

NMF and Image zipping

Step by step improvement makes perfect

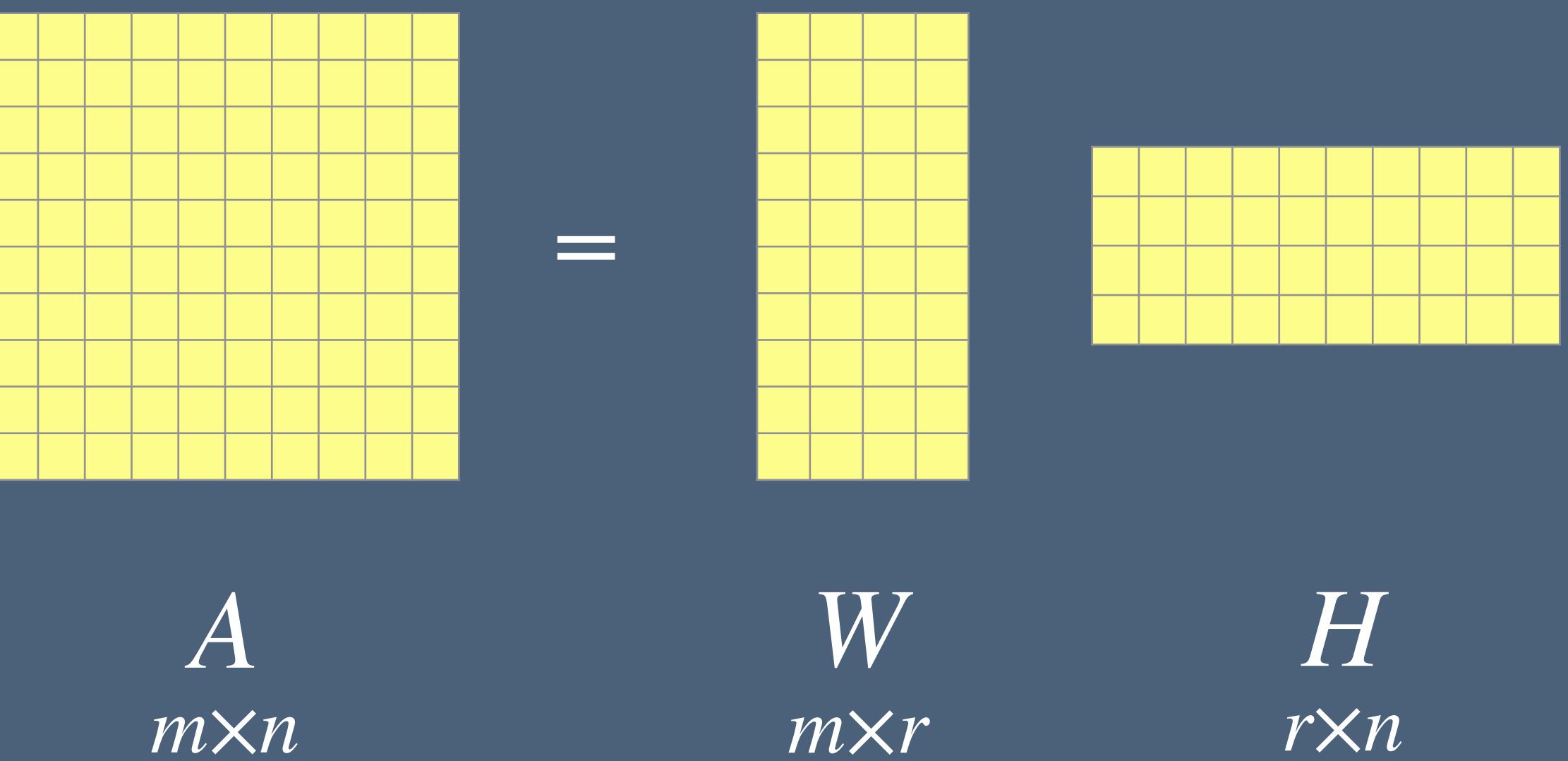
NMF

Non-negative matrix decomposition

Given A with $A_{ij} > 0$

$$\arg \min_{0 < w_{ij} \in W, 0 < h_{ij} \in H} f(W, H) = \|A - WH\|_F^2$$

F-norm: $\|M\|_F = \sqrt{\sum_{i=1}^N \sum_{j=1}^M m_{ij}^2}$



NMF

Projected Gradient Descent methods(Andersen Ang)

Algorithm 2: The PGD algorithm on NMF

Result: A solution x that approximately solves $\min_{x \in \mathcal{Q}} f(x)$

Initialization Pick initial matrices $W^0 \geq 0$ and $H^0 \geq 0$

while stopping condition is not met **do**

$$W^{k+1} = \left[W^k - t_W^k \nabla f(W^k; H^k) \right]_+ ; \quad \begin{matrix} \text{Do gradient descent on } W \\ \text{values, then project.} \end{matrix}$$
$$H^{k+1} = \left[H^k - t_H^k \nabla f(H^k; W^{k+1}) \right]_+ ; \quad \begin{matrix} \text{Do gradient descent on } H \\ \text{values, then project.} \end{matrix}$$

end

$$\arg \min_{0 < w_{ij} \in W, 0 < h_{ij} \in H} f(W, H) = \|A - WH\|_F^2$$

$$t_W^k = \frac{1}{\|HH^T\|_2}, \quad t_H^k = \frac{1}{\|WTW\|_2}$$

$$\nabla_W f(W, H) = (WH - X)H^T$$

$$\nabla_H f(W, H) = W^T(WH - X)$$

NMF

$$\arg \min_{0 < w_{ij} \in W, 0 < h_{ij} \in H} f(W, H) = \frac{1}{2} \|A - WH\|_F^2$$

Coordinate descent methods(Bindel)

$$R = A - WH$$

$$\frac{1}{2} \|A - (W + se_i e_j^T)H\|_F^2 = \frac{1}{2} \|R\|_F^2 - s \langle (e_i e_j^T), RH^T \rangle_F + \frac{1}{2} s^2 \|e_i e_j^T H\|_F^2$$

$$\begin{matrix} 1 & 2 & \dots & n \\ \textcolor{green}{1} & a_{11} & a_{12} & \dots & a_{1n} \\ 2 & a_{21} & a_{22} & \dots & a_{2n} \\ 3 & a_{31} & a_{32} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m & a_{m1} & a_{m2} & \dots & a_{mn} \end{matrix}$$

$$s = \max \left(-w_{ij}, \frac{(RH^T)_{ij}}{(HH^T)_{jj}} \right), \quad w_{ij} := w_{ij} + s, \quad R_{i,:} := R_{i,:} - sH_{j,:}$$

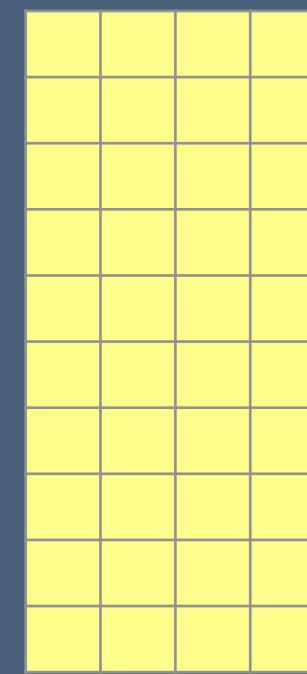
$$s = \max \left(-h_{ij}, \frac{(W^T R)_{ij}}{(W^T W)_{ii}} \right), \quad h_{ij} := h_{ij} + s, \quad R_{:,j} := R_{:,j} - sW_{:,i}$$

Application: Image Zipping

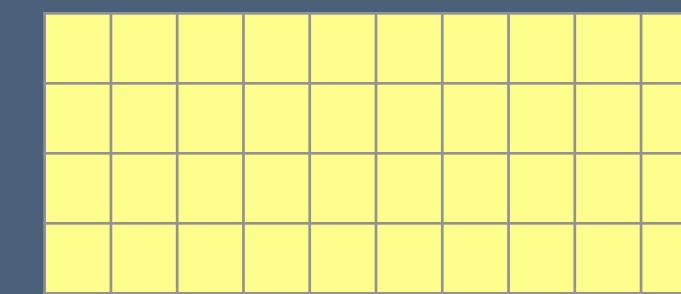


A
 600×600

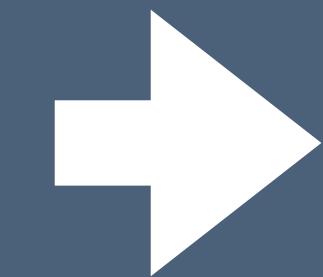
=



W
 $600 \times r$



H
 $r \times 600$



reconstruct A
 600×600

$$\frac{2 \times r \times 600}{600 \times 600} = \frac{r}{300}$$

Zip it?



	Pro.png	149 KB
	修改时间: 2024年10月16日下午8:20	
	Pro.png.zip	150 KB
	修改时间: 今天上午10:28	

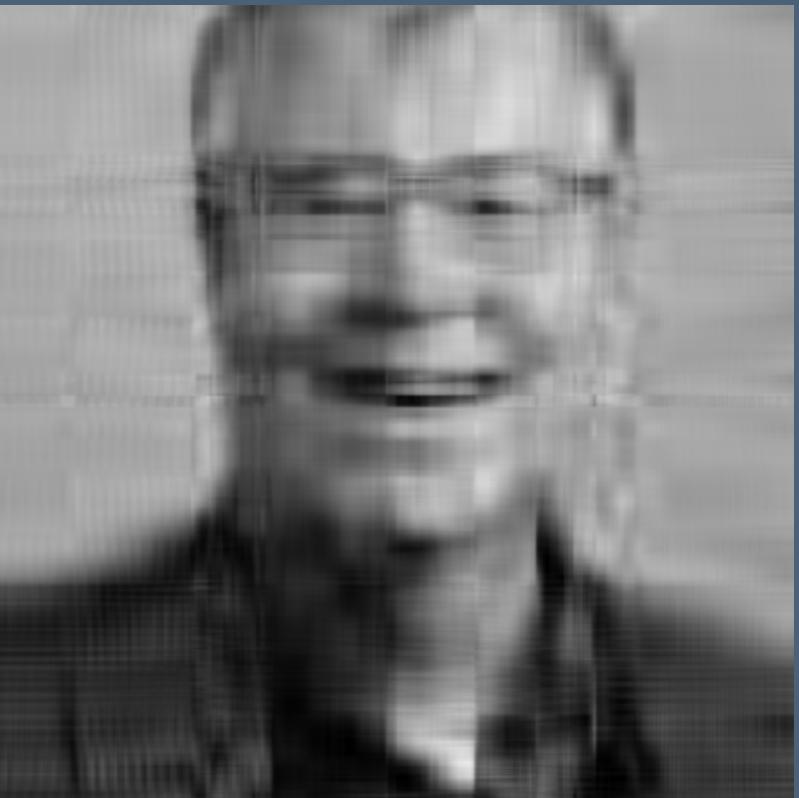
Results

PGD



$r = 10$

CD



$r = 100$



$r = 40$



$r = 200$



$r = 100$



Original



$$\begin{array}{ccc} A & = W \times H \\ m \times n & m \times r & r \times n \end{array}$$

Time(s)

$$A = W \times H$$
$$m \times n \quad m \times r \quad r \times n$$

Machine: ASUS ROG Zephyrus G14, CPU: AMD Ryzen9 7940HS, 16 RAM, GPU: RTX4060 8GB

r	10	20	30	40	50	60	70	80	90	100	150	200	250	300
GSD	25	98	223	232	231	475	373	304	639	740	653	778	848	949
CD	76	899	1437	3780	5761	2143	4942	6260	9770	5284				
<u>Lin's</u>	10	25	33	31	49	29	42	29	9	7	4	4	8	7

From Lin's method

Stopping condition

$$\|\nabla^P f(W^k, H^k)\|_F \leq \epsilon \|\nabla f(W^1, H^1)\|_F$$

Backstepping?



$$t_W^k = \frac{1}{\|HHT\|_2}, \quad t_H^k = \frac{1}{\|WTW\|_2}$$

Algorithm 4 An improved projected gradient method

1. Given $0 < \beta < 1, 0 < \sigma < 1$. Initialize any feasible \mathbf{x}^1 . Set $\alpha_0 = 1$.
2. For $k = 1, 2, \dots$

(a) Assign $\alpha_k \leftarrow \alpha_{k-1}$

(b) If α_k satisfies (13), repeatedly increase it by

$$\alpha_k \leftarrow \alpha_k / \beta$$

until either α_k does not satisfy (13) or $\mathbf{x}(\alpha_k / \beta) = \mathbf{x}(\alpha_k)$.

Else repeatedly decrease α_k by

$$\alpha_k \leftarrow \alpha_k \cdot \beta$$

until α_k satisfies (13).

- (c) Set

$$\mathbf{x}^{k+1} = P[\mathbf{x}^k - \alpha_k \nabla f(\mathbf{x}^k)].$$

From Ang's method

Exact BCD algorithm with momentum for NMF

1. (On W) Initialize W^0, ϵ , set $V^0 = W^0, \lambda^0 = 0$, then loop until stopping condition is met :

$$1.1 \quad \lambda^k = \frac{1}{2}(1 + \sqrt{1 + 4\lambda_{k-1}^2}), \quad \gamma^k = \frac{1 - \lambda^{k-1}}{\lambda^k}$$

$$1.2 \quad \text{Update: } V^k = \max \left([W^k - t_W^k \nabla_W f(W^k; H)]_{ij}, \epsilon \right)$$

$$1.3 \quad \text{Extrapolation : } W^k = (1 - \gamma^k)V^k + \gamma V^{k-1}$$

2. (On H) Initialize H^0, ϵ , set $G^0 = H^0, \lambda^0 = 0$, then loop until stopping condition is met :

$$2.1 \quad \lambda^k = \frac{1}{2}(1 + \sqrt{1 + 4\lambda_{k-1}^2}), \quad \gamma^k = \frac{1 - \lambda^{k-1}}{\lambda^k}$$

$$2.2 \quad \text{Update: } G^k = \max \left([H^k - t_H^k \nabla_H f(H^k; W)]_{ij}, \epsilon \right)$$

$$2.3 \quad \text{Extrapolation : } H^k = (1 - \gamma^k)G^k + \gamma G^{k-1}$$

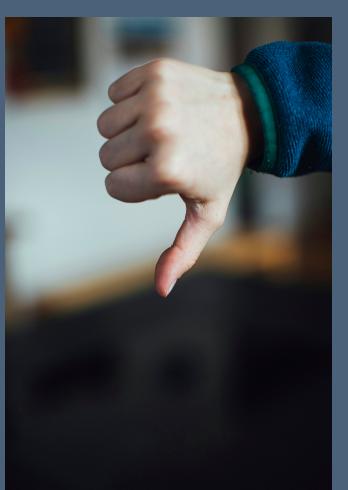
The acceleration is achieved by extrapolating current W, H by the coupling variables V, G with the extrapolation weights $\{\gamma^k\}$.

