ex1:

a) $f(x) = \dfrac{2}{(x+2)\sqrt{x+1}}$

domain: $\begin{cases} x+2 \neq 0 \\ x+1 > 0 \end{cases} \Rightarrow \begin{cases} x \neq -2 \\ x > -1 \end{cases}$

$\Rightarrow D = (-1; +\infty)$.

b) $f(x) = \dfrac{2x-1}{\sqrt{x|x-4|}}$

x và x-4 cùng dấu

domain $\begin{cases} x > 0 \\ x-4 > 0 \end{cases}$ $x \in (-\infty; +\infty)$

x và x-4 trái dấu: $\begin{cases} x > 0 \\ x-4 > 0 \end{cases} \Rightarrow x > 0$ $x \in [4; +\infty)$.

$\Rightarrow x \in (-\infty; +\infty) \cup [4; +\infty)$

c) $f(x) = \ln(x+1) - \dfrac{x}{\sqrt{x-1}}$

$\begin{cases} x+1 > 0 \\ x-1 > 0 \end{cases} \Leftrightarrow \begin{cases} x > -1 \\ x > 1 \end{cases}$

$x \in (1; +\infty)$

d) $f(x) = \dfrac{\sqrt{x^2-4x+3}}{\log(x-2)}$.

$\begin{cases} x^2-4x+3 > 0 \\ x-2 > 0 \end{cases} \Leftrightarrow \begin{cases} x < 1 \lor x > 3 \\ x > 2 \end{cases}$

1 2 3

$\Rightarrow x \in (3; +\infty)$

b) $f(x) = \dfrac{2x-1}{\sqrt{x|x-4|}}$

ĐK: $x|x-4| > 0$

TH1: $\begin{cases} x < 0 \\ -x+4 < 0 \end{cases} \Rightarrow \begin{cases} x < 0 \\ x > 4 \end{cases} \Rightarrow x \in (0; 4)$ $\Rightarrow x \in (0; 4) \cup (4; +\infty)$

TH2: $\begin{cases} x > 0 \\ x-4 > 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x > 4 \end{cases}$ $x \in (4; +\infty)$

ex2)

a) $f(x) = \dfrac{3x+5}{2x-1}$ $\quad f'(x) = \dfrac{-13}{(2x-1)^2}$

TVD : $D = \mathbb{R} \setminus \left\{ \dfrac{1}{2} \right\}$.

$\lim\limits_{x \to \frac{1}{2}^+} f(x) = \lim\limits_{x \to \frac{1}{2}^+} \dfrac{3x+5}{2x-1} = -\infty$.

$\lim\limits_{x \to \frac{1}{2}^-} f(x) = \lim\limits_{x \to \frac{1}{2}^-} \dfrac{3x+5}{2x-1} = +\infty$.

∘) TCA $x = \dfrac{1}{2}$

$\lim\limits_{x \to +\infty} \dfrac{3x+5}{2x-1} = \dfrac{3}{2}$.

$\lim\limits_{x \to -\infty} \dfrac{3x+5}{2x-1} = \dfrac{3}{2}$

∘) TCN $y = \dfrac{3}{2}$.

$f'(x) = \dfrac{-13}{(2x+1)^2} < 0.$

x	$-\infty$		1		$+\infty$
x'		$-$	$\Vert$	$-$	
y	$\frac{3}{2}$	$\searrow$	$-\infty \ \Vert \ +\infty$	$\searrow$	$3/2$

$\Rightarrow R = \left(-\infty; \dfrac{3}{2}\right) \cup \left(\dfrac{3}{2}; +\infty\right)$

b) $f(x) = x^2 - 2x \Rightarrow a = 1 > 0.$

$f'(x) \quad x = \dfrac{-b}{2a} = 1 \ ; \ y = -1$

x	$-\infty$		1		$+\infty$
y	$+\infty$	$\searrow$	-1	$\nearrow$	$+\infty$

$\Rightarrow R = [-1; +\infty)$

c) $f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

$\lim\limits_{x \to +\infty} \dfrac{x^2 - x + 1}{x^2 + x + 1} = 1$

$\lim\limits_{x \to -\infty} \dfrac{x^2 - x + 1}{x^2 + x + 1} = 1$

$\Rightarrow \cancel{x} \; y = 1.$

$x^2 + x + 1 \neq 0 \; \forall x$

$\Delta = b^2 - 4ac = -3 \Rightarrow$ PTVN

$\Rightarrow$ 0 có TCĐ.

