Exercise 1 (3p.)

a. Show theoretically that

$$\int_0^t W(s)\mathrm{d}s = \int_0^t (t-s)\mathrm{d}W(s).$$

b. Take $t = 5$ and validate by a numerical experiment the equality above.

Exercise 2 (4p.)

a. Suppose $S(t)$ follows a GBM given by:

$$\frac{\mathrm{d}S(t)}{S(t)} = \mu\mathrm{d}t + \sigma\mathrm{d}W(t),$$

with constant parameters μ, σ and Wiener process $W(t)$. Find the dynamics for process $Y_1(t) = \mu S^2(t)$.

b. Apply Itô's formula to $Y_2(t) = \mathrm{e}^{W(t)}$, where $W(t)$ is a standard Brownian motion. Is this a martingale?

Exercise 3 (3p.)

In the Lab we have discussed the concept of "standarizing" samples from a normal distribution. For example,

```
Z = np.random.normal(0.0,1.0,[NoOfPaths,NoOfSteps])
for i in range(0,NoOfSteps):
       # Making sure that samples from a normal have mean 0 and variance 1
       if NoOfPaths > 1:
           Z[:,i] = (Z[:,i] - np.mean(Z[:,i])) / np.std(Z[:,i])
```

Now, consider the problem of call option pricing under the Black-Scholes model. Discuss the impact of sample standardization on option pricing. Does it actually help to improve convergence?

NB. Please feel free to choose model parameters and particular settings for derivative pricing.

Exercise 4 (3p.)

For a Wiener process $W(t)$ consider

$$X(t) = W(t) - \frac{t}{T}W(T-t), \text{ for } 0 \le t \le T.$$

For $T = 10$, find analytically $\mathrm{Var}(X(t))$ and perform a numerical simulation to confirm your result. Is the accuracy sensitive to t?