$$\nabla_W f(W, H) = \boxed{WHH^T} - XH^T$$

$$\nabla_W^{New} f(W, H) = \nabla_W^{old} f(W, H) +$$

$$(W^{new} - W^{old})HH^T$$

$$t_W^k = \frac{1}{\|HH^T\|_2}, \quad t_H^k = \frac{1}{\|WTW\|_2}$$



Fix step size

Time(s)

$$A = W \times H$$

$$m \times n \quad m \times r \quad r \times n$$

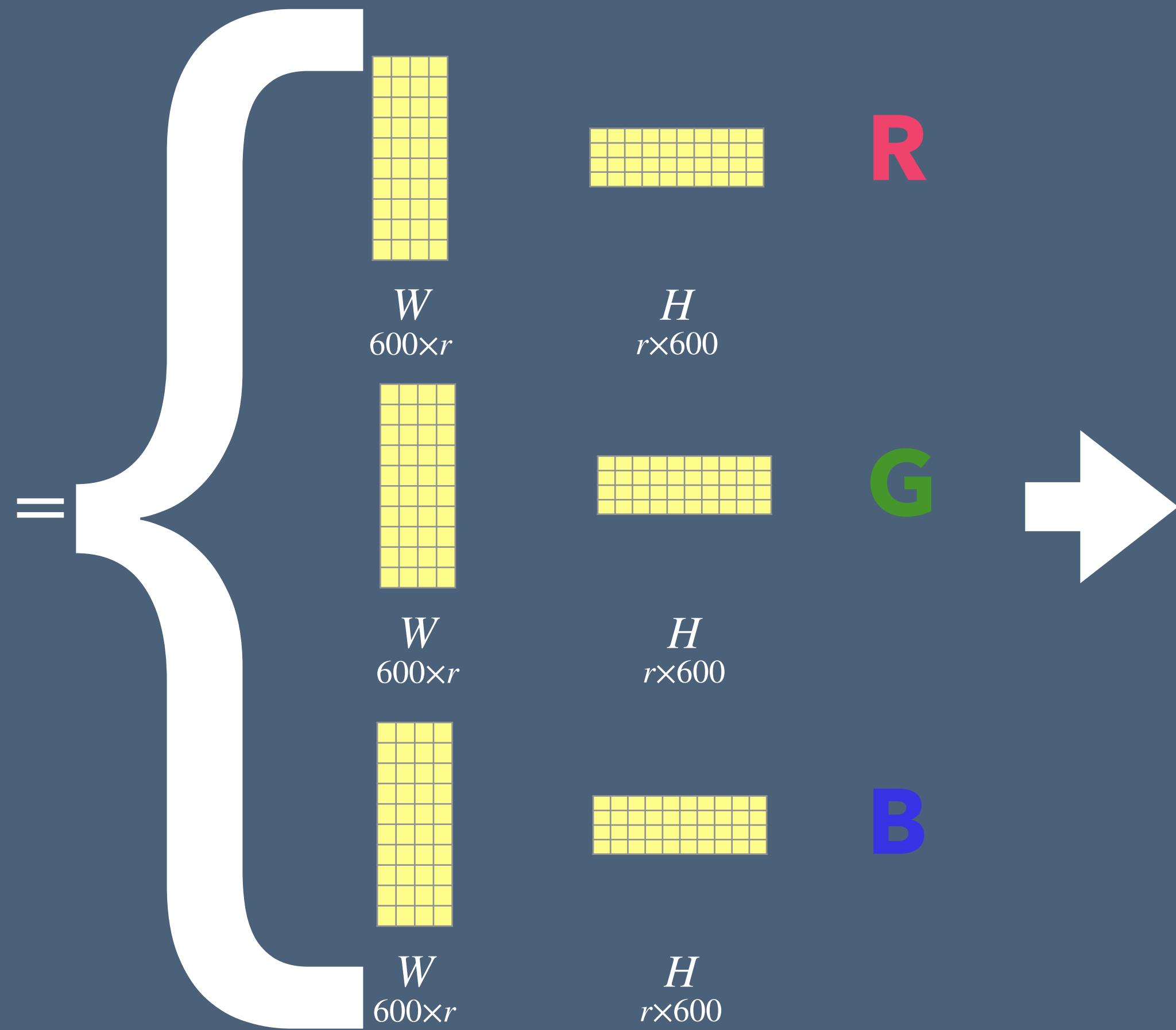
Machine: ASUS ROG Zephyrus G14, CPU: AMD Ryzen9 7940HS, 16 RAM, GPU: RTX4060 8GB

r	10	20	30	40	50	60	70	80	90	100	150	200	250	300
GSD	25	98	223	232	231	475	373	304	639	740	653	778	848	949
CD	76	899	1437	3780	5761	2143	4942	6260	9770	5284				
Lin's	10	25	33	31	49	29	42	29	9	7	4	4	8	7
Myresult	16	11	32	7	61	2.8	5.3	5	8.6	8.4	5.8	7.37	11.44	12.36

One more step



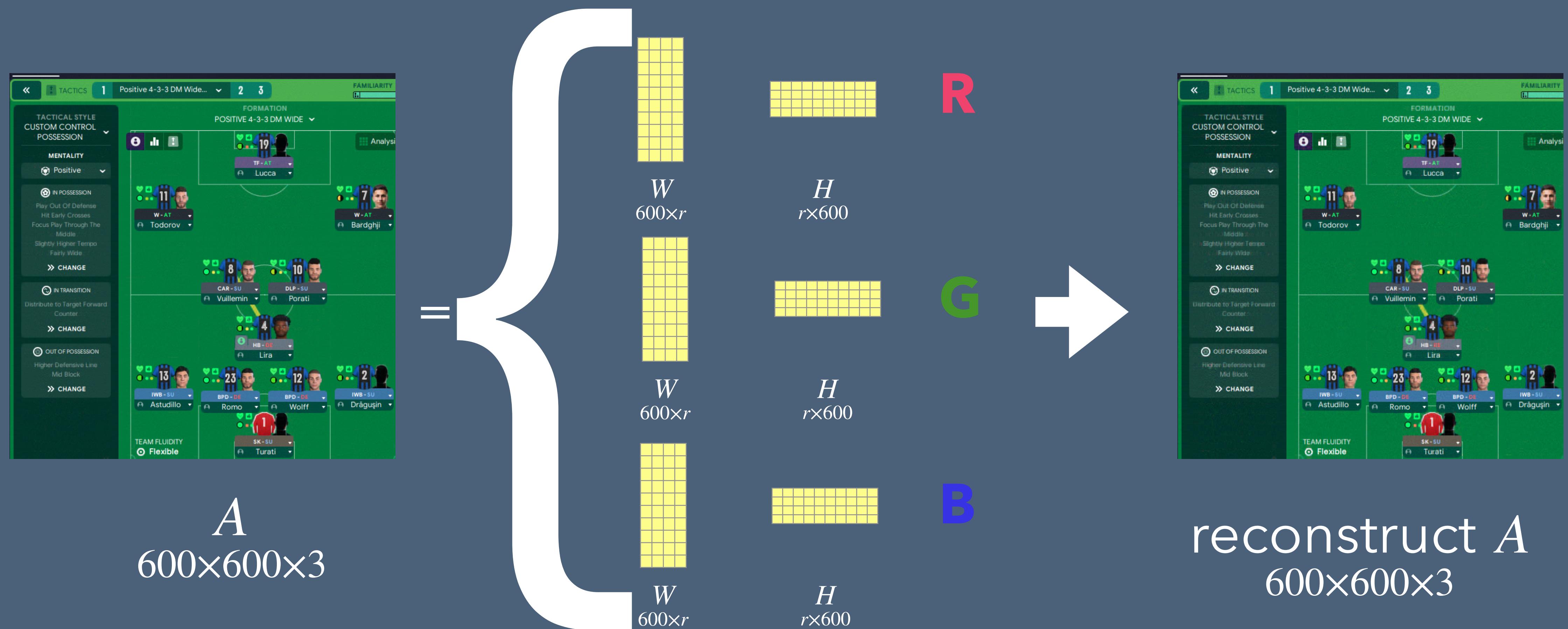
A
 $600 \times 600 \times 3$



reconstruct A
 $600 \times 600 \times 3$

$$\frac{2 \times r \times 600 \times 3}{600 \times 600 \times 3} = \frac{r}{300}$$

One more step



$$\frac{2 \times r \times 600 \times 3}{600 \times 600 \times 3} = \frac{r}{300}$$

reconstruct A
 $600 \times 600 \times 3$

Future work

$$\begin{matrix} A \\ m \times n \end{matrix} = \begin{matrix} W \times H \\ m \times r \\ r \times n \end{matrix}$$

- More datasets experiments
- Parallel computing
- The best r?

Reference

- Nonnegative Matrix Factorization via (alternating) Projected Gradient Descent, Andersen Ang.
- Projected Gradient Methods for Non-negative Matrix Factorization, Chih-Jen Lin
- Non-negative Matrix Factorization (NMF), Bindel, Summer 2018

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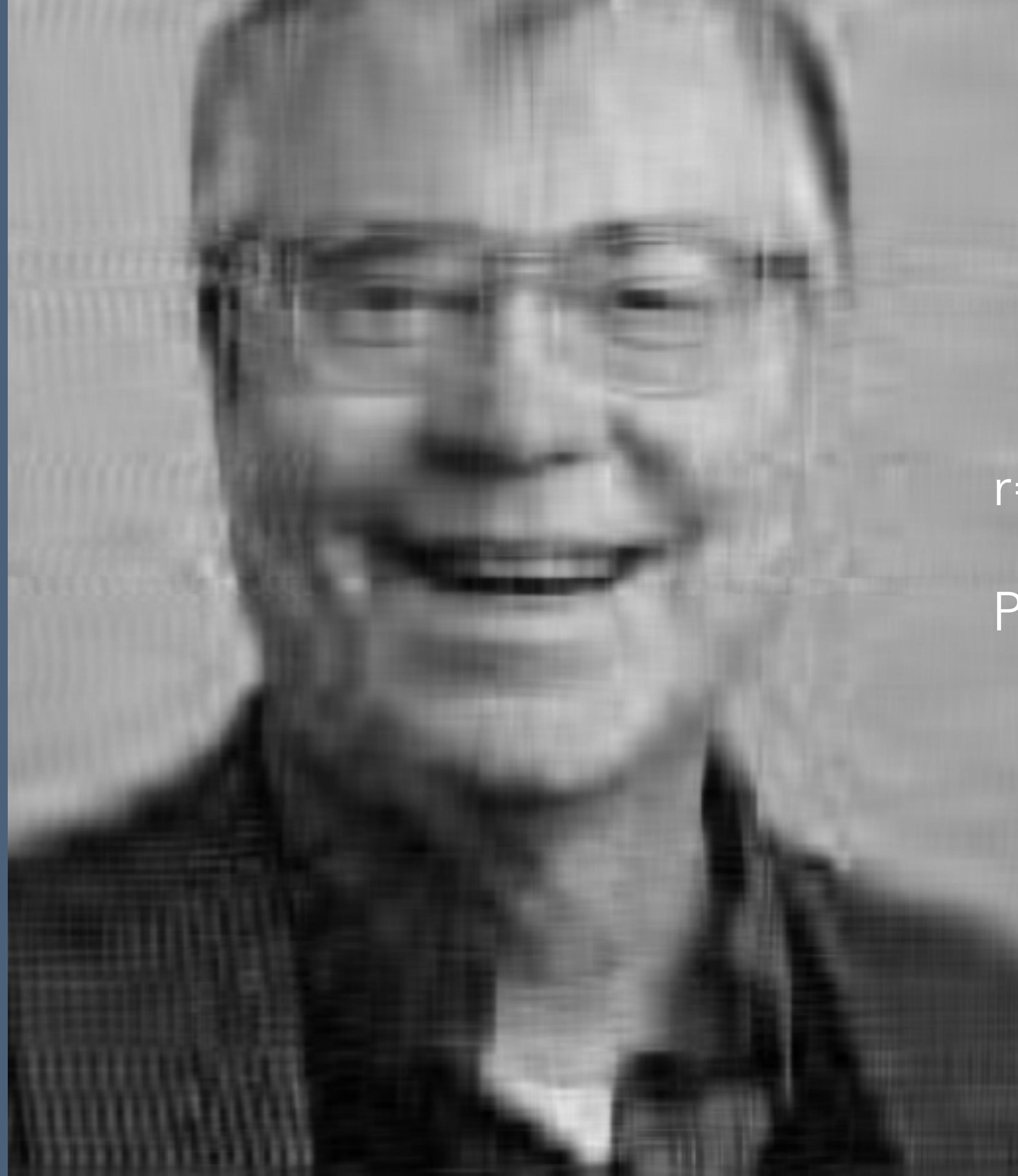
Code demo and Question time

The code and results are available on <https://github.com/GhostBlue32/Algorithm-For-Op/tree/main/MidProject/NMF>.



