# Trigonometric Identities & Formulas

*Tutorial Services – Mission del Paso Campus*

đặc tính, tính đồng nhất, đồng nhất thức        tỉ số, tỉ lệ        thương

## Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \csc x = \frac{1}{\sin x}$$

$$\cos x = \frac{1}{\sec x} \qquad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x} \qquad \cot x = \frac{1}{\tan x}$$

## Ratio or Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sin x = \cos x \, \tan x \qquad \cos x = \sin x \, \cot x$$

## Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

Note: there are only three, basic Pythagorean identities, the other forms are the same three identities, just arranged in a different order.

## Pythagorean Identities in Radical Form

$$\sin x = \pm \sqrt{1 - \cos^2 x}$$

$$\tan x = \pm \sqrt{\sec^2 x - 1}$$

$$\cos x = \pm \sqrt{1 - \sin^2 x}$$

## Confunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

## Odd-Even Identities

Also called negative angle identities

$$\text{Sin}(-x) = -\sin x \qquad \text{Csc}(-x) = -\csc x$$

$$\text{Cos}(-x) = \cos x \qquad \text{Sec}(-x) = \sec x$$

$$\text{Tan}(-x) = -\tan x \qquad \text{Cot}(-x) = -\cot x$$

chuyển pha

$$\textbf{Phase Shift} = \frac{-c}{b}$$

$$\textbf{Period} = \frac{2\pi}{b}$$

## Sum and Difference Formulas/Identities

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$
$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$
$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

## How to Find Reference Angles

góc phần tư, cung phần tư

Step 1: Determine which quadrant the angle is in

Step 2: Use the appropriate formula

Quad I     =     is the angle itself
Quad II    =    $180 - \theta$      or       $\pi - \theta$
Quad III   =    $\theta - 180$      or       $\theta - \pi$
Quad IV    =    $360 - \theta$      or       $2\pi - \theta$

## Reciprocal Identities

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$$\cos x = \frac{1}{\sec x} \qquad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x} \qquad \cot x = \frac{1}{\tan x}$$

## Ratio or Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sin x = \cos x \tan x \qquad \cos x = \sin x \cot x$$

## Pythagorean Identities

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$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

## Odd-Even Identities
### Also called negative angle identities

$$\text{Sin}\,(-x) = -\sin x \qquad \text{Csc}\,(-x) = -\csc x$$

$$\text{Cos}\,(-x) = \cos x \qquad \text{Sec}\,(-x) = \sec x$$

$$\text{Tan}\,(-x) = -\tan x \qquad \text{Cot}\,(-x) = -\cot x$$

## Sum and Difference Formulas - Identities

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$
$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

