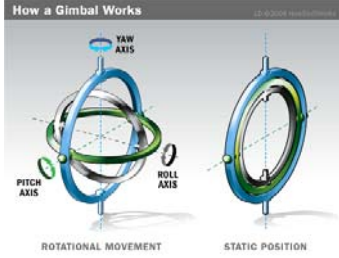


Fig. 1. Euler angle singularity on gimbal lock

The roll angle rotation and yaw angle rotation exhibit identical motions when the pitch angle is 90 degree. As previously mentioned, the yaw parameter cannot be estimated.

This problem can be resolved by introducing the parameters of  $\psi - \phi$  and  $\psi + \phi$ .

The state  $X_1$  will be 1 or -1 in this situation.

#### 3.1 Case1: pitch angle near 90 degrees

For case 1, the state  $X_1$  is expressed as  $X_1 < -(1-10^{-5})$ . The DCM from the body to the reference frame is defined as

$$C_b^n = \begin{bmatrix} 0 & \sin \phi \cos \psi - \cos \phi \sin \psi & \cos \phi \cos \psi + \sin \phi \sin \psi \\ 0 & \sin \phi \sin \psi + \cos \phi \cos \psi & \cos \phi \sin \psi - \sin \phi \cos \psi \\ -1 & 0 & 0 \end{bmatrix} \quad (17)$$

$$= \begin{bmatrix} 0 & -\sin(\psi - \phi) & \cos(\psi - \phi) \\ 0 & \cos(\psi - \phi) & \sin(\psi - \phi) \\ -1 & 0 & 0 \end{bmatrix}$$

Then, the state  $\psi - \phi$  can be defined as

$$\psi - \phi = \arctan \left[ \frac{c_{23} - c_{12}}{c_{13} + c_{22}} \right] \quad (18)$$

The differential equation of this state can not be derived directly. Thus, the differential equation of the direction cosine matrix is used for time updating the state.

$$\dot{C}_b^n = C_b^n \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \quad (19)$$

When the singularity does not occur, the DCM is calculated from the roll, pitch, and yaw angles using equation (1). When singularity occurs, the DCM is updated through equation (19), and the state  $\psi - \phi$  is calculated by equation (18).

#### 3.2 Case2: pitch angle near -90 degrees

For case 2, the state  $X_1$  is expressed as  $X_1 > (1-10^{-5})$ . The DCM from the body to the reference frame is defined as

$$C_b^n = \begin{bmatrix} 0 & -\sin \phi \cos \psi - \cos \phi \sin \psi & -\cos \phi \cos \psi + \sin \phi \sin \psi \\ 0 & -\sin \phi \sin \psi + \cos \phi \cos \psi & -\cos \phi \sin \psi - \sin \phi \cos \psi \\ 1 & 0 & 0 \end{bmatrix} \quad (20)$$

$$= \begin{bmatrix} 0 & -\sin(\psi + \phi) & -\cos(\psi + \phi) \\ 0 & \cos(\psi + \phi) & -\sin(\psi + \phi) \\ 1 & 0 & 0 \end{bmatrix}$$

Then, the state  $\psi + \phi$  can be defined as

$$\psi + \phi = \arctan \left[ \frac{-(c_{23} + c_{12})}{-(c_{13} - c_{22})} \right] \quad (21)$$

The differential equation of this state cannot be derived directly. Thus, a DCM update calculation is used.

### 4. EKF STRUCTURE WITH MODIFIED EULER EQUATION

Because the tilt angles and heading angle are separated, filtering is operated on two separated stage.