$r=10$
PGD

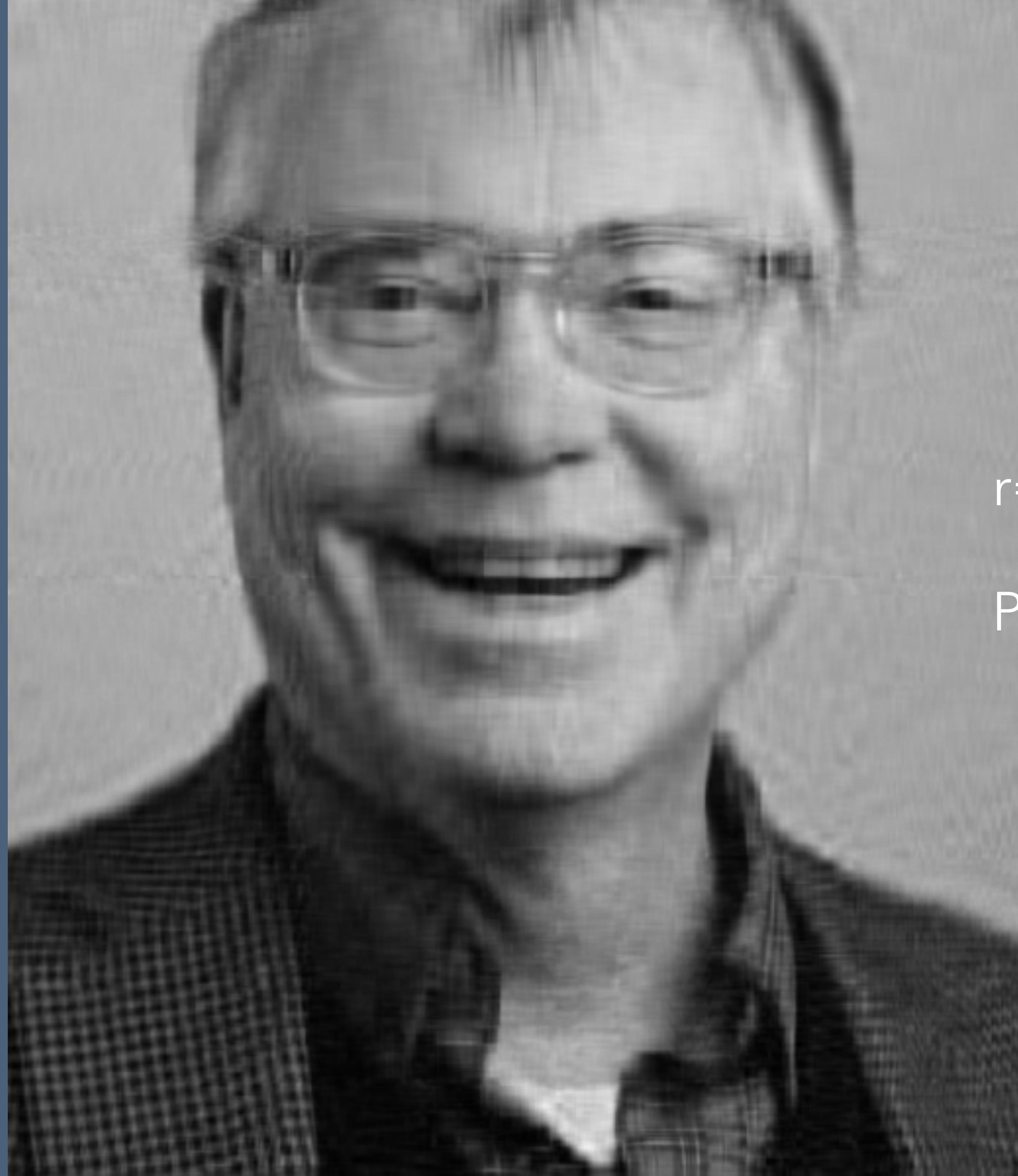




r=20

PGD

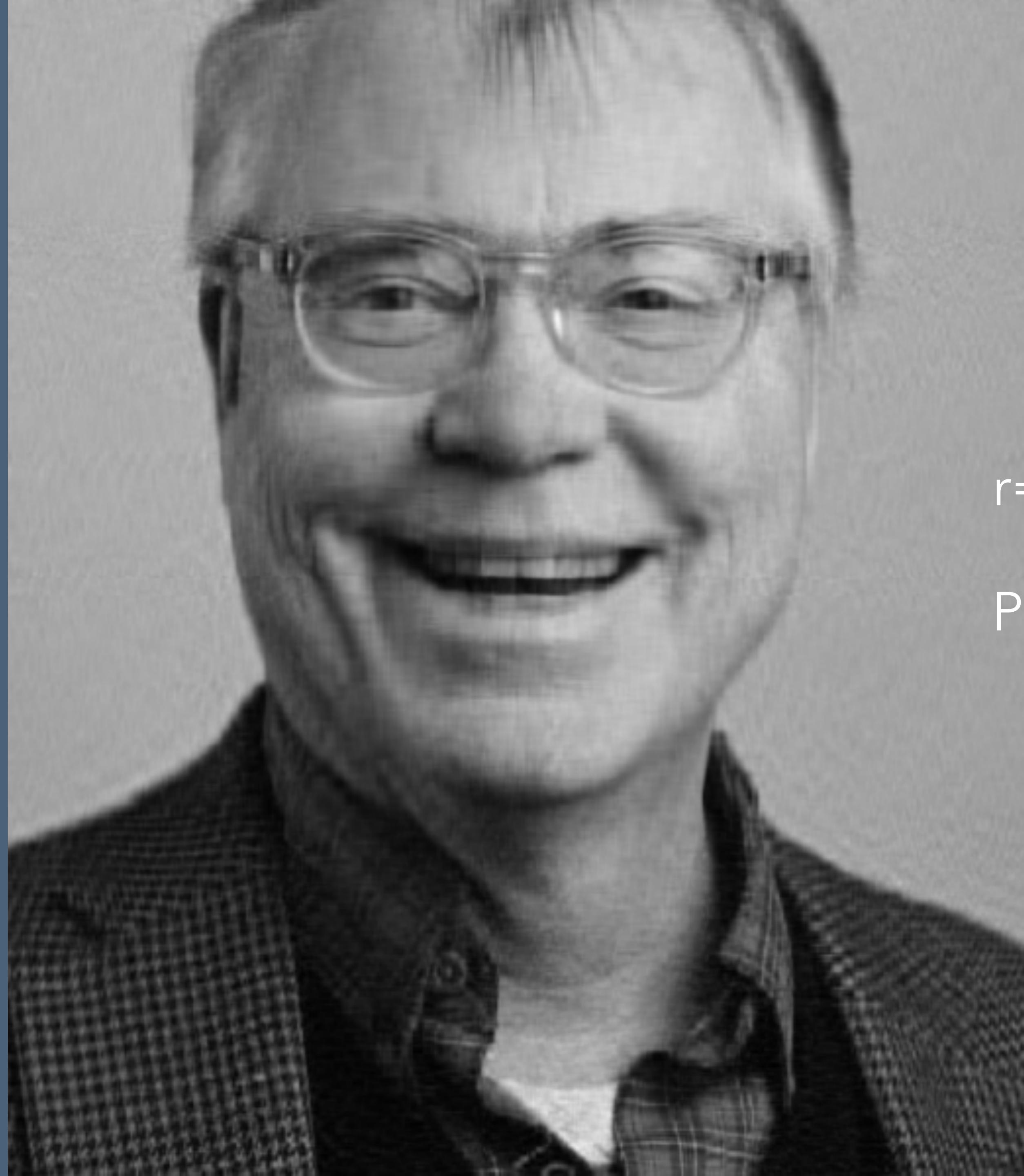




$r=40$

PGD





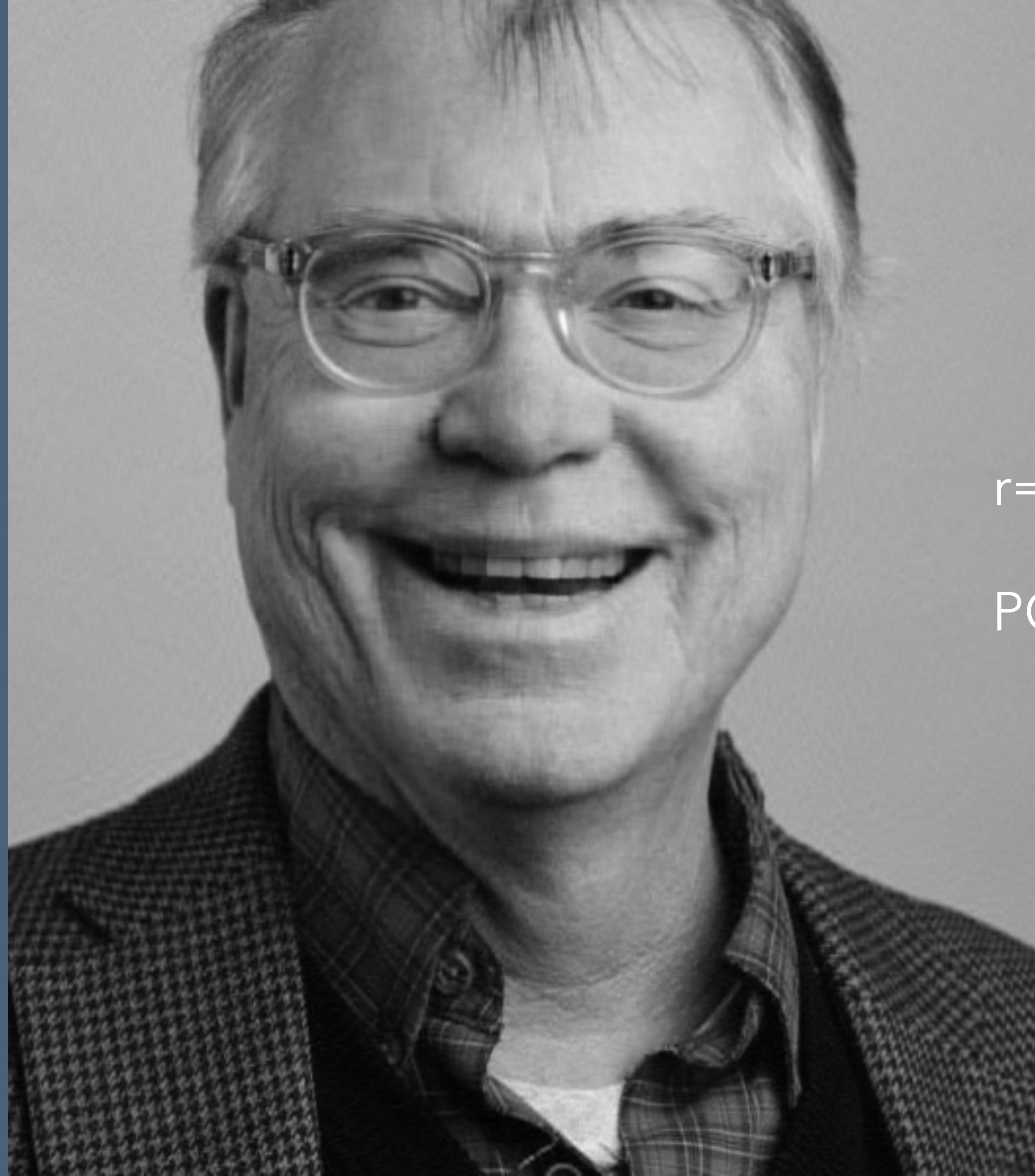
r=70
PGD



A black and white portrait of a middle-aged man with light-colored hair and glasses, smiling broadly. He is wearing a dark, textured jacket over a plaid shirt.

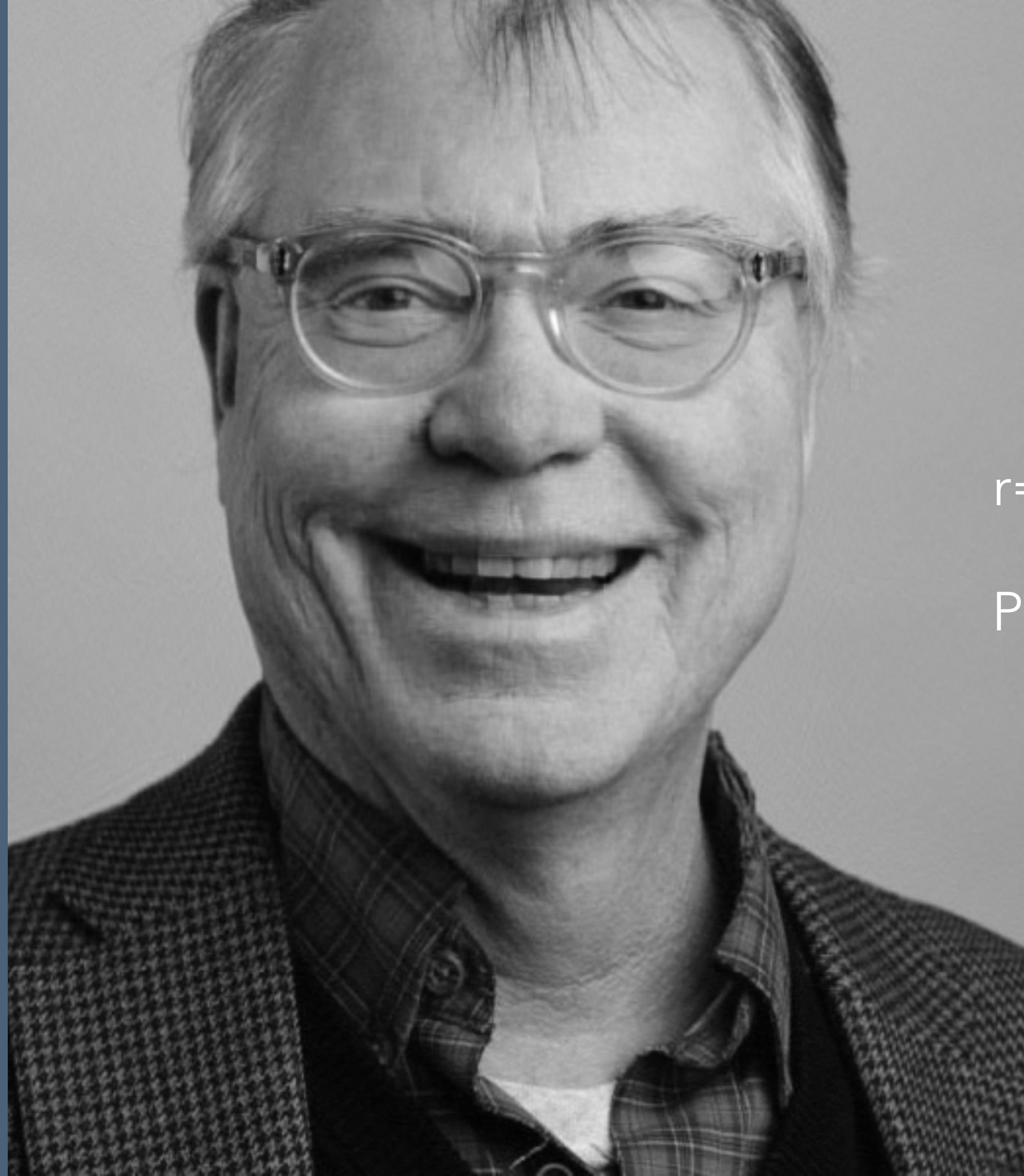
$r=100$
PGD





$r=150$
PGD

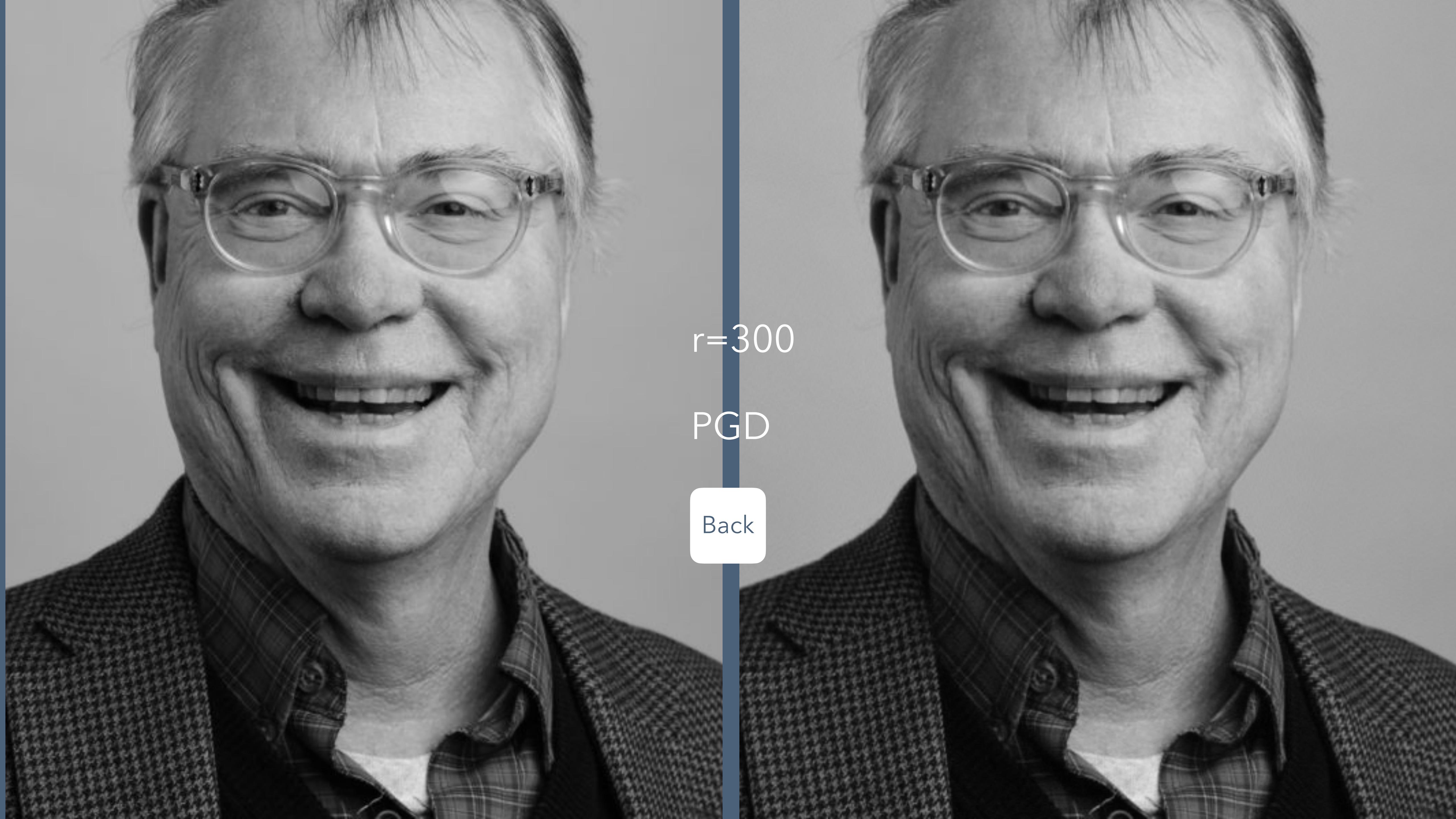




$r=200$

PGD



A black and white portrait of a middle-aged man with light-colored hair and glasses, smiling warmly at the camera. He is wearing a dark, textured jacket over a plaid shirt. The background is a plain, light color.