$y' = f'(x) = \dfrac{2x^2 - 2}{(x^2 + x + 1)^2} = 0 \Rightarrow \begin{cases} x = 1 \\ x = -1 \end{cases}$

x		-1			1	
y'	$+$	0	$-$	$-$	0	$+$
y	$\nearrow$	3	$\searrow$	$\searrow$	$\frac{1}{3}$	$\nearrow$

2) $R = (-\infty; 1) \cup (1; +\infty)$.

ex3:

a $f(x) = \dfrac{|x-1| + |x+1|}{x^3}$.

~~TH1~~: $\begin{cases} x-1 \geq 0 \\ x+1 \end{cases}$ $x \geq 0$

~~TH1~~ : $x \geq 0$.

$\Rightarrow f(x) = \dfrac{x-1+x+1}{x^3} = \dfrac{2x}{x^3} = \dfrac{2}{x^2}$

$f(-x) = \cancel{\dfrac{2}{(x)}}\ \dfrac{2}{(-x)^2} = \dfrac{2}{x^2}$ → even.

c) $\cancel{b)}$ $f(x) = \ln(x + \sqrt{1+x^2})$

$\cancel{\text{thay}}$ $f(-x) = \ln(-x + \sqrt{1+(-x)^2}) = \ln(-x + \sqrt{1+x^2})$

So $\begin{cases} f(-x) \neq f(x) \\ f(-x) \neq -f(x) \end{cases}$

$\Rightarrow$ neither.

b) $f(x) = \frac{1}{2}(a^x + a^{-x})$

$f(-x) = \frac{1}{2}(a^{-x} + a^x) = f(x) \rightarrow$ even.

d) $f(x) = \lg \frac{1+x}{1-x}$ đk $\cancel{(x > 1)}$ $\cancel{x \neq 1}$

$x < 0 \rightarrow$ 4°d xd

• $x > 0$ $\cancel{= \lg 1+x}$

Thì $f(-x) = \log \frac{1-x}{1+x} \neq \begin{cases} f(x) \\ -f(x) \end{cases} \Rightarrow$ neither.

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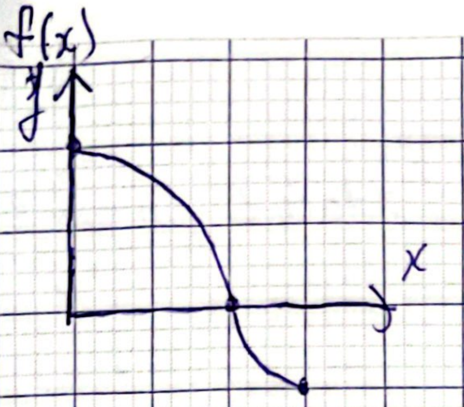
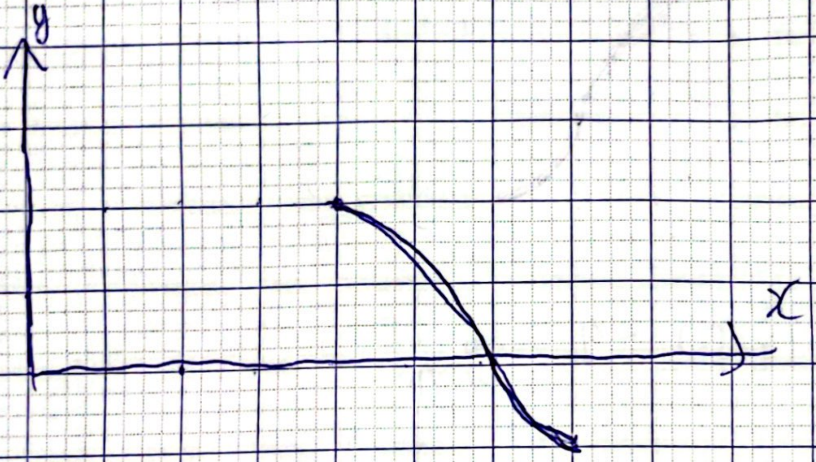
ex4:



x ~~tăng~~ 4.