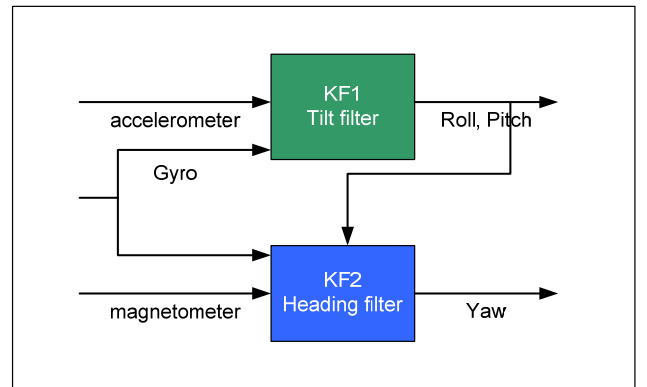


Fig. 2. structure of attitude estimation

Generally speaking, the tilt angles, which are roll and pitch, are more important to the stability of a vehicle than the heading angle. Because the heading angle can easily be affected by a magnetometer disturbance such as motor rotation, heading angle fluctuations can cause serious failures. But the stability of vehicle is guaranteed using the proposed algorithm.

#### 4.1 KF1 : tilt angle filtering

The tilt side filter, which is shown as KF1, uses an accelerometer signal. KF1 consists of a linear filter model and is described by following equation:

$$\hat{x}_k^- = F_k x_{k-1}$$

where,

$$F_k = e^{W \Delta t}$$

And the error covariance matrix is

$$P_k^- = F_k P_{k-1} F_k^T + G_k Q_{k-1} G_k^T$$

The measurement update is

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}$$

$$\hat{x}_k = \hat{x}_k^- + K_k (y_k - H_k \hat{x}_k^-)$$

$$P_k = (I - K_k H_k) P_k^-$$

where,

$$G_k = \begin{bmatrix} 0 & X_3 & -X_2 \\ -X_3 & 0 & X_1 \\ X_2 & -X_1 & 0 \end{bmatrix}$$

$$H_k = -g I_{3 \times 3}$$

and  $Q_{k-1}$  is a gyro measurement noise covariance matrix,  $R_{k-1}$  is a accelerometer measurement noise covariance matrix, and measurement  $y_k$  is the accelerometer measurement vector.

#### 4.2 KF2 for a non-singular case

In order to estimate the heading angle, an extended Kalman filter is used for the heading filter given by KF2. KF2 requires a two-mode structure for a non-singular case and singular case. The magnetometer signal is used as a measurement for both cases. Although the states of KF1 are included in the KF2 models, these parameters are assumed as constants.

For a non-singular case, the yaw angle is considered a scalar state. Then, the propagation equation is equation (14), and error covariance matrix of KF2 ( $P_{k,2}$ ) is updated as

$$P_{k,2}^- = P_{k,2} + G_{k,2} Q_{k-1} G_{k,2}^T \quad (29)$$

$$\text{Where, } G_{k,2} = \begin{bmatrix} 0 & \frac{X_2}{X_2^2 + X_3^2} & \frac{X_3}{X_2^2 + X_3^2} \end{bmatrix} \quad (30)$$

The measurement equation contains highly non-linear terms shown as equation (16).

$$y_k = h(X_1, X_2, X_3, \psi) = H_b \quad (31)$$

The error covariance matrix of KF2 is updated as

$$P_{k,2} = (I - K_k H_{k,2}) P_{k,2}^- \quad (32)$$

where

$$H_{k,2} = \frac{\partial H_b}{\partial \psi} = \begin{bmatrix} -H_N \cos \theta \sin \psi + H_E \cos \theta \cos \psi \\ H_N (-\sin \phi \sin \theta \sin \psi - \cos \phi \cos \psi) + H_E (\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) \\ H_N (-\cos \phi \sin \theta \sin \psi + \sin \phi \cos \psi) + H_E (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \end{bmatrix} \quad (33)$$

#### (22) 4.3 KF2 for a singular case

If the pitch angle is not a singular value, states of KF2 are initialized at every time step through by non-singular computation. When the singularity occurs, the filter structure will be changed to a singular mode.