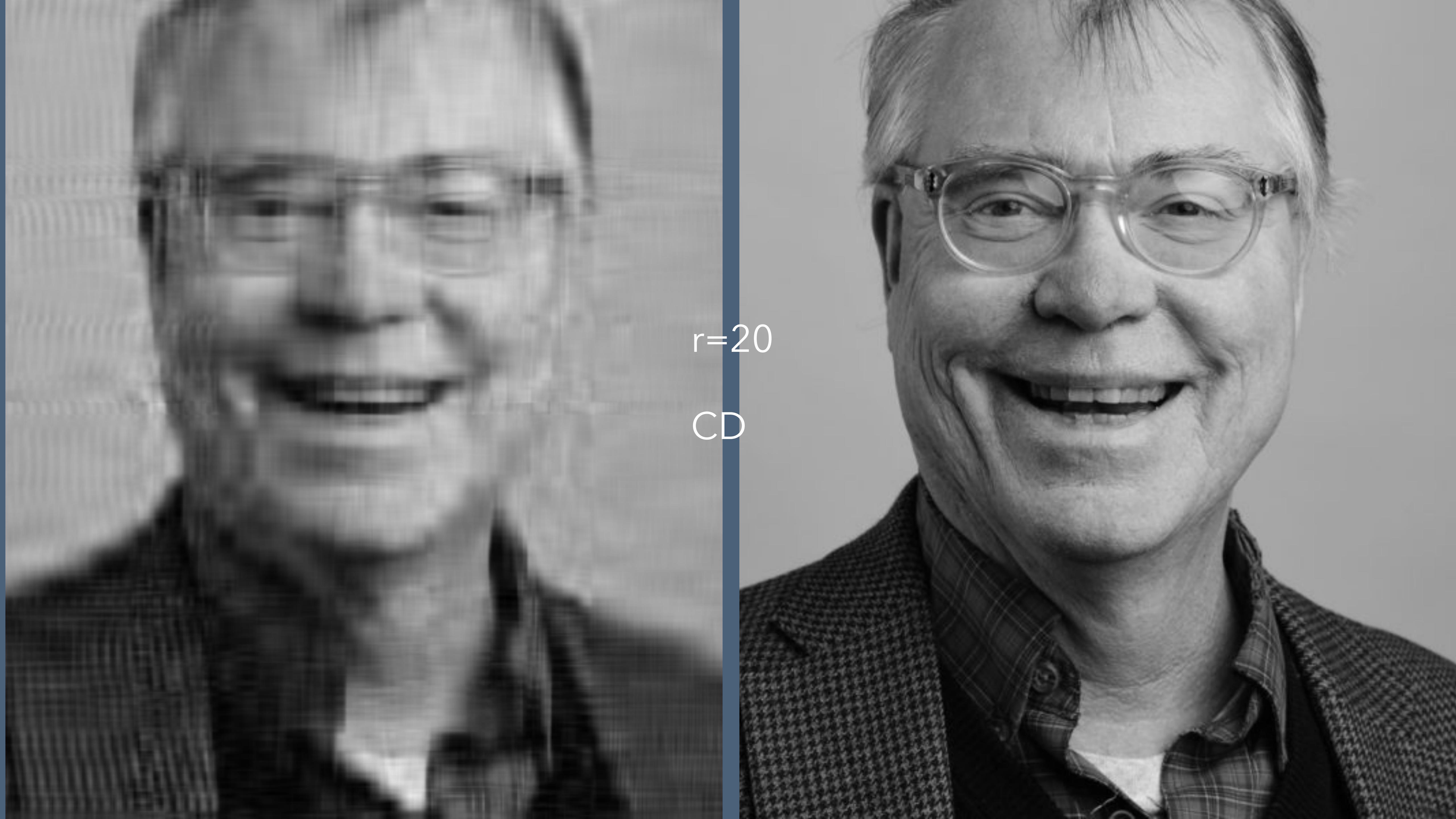
$r=300$

PGD

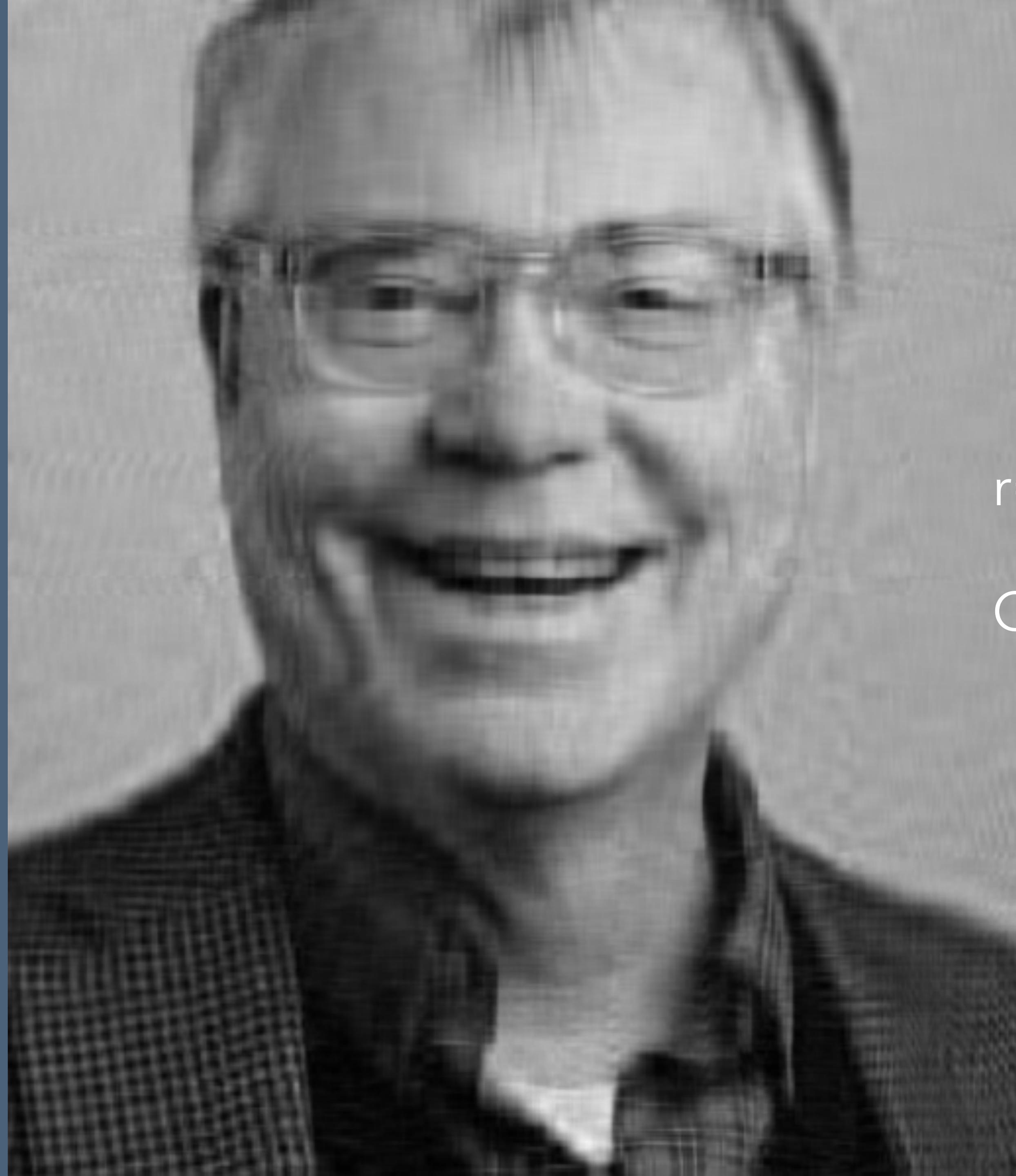
Back

A black and white portrait of a middle-aged man with light-colored hair and glasses, smiling warmly at the camera. He is wearing a dark, textured jacket over a plaid shirt. The background is plain and light.

$r=10$
CD

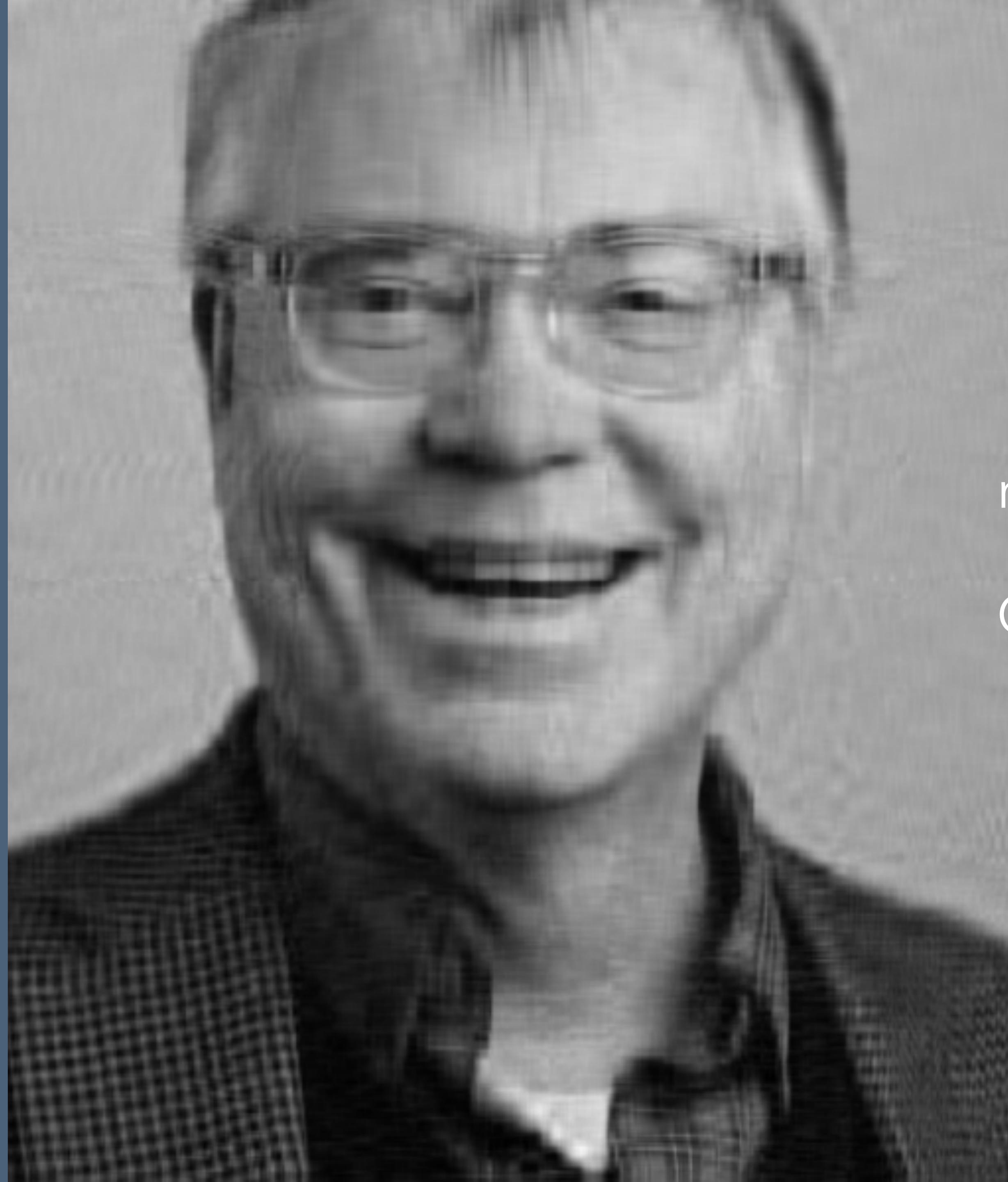
A black and white photograph of a middle-aged man with light-colored hair and glasses, smiling warmly at the camera. He is wearing a dark, textured jacket over a plaid shirt. The background is plain and light.

$r=20$
CD



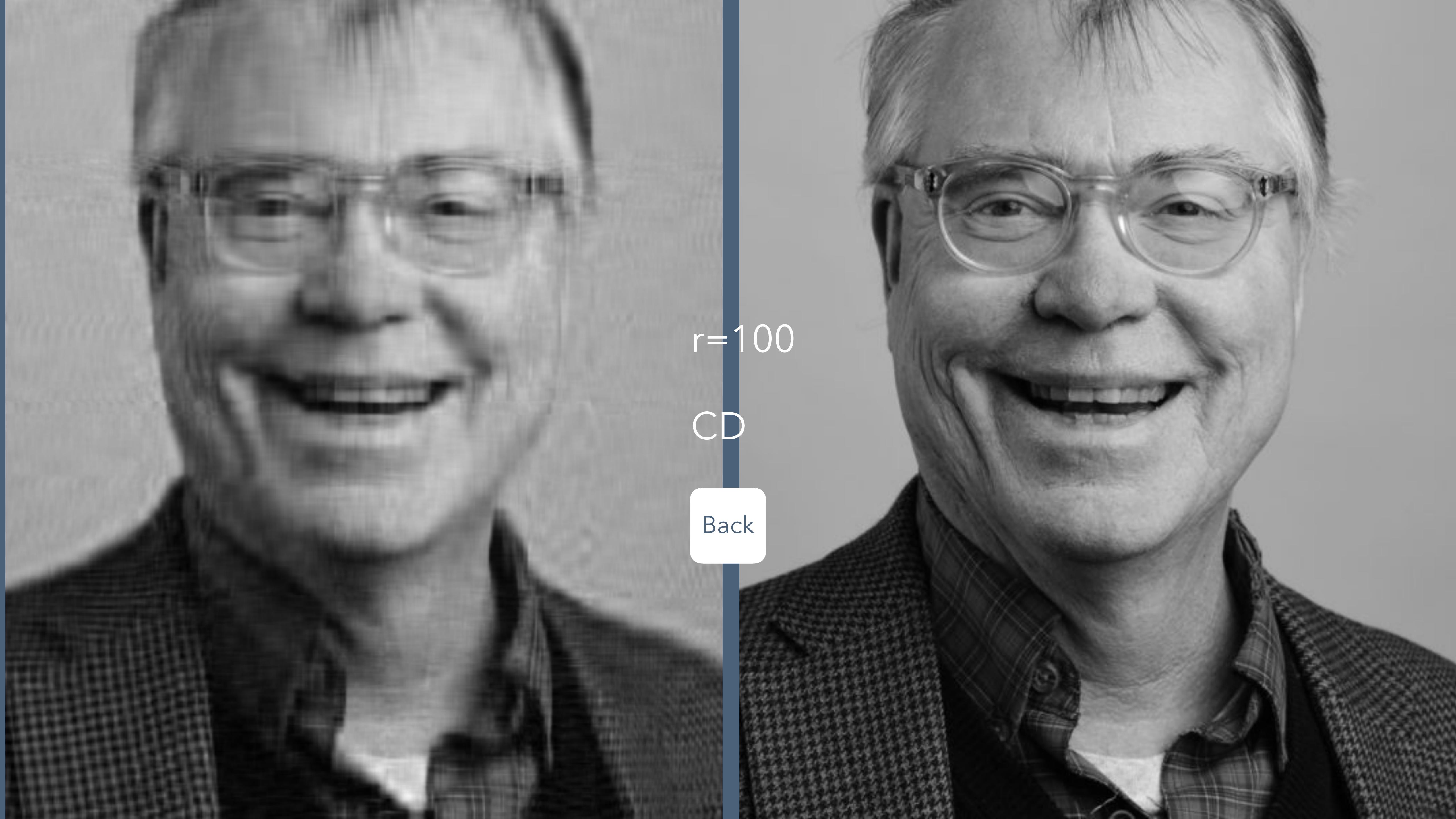
r=40
CD





r=80

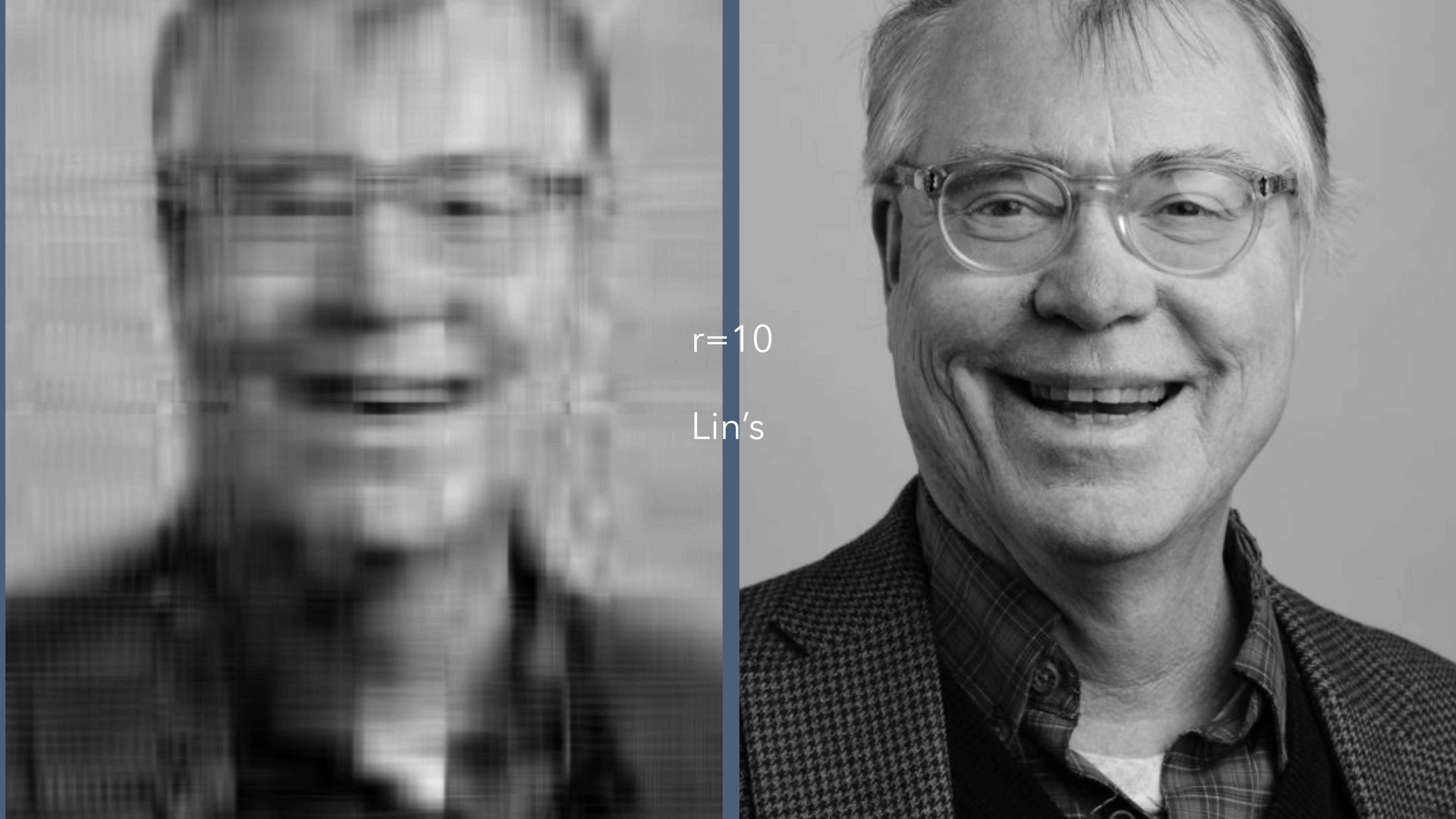
CD

A black and white portrait of a middle-aged man with light-colored hair and glasses, smiling broadly. He is wearing a dark, textured jacket over a plaid shirt. The background is plain and light.