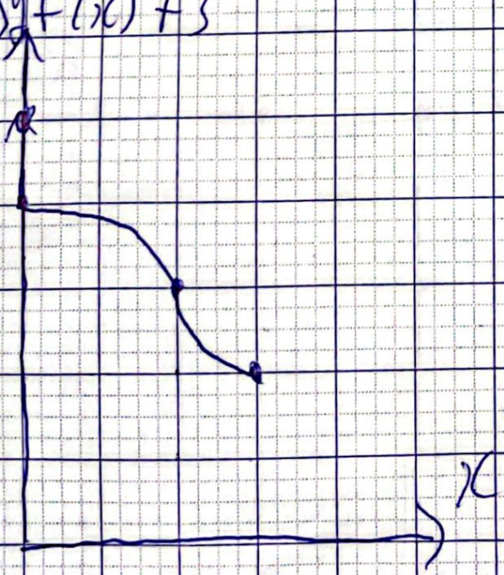
a) f(x-4)

x tăng 4



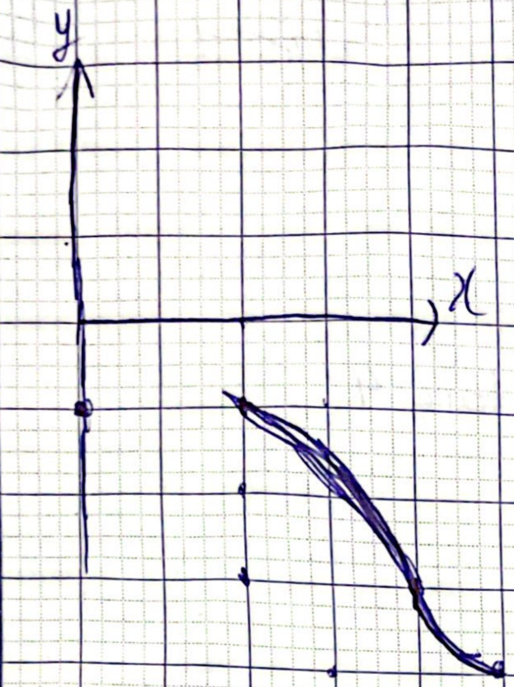
b) f(x) + 3

y tăng 3

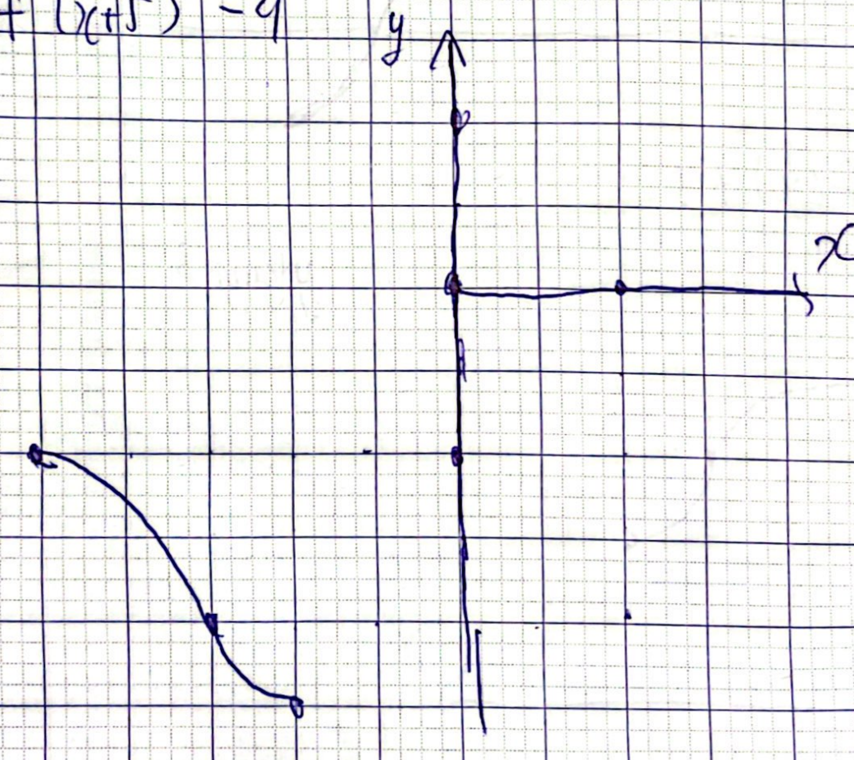


c) $f(x-2)-3$

x tăng 2 y giảm 3



