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Stock index forecasting based on a hybrid model

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ABSTRACT

1 Forecasting the stock market price index is a challenging task. The exponential smoothing model (ESM), autoregressive integrated moving average model (ARIMA), and the back propagation neural network (BPNN) can be used to make forecasts based on time series. In this paper, a hybrid approach combining 2 ESM, ARIMA, and BPNN is proposed to be the most advantageous of all three models. The weight of the proposed hybrid model (PHM) is determined by genetic algorithm (GA). The closing of the Shenzhen Integrated Index (SZII) and opening of the Dow Jones Industrial Average Index (DJIAI) are used as 4 illustrative examples to evaluate the performances of the PHM. Numerical results show that the proposed model outperforms all traditional models, including ESM, ARIMA, BPNN, the equal weight hybrid model (EWH), and the random walk model (RWM). 6

1. Introduction

1 The stock market has become a popular investment channel in recent years due to the low return rates of other investment instruments. The stock price index prediction is in the interest of both private and institution investors. However, making accurate forecasts of this type is a challenging task due to the inherently 2 noisy and non-stationary nature of stock prices [1,2]. Stock prices are affected by many macro-economical factors such as political events, firms' policies, general economic conditions, commodity price indexes, interest and exchange rates, and investors' expectations and psychological factors.

III Many studies on stock price predictions have been conducted over the past two decades. The forecasting techniques used in the literature can be classified into two categories: statistical models and artificial intelligence models (AI). The statistical models include exponential smoothing, autoregressive integrated moving average (ARIMA), and generalized autoregressive conditional heteroskedasticity (GARCH) volatility [3]. These models are based on the assumption that a linear correlation structure exists among time series values. Therefore, non-linear patterns cannot be captured by these models. To overcome this limitation, AI models, mainly artificial neural networks (ANNs) and genetic algorithm (GA), have been utilized to improve stock price forecasts with purely non-linear time series [4–11]. ANNs are the data-driven and non-parametric models which can capture subtle, unknown functional relationships among the empirical data.

II Various network architectures and learning algorithms have been developed in the literature. The back propagation neural network (BPNN) is feed-forward network and is probably the most commonly used class of neural network in financial time series forecasting and business [12–14]. In this paper, we adopt the BPNN. The GA is a tool for determining the optimal parameters in the forecasting model. However, stock price time series are often neither purely linear nor purely non-linear; thus, using a statistical model alone or using an AI model alone is not adequate in making forecasts with a stock price time series.

III In the literature, it has been verified that no single method or model works well in all situations [15,16]. In general, it is more effective to combine individual models for making forecasts [17–19], thus, there was a rapid development of hybrid models. For example, Sharaf proposed a short-term load forecasting model based on neural networks and fuzzy logic and Huang et al. presented a Grey–Markov forecasting model to predict the electric-power demand in China [20–22]. Pao applied hybrid non-linear models to energy consumption forecasting in Taiwan [23]. Zhang and Dong developed an adaptive neural-wavelet model for short-term load forecasts in the competitive electricity market [24]. El-Keib, Maia and Goncalves proposed the hybrid model for electric load forecasting [25,26].

1 In this paper, we propose a hybrid model to capture the linear and non-linear characteristics in a stock price time series. Our approach, called the proposed hybrid model (PHM), is to combine the exponential smoothing model (ESM), ARIMA, and BPNN. The ESM and ARIMA mainly model the linear relationships while the

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BPNN captures the non-linear patterns well, as the weights of these models are determined by the GA. To evaluate the performance of the proposed approach, we use the closing index of the Shenzhen Integrated Index (SZII-China) and opening index of the Dow Jones Industrial Average Index (DJIAI-USA) as the illustrative examples. These examples show that the PHM outperforms all component models used alone and the two special cases of the equal weights hybrid model (EWH) and the random walk model (RWM). In addition, our model is shown to be more robust with regard to the possible structure changes in the data.

The paper is organized as follows. Section 2 introduces the component models of ESM, ARIMA, and BPNN briefly. The hybrid methodology is described in Section 3. Section 4 presents the experimental results based on the real datasets. Finally, Section 5 concludes the paper with a summary and some future research directions.

2. Individual forecasting models used in the hybrid model

2.1. Exponential smoothing model (ESM)

The exponential smoothing method is relatively simple but robust approach to time-series based forecasting [26] and was first developed by Trigg and Leach [27,30]. This classical method applies to both homoscedastic and heteroscedastic time series cases. A homoscedastic case is equivalent to an ARIMA process [28]. However, a heteroscedastic case is not equivalent to any ARIMA process. Thus, exponential smoothing can be used in a wider class of models than the ARIMA class [29]. With an ESM, the forecast for the future period is made based on the following recursion:

$$F_{t+1} = \alpha x_t + (1-\alpha)F_t = F_t + \alpha(x_t - F_t) = F_t + \alpha e_t \tag{1}$$

where x_t and F_t represent the actual and the forecasted values, respectively, at time t; α is the exponential smoothing parameter between 0 and 1, determining the weight change rate; and e_t is the forecast error at time t and $e_t = x_t - F_t$.