(24) If the pitch is near 90 deg, the state is defined as  $\psi - \phi$ . If the pitch is near -90 deg, state is defined as  $\psi + \phi$ . The time propagation of the states is computed through the DCM update as mentioned in section 3. However, the error covariance matrix  $P_{k,3}$ , which is the error covariance of  $\psi + \phi$  or  $\psi - \phi$ , cannot be easily computed. When pitch is not singular,  $P_{k,3}$  can be assumed as

$$P_{k,3} = P_\phi + P_\psi \approx P_k(2,2) + P_{k,2} \quad (34)$$

Because the error of  $X_2$  is small, the error of  $X_2$  is assumed to the error of the roll angle. Thus  $P_{k,3}$  must be initialized through equation (34) at every step during the non-singular mode.

The DCM matrix is used to update the error covariance. Equation (19) can be given in a similar manner to equation (34).

$$\begin{bmatrix} \dot{c}_{11} & \dot{c}_{12} & \dot{c}_{13} \\ \dot{c}_{21} & \dot{c}_{22} & \dot{c}_{23} \\ \dot{c}_{31} & \dot{c}_{32} & \dot{c}_{33} \end{bmatrix} = \begin{bmatrix} \cdots & -\omega_z c_{11} + \omega_x c_{13} & \omega_y c_{11} - \omega_x c_{12} \\ \cdots & -\omega_z c_{21} + \omega_x c_{23} & \omega_y c_{21} - \omega_x c_{22} \\ \cdots & \cdots & \cdots \end{bmatrix} \quad (34)$$

Then, differentiation of the equation (18) is expressed as

$$\begin{aligned} \dot{\psi} - \dot{\phi} &= \left( \arctan \left[ \frac{c_{23} - c_{12}}{c_{13} + c_{22}} \right] \right)' \\ &= \begin{bmatrix} -1 & \frac{(c_{13} + c_{22})c_{21} - (c_{23} - c_{12})c_{11}}{(c_{13} + c_{22})^2 + (c_{23} - c_{12})^2} & \frac{(c_{13} + c_{22})c_{11} + (c_{23} - c_{12})c_{21}}{(c_{13} + c_{22})^2 + (c_{23} - c_{12})^2} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \\ &\equiv G_{k,3-1} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \end{aligned} \quad (35)$$

The differentiation of the equation (21) is expressed as

$$\begin{aligned} \dot{\psi} + \dot{\phi} &= \left( \arctan \left[ \frac{c_{23} + c_{12}}{c_{13} - c_{22}} \right] \right)' \\ &= \begin{bmatrix} 1 & \frac{(c_{13} - c_{22})c_{21} - (c_{23} + c_{12})c_{11}}{(c_{13} - c_{22})^2 + (c_{23} + c_{12})^2} & \frac{-(c_{13} - c_{22})c_{11} - (c_{23} + c_{12})c_{21}}{(c_{13} - c_{22})^2 + (c_{23} + c_{12})^2} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \\ &\equiv G_{k,3-2} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \end{aligned} \quad (36)$$

Using the G matrices, the update of the error covariance matrix is given by

$$P_{k,3}^- = \begin{cases} P_{k,3} + G_{k,3-1} Q_{k-1} G_{k,3-1}^T & (X_1 < -(1-10^{-5})) \\ P_{k,3} + G_{k,3-2} Q_{k-1} G_{k,3-2}^T & (X_1 < +(1-10^{-5})) \end{cases} \quad (37)$$

The measurement equation of equation (31) is expressed in the following for pitch angle near 90 deg.

$$H_b = \begin{bmatrix} -H_D \\ H_N (\sin \phi \cos \psi - \cos \phi \sin \psi) + H_E (\sin \phi \sin \psi + \cos \phi \cos \psi) \\ H_N (\cos \phi \cos \psi + \sin \phi \sin \psi) + H_E (\cos \phi \sin \psi - \sin \phi \cos \psi) \end{bmatrix} \quad (38)$$

$$= \begin{bmatrix} -H_D \\ -H_N \sin(\psi - \phi) + H_E \cos(\psi - \phi) \\ H_N \cos(\psi - \phi) + H_E \sin(\psi - \phi) \end{bmatrix}$$