d) $f(x+5)-4$

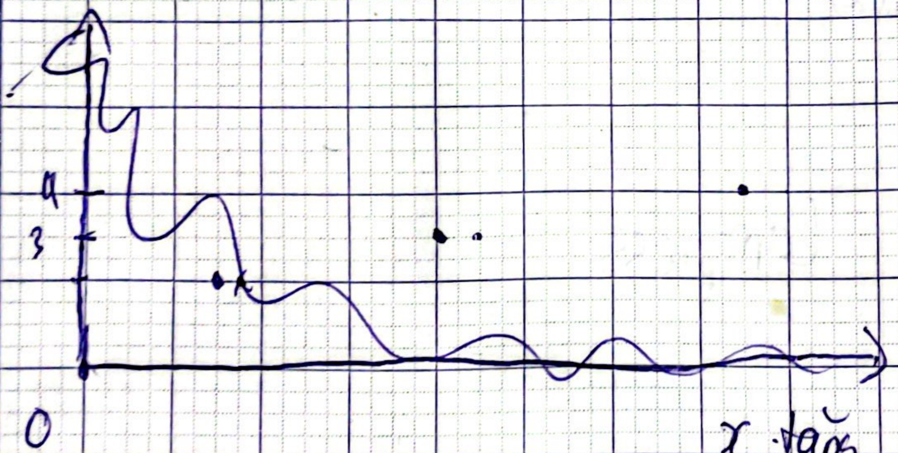


x giảm 5 y giảm 4

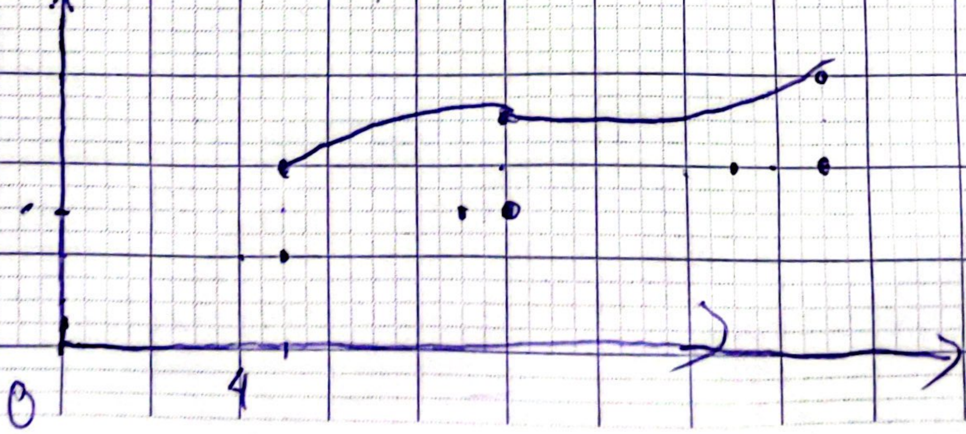
Ex 5)

$f(x) = \sqrt{x}$

$\cancel{f(x)} = f(x-1)+2 = \sqrt{x}$



x tăng 1 y tăng 2.