2.2. Autoregressive integrated moving average model (ARIMA)

Introduced by Box and Jenkins [31], the ARIMA model has been one of the most popular approaches to time-series forecasting. As a classical model, we give a brief description. In an ARIMA, the future value of a variable is assumed to be a linear function of several past observations plus random errors. The linear function is based upon three parametric components: auto-regression (AR), integration (I), and moving average (MA) [31] and can be denoted by ARIMA(p,d,q), where p is the number of autoregressive terms, d is the number of non-seasonal differences, and q is the number of lagged forecast errors in the prediction equation. For example, the autoregressive or ARIMA (p,0,0) model is represented as follows:

$$y_t = \theta_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t \tag{2}$$

where y_t and ε_t are the actual value and random error at time period t, respectively; θ_0 is the intercept; $\phi_i(i=1,2,\ldots,p)$ are a finite set of parameters, determined by linear regression; p is an integer and often referred to as the orders of the autoregressive model; and ε_ts are random errors and assumed to be independently and identically distributed with a mean of zero and a constant variance of σ^2. This time series depends only on p past values of itself and a random term ε_t. Another example is the moving average or ARIMA (0, 0, q) model, which is represented as

$$y_t = \mu - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q} + \varepsilon_t \tag{3}$$

where q is an integer and often referred to as orders of the moving average model; $\theta_j(j=1,2,\ldots,q)$s are the weight parameters; and μ is the mean of the series. This time series depends only on q past random terms and a present random term ε_t. As the third example, an ARIMA (p, 0, q) or ARMA (p, q) is a model for a time series that depends on p past values of itself and on q past random terms ε_t. This model has the form as follows:

$$y_t = \theta_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \mu - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q} + \varepsilon_t \tag{4}$$

Finally, ARIMA (p, d, q) is a more general ARIMA discussed by Ortega et al. [32,34].

In an ARIMA model building process, determining p, d, and q is very important and is typically repeated several times until a satisfactory model is finally selected. Pang used the Logistic Regression model, First-order Autoregressive Model (ARIMA (1, 0, 0)), and Second-order Autoregressive Model (ARIMA(2, 0, 0)) to predict the volatility of the Shenzhen Stock Market. The results show that the predicted result of the ARIMA(1, 0, 0) model is the best [33]. Considering the inherently noisy and non-stationary nature of stock prices, in this paper, we select the $p=1$, $d=2$, and $q=0$. That is, ARIMA (1, 2, 0).

2.3. Back propagation neural network model (BPNN)

To model a time series with non-linear structures, a three-layer feed-forward back propagation network is commonly used [34]. The back propagation process determines the weights for connections among the nodes based on data training, producing a least-mean-square error measure of the actual or desired and the estimated values from the output of the neural network. First, the connection weights are assigned initial values. Secondly, the error between the predicted and actual output values is back propagated via the network to update the weights. The supervised learning procedure then attempts to minimize the error between the desired and forecasted outputs [35].

This network architecture consists of a hidden layer of neurons with non-linear transfer functions and an output layer of neurons with linear transfer functions. A schematic diagram of a back propagation network is given in Fig. 1, where $x_j(j=1,2,\ldots,n)$ represents the input variables; $z_i(i=1,2,\ldots,m)$ represents the outputs of neurons in the hidden layer; and $y_t(t=1,2,\ldots,l)$ represents the outputs of the neural network [36]. Theoretically,

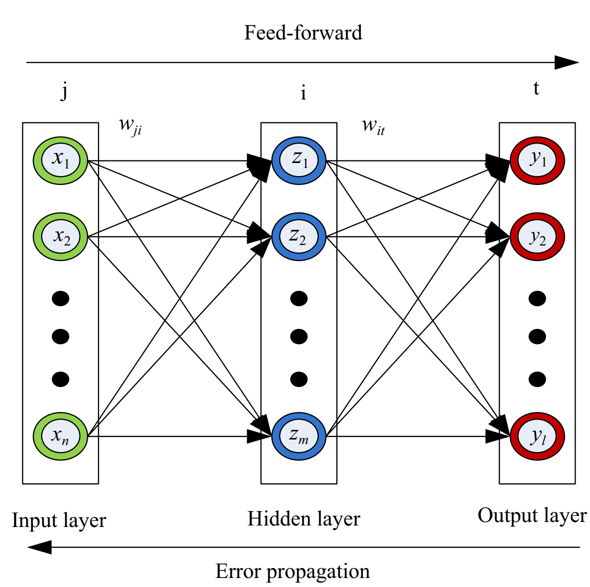

Fig. 1. A three-layer feed-forward back propagation neural network.

neural networks can simulate any kind of data pattern given a sufficient training. A neural network must be trained to determine the weights which will produce the correct outputs. The training process is described by the following two steps to update these weighted values [37]:

(I) Hidden layer stage: The outputs of all neurons in the hidden layer are calculated by the following equations:

$$net_i = \sum_{j=0}^{n} w_{ji} x_j \quad \nu i = 1,2,\ldots,m \tag{5}$$

$$z_i = f_H(net_i) \quad i = 1,2,\ldots,m \tag{6}$$

where net_i is the activation value of the ith node, z_i is the output of the hidden layer, and f_H is called the activation function of a node, usually a sigmoid function as follows:

$$f_H(x) = \frac{1}{1+\exp(-x)} \tag{7}$$

(II) Output stage: The outputs of all neurons in the output layer are given as follows:

$$y_t = f_t\left(\sum_{i=0}^{m} w_{it} z_i\right) \quad t = 1,2,\ldots,l \tag{8}$$

where $f_t(t=1,2,\ldots,l)$ is the activation function, usually a line function. All weights are assigned with random values initially, and are modified by the delta rule according to the learning samples traditionally.