If the pitch is near -90 deg, the magnetic vector is

$$H_b = \begin{bmatrix} H_D \\ -H_N (\sin \phi \cos \psi + \cos \phi \sin \psi) - H_E (\sin \phi \sin \psi - \cos \phi \cos \psi) \\ -H_N (\cos \phi \cos \psi - \sin \phi \sin \psi) - H_E (\cos \phi \sin \psi + \sin \phi \cos \psi) \end{bmatrix} \quad (39)$$

$$= \begin{bmatrix} H_D \\ -H_N \sin(\psi + \phi) + H_E \cos(\psi + \phi) \\ -H_N \cos(\psi + \phi) - H_E \sin(\psi + \phi) \end{bmatrix}$$

## 5. EXPERIMENTAL RESULTS

Experiments were conducted using MTi, which is a commercial IMU from Xsens. The attitude output of MTi exhibited an attitude performance specification of 0.5 deg in static situations and 2 degrees in dynamic situations. MTi also provided calibrated sensor outputs that were used as the IMU. In order to test the proposed EKF attitude calculation method, the attitude result was compared to the attitude results obtained from quaternion based Kalman filter in Sabatini's paper and the attitude output of MTi as a reference.

Tests were performed using a jig.



Fig. 3. MTi and jig for attitude test

The purpose of this paper is to show that an Euler based algorithm is similar or better than quaternion based algorithm. Thus, any disturbance removal algorithm or any other signal processing method was not used. The only difference with the filter model is the point of comparison.

In this test, the rate table can not be used for a reference, because it generates a magnetic disturbance which is induced by motor and steel body. Thus, the jig which is operated manually was used for tests. In this case, an exact reference did not exist. In order to compare the results, the attitude result of MTi was used. MTi result was fairly reliable and more accurate than the simple Kalman filtering; because it compensated for several measurement disturbances using

algorithms similar to those presented by Kang et al. or Sabatini or Rotenberg et al. Regardless of the algorithm MTi used, the attitude can be an appropriate reference.

The first test was an arbitrary motion. The roll was rotated twice and the yaw was rotated twice.

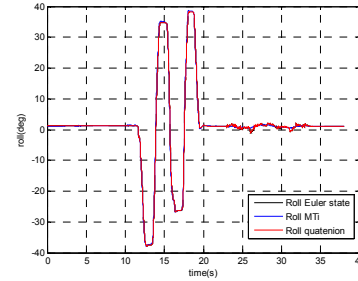


Fig. 4. Roll result of arbitrary motion test

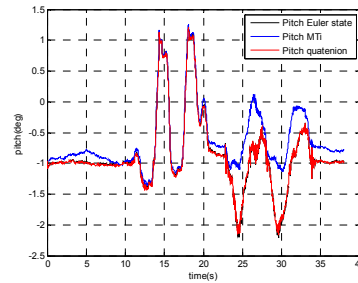


Fig. 5. Pitch result of arbitrary motion test

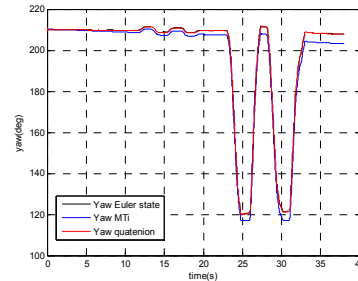


Fig. 6. Yaw result of arbitrary motion test

This result indicated that the performances of the quaternion model and the Euler model were very similar because the initial filter parameter and process and measurement noise matrices were set to the same value.

Subsequently, a singularity test was conducted. First, the AHRS was rotated 90-degrees about the y-axis; consequently, the pitch angle became 90-degrees. Then, the AHRS was rotated 90-deg about the x-axis. For the singular case, the Euler result was displayed through a roll "freezing" method described in Titterton's book.