ex(6) $f(x) = \sqrt{x}$ $g(x) = \sqrt{2-x}$

a) $f o g(x) \Rightarrow f(g(x)) = \sqrt[4]{2-x}$

b) $g o f(x) \Rightarrow g(f(x)) = \sqrt{2-\sqrt{x}}$

c) $g \circ g(x) \Rightarrow g(g(x)) = \sqrt{2-\sqrt{2-x}}$

c) $f o f(x) \Rightarrow f(f(x)) = \sqrt[4]{x}$

P) or P)

7)

a) $$f\left(x+\frac{1}{x}\right) = \frac{(x+1/x)^2 + x + 1/x + 1}{x+1/x}$$

$$= \frac{x^2 + 2 + 1/x^2 + x + 1/x + 1}{x + 1/x}$$

$$= \frac{\frac{x^4+1}{x^2} + \frac{x^2+1}{x} + 3}{\frac{x^2+1}{x}}$$

$$= \frac{\frac{x^4+1+x^3+x+3x^2}{x^2}}{\frac{x^2+1}{x}}$$

$$= \frac{x^4+x^3+3x^2+x+1}{x^2} \cdot \frac{x}{x^2+1}$$

$$= \frac{x^4+x^3+3x^2+x+1}{x(x^2+1)}$$

b) $$f(2x-1) = \frac{(2x-1)^2 + 2x-1+1}{2x-1}$$

$$= \frac{4x^2-2x+1}{2x-1}$$

$$= \frac{4x^2}{2x-1} - \frac{2x-1}{2x-1} = \frac{4x^2}{2x-1} - 1$$

c) $f \circ f\ (x) \Rightarrow f(f(x)) = \forall x$

~~ex 8)~~ ex 8)

a) $f(g(1))$

$g(1) = 6 \Rightarrow f(6) = 5.$

b) $g(f(1))$

$f(1) = 3 \Rightarrow g(3) = 2$

c) $f(f(1))$

$f(1) = 3 \Rightarrow f(3) = 4.$

d) $g(g(1))$

$g(1) = 6 \Rightarrow g(6) = 3.$

e) $(g \circ f)(3) \Rightarrow g(f(3))$

$f(3) = 4 \Rightarrow g(4) = 1.$

f) $(g \circ f)(6) \Rightarrow g(f(6))$

$f(6) = 5 \Rightarrow g(5) = 2.$

The strive the luckier

ex 9:

a. $\lim_{x\to3} \frac{x^2+x-12}{x-3} \overset{\lim_{x\to3}}{=} \frac{(x-3)(x+4)}{x-3} \overset{\lim_{x\to3}}{=} x+4 = 7$

b) $\lim_{x\to1} \frac{x^6-1}{x^{10}-1} \overset{\lim_{x\to1}}{=} \frac{(x^3)^2-1}{(x^5)^2-1} \overset{\lim_{x\to1}}{=} \frac{(x^3-1)(x^3+1)}{(x^5+1)(x^5-1)}$

x^3-1 $\cancel{x}=x$ $\begin{array}{c|cccc} & 1 & 0 & 0 & -1 \\ 1 \to & 1 & 1 & 1 & 0 \end{array}$ $\to x^3-1 = (x-1)(x^2+x+1)$

$x^5-1 =$ $\begin{array}{c|cccccc} & 1 & 0 & 0 & 0 & 0 & -1 \\ 1\to & 1 & 1 & 1 & 1 & 1 & 0 \end{array}$ $\to x^5-1 = (x-1)(x^4+x^3+x^2+x+1)$

$\to \lim_{x\to1} \frac{x^6-1}{x^{10}-1} = \lim_{x\to1} \frac{(x-1)(x^2+x+1)(x^3+1)}{(x-1)(x^4+x^3+x^2+x+1)(x^5+1)}$