In this paper, the three-layer feed-forward back propagation network, which is a $12 \times 9 \times 12$ network architecture, is applied to create the proposed forecasting model.

3. Hybrid forecasting model

3.1. Theory of the hybrid forecasting

How to combine different forecasting techniques is a widely investigated issue in the literature. For example, Armstrong's meta-analysis revealed that combining various techniques is more useful for short range forecasting [38,39]. Timmermann showed that using simple averages may work as well as more sophisticated approaches [40]. Nevertheless, there are situations where one method produces more accurate forecasts than another. If such cases can be identified in advance, simple averages would not be sufficient [40]. The hybrid forecasting method is based on a certain linear combination of various results from different forecast models. In a forecasting problem, we assume that the actual value in period t is $y_t(t=1,2,\ldots,n)$ and consider m forecasting models of different types. Let the forecasted value in period t by model i be $f_{it}(i=1,2,\ldots,m)$, then the corresponding forecast error is $e_{it} = y_t - f_{it}$. Let the weight vector be $W = [w_1, w_2, \ldots, w_m]^T$, the forecasted value from the hybrid model is computed as follows [41,42]:

$$\hat{y}_t = \sum_{i=1}^{m} w_i f_{it} \quad (t = 1,2,\ldots,n) \tag{9}$$

$$\sum_{i=1}^{m} w_i = 1 \tag{10}$$

Eq. (9) can also be expressed in the matrix form as

$$\hat{Y} = FW \tag{11}$$

where $\hat{Y} = [\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_n]^T$, $\mathrm{F} = [f_{it}]_{n \times m}$.

The forecast error of the hybrid model can be written as

$$e_t = y_t - \hat{y}_t = \sum_{i=1}^{m} w_i y_t - \sum_{i=1}^{m} w_i f_{it} = \sum_{i=1}^{m} w_i (y_t - f_{it}) = \sum_{i=1}^{m} w_i e_{it} \tag{12}$$

We propose a hybrid model of combining ESM, ARIMA and BPNN. That is, Eq. (9) becomes

$$\hat{Y}_{Combined(t)} = w_1 \hat{Y}_{ESM(t)} + w_2 \hat{Y}_{ARIMA(t)} + w_3 \hat{Y}_{BPNN(t)} \quad (t = 1,2,\ldots,n) \tag{13}$$

where $\hat{Y}_{Combined(t)}$, $\hat{Y}_{ESM(t)}$, $\hat{Y}_{ARIMA(t)}$ and $\hat{Y}_{BPNN(t)}$ are forecasting values in period t for the hybrid, ESM, ARIMA, and BPNN model, respectively, and $w_i(i=1,2,3)$s are weights assigned to ESM, ARIMA, and BPNN model, respectively, with $\sum_{i=1}^{3} w_i = 1, 0 \le w_i \le 1$.

3.2. Determining the weights in the hybrid model

Determining the weight for each individual model is the key step in developing a hybrid forecasting model. The simplest method of combining the three forecasts is to set $w_1 = w_2 = w_3 = 1/3$ in Eq. (13). However, in most cases, the equal weights cannot generate accurate forecasting results. Thus, we propose to use the GA to determine the weights. The flow chart of the PHM is given in Fig. 2.

The genetic algorithm is a tool for solving the optimization problems such as determining a set of optimal weights for our hybrid model. There are many differences between GA and other conventional optimization methods [43], which make GA more efficient in searching for optimal solutions in our proposed model.

(1) The GA computes strings by encoded and decoded discrete points instead of the original parameter values. Thus, GAs can handle the discontinuity or non-differentiability function problems that traditional calculus methods fail to work. This characteristic also allows GAs to fit computer logic operations better due to the adaptation of binary strings.
(2) There is no need for prior information because the primary population is randomly generated. A fitness function is used for evaluation of the GA solution.