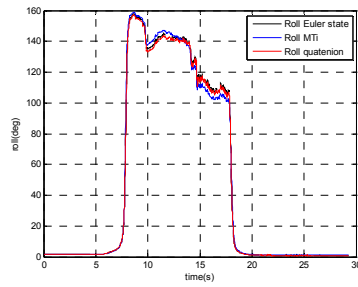


Fig. 7. Roll result of singular motion test

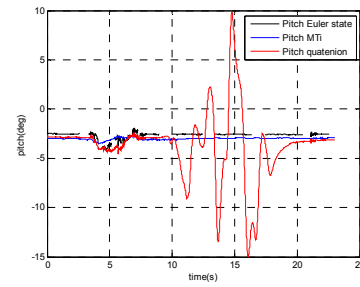


Fig. 8. Pitch result on magnetic disturbance

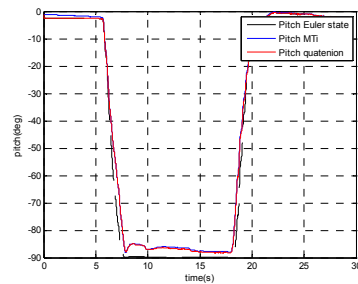


Fig. 8. Pitch result of singular motion test

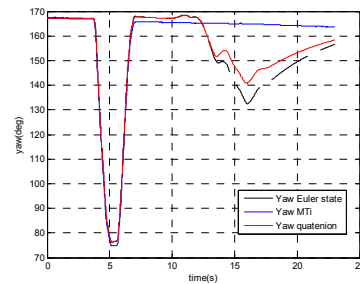


Fig. 9. Yaw result on magnetic disturbance

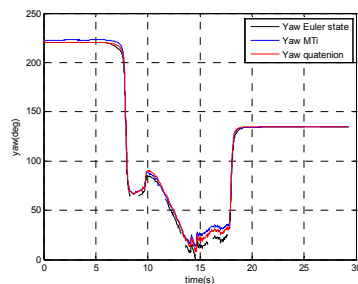


Fig. 9. Yaw result of arbitrary motion test

In this orientation, the general Euler equation produced a singularity problem and could not calculate the attitude of motion. However, proposed algorithm exhibited a similar result to the quaternion based method. Thus, singularity was resolved through the proposed algorithm.

The third test was magnetic disturbance test.

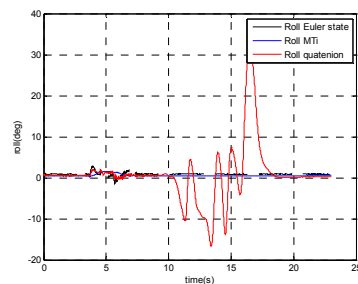


Fig. 7. Roll result on magnetic disturbance

Magnetic disturbance was induced by a magnet for 10 seconds. The filter did not contain a disturbance removal algorithm; consequently, disturbance affected the attitude result. The results acquired from the Euler based model differed from that of the quaternion model. The roll and pitch angle errors were produced by magnetic disturbances, which resulted because the tilt angle of the quaternion model was dependent on the yaw angle error. However, the proposed algorithm did not produce roll and pitch angle errors because the algorithm used a magnetic vector to compensate only for the yaw angle. The tilt angle was completely separated from the yaw angle.

## 6. CONCLUSIONS AND FUTURE WORK

In this paper, the Euler angle based attitude estimation method was proposed. The existing approaches used quaternion algorithms in order to avoid the singularity problem. However, this paper resolved the problem using a two-stage Kalman filter structure and introducing new parameters,  $\psi - \phi$  and  $\psi + \phi$ . Based on the tests, the results verified the algorithm and proved that the performance on general motion and singular motion were also similar to the quaternion model. However, the proposed algorithm exhibited good characteristics on attitude error separation. These characteristics prevented heavy magnetic disturbance influences; thus, an aircraft could be protected from unwanted attitude control.