$= \lim_{x\to1} \frac{(x^2+x+1)(x^3+1)}{(x^5+1)(x^4+x^3+x^2+x+1)} = \frac{3}{5}.$

c) $\lim_{x\to0} \frac{\tan 3x + 2x}{\tan 5x - \sin x}$

Đạo hàm tử và mẫu: $(\tan 3x + 2x)' = \frac{3}{\cos^2 x} + 2$

$(\tan 5(x) - \sin x)' = \frac{5}{\cos^2 x} - \cos x.$

$\to \lim_{x\to0} \frac{\frac{3}{\cos^2 x} + 2}{\frac{5}{\cos^2 x} - \cos x} = \frac{3 + 2\cos^2 x}{5 - \cos^3 x} = \frac{5}{4}.$

d) $\lim_{h \to 0} \dfrac{(3+h)^2 - 9}{h}$ x

gọi $t = 3+h \Rightarrow h = t-3$

$\Rightarrow \lim_{t \to 3} \dfrac{t^2 - 9}{t-3} = \lim_{t \to 3} t + 3 = 6.$

e) $\lim_{t \to 0} \dfrac{\sqrt{t^2+9} - 3}{t^2}$

gọi $x = \sqrt{t^2+9} \Rightarrow x^2 = t^2 + 9 \Rightarrow t^2 = x^2 - 9.$

$\Rightarrow \lim_{x \to 3} \dfrac{x-3}{x^2-9} = \lim_{x \to 3} \dfrac{1}{x+3} = \dfrac{1}{6}.$

f) $\lim_{x \to \infty} \dfrac{x^2 + x - 12}{x^3 - 3}$

Chia tử và mẫu cho x^3 $\Rightarrow \lim_{x \to \infty} \dfrac{\frac{1}{x} + \frac{1}{x^2} - \frac{12}{x^3}}{1 - \frac{3}{x^3}}$.

Ta có: $\lim_{x \to \infty} \dfrac{1}{x^\alpha} = 0 \Rightarrow \lim_{x \to \infty} \dfrac{x^2 + x - 12}{x^3 - 3} = 0.$

e) $\lim_{x \to 0} \dfrac{\tan 3x + 2x}{\tan 5x - \sin x}$

Chia tử mẫu cho x $\Rightarrow \lim_{x \to 0} \dfrac{\frac{\tan 3x}{x} + 2}{\frac{\tan 5x}{x} - \frac{\sin x}{x}} = \dfrac{3+2}{5-1} = \dfrac{5}{4}.$

$\lim_{x \to 0} \dfrac{\sin(kx)}{x} = k.$ $\lim_{x \to 0} \dfrac{\tan(kx)}{x} = k$

g) $\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right)$.

vì $x \to 0^-$ $\Rightarrow |x| = -x$

$\Rightarrow \lim_{x \to 0^-} \left(\frac{1}{x} + \frac{1}{x} \right) = \lim_{x \to 0^-} \frac{2}{x} = -\infty$.

h) $\lim_{x \to 1} \frac{x^2 - 1}{|x-1|}$

vì $x \to 1$ thì $x - 1$ ~~tiến đ~~ $\to 0 \Rightarrow |x-1| = x - 1$.

$\Rightarrow \lim_{x \to 1^+} \frac{(x-1)(x+1)}{x-1} = \lim_{x \to 1} x + 1 = 2$.

$\lim_{x \to 1^-} -(x+1) = -2$ $\Rightarrow$ k⁰ tồn tại

ex12)

ex 10:

(a) $D = [-3;3)$ $-3 \leq x \leq 3$

$R = [-2;2)$ $-2 \leq y \leq 2$

(b) $D = [-3;3]$ ~~$\frac{1}{2}$~~

Domain x.

Range y.

$R = [-2;3)$

11)

a) $\lim\limits_{x \to 0^+} f(x) = 0$

$\lim\limits_{x \to 0} f(x) = 2.$

$\lim\limits_{x \to 0^-} f(x) = 2$

b) tại $x = 0$ và $x = 2$ và $x = 4$ đ tròn rỗng

$\Rightarrow$ hS ko ltục

$x \neq \{2\}$ $\quad x \to 1$

a) $f(x) = \dfrac{2x^2 + x - 1}{x - 2}$.

$D = R \setminus \{2\}$.

$\lim\limits_{x \to 2^+} \dfrac{2x^2 + x - 1}{x - 2} = +\infty$. $\quad \lim\limits_{x \to 2^-} \dfrac{2x^2 + x - 1}{x - 2} = -\infty$.