$r=100$

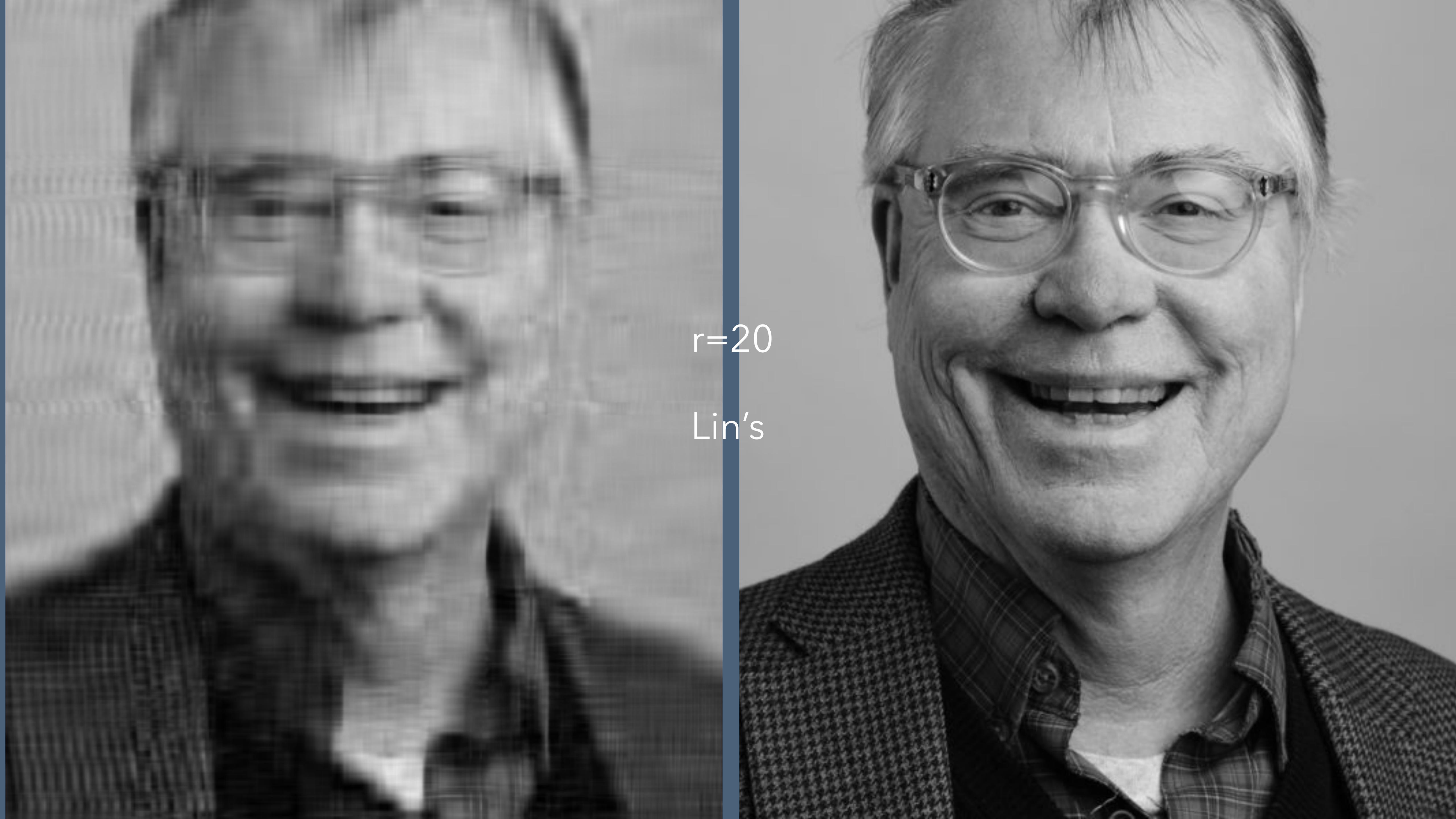
CD

Back



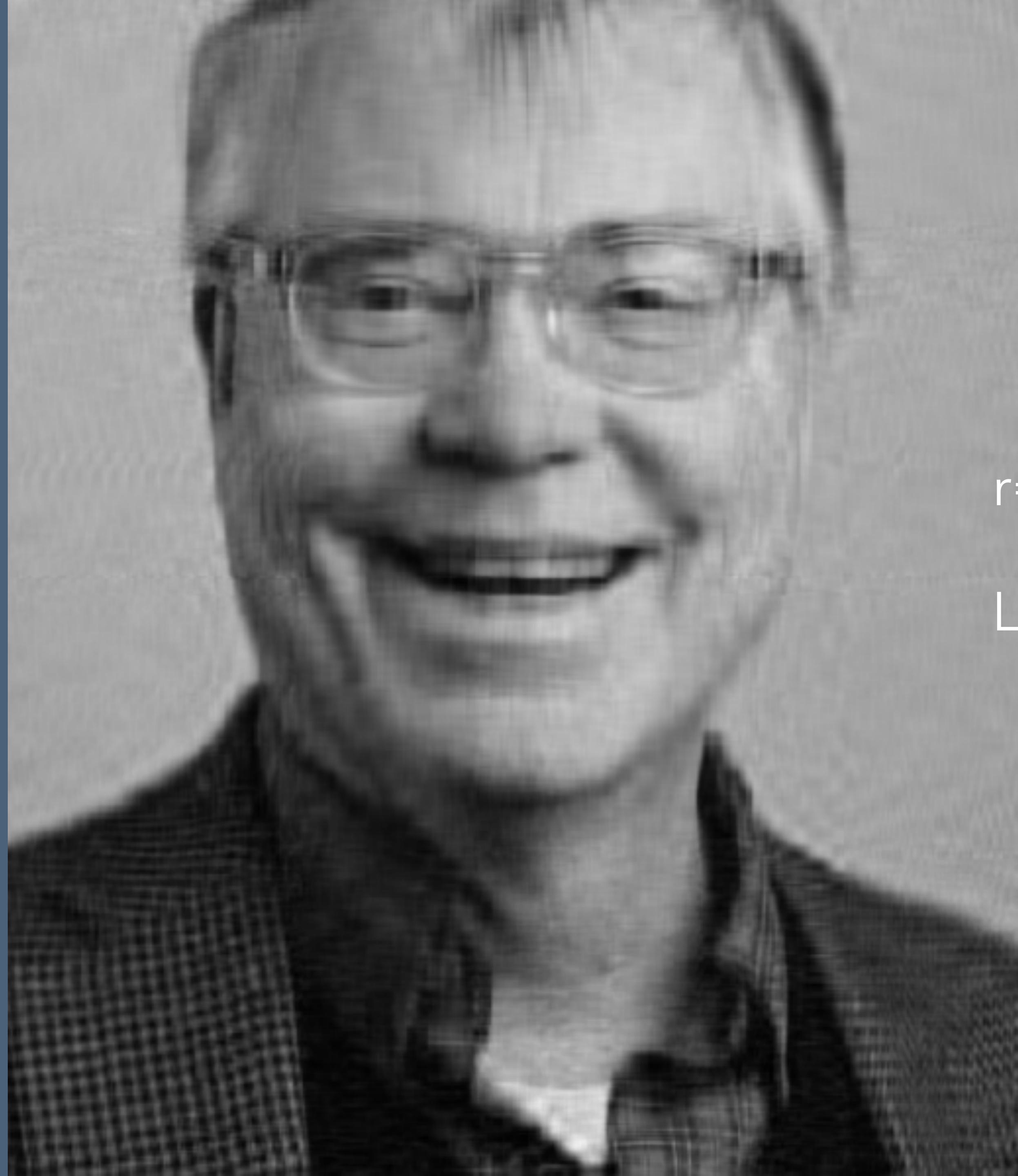
r=10

Lin's



r=20

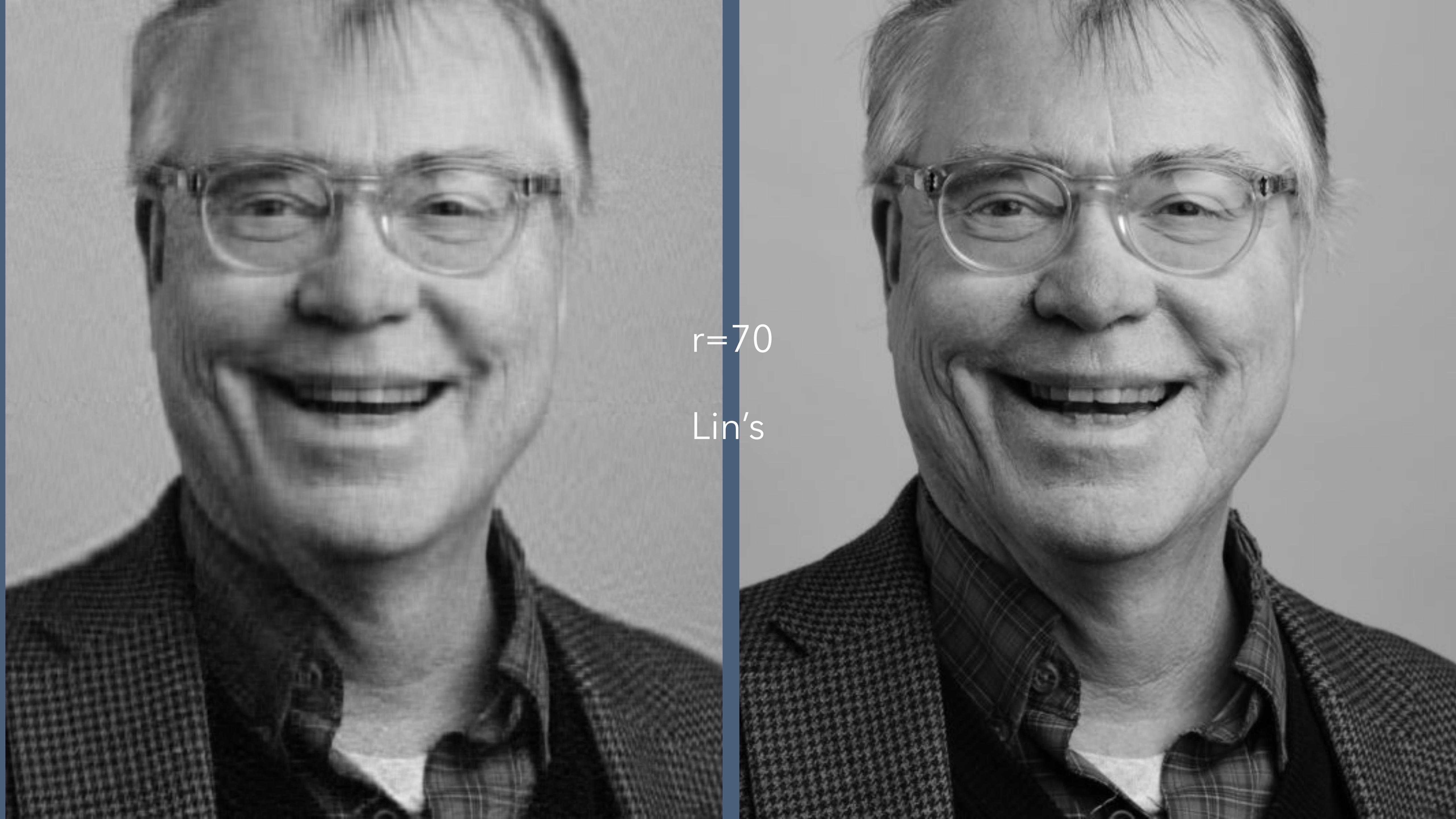
Lin's



r=40

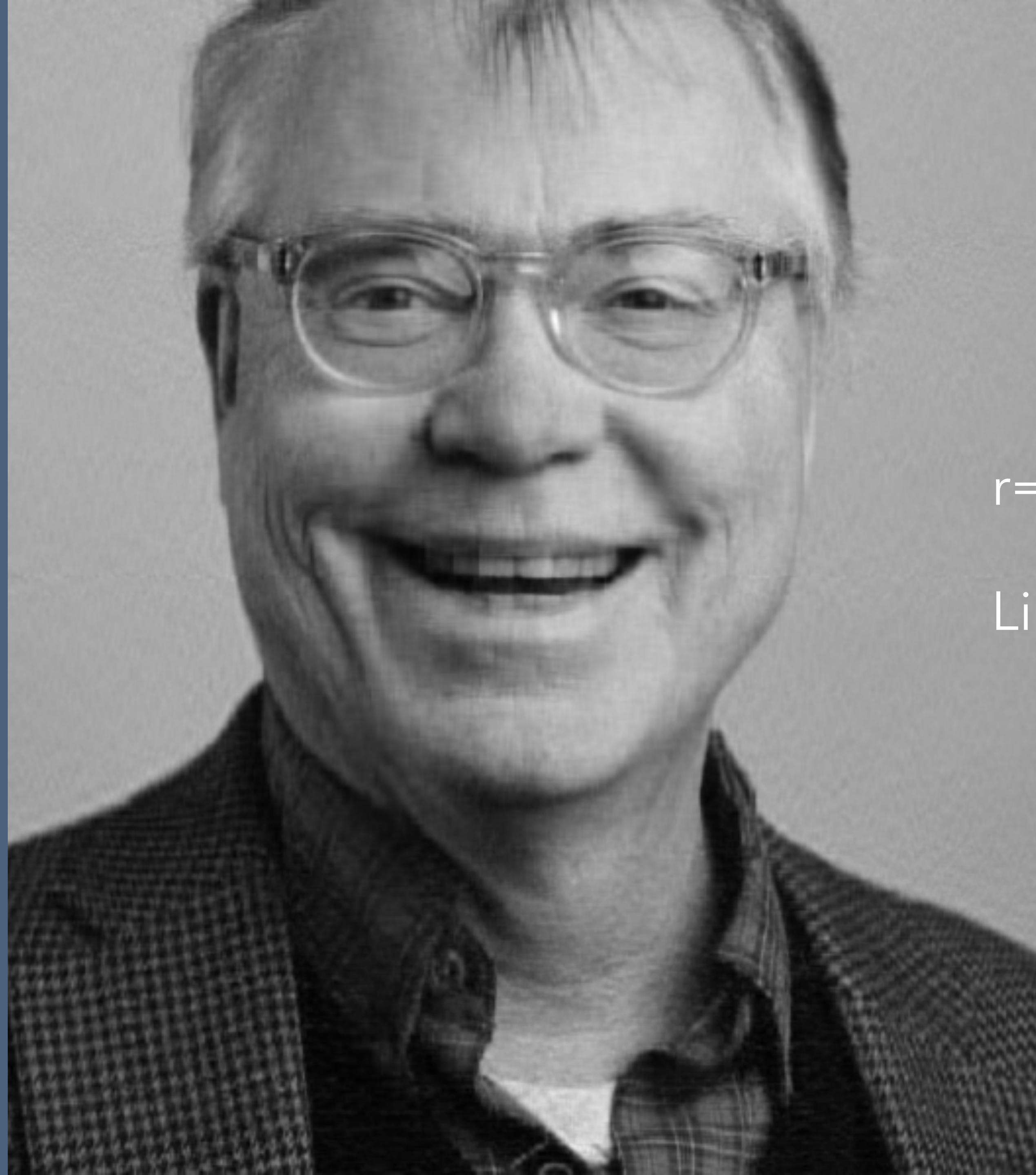
Lin's





$r=70$

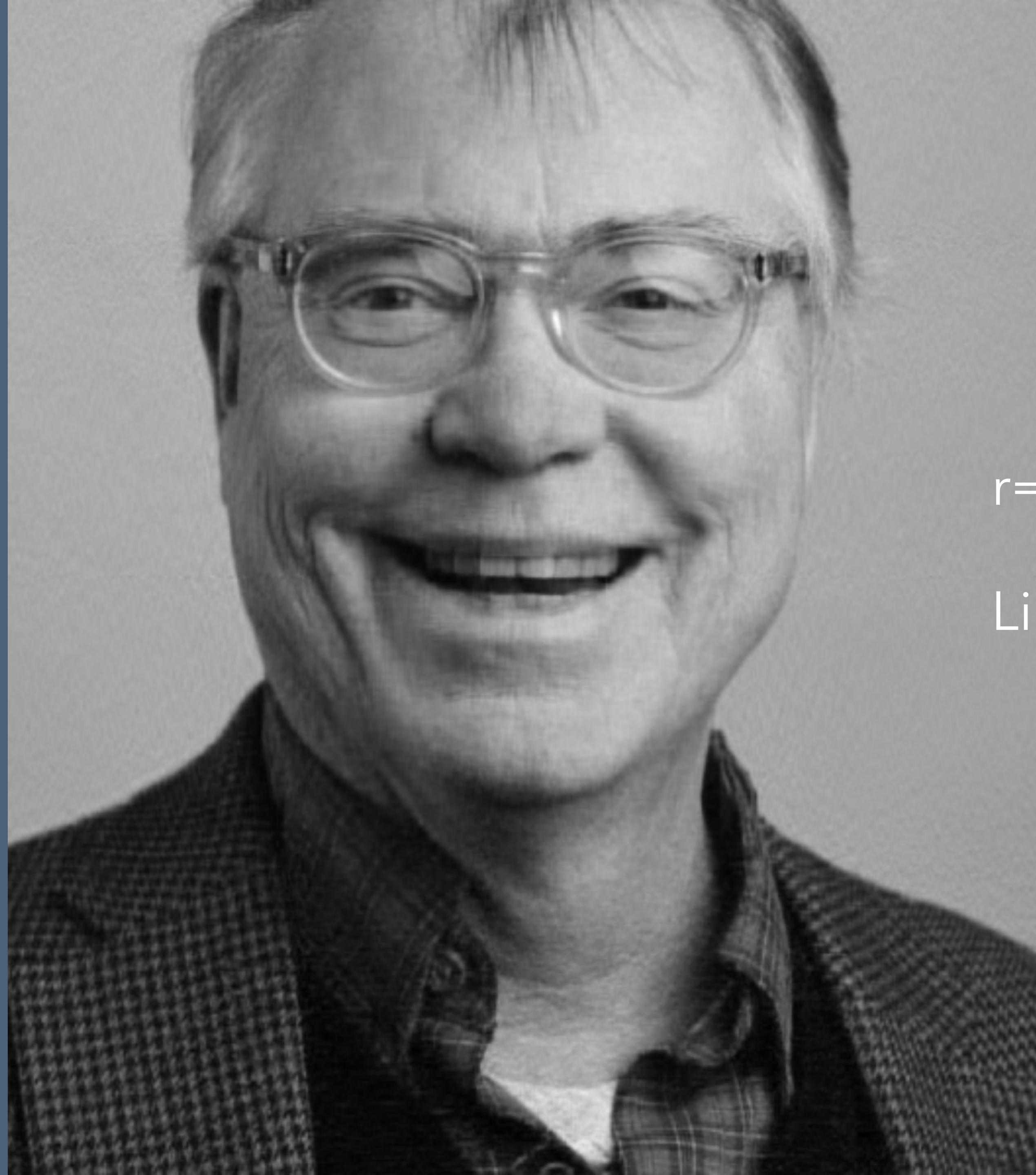
Lin's



r=100

Lin's

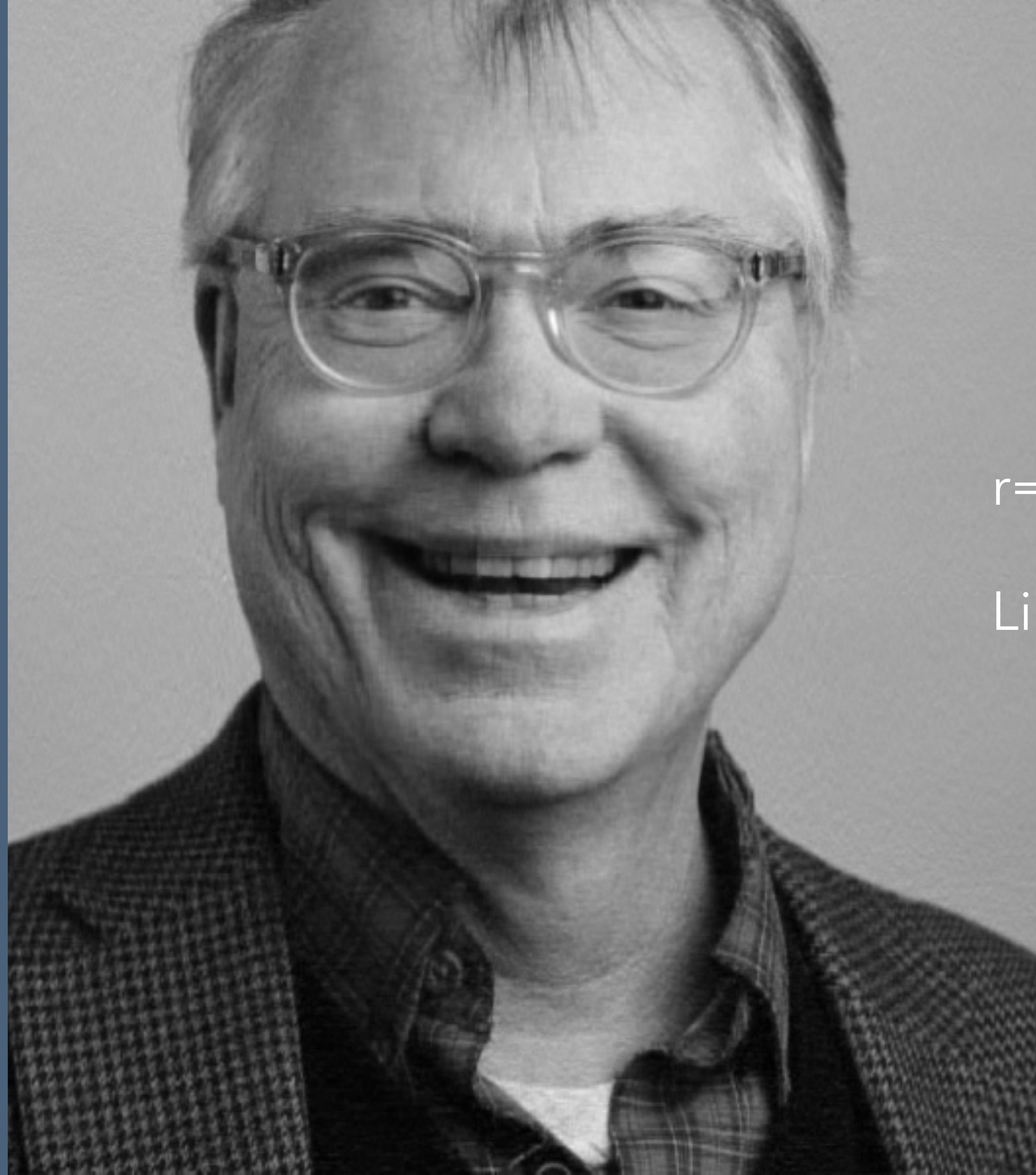




r=150

Lin's

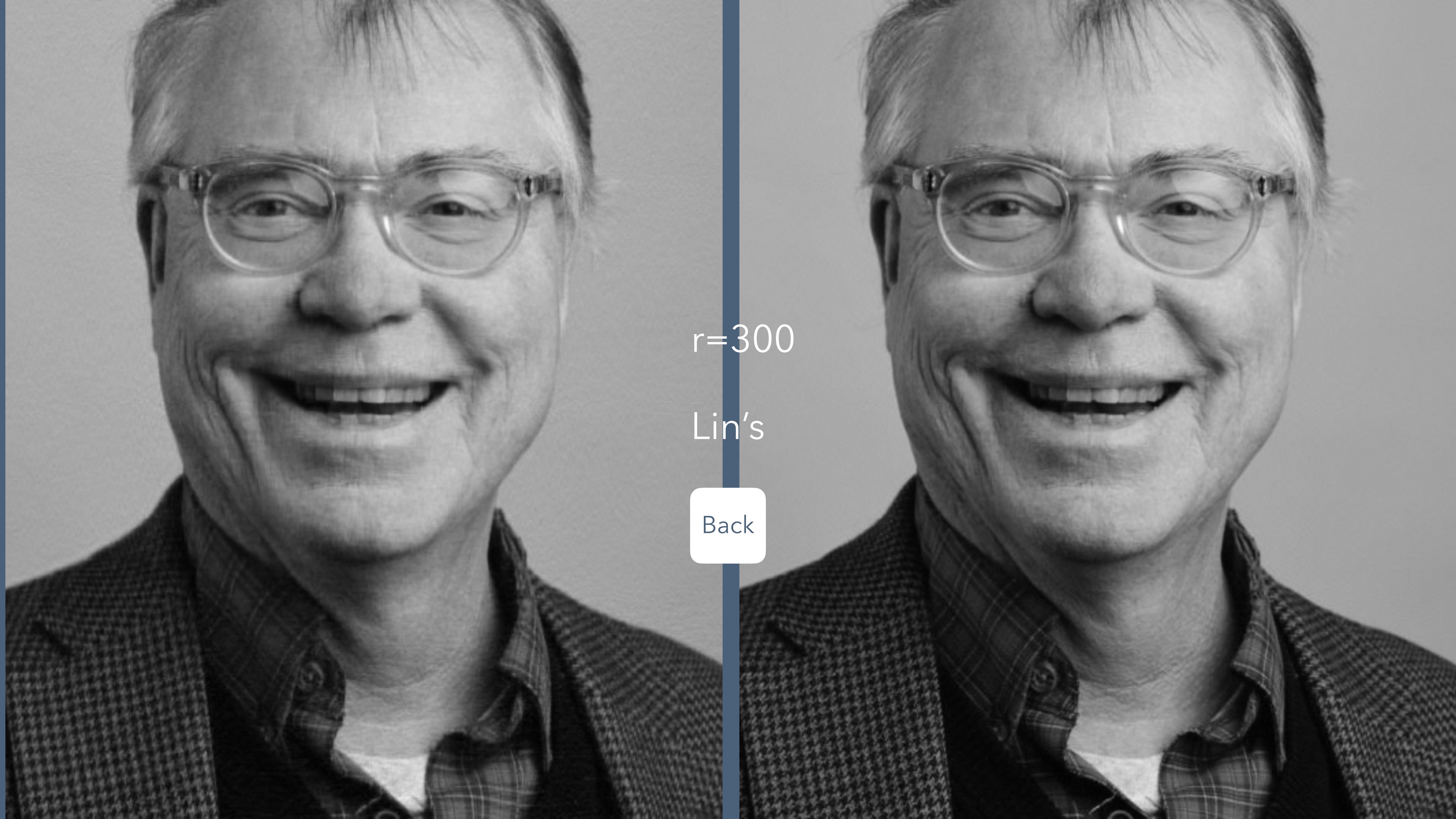




r=200

Lin's

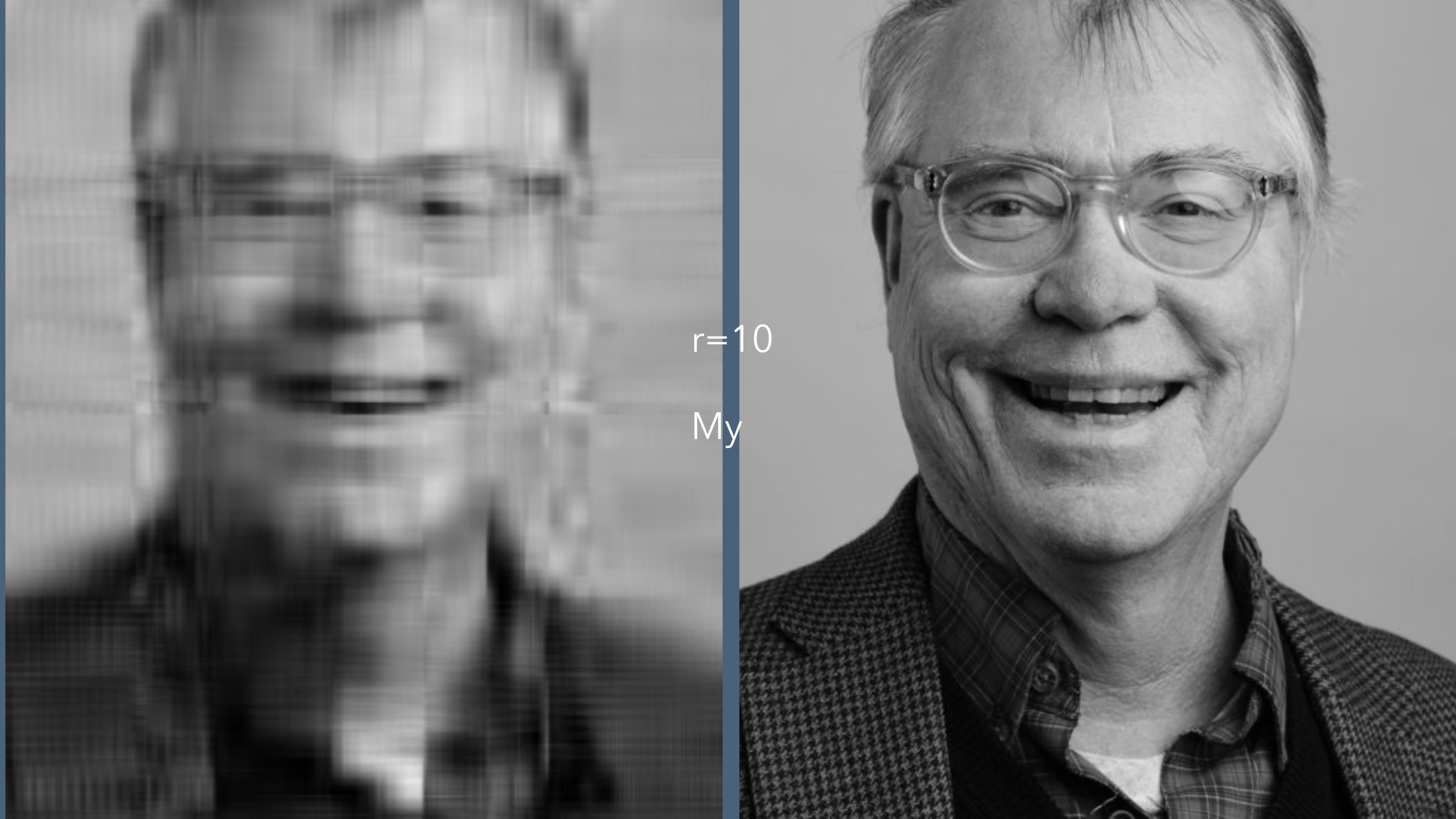




r=300