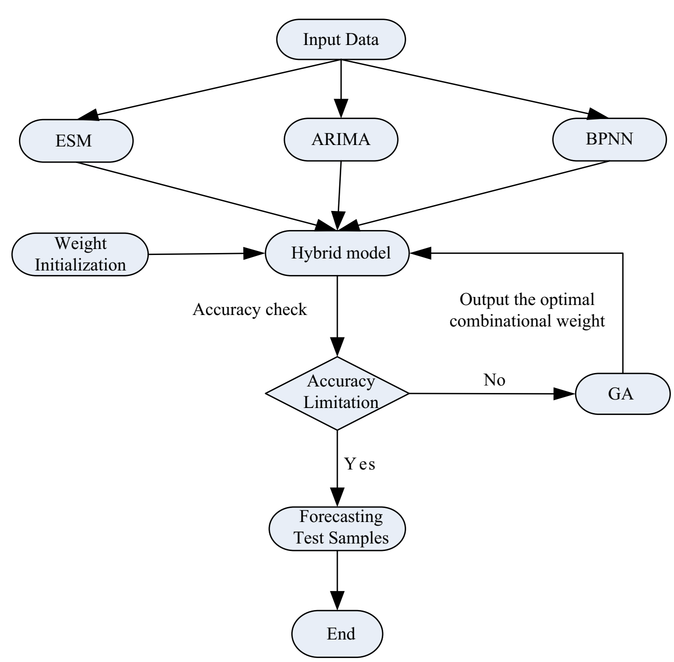

Fig. 2. The flow chart of the PHM.

(3) The searching process of the GA depends on the initialization, selection, and reproduction (crossover and mutation) which all involve random factors. Therefore, even under an identical parameter setting, every single execution of the GA will be a stand-alone searching process which may have different results. This feature makes the GA a search heuristic that mimics the process of natural evolution.

3.3. Evaluation criteria

To evaluate the forecasting performance of the hybrid model, we use the mean absolute error (MAE), root mean-square error (RMSE), mean absolute percentage error (MAPE), mean error (ME), and directional accuracy (DA). These measures are as follows:

$$MAE = T^{-1}\sum_{t=1}^{T}\left|Y_{(t)}-\widehat{Y}_{(t)}\right| \tag{14}$$

$$RMSE = \left(T^{-1}\sum_{t=1}^{T}(Y_{(t)}-\widehat{Y}_{(t)})^2\right)^{1/2} \tag{15}$$

$$MAPE = T^{-1}\sum_{t=1}^{T}\left|(Y_{(t)}-\widehat{Y}_{(t)})/Y_{(t)}\right| \tag{16}$$

$$ME = T^{-1}\sum_{t=1}^{T}(Y_{(t)}-\widehat{Y}_{(t)}) \tag{17}$$

$$DA = \frac{100}{T}\sum_{t=1}^{T} d_t \tag{18}$$

where $d_t = \begin{cases} 1 & (Y_{(t)}-Y_{(t-1)})(\widehat{Y}_{(t)}-\widehat{Y}_{(t-1)}) \geq 0 \\ 0 & \text{otherwise} \end{cases}$, $Y_{(t)}$ and $\widehat{Y}_{(t)}$ are the actual and prediction values, at time t, respectively, and T is the sample size. Note that MAE, RMSE, MAPE, and ME are measures of the deviation between actual and prediction values. Therefore, the forecasting performance is better when the values of these measures are smaller. However, if the results are not consistent among these criterions, we choose the MAPE, suggested by Makridakis (1993), as the benchmark as MAPE is relatively more stable than other criteria. In addition, DA provides the correctness of the predicted direction and can also be utilized to evaluate the prediction accuracy. The higher the DA value is, the better forecasts that are made.

4. Experimentation design and results

4.1. Data sets

In this paper, our purpose is to predict the trend of the stock price index by the PHM. To test the PHM, we use the monthly SZII closing index from China and monthly DJIAI opening index from the US. The SZII closing index dataset covers the period from January 1993 to December 2010 (that is the latest data we can get at the time of writing this paper) and the time series plot is shown in Fig. 3. There are a total of 216 values in the dataset. The first 168 values (about 75% of the sample) are used as the training sample and the remaining 48 values are used as the testing sample. The DJIAI opening index dataset contains stock opening prices from January 1991 to December 2010 and the time series plot is shown in Fig. 4. There are total of 240 values in the dataset. The first 180 values (again about 75% of the sample) are used as the training sample and the remaining 60 values are used as the testing sample.

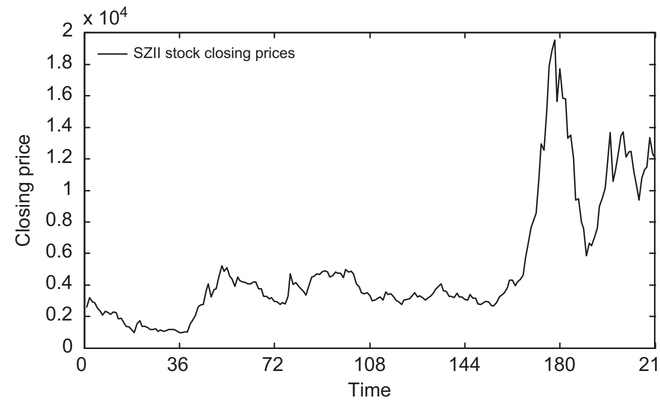

Fig. 3. The monthly SZII stock closing prices from January, 1993 to December, 2010.