$\Rightarrow$ f(x) lt ở tất cả các giá trị trừ 2. $\Rightarrow$ $D = R \setminus \{2\}$.

b) $f(x) = \dfrac{x-9}{\sqrt{4x^2+4x+1}} = \dfrac{x-9}{\sqrt{(2x+1)^2}} = \dfrac{x-9}{|2x+1|}$

TXĐ : $4x^2+4x+1 \neq 0 \Leftrightarrow x \neq \dfrac{-1}{2} \Rightarrow D = \mathbb{R} \setminus \left\{\dfrac{-1}{2}\right\}$.

$\lim\limits_{x \to \frac{-1}{2}^+} \dfrac{x-9}{|2x+1|} = +\infty \quad \lim\limits_{x \to \frac{-1}{2}^-} \dfrac{x-9}{|2x+1|} = -\infty$.

$\Rightarrow$ hs lt trên D $= \mathbb{R} \setminus \left\{\dfrac{-1}{2}\right\}$.

c) $f(x) = \ln(2x+5)$

TXĐ: $2x+5 > 0 \Rightarrow x > \dfrac{-5}{2}$.

$\Rightarrow$ hs lt khi $D = \left(\dfrac{-5}{2}; +\infty\right)$.

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ex13)

a) $f(x) = \begin{cases} x^2 - m^2 & ; x < 4 \\ mx + 20 & ; x \geq 4 \end{cases}$

$\lim\limits_{x \to 4^-} x^2 - m = \cancel{\lim\limits_{x \to 4^-}} 16 - m$

$\lim\limits_{x \to 4^+} mx + 20 = \cancel{\lim\limits_{x \to 4^-}} 4m + 20$

hslt kh: $\lim\limits_{x \to 4^-} f(x) = \lim\limits_{x \to 4^+} f(x) \Rightarrow 16 - m = 4m + 20. \Rightarrow m = \dfrac{-4}{5}$

b) $f(x) = \begin{cases} mx^2 + 2x. & ; x < 2. \\ x^3 - mx & , x \geq 2. \end{cases}$

$\lim\limits_{x \to 2^-} mx^2 + 2x = \cancel{\lim\limits_{x \to 2^-}} 4m + 4$

$\lim\limits_{x \to 2^+} x^3 - mx = 8 - 2m$

Để hslt $\Rightarrow \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) \Rightarrow 4m + 4 = 8 - 2m \Rightarrow m = \dfrac{2}{3}$

c) $f(x) = \begin{cases} \dfrac{e^{2x}-1}{x}, & x \neq 0. \\ m \; ; & x = 0. \end{cases}$

By quy tắc hopital => $f(x) = \dfrac{e^{2x}-1}{x} = \dfrac{2e^{2x}}{1}$ =>

=> đạo hàm tử và mẫu

=> $\lim\limits_{x\to 0} \dfrac{e^{2x}-1}{x} = \dfrac{2e^{2x}}{1} = 2$

$\lim\limits_{x\to 0} m$ => ~~m~~ Để hàm số lt $\lim\limits_{x\to 0^-} = \lim\limits_{x\to 0^+}$ => $m = 2$.

d) $f(x) = \begin{cases} \dfrac{x^2-1}{x-1} \; ; & x \neq 1. \\ m+1, & x = 1 \end{cases}$

$\lim\limits_{x\to 1} \dfrac{x^2-1}{x-1} = \lim\limits_{x\to 1} x+1 = 2$

Để hs lt <=> $m+1 = 2$ => $m = 1$.

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ex(4)

$$f(x) = \begin{cases} x+2 & , x<0 \\ 2x^2 & ; 1 \geq x \geq 0 \\ 2-x & ; x>1. \end{cases}$$

$$\left.\begin{aligned} &\lim_{x\to 0^-} x+2 = 2. \\ &\lim_{x\to 0^+} 2x^2 = 2 \end{aligned}\right\} \Rightarrow \text{hs lt tại } x=0$$

$$\left.\begin{aligned} &\lim_{x\to 1^+} 2-x = 1. \\ &\lim_{x\to 1^-} 2x^2 = 2 \end{aligned}\right\} \Rightarrow \text{hs k}^0 \text{ lt tại } x=1.$$