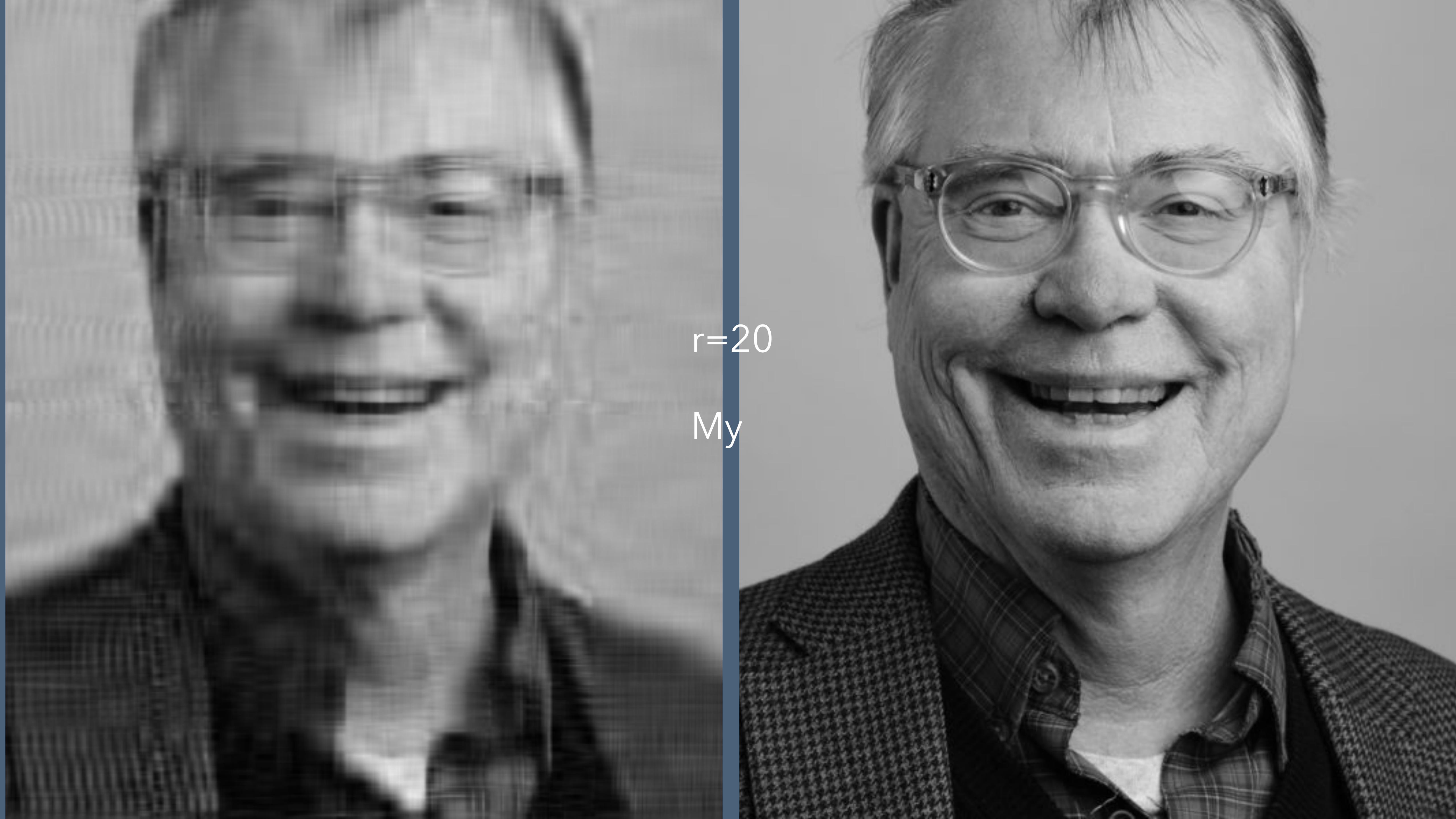
Lin's

Back

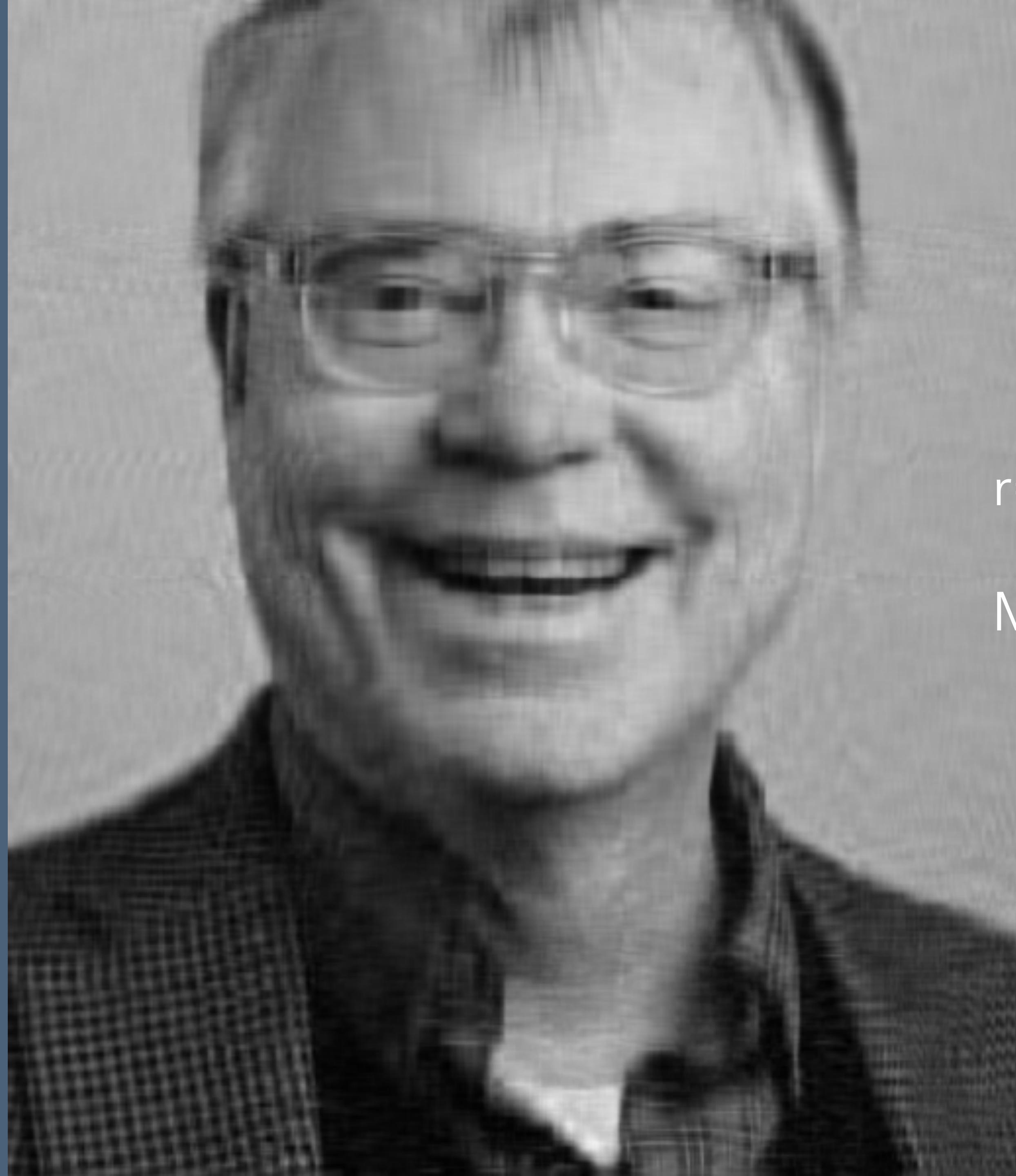
A black and white portrait of a middle-aged man with light-colored hair and glasses, smiling warmly at the camera. He is wearing a dark, textured jacket over a plaid shirt. The background is plain and light.

$r=10$

My

A black and white photograph of a middle-aged man with light-colored hair and glasses, smiling warmly at the camera. He is wearing a dark, textured jacket over a plaid shirt. The background is plain and light.