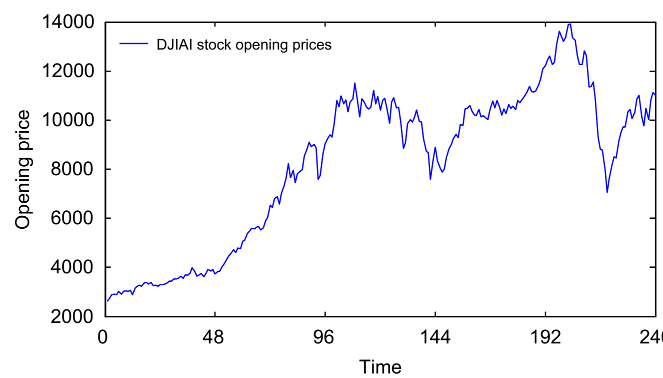

Fig. 4. The monthly DJIAI stock opening prices from January, 1991 to December, 2010.

4.2. Forecasting results

The neural network toolbox and genetic algorithm toolbox of MATLAB software are utilized in the PHM. The original data are scaled into the range of [0, 1.0] when building the BPNN forecasting model.

Since one hidden layer network is sufficient to model any complex system with desired accuracy [44], the three-layer feed-forward back propagation network with only one hidden layer is used in the PHM. The input layer has 12 nodes. Since there are no general rules for choosing the number of the hidden nodes, it is set to be 7, 8, 9, and 10. The output layer has 12 nodes. As the learning rate is important in training, learning rates of 0.01, 0.02, 0.03, and 0.04 are tested during the training process. The convergence criterion for training the BPNN is the maximum of 10,000 iterations. The network topology with the minimum testing error is considered as the optimal network. The testing results of the BPNN with combinations of different hidden nodes and learning rates are summarized in Table 1. From Table 1, it is observed that the {12-9-12} topology (12 nodes in each of the input and output layers and 9 nodes in the hidden layer) with a learning rate of 0.02 gives the minimum testing error and hence is the best topology setup for the BPNN in forecasting the SZII closing index. The GA is used to determine a set of optimal weights for our PHM. The convergence criterion for training the GA is the maximum of 10,000 iterations.

To compare the performances of different models, we apply ESM, ARIMA, BPNN, and the PHM to forecast the SZII closing index and DJIAI opening index with the two real datasets, respectively. In forecasting the SZII closing index, the comparisons between each of the ESM, ARIMA, BPNN and the PHM are shown in Figs. 5–8 in terms of absolute error defined as 'forecasting value minus actual value' and relative error defined as 'the ratio of absolute error to the actual value'.

In Figs. 5 and 6, for ESM and ARIMA models, it has been observed that the fitting values (or forecasted values) approximately describe the characteristics of the SZII closing index time series. However, the absolute errors and relative errors of the ESM and ARIMA are much larger than those of the hybrid model. This observation may be due to the fact that the ESM and ARIMA models are based on the linear assumption, but the closing index data contain both linear and non-linear components.

The BPNN fitting values based on the SZII closing index are shown in Fig. 7. Note that the maximum absolute error exceeds –1500 and the maximum relative error exceeds -0.5. These large errors may result from fact that the BPNN model is based on the non-linear assumption but the closing index data contain both linear and non-linear components.

Table 1
Model selection results of the BPNN forecasting model.

Number of nodes in the hidden layer	Learning rate	Training error	Testing error
7	0.01	0.016087	0.013231
	0.02	0.016098	0.013246
	0.03	0.016351	0.013475
	0.04	0.016472	0.013814
8	0.01	0.016927	0.013952
	0.02	0.016532	0.013427
	0.03	0.016491	0.013528
	0.04	0.016303	0.013437
9	0.01	0.016836	0.013783
	0.02	0.016012	0.013129
	0.03	0.016438	0.013583
	0.04	0.016804	0.013641
10	0.01	0.016528	0.013489
	0.02	0.016862	0.013724
	0.03	0.016218	0.013546
	0.04	0.016142	0.013335

Fig. 8 obviously shows that the PHM captures both the linear and non-linear features of the SZII closing index data. The relative error is in a small range including zero, which indicates that compared with other models, the PHM can describe the stock index time series more accurately.

To evaluate the forecasting performances of the PHM, we compare the ESM, ARIMA, BPNN, EWH, RWM, and PHM with the SZII closing index dataset. Table 2 presents the five performance measures of MAE, RMSE, MAPE, ME, and DA for the six models. Obviously, except for the ME, the PHM has much less errors than the other five models. Moreover, compared to the ESM, ARIMA, BPNN, EWH, and RWM, the PHM has the highest DA ratio. The DA provides a good measure of the consistency in predicting the price direction. Thus, the PHM provides better forecasting results than the other models in terms of prediction errors or accuracy.

The PHM also performs well in forecasting the DJIAI opening index. Table 3 reports the DJIAI opening index forecasting results using the six models. Again, except for the ME, Table 3 shows that the PHM has the smallest MAE, RMSE and MAPE values and the highest DA ratio. Thus, the PHM can produce better forecasts in the DJIAI opening index case.

