Problem 1 (Probability Theory).

1. Calculate the covariance between X and Y in the following data.

X	Y
9	1
3	-9
1	-1
4	3
10	-9
5	-1
9	-3
3	-10
10	-4
3	-2

2. What would be the new value for covariance if:
   - Column X was multiplied by 0.5?
   - Column Y was replace by a value $2c - 0.5$, where c is a constant that is fixed for all Y.

Problem 2 (Probabilities). What are the canonical parameters associated with the following probability distributions?

- Uniform
- Binomial
- Gaussian
- Bernoulli
- Exponential

Problem 3 (Probabilities). Let X_1 and X_2 be two independent binary random variables with $\Pr(X_1 = 1) = 0.8$ and $\Pr(X_2 = 1) = 0.5$. Let Y be their sum, $Y = X_1 + X_2$.

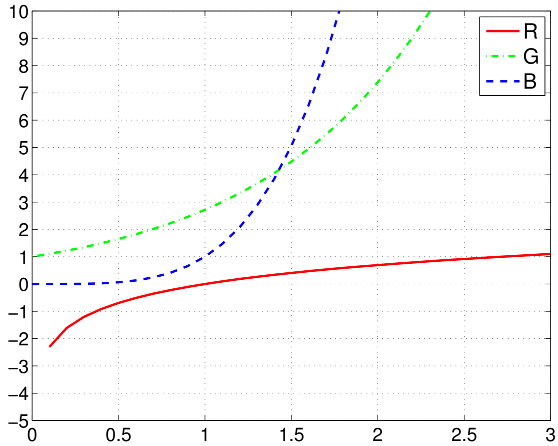
1. What is $\mathbb{E}(X_1)$?
2. What is $\mathbb{E}(Y)$?
3. What is $\mathbb{E}(Y \mid X_2 = 0)$?

Problem 4 (Analysis). What do the following functions converge to?

1. $f(x) = x/(1-x)$ for $x \to \infty$
2. $f(x) = \frac{x^2-4}{x-2}$ for $x \to 2$
3. $f(x) = (n+1)x/x^n$ for $n > 1$ and for $x \to \infty$
4. $f(k) = \sum_{i=1}^{k} (1/2)^k$ for $k \to \infty$

Problem 5 (Analysis).
Mark which of the following three functions correspond to which of the lines in the plot below.

1. $f(x) = \mathrm{e}^x$
2. $f(x) = x^4$
3. $f(x) = \log_{\mathrm{e}}(x)$



Problem 6 (Combinatronics).

1. How many different subsets does a set of size n have?
2. How many different subsets of size k does a set of size n have?

Problem 7 (Linear Algebra).
How many solutions are there for each of the given system equations? Why?

a) $\begin{aligned} 2x + \ \ y &= 1 \\ 5x - 2y &= 3 \end{aligned}$

b) $\begin{aligned} 3u - 5v + 2w &= 10 \\ 2u - 3v + 7w &= 23 \end{aligned}$

c) $\begin{aligned} 3p + \ 5q - \ r &= 1 \\ -p + \ 4q + 2r &= 9 \\ 6p + 10q - 2r &= 3 \end{aligned}$

Problem 8 (Linear Algebra). Consider the following vectors.

1. What are the dimensions of each vector?

   (a) $\begin{pmatrix} 4 \\ 2 \\ -5 \end{pmatrix}$ (c) $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ (e) $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

   (b) $\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ 1 \\ 4 \\ 2 \end{pmatrix}$ (f) $\begin{pmatrix} -1 \\ 1 \\ 1 \\ -2 \end{pmatrix}$

2. Perform the following vector operations using the vectors specified in Part. 1.

   (a) $1a + 1b$
   (b) $4 * 1e$
   (c) $1d \cdot 1f$
   (d) $||1d||$

3. Which pair of vectors in Part. 1 are orthogonal?

Problem 9 (Programming).
Familiarize yourself with the R programming language. Go through **2.3 Lab: Introduction to R** (ISLR p. 42–51).

1. Download the dataset `ozone.RData` from the course website. (*hint:* use the `load()` command). This file contains 3 objects: `ozone` (the data table), `trainset` (the row indices for the training set) and `testset` (the row indices of the test set). Inspect the structure of the objects using `ls(), str(), summary(), dim(), length(),range(), colnames()`. Identify the column names corresponding to each of the data types mentioned in the introduction. How many observations do you have (in total, in the training set, in the testset)?

2. What is the range of each input variable? What is the mean and standard deviation of each variable? *useful functions:* `range(), apply(), mean(), sd()`

3. Create scatterplots for every pair of features in the dataset. Calculate the Pearson correlation coefficients for each pair of datatypes. In general, what is the range of the Pearson correlation coefficient? What does a correlation coefficient of 0 tell you about the relationship between two variables? What trends do you observe in the data according to the correlation coefficient? Can you see them directly from the plot (visually)? *useful functions:* `plot(), pairs(), cor()`

4. Implement a function `rss` that computes the Residual Sum of Squares (RSS) between a *vector* of predicted values and a vector of true values. (*hint:* $RSS = sum((predicted - true)^2)$)