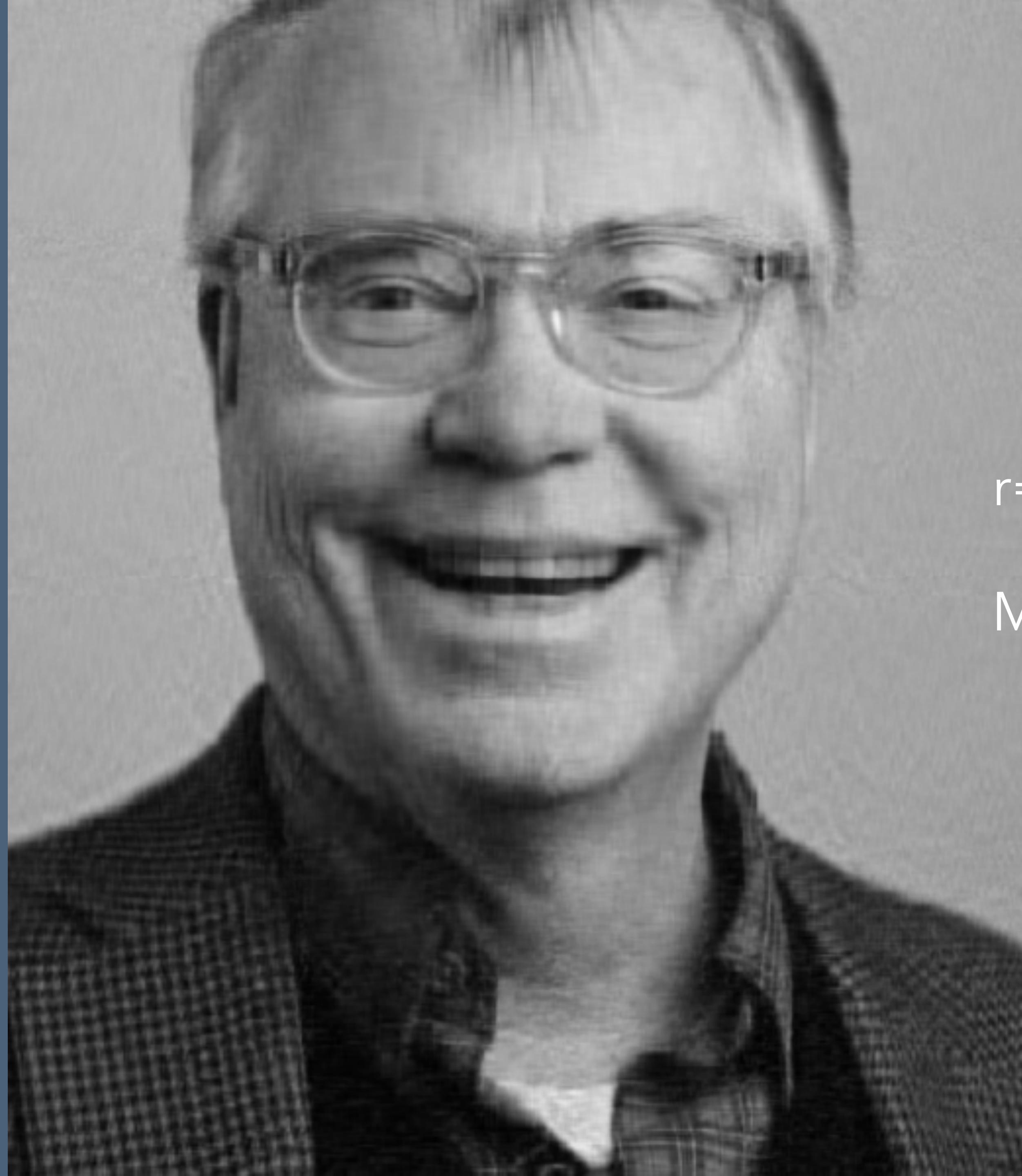
$r=20$
My



r=40

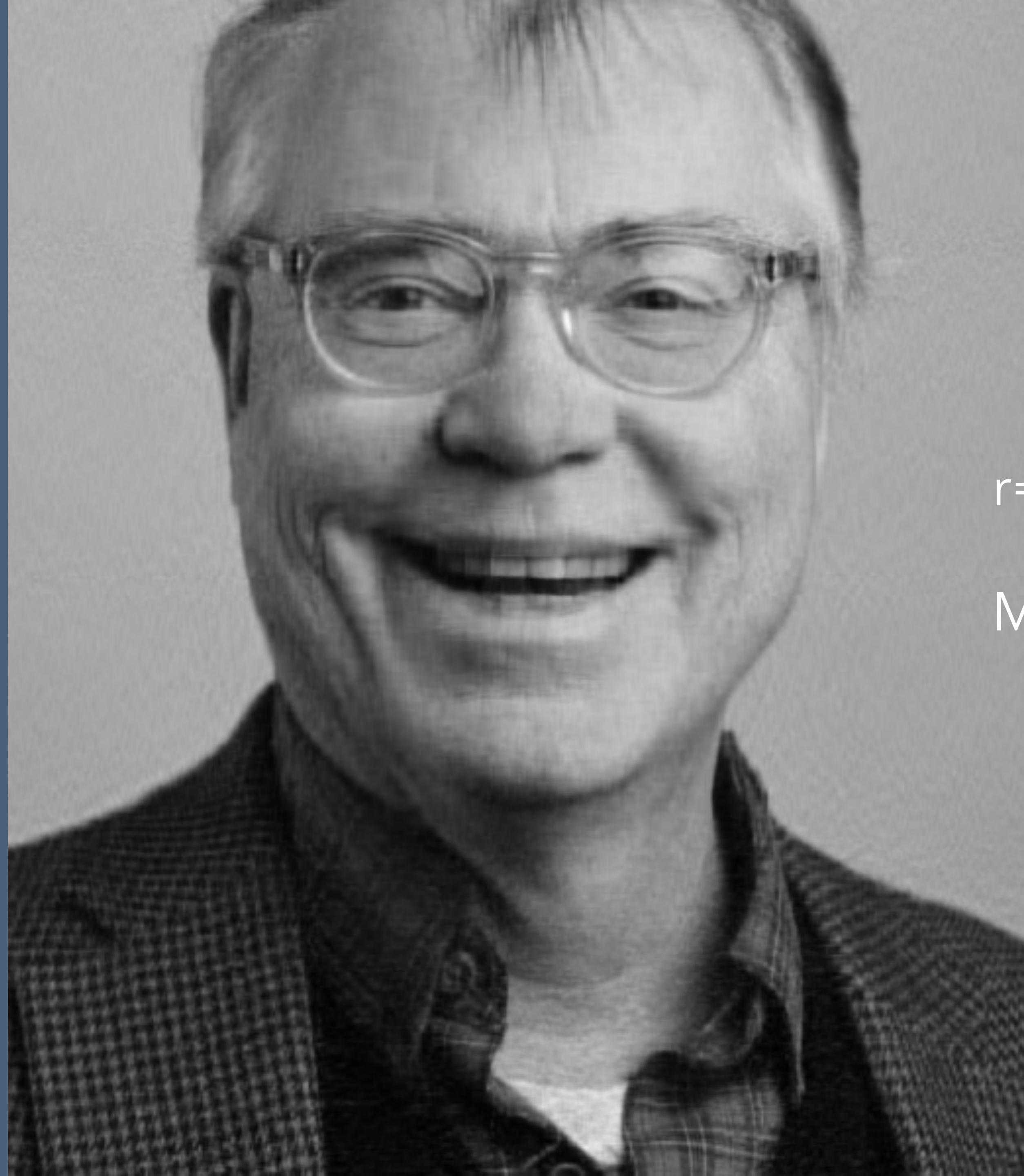
My





r=70
My





r=100

My





r=150

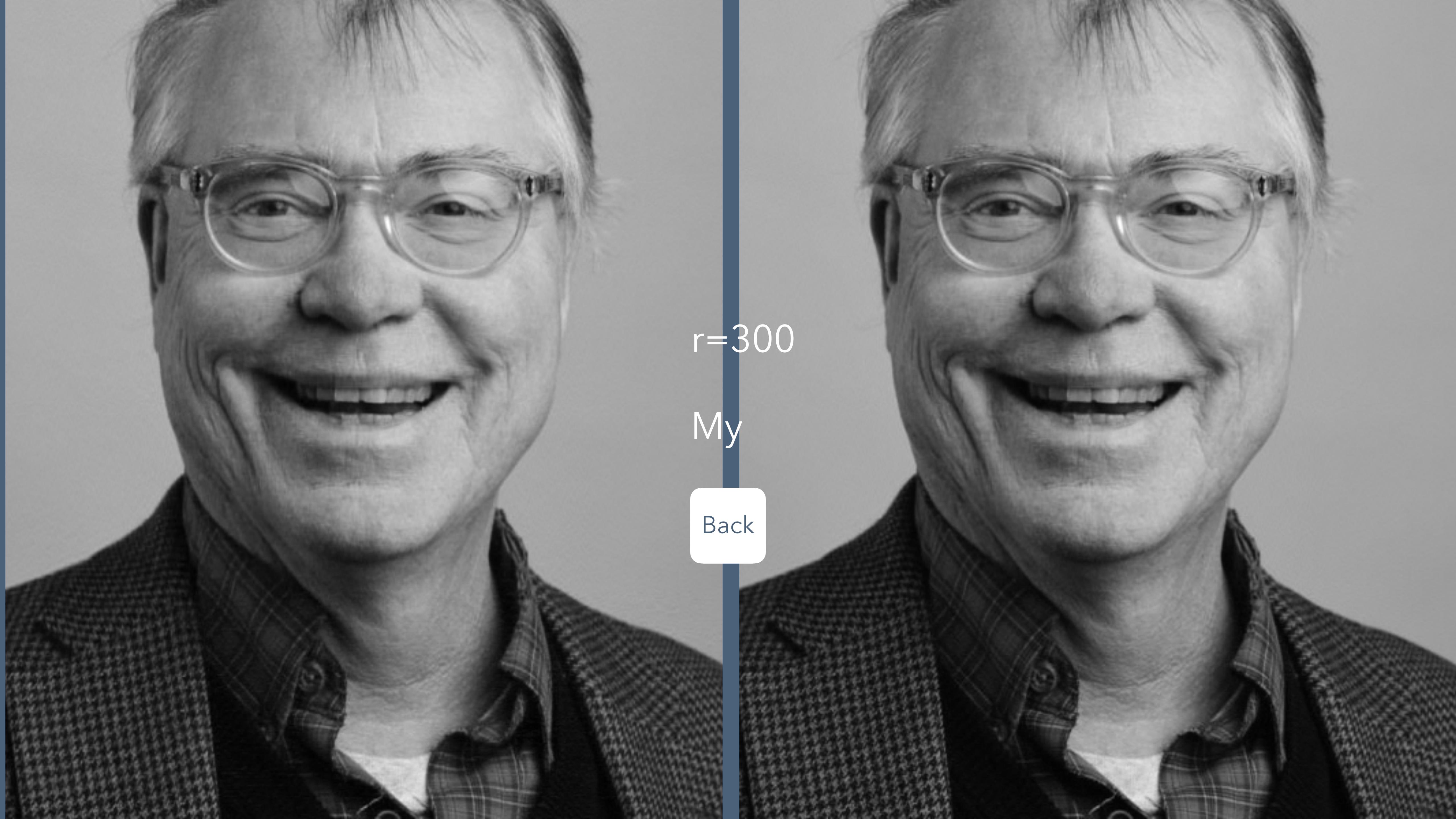
My





r=200
My

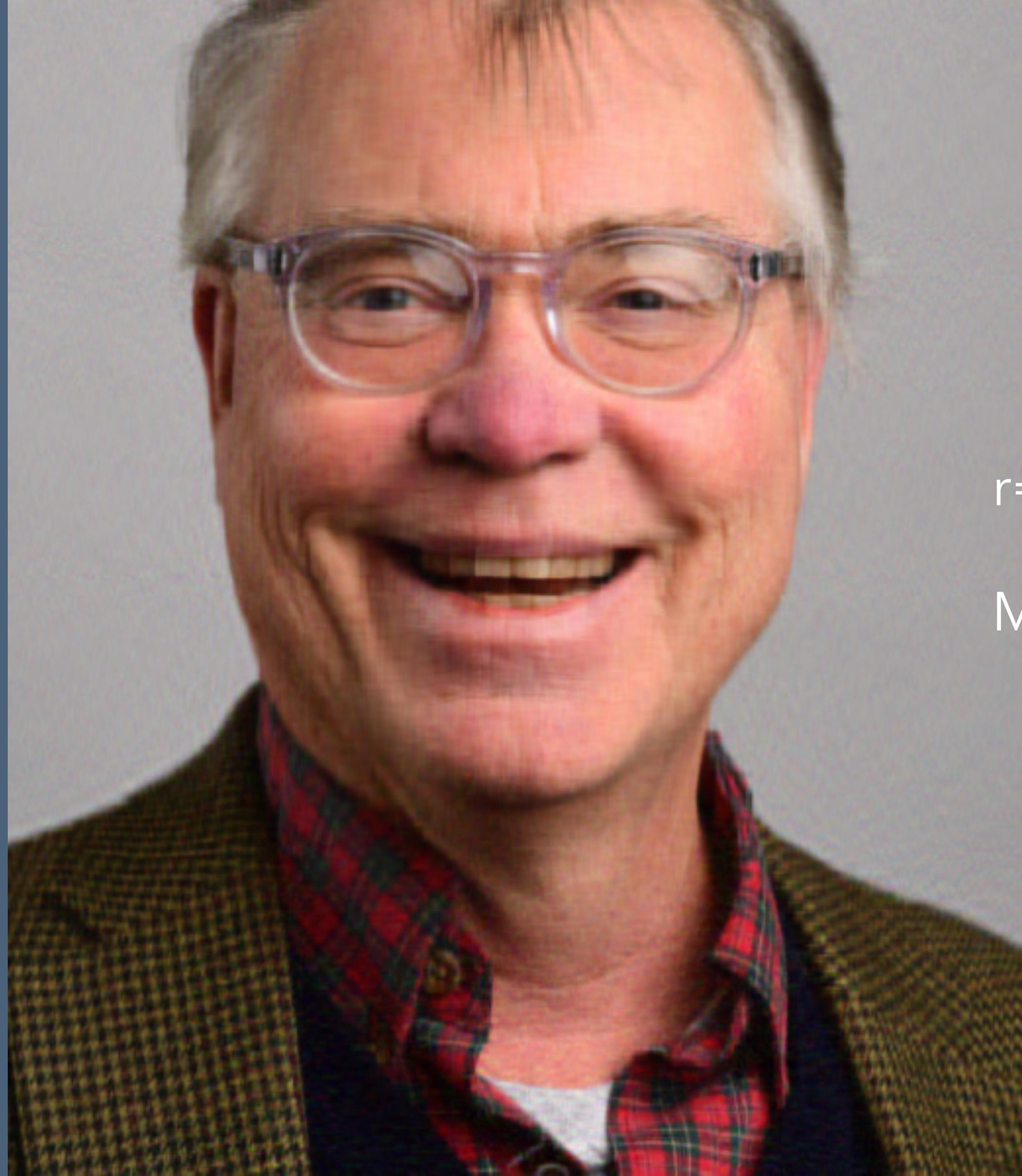


A black and white portrait of a middle-aged man with light-colored hair and glasses, smiling warmly at the camera. He is wearing a dark, textured jacket over a plaid shirt. The background is a plain, light color.