4.3. Robustness of hybrid model

To evaluate the robustness of the PHM, we test the performances of the ESM, ARIMA, BPNN, EWH, RWM, and PHM using different ratios of training dataset to sample sizes. Four relative ratios of 60%, 70%, 80%, and 90% are considered. The prediction results from the six models for the SZII closing index and DJIAI opening index are summarized in Table 4. Except for the ME, we observe again that the PHM outperforms the other five models for all four different ratios in terms of all criteria. Therefore, the PHM is quite robust in terms of producing the more accurate forecasting results. The reason for this robustness is because PHM can effectively capture both the linear and non-linear characteristics

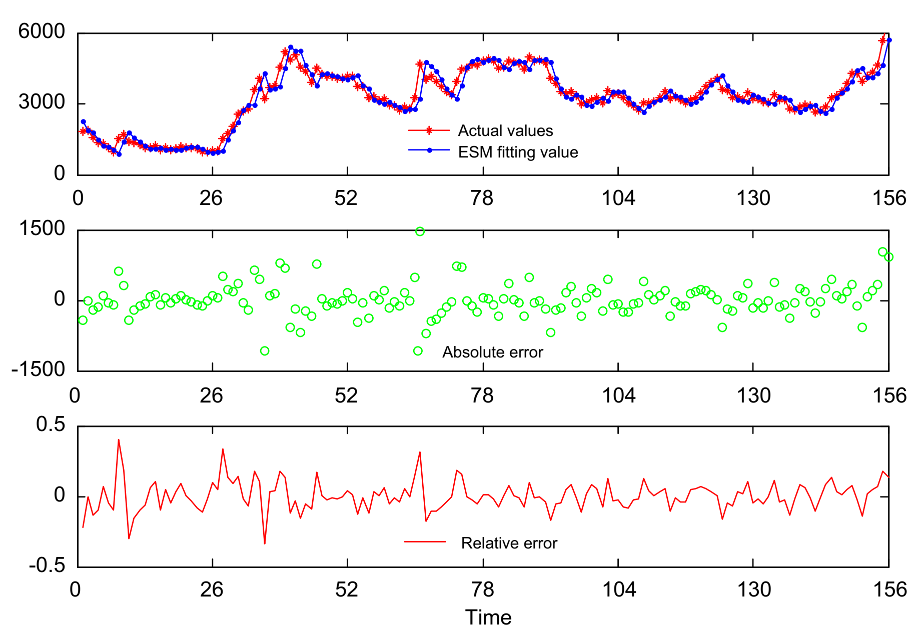

Fig. 5. Comparison of the fitting values by ESM model and the actual values and the corresponding errors.

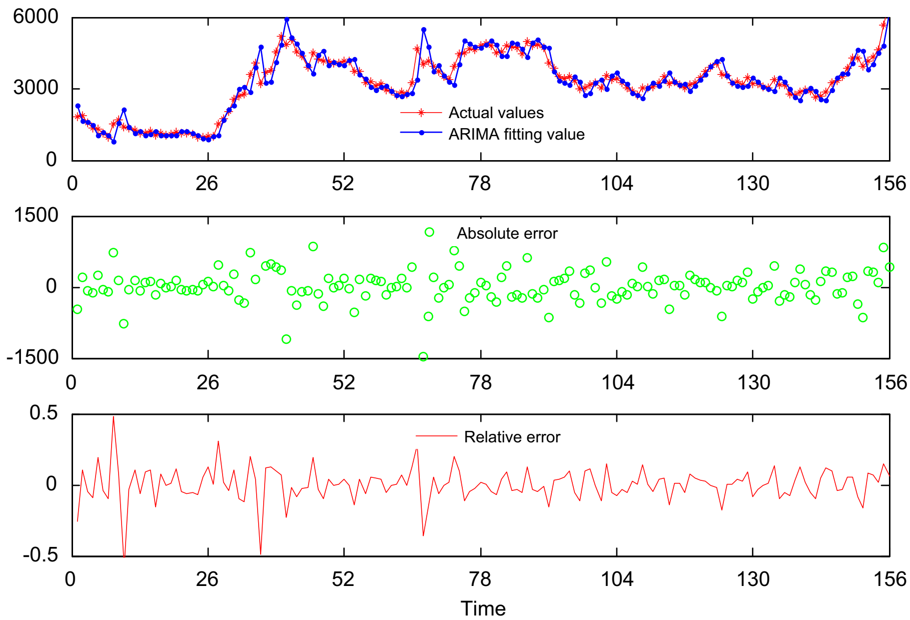

Fig. 6. Comparison of the fitting values by ARIMA model and the actual values and the corresponding errors.