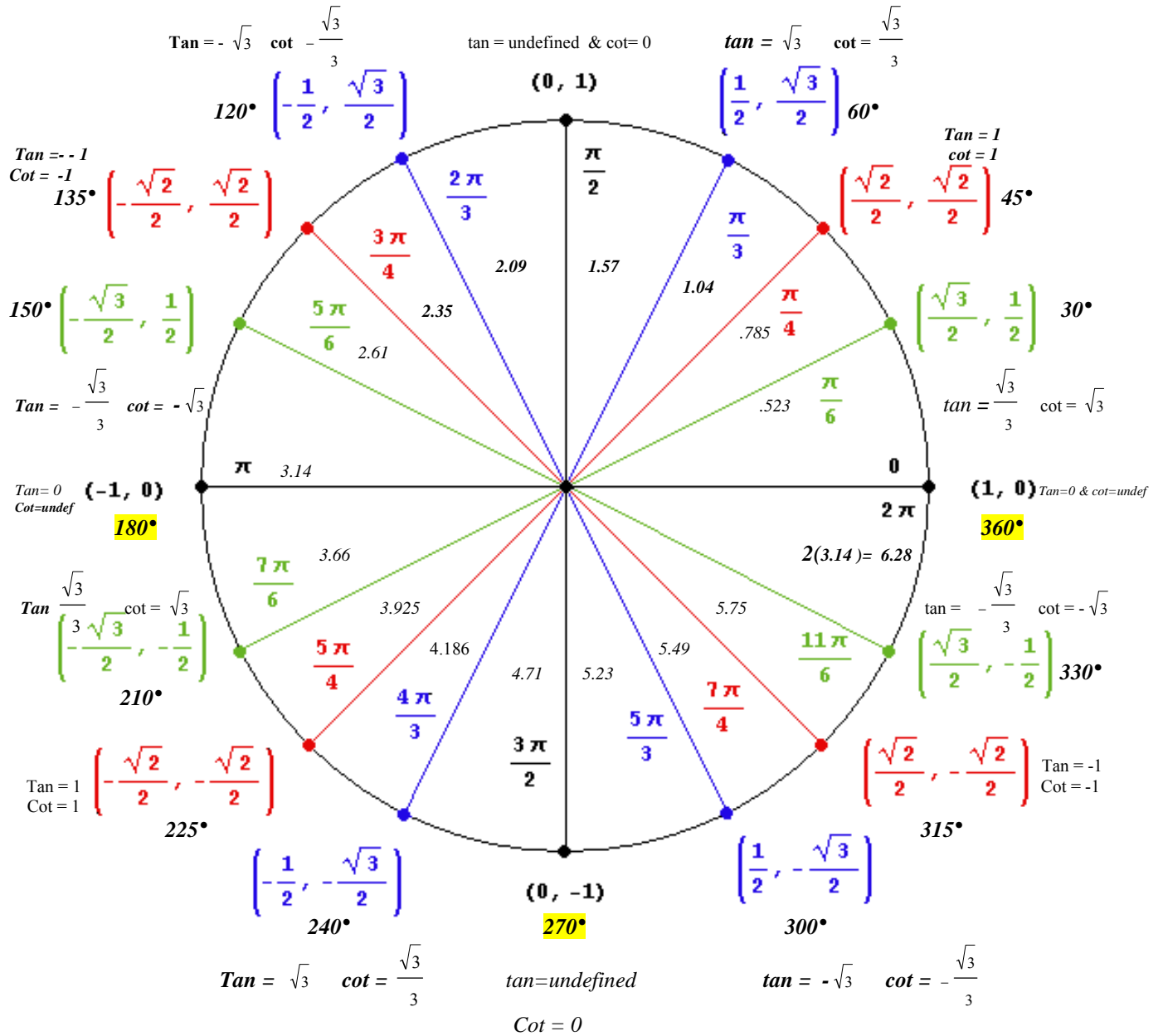$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$
$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

# The Unit Circle

## 90°

Tan = - $\sqrt{3}$   cot  $-\dfrac{\sqrt{3}}{3}$        tan = undefined  & cot= 0        tan = $\sqrt{3}$    cot = $\dfrac{\sqrt{3}}{3}$

120°  $\left[-\dfrac{1}{2},\ \dfrac{\sqrt{3}}{2}\right]$        (0, 1)        $\left[\dfrac{1}{2},\ \dfrac{\sqrt{3}}{2}\right]$ 60°

Tan =- - 1
Cot = -1
135°  $\left[-\dfrac{\sqrt{2}}{2},\ \dfrac{\sqrt{2}}{2}\right]$        Tan = 1  cot = 1   $\left[\dfrac{\sqrt{2}}{2},\ \dfrac{\sqrt{2}}{2}\right]$ 45°

$\dfrac{\pi}{2}$

$\dfrac{2\pi}{3}$    $\dfrac{\pi}{3}$

$\dfrac{3\pi}{4}$   2.09   1.57   1.04   $\dfrac{\pi}{4}$

150°  $\left[-\dfrac{\sqrt{3}}{2},\ \dfrac{1}{2}\right]$        $\dfrac{5\pi}{6}$   2.35   .785   $\left[\dfrac{\sqrt{3}}{2},\ \dfrac{1}{2}\right]$ 30°
2.61

Tan = $-\dfrac{\sqrt{3}}{3}$   cot = - $\sqrt{3}$        .523   $\dfrac{\pi}{6}$        tan = $\dfrac{\sqrt{3}}{3}$   cot = $\sqrt{3}$

$\pi$   3.14        0

Tan= 0  (-1, 0)                (1, 0) Tan=0 & cot=undef
Cot=undef
180°        2$\pi$        360°

$\dfrac{7\pi}{6}$   3.66   2(3.14 )= 6.28

Tan $\dfrac{\sqrt{3}}{3}$  cot = $\sqrt{3}$        3.925   5.75        tan = $-\dfrac{\sqrt{3}}{3}$   cot = - $\sqrt{3}$
$\left[-\dfrac{\sqrt{3}}{2},\ -\dfrac{1}{2}\right]$        $\dfrac{5\pi}{4}$   4.186   4.71   5.23   5.49   $\dfrac{11\pi}{6}$   $\left[\dfrac{\sqrt{3}}{2},\ -\dfrac{1}{2}\right]$ 330°
210°        $\dfrac{4\pi}{3}$        $\dfrac{5\pi}{3}$   $\dfrac{7\pi}{4}$