Future work will develop an adaptive fusion algorithm for IMU sensors to remove disturbances which are contained in the accelerometer and magnetometer outputs.

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## REFERENCES

- John L. Crassidis, F. Landis Markley, and Yang Cheng, "A Survey of Nonlinear Attitude Estimation Methods," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 1, 2007.
- Chul Woo Kang and Chan Gook Park, "Attitude Estimation with Accelerometers and Gyros Using Fuzzy Tuned Kalman Filter," *ECC '09*, Budapest, Hungary, August 23-26, 2009.
- Derek B. Kingston and Randal W. Beard, "Real-Time Attitude and Position Estimation for Small UAVs Using Low-Cost Sensors," *AIAA 3rd "Unmanned Unlimited" Technical Conference, Workshop and Exhibit*, Chicago, IL; USA; 20-23 Sept. 2004. pp. 1-9. 2004.
- N. Miller, O.C. Jenkins, M. Kallmann, M. J. Mataric, "Motion capture from inertial sensing for untethered humanoid teleoperation," *Humanoid Robots, 2004 4th IEEE/RAS International Conference on*, vol. 2, pp. 547-565, Nov. 2004.
- Jong Nam Lim, "Design of Attitude Estimation System for Micro Aerial Vehicle," *Thesis, of Mechanical and Aerospace Engineering*, Seoul National University., Korea, Seoul, 1993.
- H. Hong, J. G Lee, C. G. Park, H. S. Han, "A leveling algorithm for an underwater vehicle using extended Kalman filter," *Proceedings of the IEEE 1998 Position Location and Navigation Symposium*, Palm Springs, California, U. S. A., April 20-23, pp. 280-285, 1998.
- E. R. Bachmann, I. Duman, U. Usta, R. B. McGhee, X. Yun, and M. J. Zyda. "Orientation tracking for humans and robots using inertial sensors," *International Symposium on Computational Intelligence in Robotics and Automation*, pp. 187-194, 1999.
- D. Rotenberg, H. Luinge , C. Baten and P. Veltink "Compensation of magnetic disturbances improves inertial and magnetic sensing of human body segment orientation," *IEEE Trans. Neural Syst. Rehabil. Eng.*, vol. 13, pp. 395, Sep. 2005.
- Daniel Roetenberg, Per J. Slycke, and Peter H. Veltink, "Ambulatory Position and Orientation Tracking Fusing Magnetic and Inertial Sensing," *IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING*, VOL. 54, NO. 5, MAY 2007
- Xiaoping Yun: Mariano Lizarraga, Eric R. Bachmann and Robert B. McGhee, "An Improved Quaternion-Based Kalman Filter for Real-Time Tracking of Rigid Body Orientation," *Proceedings of the 2003 IEEE/RSJ Intl. Conference on Intelligent Robots and Systems*, Las Vegas. Nevada, October 2003
- Chul Woo Kang, Young Min Yoo, Chan Gook Park, "Performance Improve of Attitude Estimation Using Modified Euler Angle Based Kalman Filter," *Journal of Institute of Control, Robotics and Systems*, Vol. 14, No. 9, September 2008.
- A. Gelb, *Applied Optimal Estimation*, ed. MIT Press, 1992.
- D. H. Titterton and J. L. Weston, "Strapdown Inertial Navigation Technology," Stevenage, U.K.: Peregrinus, 1997.
- G Welch and G Bishop,"An Introduction to Kalman Filters" *SIGGRAPH*, 2001.
- A. Sabatini, "Quaternion-based extended kalman filter for determination orientation by inertial and magnetic sensing," *IEEE Trans. Biomedical Eng.*, Vol. 53, July 2006.
- Hanspeter Schaub and John L. Junkins, "Stereographic Orientation Parameters for Attitude Dynamics: a Generation of the Rodrigues Parameters," *Journal of the Astronautical Sciences*, Vol. 44, no. 1, pp. 1-19. Jan.-Mar. 1996.