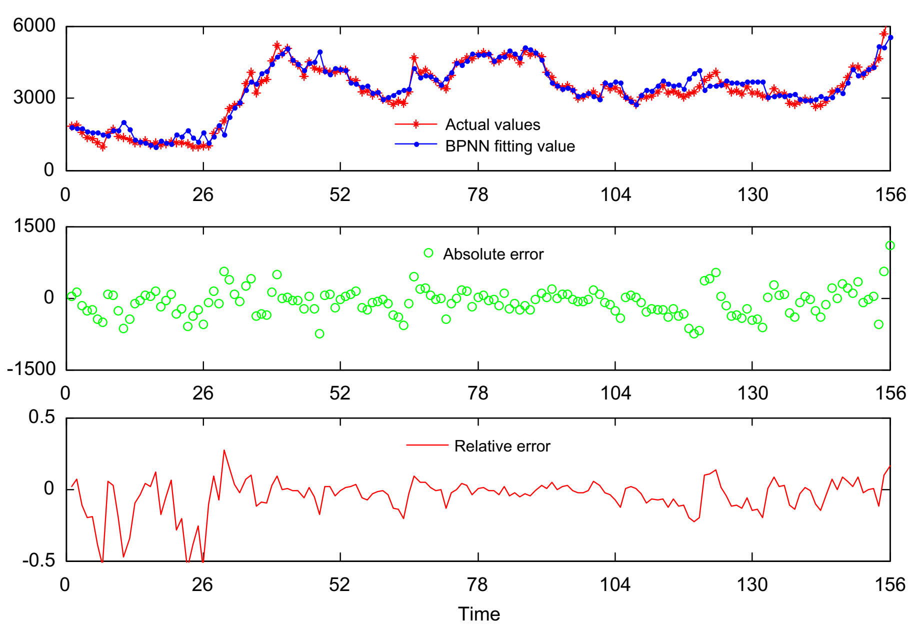

Fig. 7. Comparison of the fitting values by BPNN model and the actual values and the corresponding errors.

in stock price time series. Considering the superior performance of the PHM forecasts, investors could develop a simple PHM-guided trading strategy which allocates more assets to the stock index fund when there is a predicted up-trend and allocate more assets to the bonds when there is a predicted down-trend.

5. Conclusions, implications and extensions

In real situations, the dynamics of stock index time series is complex and unknown. Using a single classical model cannot produce accurate forecasts for stock price indexes. In this paper, a hybrid method combining linear ESM, ARIMA and non-linear BPNN techniques was proposed and applied to the two real stock price datasets. The main idea of the hybrid model is to capture different forms of relationships in time series data more effectively. The SZII closing index and DJIAI opening index are used for evaluating the PHM performances. We have compared the PHM with the ESM, ARIMA, BPNN, EWH, and RWM and showed that the PHM can outperform all other models. Thus, our study indicates that by combining different models we may develop a powerful hybrid model to generate more accurate forecasts for an extremely complicated stock price time series. It is worth noting that the hybrid forecasting models are indeed powerful

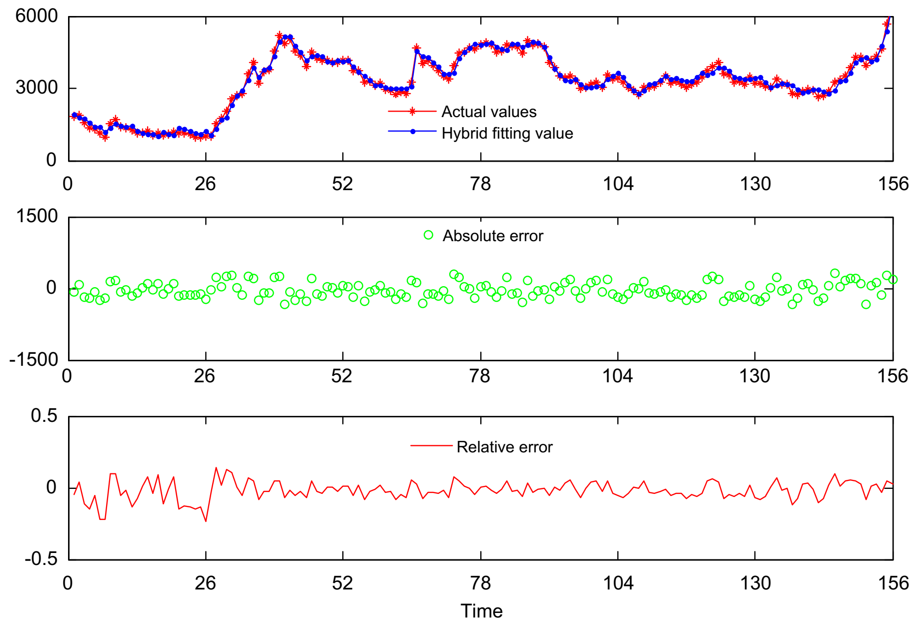

Fig. 8. Comparison of the fitting values by PHM and the actual values and the corresponding errors.

Table 2
The SZII closing index forecasting results using PHM, ESM, ARIMA, BPNN, EWM, and RWM.

Errors and directional accuracy	Models					
	ESM	ARIMA	BPNN	EWH	PHM	RWM
MAE	3991.5	3615.8	4453.6	3420.3	2910.8	4506.2
RMSE	5562.4	4829.6	5195.3	4321.7	3761.5	5637.4
MAPE	40.51%	36.85%	34.97%	31.44%	26.06%	39.52%
ME	1524.7	1105.4	−1305.1	441.7	446.1	2215.3
DA	60.72%	75.33%	77.85%	80.15%	83.91%	74.28%

Table 3
The DJIAI opening index forecasting results using PHM, ESM, ARIMA, BPNN, EWH, and RWM.