Tan = 1   $\left[-\dfrac{\sqrt{2}}{2},\ -\dfrac{\sqrt{2}}{2}\right]$        $\dfrac{3\pi}{2}$        $\left[\dfrac{\sqrt{2}}{2},\ -\dfrac{\sqrt{2}}{2}\right]$   Tan = -1
Cot = 1                Cot = -1
225°        315°

$\left[-\dfrac{1}{2},\ -\dfrac{\sqrt{3}}{2}\right]$        (0, -1)        $\left[\dfrac{1}{2},\ -\dfrac{\sqrt{3}}{2}\right]$
240°        270°        300°

Tan = $\sqrt{3}$    cot = $\dfrac{\sqrt{3}}{3}$        tan=undefined        tan = - $\sqrt{3}$   cot = $-\dfrac{\sqrt{3}}{3}$

Cot = 0

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*Definition of Trigonometric Functions concerning the Unit Circle*

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r} \qquad \csc\theta = \frac{hyp}{opp} = \frac{r}{y}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r} \qquad \sec\theta = \frac{hyp}{adj} = \frac{r}{x}$$

$$\tan\theta = \frac{opp}{adj} = \frac{y}{x} \qquad \cot\theta = \frac{adj}{opp} = \frac{x}{y}$$
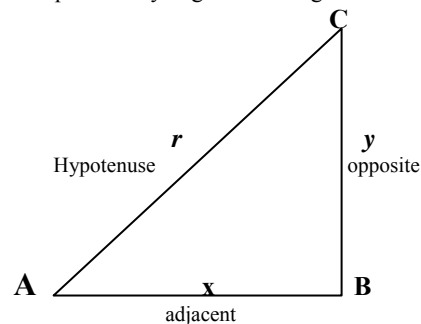
# Right Triangle Definitions of Trigonometric Functions

Note: sin & cos are complementary angles, so are tan & cot and sec & cos, and the sum of complementary angles is 90 degrees.

$$\sin \theta = \frac{opp}{hyp} = \frac{y}{r} \qquad\qquad \csc \theta = \frac{hyp}{opp} = \frac{r}{y}$$

$$\cos \theta = \frac{adj}{hyp} = \frac{x}{r} \qquad\qquad \sec \theta = \frac{hyp}{adj} = \frac{r}{x}$$

$$\tan \theta = \frac{opp}{adj} = \frac{y}{x} \qquad\qquad \cot \theta = \frac{adj}{opp} = \frac{x}{y}$$

**Adjacent =** is the side adjacent to the angle in consideration. So if we are considering Angle A, then the adjacent side is CB

## Trigonometric Values of Special Angles

| Degrees | 0° | 30° | 45° | 60° | 90° | 180° | 270° |
|---|---|---|---|---|---|---|---|
| Radians | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ |
| sinθ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | 0 | -1 |
| cosθ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | -1 | 0 |
| tanθ | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | undefined | 0 | undefined |

**To Convert _Degrees to Radians_, Multiply by** $\dfrac{\pi \, rad}{180 \, deg}$

**To Convert _Radians to Degrees_, Multiply by** $\dfrac{180 \, deg}{\pi \, rad}$

## Vocabulary
- Cotangent Angles - are two angles with the same terminal side
- Reference Angle - is an acute angle formed by terminal side of angle(α) with x-axis

### Double Angle Identities

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2\cos^2 A - 1$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

### Half Angle Identities

$$\sin \frac{A}{2} = \pm\sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm\sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A}$$

$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$$

### Power Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

### Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}\left[\cos(u - v) - \cos(u + v)\right]$$

$$\cos u \cos v = \frac{1}{2}\left[\cos(u - v) - \cos(u + v)\right]$$

$$\sin u \cos v = \frac{1}{2}\left[\sin(u + v) + \sin(u - v)\right]$$

$$\cos u \sin v = \frac{1}{2}\left[\sin(u + v) - \sin(u - v)\right]$$

### Sum-to-Product Formulas

$$\sin x + \sin y = 2\sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

### Law of Sines
Solving Oblique Triangles using sine: AAS,  ASA,  SSA,  SSS,  SAS

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

### Law of Cosines
*Cosine: SAS, SSS*

**Standard Form**

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = b^2 + a^2 - 2ab\cos C$$

**Alternative Form**

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

### Finding the Area of non-90degree Triangles

*Area of an Oblique Triangle*

$$area = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$$

*Heron's Formula*

Step 1: Find "s"  $\quad s = \frac{(a + b + c)}{2}$

Step 2: Use the formula  $\quad area = \sqrt{s(s - a)(s - b)(s - c)}$