$r=300$

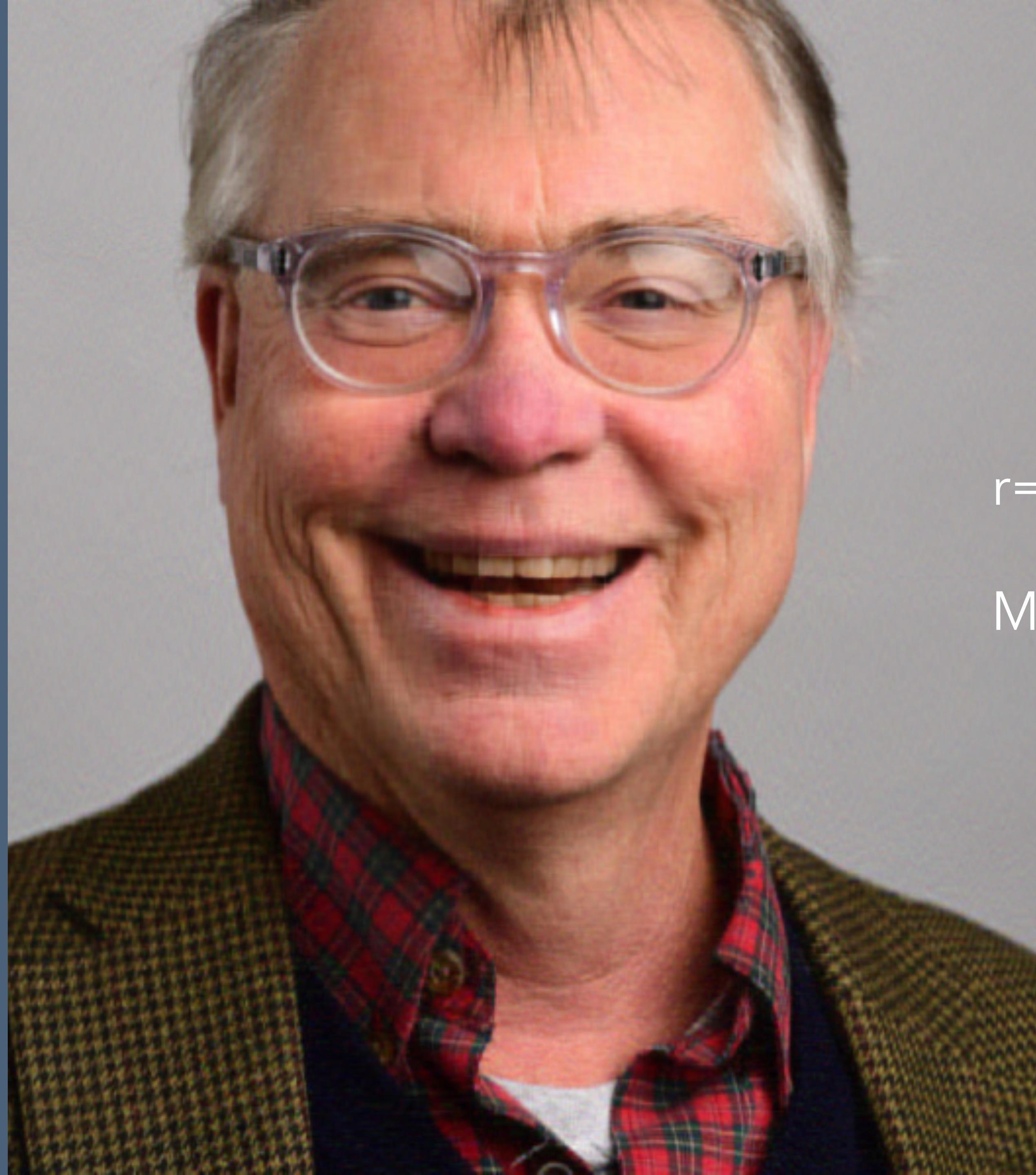
My

Back



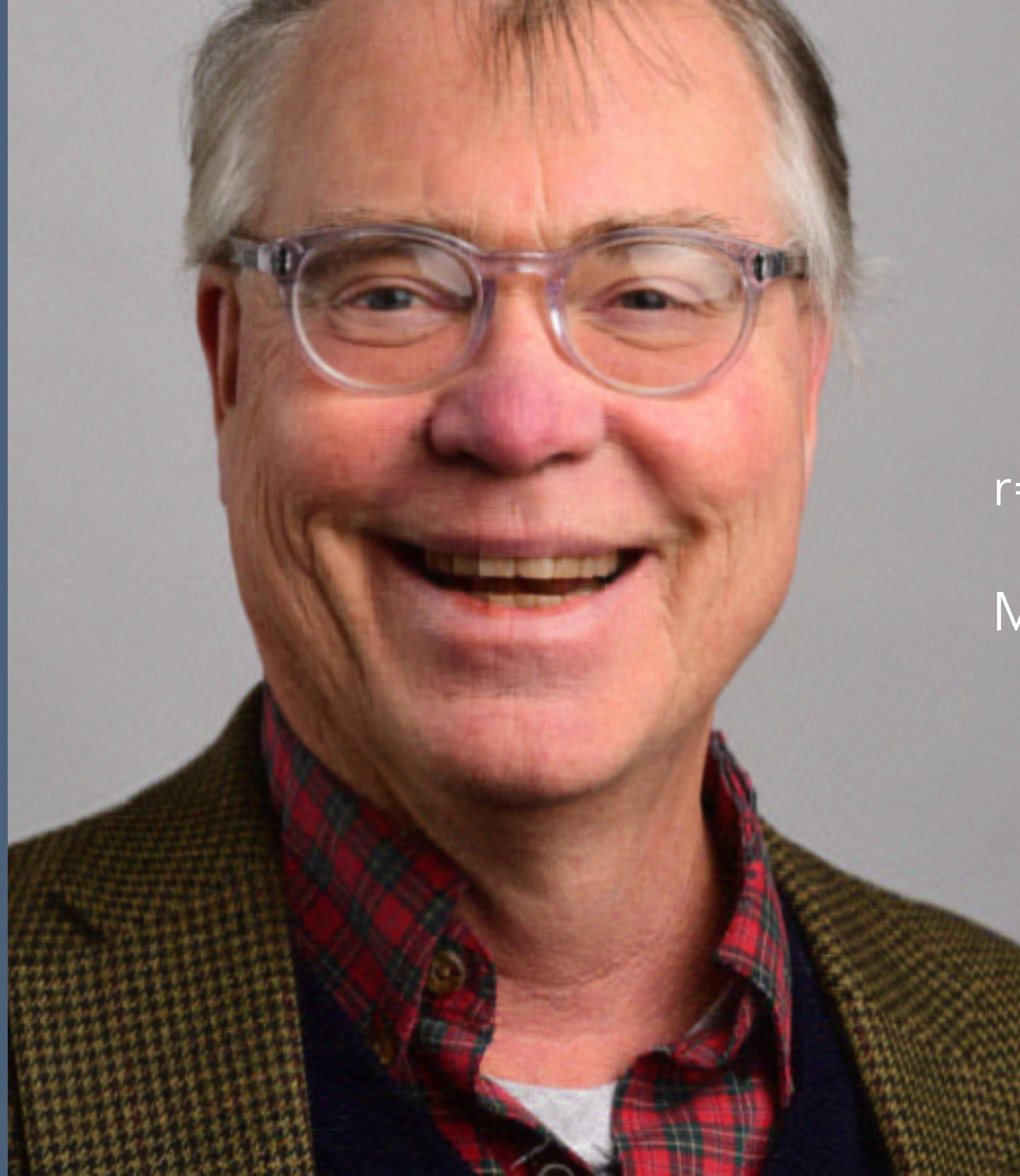
r=100
My





r=150
My

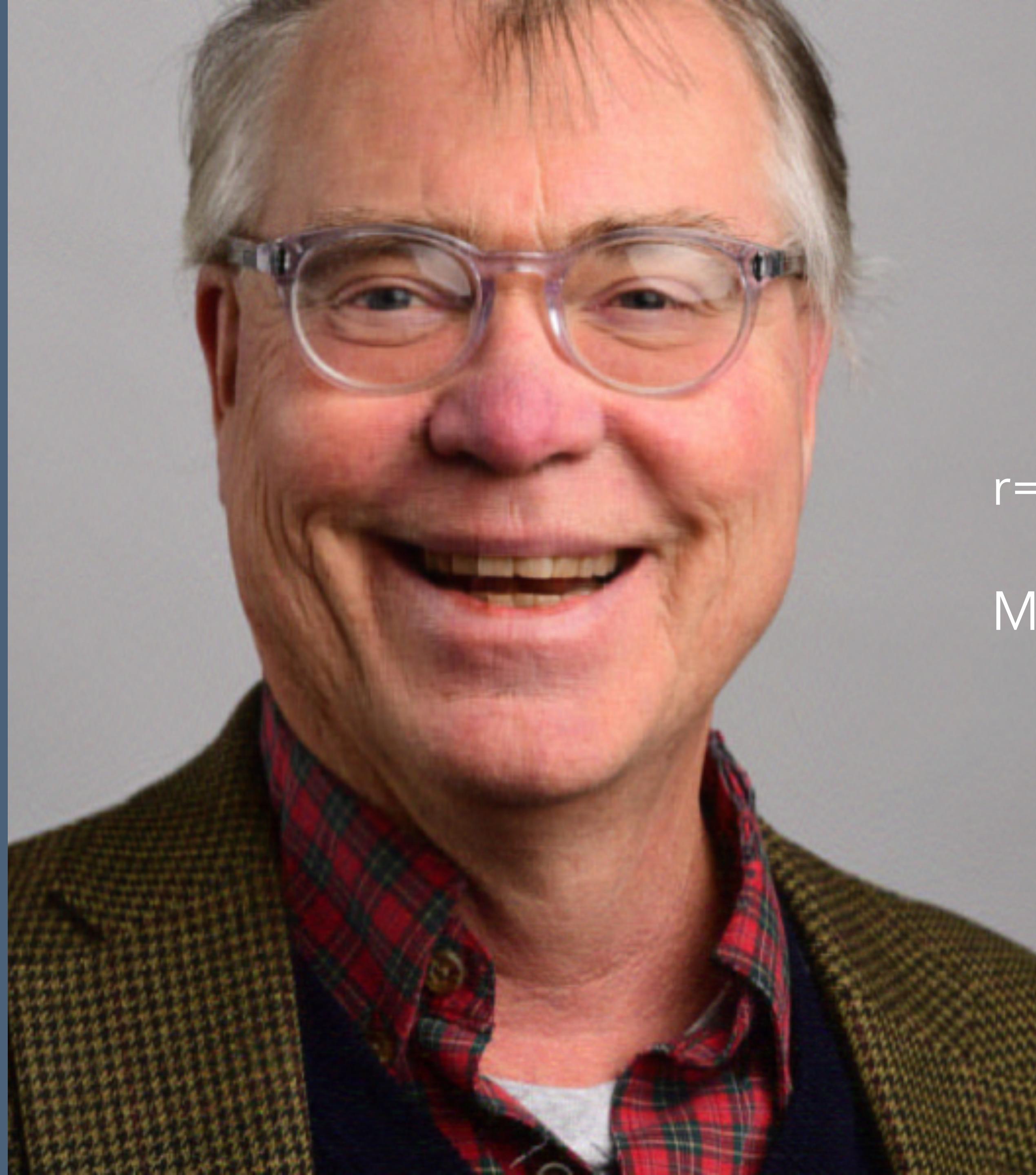


A close-up portrait of a middle-aged man with light brown hair and glasses, smiling broadly. He is wearing a red and green plaid shirt under a dark green textured jacket.

$r=175$

A close-up portrait of the same man, showing a slightly different expression or lighting. The text "My" is visible above the left side of his face.

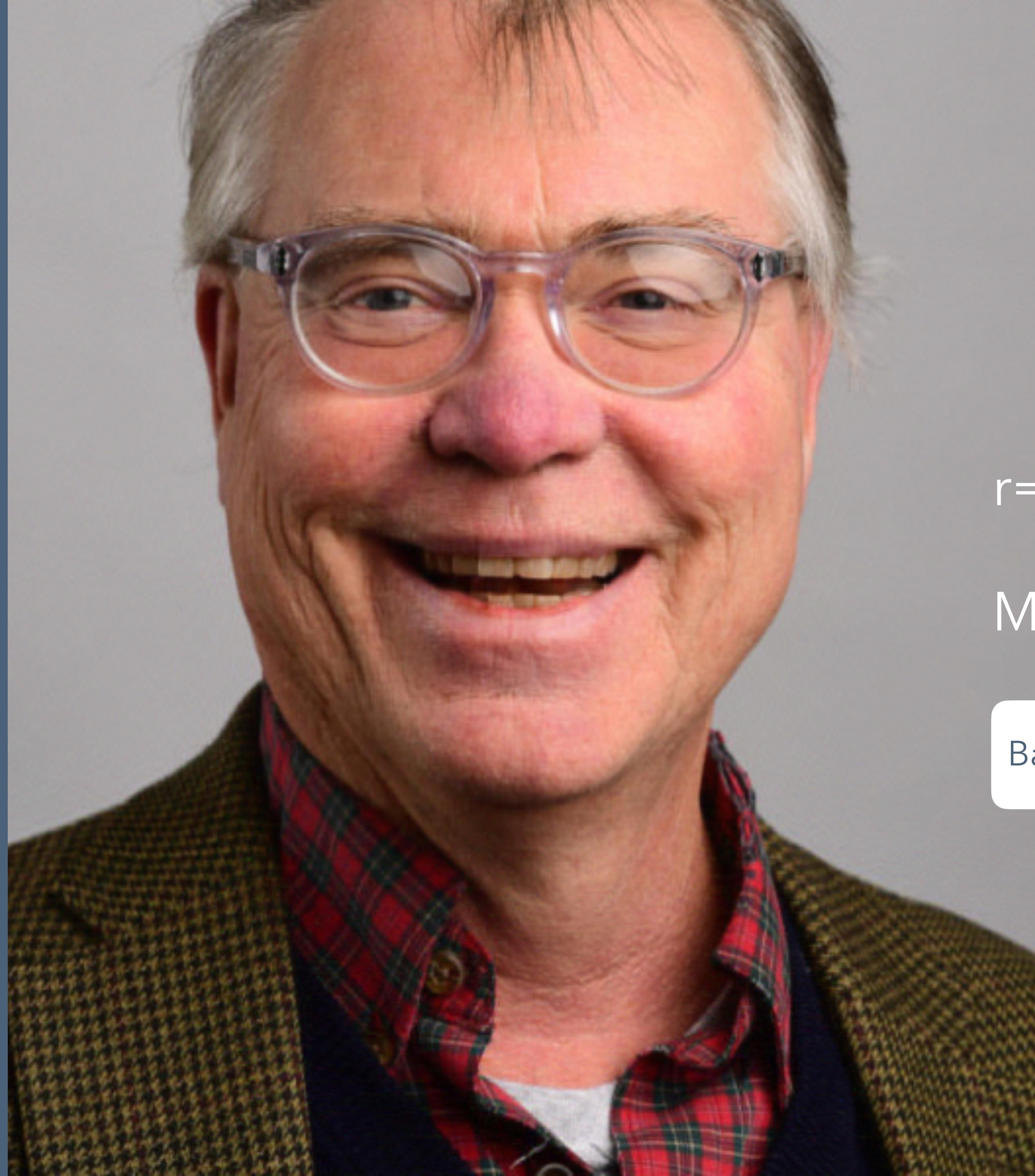
My



r=200

My





$r=300$

My

Back