Errors and directional accuracy	Models					
	ESM	ARIMA	BPNN	EWH	PHM	RWM
MAE	5142.1	4990.5	4895.1	4407.2	3286.1	5870.5
RMSE	5984.2	5715.4	5821.4	5061.4	4356.6	6178.2
MAPE	47.61%	41.30%	38.97%	34.71%	30.53%	51.29%
ME	3035.6	−2831.2	2098.5	767.6	804.0	3124.5
DA	46.51%	58.17%	56.98%	61.54%	70.16%	60.34%

Management Science (MS) tools for practitioners. Some other examples include forecasting models for local traffic flow parameters (see Hong et al. [45]) and forecasting models for prices of agricultural commodities (see Ribeiro and Oliveira [46]). The implication of the wide applications of hybrid models is that the forecasting quality can be improved in many areas where datasets reveal the complex relationships among variables.

Although we illustrate the effectiveness of using the hybrid model in this paper, there certainly exist limitations. In this study, only the monthly SZII closing index and DJIAI opening index are used as illustrative examples to evaluate the performances of the PHM. As the monthly data reflect the relatively long-term trend of the stock price index, the forecasts generated from the PHM provide useful information for the long- or medium- term investors. A possible future research topic is to develop a hybrid model for forecasting with the daily or even hourly data where how to improve the computation time becomes a more critical issue. Such a model is more important for the short-term investors. Another direction is to explore the possibility of combining other forecasting tools, like support vector regression (SVR) and multivariate adaptive regression splines (MARS) to further improve time-series forecasting. Moreover, a further

Table 4
Robustness evaluation of ESM, ARIMA, BPNN, EWH, PHM and RWM by different training and testing sample sizes.

Relative ratio (%)	Models	Testing data									
		SZII					DJIAI				
		MAE	RMSE	MAPE	ME	DA	MAE	RMSE	MAPE	ME	DA
60	ESM	4658.1	6093.3	44.25%	2013.2	57.62%	6124.3	7855.6	54.33%	3298.1	38.54%
	ARIMA	4590.8	5995.2	42.59%	1721.1	70.86%	5865.2	7275.1	49.71%	−2711.4	53.48%
	BPNN	4745.6	6268.1	43.53%	−1872.5	76.55%	5912.5	7546.4	52.36%	2356.6	58.75%
	EWH	4543.2	5776.5	39.24%	752.1	78.23%	4925.1	7033.2	44.19%	979.2	59.08%
	PHM	4483.5	5525.3	39.15%	500.1	79.86%	4876.1	6569.8	43.51%	1024.2	65.81%
	RWM	4921.3	6692.5	45.54%	2733.2	71.12%	6783.3	8168.5	56.39%	3055.2	57.12%
70	ESM	4391.3	5862.2	43.75%	1875.1	56.89%	5693.1	6784.4	51.24%	3178.7	45.15%
	ARIMA	4046.8	5349.7	40.43%	1560.3	72.95%	5329.7	6547.2	45.95%	−2844.3	58.61%
	BPNN	4537.7	5996.5	41.69%	−1694.3	71.52%	5268.8	6125.4	40.24%	2157.1	57.67%
	EWH	4181.6	4498.3	37.56%	654.3	73.86%	4134.2	5424.6	38.49%	905.1	60.62%
	PHM	3983.2	4127.6	36.48%	646.9	80.65%	3952.5	4734.3	36.52%	861.0	70.25%
	RWM	4880.4	6598.2	43.89%	2431.6	72.38%	6413.6	6991.5	55.10%	2873.6	59.18%
80	ESM	4952.6	6238.4	52.21%	1633.5	62.03%	5437.5	6225.4	53.26%	2989.5	46.62%
	ARIMA	4567.3	6062.1	42.88%	1284.7	75.66%	5109.3	5892.8	44.19%	−2515.2	59.23%
	BPNN	4761.9	5548.3	39.01%	−1289.4	78.01%	5430.1	6043.2	42.53%	2213.1	58.02%
	EWH	4312.7	5215.9	34.63%	462.8	82.09%	4591.5	5192.3	39.76%	725.4	63.47%
	PHM	3436.3	4722.5	32.25%	502.2	82.71%	4298.3	4967.7	39.54%	949.9	70.97%
	RWM	5413.2	6311.7	55.47%	2179.5	75.72%	6059.7	6416.5	58.91%	2935.8	62.45%
90	ESM	5267.4	6432.3	54.72%	1744.8	63.79%	5981.4	6877.3	56.46%	3076.4	48.81%
	ARIMA	4689.2	6325.4	43.69%	1355.6	77.13%	5475.1	6343.5	45.58%	−2449.6	60.25%
	BPNN	5102.1	5714.6	41.28%	−1005.7	69.75%	5729.4	5947.3	40.27%	1989.2	57.96%
	EWH	4153.4	5582.9	39.78%	491.2	77.91%	4273.9	5244.8	39.60%	815.6	61.44%
	PHM	3953.3	5231.1	37.72%	725.1	84.12%	4056.7	5186.4	38.51%	875.0	73.14%
	RWM	5309.7	6587.5	55.96%	2031.1	73.84%	6793.0	6907.2	49.84%	3314.2	61.39%

extension of our study is to test the hybrid model on the datasets for individual stocks to examine the robustness of the model.

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